

## THE BETA FUNCTION

*Definition:*  $B(m, n) = \int_0^1 t^{m-1} (1-t)^{n-1} dt \quad m > 0, n > 0$

*Relationship with Gamma function:*  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

*Properties:*

$$B(m, n) = B(n, m)$$

$$B(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$$

$$B(m, n) = \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}} dt$$

$$B(m, n) = r^n (r+1)^m \int_0^1 \frac{t^{m-1} (1-t)^{n-1}}{(r+t)^{m+n}} dt$$