THE ERROR FUNCTION

Definition: $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

Series: $erf(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + \cdots \right)$

Property: $\operatorname{erf}(x) = -\operatorname{erf}(-x)$

Relationship with Normal Probability Function $f(t)$: $\int_0^x f(t)\, dt = \frac{1}{2} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)$ To evaluate erf(2.3), one proceeds as follows: For $\frac{x}{\sqrt{2}} = 2.3$, one finds $x = (2.3)(\sqrt{2}) = 3.25$. In the normal probability function table (page A-104), one finds the entry 0.4994 opposite the value 3.25. Thus $\operatorname{erf}(2.3) = 2(0.4994) = 0.9988$.

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2}\, dt$$

is known as the complementary error function.