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# Concrete Design and Construction

Concrete made with portland cement is widely used as a construction material because of its many favorable characteristics. One of the most important is a large strength-cost ratio in many applications. Another is that concrete, while plastic, may be cast in forms easily at ordinary temperatures to produce almost any desired shape. The exposed face may be developed into a smooth or rough hard surface, capable of withstanding the wear of truck or airplane traffic, or it may be treated to create desired architectural effects. In addition, concrete has high resistance to fire and penetration of water.

But concrete also has disadvantages. An important one is that quality control sometimes is not so good as for other construction materials because concrete often is manufactured in the field under conditions where responsibility for its production cannot be pinpointed. Another disadvantage is that concrete is a relatively brittle material—its tensile strength is small compared with its compressive strength. This disadvantage, however, can be offset by reinforcing or prestressing concrete with steel. The combination of the two materials, reinforced concrete, possesses many of the best properties of each and finds use in a wide variety of constructions, including building frames, floors, roofs, and walls; bridges; pavements; piles; dams; and tanks.

## 8.1 Important Properties of Concrete

Characteristics of portland cement concrete can be varied to a considerable extent by controlling its ingredients. Thus, for a specific structure, it is economical to use a concrete that has exactly the characteristics needed, though weak in others. For example, concrete for a building frame should have high compressive strength, whereas concrete for a dam should be durable and watertight, and strength can be relatively small. Performance of concrete in service depends on both properties in the plastic state and properties in the hardened state.

### 8.1.1 Properties in the Plastic State

**Workability** is an important property for many applications of concrete. Difficult to evaluate, workability is essentially the ease with which the ingredients can be mixed and the resulting mix handled, transported, and placed with little loss in homogeneity. One characteristic of workability that engineers frequently try to measure is consistency, or fluidity. For this purpose, they often make a slump test.

In the slump test, a specimen of the mix is placed in a mold shaped as the frustum of a cone, 12 in high, with 8-in-diameter base and 4-in-diameter top (ASTM Specification C143). When the mold is removed, the change in height of the specimen is measured. When the test is made in accordance with the ASTM Specification, the change in height may be taken as the slump. (As measured by this test, slump decreases as temperature increases; thus the temperature of the mix at time of test should be specified, to avoid erroneous conclusions.)

Tapping the slumped specimen gently on one side with a tamping rod after completing the test may give additional information on the cohesiveness, workability, and placeability of the mix ("Concrete Manual," Bureau of Reclamation, Government Printing Office, Washington, DC 20402 (www.gpo.gov)). A well-proportioned, workable mix settles slowly, retaining its original identity. A poor mix crumbles, segregates, and falls apart.

Slump of a given mix may be increased by adding water, increasing the percentage of fines (cement or aggregate), entraining air, or incorporating an admixture that reduces water requirements. But these changes affect other properties of the concrete, sometimes adversely. In general, the slump specified should yield the desired consistency with the least amount of water and cement.

## 8.1.2 Properties in the Hardened State

**Strength** is a property of concrete that nearly always is of concern. Usually, it is determined by the ultimate strength of a specimen in compression, but sometimes flexural or tensile capacity is the criterion. Since concrete usually gains strength over a long period of time, the compressive strength at 28 days is commonly used as a measure of this property. In the United States, it is general practice to determine the compressive strength of concrete by testing specimens in the form of standard cylinders made in accordance with ASTM Specification C192 or C31. C192 is intended for research testing or for selecting a mix (laboratory specimens). C31 applies to work in progress (field specimens). The tests should be made as recommended in ASTM C39. Sometimes, however, it is necessary to determine the strength of concrete by taking drilled cores; in that case, ASTM C42 should be adopted. (See also American Concrete Institute Standard 214, "Recommended Practice for Evaluation of Strength Test Results of Concrete." (www.aci-int.org))

The 28-day compressive strength of concrete can be estimated from the 7-day strength by a formula proposed by W. A. Slater (*Proceedings of the American Concrete Institute*, 1926):

$$S_{28} = S_7 + 30\sqrt{S_7} \tag{8.1}$$

where $S_{28}$ = 28-day compressive strength, psi

$S_7$ = 7-day strength, psi

Concrete may increase significantly in strength after 28 days, particularly when cement is mixed with fly ash. Therefore, specification of strengths at 56 or 90 days is appropriate in design.

Concrete strength is influenced chiefly by the water-cement ratio; the higher this ratio, the lower the strength. In fact, the relationship is approximately linear when expressed in terms of the variable $C/W$, the ratio of cement to water by weight: For a workable mix, without the use of water reducing admixtures

$$S_{28} = 2700\frac{C}{W} - 760 \tag{8.2}$$

Strength may be increased by decreasing water-cement ratio, using higher-strength aggregates, grading the aggregates to produce a smaller percentage of voids in the concrete, moist curing the concrete after it has set, adding a pozzolan, such as fly ash, incorporating a superplasticizer admixture, vibrating the concrete in the forms, and sucking out excess water with a vacuum from the concrete in the forms. The short-time strength may be increased by using Type III (high-early-strength) portland cement (Art. 5.6) and accelerating admixtures, and by increasing curing temperatures, but long-time strengths may not be affected. Strength-increasing admixtures generally accomplish their objective by reducing water requirements for the desired workability. (See also Art. 5.6.)

Availability of such admixtures has stimulated the trend toward use of high-strength concretes. Compressive strengths in the range of 20,000 psi have been used in cast-in-place concrete buildings.

**Tensile Strength**, $f_{ct}$, of concrete is much lower than compressive strength. For members subjected to bending, the modulus of rupture $f_r$ is used in design rather than the concrete tensile strength. For normal weight, normal-strength concrete, ACI specifies $f_r = 7.5\sqrt{f'_c}$.

The stress-strain diagram for concrete of a specified compressive strength is a curved line (Fig. 8.1). Maximum stress is reached at a strain of 0.002 in/in, after which the curve descends.

**Modulus of elasticity** $E_c$ generally used in design for concrete is a secant modulus. In ACI 318, "Building Code Requirements for Reinforced Concrete," it is determined by

$$E_c = w^{1.5} 33\sqrt{f'_c}, \text{ psi} \tag{8.3a}$$

where $w_c$ = density of concrete lb/ft$^3$

$f'_c$ = specified compressive strength at 28 days, psi

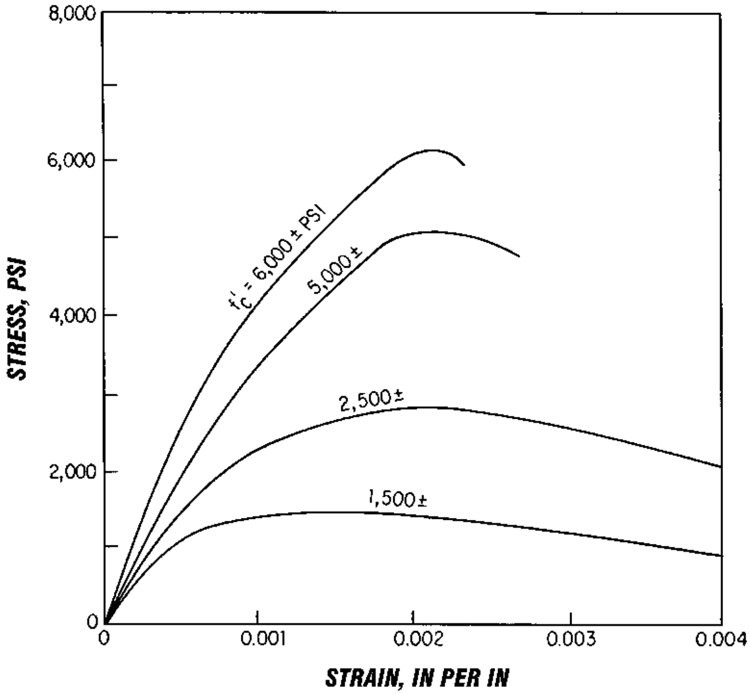



**Fig. 8.1** Stress-strain curves for concrete.

This equation applies when 90 pcf $< w_c <$ 155 pcf. For normal-weight concrete, with $w = 145$ lb/ft$^3$,

$$E_c = 57{,}000\sqrt{f'_c}, \text{ psi} \qquad (8.3b)$$

The modulus increases with age, as does the strength. (See also Art. 5.6)

**Durability** is another important property of concrete. Concrete should be capable of withstanding the weathering, chemical action, and wear to which it will be subjected in service. Much of the weather damage sustained by concrete is attributable to freezing and thawing cycles. Resistance of concrete to such damage can be improved by using appropriate cement types, lowering w/c ratio, providing proper curing, using alkali-resistant aggregates, using suitable admixtures, using an air-entraining agent, or applying a protective coating to the surface.

Chemical agents, such as inorganic acids, acetic and carbonic acids, and sulfates of calcium, sodium, magnesium, potassium, aluminum, and iron, disintegrate or damage concrete. When contact between these agents and concrete may occur, the concrete should be protected with a resistant coating. For resistance to sulfates, Type V portland cement may be used (Art. 5.6). Resistance to wear usually is achieved by use of a high-strength, dense concrete made with hard aggregates.

**Watertightness** is an important property of concrete that can often be improved by reducing the amount of water in the mix. Excess water leaves voids and cavities after evaporation, and if they are interconnected, water can penetrate or pass through the concrete. Entrained air (minute bubbles) usually increases watertightness, as does prolonged thorough curing.

**Volume change** is another characteristic of concrete that should be taken into account. Expansion due to chemical reactions between the ingredients of concrete may cause buckling and drying shrinkage may cause cracking.

Expansion due to alkali-aggregate reaction can be avoided by selecting nonreactive aggregates. If reactive aggregates must be used, expansion may be reduced or eliminated by adding pozzolanic material, such as fly ash, to the mix. Expansion due to heat of hydration of cement can be reduced by keeping cement content as low as possible, using Type IV cement (Art. 5.6), and chilling the aggregates, water, and concrete in the forms. Expansion due to increases in air temperature may be decreased by producing concrete with a lower coefficient of expansion, usually by using coarse aggregates with a lower coefficient of expansion.

Drying shrinkage can be reduced principally by cutting down on water in the mix. But less cement also will reduce shrinkage, as will adequate moist curing. Addition of pozzolans, however, unless enabling a reduction in water, may increase drying shrinkage.

Autogenous volume change, a result of chemical reaction and aging within the concrete and usually shrinkage rather than expansion, is relatively independent of water content. This type of shrinkage may be decreased by using less cement, and sometimes by using a different cement.

Whether volume change will damage the concrete often depends on the restraint present. For example, a highway slab that cannot slide on the subgrade while shrinking may crack; a building floor that cannot contract because it is anchored to relatively stiff girders also may crack. Hence, consideration should always be given to eliminating restraints or resisting the stresses they may cause.

**Creep** is strain that occurs under a sustained load. The concrete continues to deform, but at a rate that diminishes with time. It is approximately proportional to the stress at working loads and increases with increasing water-cement ratio. It decreases with increase in relative humidity. Creep increases the deflection of concrete beams and scabs and causes loss of prestress.

**Density** of ordinary sand-and-gravel concrete usually is about 145 lb/ft$^3$. It may be slightly lower if the maximum size of coarse aggregate is less than $1\frac{1}{2}$ in. It can be increased by using denser aggregate, and it can be decreased by using lightweight aggregate, increasing the air content, or incorporating a foaming, or expanding, admixture.

(J. G. MacGregor, "Reinforced Concrete," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); M. Fintel, "Handbook of Concrete Engineering," 2nd ed., Van Nostrand Reinhold, New York.)

## 8.2 Lightweight Concretes

Concrete lighter in weight than ordinary sand-and-gravel concrete is used principally to reduce dead load, or for thermal insulation, nailability, or fill. Structural lightweight concrete must be of sufficient density to satisfy fire ratings.

Lightweight concrete generally is made by using lightweight aggregates or using gas-forming or foaming agents, such as aluminum powder, which are added to the mix. The lightweight aggregates are produced by expanding clay, shale, slate, diatomaceous shale, perlite obsidian, and vermiculite with heat and by special cooling of blast-furnace slag. They also are obtained from natural deposits of pumice, scoria, volcanic cinders, tuff, and diatomite, and from industrial cinders. Usual ranges of weights obtained with some lightweight aggregates are listed in Table 8.1.

Production of lightweight-aggregate concretes is more difficult than that of ordinary concrete because aggregates vary in absorption of water, specific gravity, moisture content, and amount and grading of undersize. Frequent unit-weight and slump tests are necessary so that cement and water content of the mix can be adjusted, if uniform results are to be obtained. Also, the concretes usually tend to be harsh and difficult to place and finish because of the porosity and angularity of the aggregates. Sometimes, the aggregates may float to the surface. Workability can be improved by increasing the percentage of fine aggregates or by using an air-entraining admixture to incorporate from 4 to 6% air. (See also ACI 211.2, "Recommended Practice for Selecting Proportions for Structural Lightweight Concrete," American Concrete Institute (www.aci-int.org).)

To improve uniformity of moisture content of aggregates and reduce segregation during stockpiling and transportation, lightweight aggregate

**Table 8.1** Approximate Weights of Lightweight Concretes

| Aggregate | Concrete Weight, lb/ft$^3$ |
|---|---|
| Cinders: | |
| Without sand | 85 |
| With sand | 110–115 |
| Shale or clay | 90–110 |
| Pumice | 90–100 |
| Scoria | 90–110 |
| Perlite | 50–80 |
| Vermiculite | 35–75 |

should be wetted 24 h before use. Dry aggregate should not be put into the mixer because the aggregate will continue to absorb moisture after it leaves the mixer and thus cause the concrete to segregate and stiffen before placement is completed. Continuous water curing is especially important with lightweight concrete.

Other types of lightweight concretes may be made with organic aggregates, or by omission of fines, or gap grading, or replacing all or part of the aggregates with air or gas. Nailing concrete usually is made with sawdust, although expanded slag, pumice, perlite, and volcanic scoria also are suitable. A good nailing concrete can be made with equal parts by volume of portland cement, sand, and pine sawdust, and sufficient water to produce a slump of 1 to 2 in. The sawdust should be fine enough to pass through a $\frac{1}{4}$-in screen and coarse enough to be retained on a No. 16 screen. (Bark in the sawdust may retard setting and weaken the concrete.) The behavior of this type of concrete depends on the type of tree from which the sawdust came. Hickory, oak, or birch may not give good results ("Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, DC, 20402 (www.gpo.gov)). Some insulating lightweight concretes are made with wood chips as aggregate.

For no-fines concrete, 20 to 30% entrained air replaces the sand. Pea gravel serves as the coarse aggregate. This type of concrete is used where low dead weight and insulation are desired and strength is not important. No-fines concrete may weigh from 105 to 118 lb/ft$^3$ and have a compressive strength from 200 to 1000 psi.

A porous concrete may be made by gap grading or single-size aggregate grading. It is used where drainage is desired or for light weight and low conductivity. For example, drain tile may be made with a No. 4 to $\frac{3}{8}$- or $\frac{1}{2}$-in aggregate and a low water-cement ratio. Just enough cement is used to bind the aggregates into a mass resembling popcorn.

Gas and foam concretes usually are made with admixtures. Foaming agents include sodium lauryl sulfate, alkyl aryl sulfonate, certain soaps, and resins. In another process, the foam is produced by the type of foaming agents used to extinguish fires, such as hydrolyzed waste protein. Foam concretes range in weight from 20 to 110 lb/ft$^3$.

Aluminum powder, when used as an admixture, expands concrete by producing hydrogen bubbles. Generally, about $\frac{1}{4}$ lb of the powder per bag of cement is added to the mix, sometimes with an alkali, such as sodium hydroxide or trisodium phosphate, to speed the reaction.

The heavier cellular concretes have sufficient strength for structural purposes, such as floor slabs and roofs. The lighter ones are weak but provide good thermal and acoustic insulation or are useful as fill; for example, they are used over structural floor slabs to embed electrical conduit.

(ACI 213R, "Guide for Structural Lightweight-Aggregate Concrete," and 211.2 "Recommended Practice for Selecting Proportions for Structural Lightweight Concrete," American Concrete Institute, 38800 Country Club Drive Farmington Hills, MI, 48331 (www.aci-int.org).)

## 8.3 Heavyweight Concretes

Concrete weighing up to about 385 lb/ft$^3$ can be produced by using heavier-than-ordinary aggregate. Theoretically, the upper limit can be achieved with steel shot as fine aggregate and steel punchings as coarse aggregate. (See also Art. 5.6.) The heavy concretes are used principally in radiation shields and counterweights.

Concrete made with barite develops an optimum density of 232 lb/ft$^3$ and compressive strength of 6000 psi; with limonite and magnetite, densities from 210 to 224 lb/ft$^3$ and strengths of 3200 to 5700 psi; with steel punchings and sheared bars as coarse aggregate and steel shot for fine aggregate, densities from 250 to 288 lb/ft$^3$ and strengths of about 5600 psi. Gradings and mix proportions are similar to those used for conventional concrete. These concretes usually do not have good resistance to weathering or abrasion.

# Structural Concrete

## 8.4 Proportioning and Mixing Concrete

Components of a mix should be selected to produce a concrete with the desired characteristics for the service conditions and adequate workability at the lowest cost. For economy, the amount of cement should be kept to a minimum. Generally, this objective is facilitated by selecting the largest-size coarse aggregate consistent with job requirements and good gradation, to keep the volume of voids small. The smaller this volume, the less cement paste needed to fill the voids.

The water-cement ratio, for workability, should be as large as feasible to yield a concrete with the desired compressive strength, durability, and watertightness and without excessive shrinkage. Water added to a stiff mix improves workability, but an excess of water has deleterious effects (Art. 8.1).

### 8.4.1 Proportioning Concrete Mixes

A concrete mix is specified by indicating the weight, in pounds, of water, cement, sand, coarse aggregate, and admixture to be used per cubic yard of mixed concrete. In addition, type of cement, fineness modulus of the aggregates, and maximum sizes of aggregates should be specified. (In the past, one method of specifying a concrete mix was to give the ratio, by weight, of cement to sand to coarse aggregate; for example, 1 : 2 : 4; plus the minimum cement content per cubic yard of concrete.)

Because of the large number of variables involved, it usually is advisable to proportion concrete mixes by making and testing trial batches. A start is made with the selection of the water-cement ratio. Then, several trial batches are made with varying ratios of aggregates to obtain the desired workability with the least cement. The aggregates used in the trial batches should have the same moisture content as the aggregates to be used on the job. The amount of mixing water to be used must include water that will be absorbed by dry aggregates or must be reduced by the free water in wet aggregates. The batches should be mixed by machine, if possible, to obtain results close to those that would be obtained in the field. Observations should be made of the slump of the mix and appearance of the concrete. Also, tests should be made to evaluate compressive strength and other desired characteristics. After a mix has been selected, some changes may have to be made after some field experience with it.

Table 8.2 estimates the 28-day compressive strength that may be attained with various water-cement ratios, with and without air entrainment. Note that air entrainment permits a reduction of water, so a lower water-cement ratio for a given workability is feasible with air entrainment.

Table 8.3 lists recommended maximum sizes of aggregate for various types of construction. These tables may be used with Table 8.4 for proportioning concrete mixes for small jobs where time or other conditions do not permit proportioning by the trial-batch method. Start with mix B in Table 8.4 corresponding to the selected maximum size of aggregate. Add just enough water for the desired

**Table 8.2** Estimated Compressive Strength of Concrete for Various Water-Cement Ratios*

| Water-Cement Ratio by Weight | 28-day Compressive Strength: Air-Entrained Concrete | 28-day Compressive Strength: Non-Air-Entrained Concrete |
|---|---|---|
| 0.40 | 4,300 | 5,400 |
| 0.45 | 3,900 | 4,900 |
| 0.50 | 3,500 | 4,300 |
| 0.55 | 3,100 | 3,800 |
| 0.60 | 2,700 | 3,400 |
| 0.65 | 2,400 | 3,000 |
| 0.70 | 2,200 | 2,700 |

* "Concrete Manual," U.S. Bureau of Reclamation.

**Table 8.3** Recommended Maximum Sizes of Aggregate*

| Minimum Dimension of Section, in | Maximum Size, in, of Aggregate for: Reinforced-Concrete Beams, Columns, Walls | Heavily Reinforced Slabs | Lightly Reinforced or Unreinforced Slabs |
|---|---|---|---|
| 5 or less | — | $\frac{3}{4}$–$\frac{1}{2}$ | $\frac{3}{4}$–$\frac{1}{2}$ |
| 6–11 | $\frac{3}{4}$–$1\frac{1}{2}$ | $1\frac{1}{2}$ | $1\frac{1}{2}$–3 |
| 12–29 | $1\frac{1}{2}$–3 | 3 | 3–6 |
| 30 or more | $1\frac{1}{2}$–3 | 3 | 6 |

* "Concrete Manual," U.S. Bureau of Reclamation.

**Table 8.4** Typical Concrete Mixes*

| Maximum Size of Aggregate, in | Mix Designation | Bags of Cement per yd$^3$ of Concrete | Aggregate, lb per Bag of Cement: Sand, Air-Entrained Concrete | Sand, Concrete without Air | Gravel or Crushed Stone |
|---|---|---|---|---|---|
| ½ | A | 7.0 | 235 | 245 | 170 |
| | B | 6.9 | 225 | 235 | 190 |
| | C | 6.8 | 225 | 235 | 205 |
| ¾ | A | 6.6 | 225 | 235 | 225 |
| | B | 6.4 | 225 | 235 | 245 |
| | C | 6.3 | 215 | 225 | 265 |
| 1 | A | 6.4 | 225 | 235 | 245 |
| | B | 6.2 | 215 | 225 | 275 |
| | C | 6.1 | 205 | 215 | 290 |
| 1½ | A | 6.0 | 225 | 235 | 290 |
| | B | 5.8 | 215 | 225 | 320 |
| | C | 5.7 | 205 | 215 | 345 |
| 2 | A | 5.7 | 225 | 235 | 330 |
| | B | 5.6 | 215 | 225 | 360 |
| | C | 5.4 | 205 | 215 | 380 |

* "Concrete Manual," U.S. Bureau of Reclamation.

workability. If the mix is undersanded, change to mix A; if oversanded, change to mix C. Weights are given for dry sand. For damp sand, increase the weight of sand 10 lb, and for very wet sand, 20 lb, per bag of cement.

## 8.4.2 Admixtures

These may be used to modify and control specific characteristics of concrete. Major types of admixtures include set accelerators, water reducers, air entrainers, and waterproofing compounds. In general, admixtures are helpful in improving concrete workability. Some admixtures, if not administered properly, could have undesirable side effects. Hence, every engineer should be familiar with admixtures and their chemical components as well as their advantages and limitations. Moreover, admixtures should be used in accordance with manufacturers' recommendations and, if possible, under the supervision of a manufacturer's representative. Many admixtures are covered by ASTM specifications.

**Accelerating admixtures** are used to reduce the time of setting and accelerating early strength development and are often used in cold weather, when it takes too long for concrete to set naturally. The best-known accelerator is calcium chloride, but it is not recommended for use in prestressed concrete, in reinforced concrete containing embedded dissimilar metals, or where progressive corrosion of steel reinforcement can occur. Nonchloride, noncorrosive accelerating admixtures, although more expensive than calcium chloride, may be used instead.

**Water reducers** lubricate the mix. Most of the water in a normal concrete mix is needed for workability of the concrete. Reduction in the water content of a mix may result in either a reduction in the water-cement ratio (w/c) for a given slump and cement content or an increased slump for the same w/c and cement content. With the same cement content but less water, the concrete attains greater strength. As an alternative, reduction of the quantity of water permits a proportionate decrease in cement and thus reduces shrinkage of the hardened concrete. An additional advantage of a water-reducing admixture is easier placement of concrete. This, in turn, helps the workers and reduces the possibility of honeycombed concrete. Some water-reducing admixtures also act as retarders of concrete set, which is helpful in hot weather and in integrating consecutive pours of concrete.

**High-range water-reducing admixtures,** also known as superplasticizers, behave much like conventional water-reducing admixtures. They help the concrete achieve high strength and water reduction without loss of workability. Superplasticizers reduce the interparticle forces that exist between cement grains in the fresh paste, thereby increasing the paste fluidity. However, they differ from conventional admixtures in that superplasticizers do not affect the surface tension of water significantly, as a result of which, they can be used at higher dosages without excessive air entrainment.

**Air-entraining agents** entrain minute bubbles of air in concrete. This increases resistance of concrete to freezing and thawing. Therefore, air-entraining agents are extensively used in exposed concrete. Air entrainment also affects properties of fresh concrete by increasing workability.

**Waterproofing chemicals** may be added to a concrete mix, but often they are applied as surface treatments. Silicones, for example, are used on hardened concrete as a water repellent. If applied properly and uniformly over a concrete surface, they can effectively prevent rainwater from penetrating the surface. (Some silicone coatings discolor with age. Most lose their effectiveness after a number of years. When that happens, the surface should be covered with a new coat of silicone for continued protection.) Epoxies also may be used as water repellents. They are much more durable, but they also may be much more costly. Epoxies have many other uses in concrete, such as protection of wearing surfaces, patching compounds for cavities and cracks, and glue for connecting pieces of hardened concrete.

Miscellaneous types of admixtures are available to improve properties of concrete either in the plastic or the hardened state. These include polymer-bonding admixtures used to produce modified concrete, which has better abrasion resistance, better resistance to freezing and thawing, and reduced permeability; dampproofing admixtures; permeability-reducing admixtures; and corrosion-inhibiting admixtures.

### 8.4.3 Mixing Concrete Mixes

Components for concrete generally are stored in batching plants before being fed to a mixer. These plants consist of weighing and control equipment and hoppers, or bins, for storing cement and aggregates. Proportions are controlled by manually operated or automatic scales. Mixing water is measured out from measuring tanks or with the aid of water meters.

Machine mixing is used wherever possible to achieve uniform consistency of each batch. Good results are obtained with the revolving-drum-type mixer, commonly used in the United States, and countercurrent mixers, with mixing blades rotating in the direction opposite to that of the drum.

Mixing time, measured from the time the ingredients, including water, are in the drum, should be at least 1.5 min for a 1-yd$^3$ mixer, plus 0.5 min for each cubic yard of capacity over 1 yd$^3$. But overmixing may remove entrained air and increase fines, thus requiring more water to maintain workability, so it is advisable also to set a maximum on mixing time. As a guide, use three times the minimum mixing time.

Ready-mixed concrete is batched in central plants and delivered to various job-sites in trucks, usually in mixers mounted on the trucks. The concrete may be mixed en route or after arrival at the site. Though concrete may be kept plastic and workable for as long as 1½ h by slow revolving of the mixer, better control of mixing time can be maintained if water is added and mixing started after arrival of the truck at the job, where the operation can be inspected.

(ACI 212.2, "Guide for Use of Admixtures in Concrete," ACI 211.1, "Recommended Practice for Selecting Proportion for Normal and Heavyweight Concrete," ACI 213R, "Recommended Practice for Selecting Proportions for Structural Lightweight Concrete," and ACI 304, "Recommended Practice for Measuring, Mixing, Transporting, and Placing Concrete," American Concrete Institute, 38800 Country Club Drive Farmington Hills, MI 48331; G. E. Troxell, H. E. Davis, and J. W. Kelly, "Composition and Properties of Concrete," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); D. F. Orchard, "Concrete Technology," John Wiley & Sons, Inc., New York; M. Fintel, "Handbook of Concrete Engineering," 2nd ed., Van Nostrand Reinhold, New York.)

## 8.5 Concrete Placement

When concrete is discharged from the mixer, precautions should be taken to prevent segregation because of uncontrolled chuting as it drops into buckets, hoppers, carts, or forms. Such segregation is less likely to occur with tilting mixers than with nontilting mixers with discharge chutes that let the concrete pass in relatively small streams. To prevent segregation, a baffle, or better still, a section of downpipe should be inserted at the end of the chutes so that the concrete will fall vertically into the center of the receptacle.

### 8.5.1 Concrete Transport and Placement Equipment

**Steel buckets**, when selected for the job conditions and properly operated, handle and place concrete very well. But they should not be used if they have to be hauled so far that there will be noticeable separation, bleeding, or loss of slump exceeding 1 in. The discharge should be controllable in amount and direction.

**Rail cars and trucks** sometimes are used to transport concrete after it is mixed. But there is a risk of stratification, with a layer of water on top, coarse aggregate on the bottom. Most effective prevention is use of dry mixes and air entrainment. If stratification occurs, the concrete should be remixed either as it passes through the discharge gates or by passing small quantities of compressed air through the concrete en route.

**Chutes** frequently are used for concrete placement. But the operation must be carefully controlled to avoid segregation and objectionable loss of slump. The slope must be constant under varying loads and sufficiently steep to handle the stiffest concrete to be placed. Long chutes should be shielded from sun and wind to prevent evaporation of mixing water. Control at the discharge end is of utmost importance to prevent segregation. Discharge should be vertical, preferably through a short length of downpipe.

**Tremies**, or elephant trunks, deposit concrete under water. Tremies are tubes about 1 ft or more in diameter at the top, flaring slightly at the bottom. They should be long enough to reach the bottom. When concrete is being placed, the tremie is always kept full of concrete, with the lower end immersed in the concrete just deposited. The tremie is raised as the level of concrete rises. Concrete should never be deposited through water unless confined.

**Belt conveyors** for placing concrete also present segregation and loss-of-slump problems. These may be reduced by adopting the same precautions as for transportation by trucks and placement with chutes.

**Sprayed concrete** (shotcrete or gunite) is applied directly onto a form by an air jet. A "gun," or mechanical feeder, mixer, and compressor comprise the principal equipment for this method of placement. Compressed air and the dry mix are fed to the gun, which jets them out through a nozzle equipped with a perforated manifold. Water flowing through the perforations is mixed with the dry mix before it is ejected. Because sprayed concrete can be placed with a low water-cement ratio, it usually has high compressive strength. The method is especially useful for building up shapes without a form on one side.

**Pumping** is a suitable method for placing concrete, but it seldom offers advantages over other methods. Curves, lifts, and harsh concrete reduce substantially maximum pumping distance. For best performance, an agitator should be installed in the pump feed hopper to prevent segregation.

**Barrows** are used for transporting concrete very short distances, usually from a hopper to the forms. In the ordinary wheelbarrow, a worker can move $1\frac{1}{2}$ to 2 $ft^3$ of concrete 25 ft in 3 min.

**Concrete carts** serve the same purpose as wheelbarrows but put less load on the transporter. Heavier and wider, the carts can handle 4.5 $ft^3$. Motorized carts with $\frac{1}{2}$-$yd^3$ capacity also are available.

Regardless of the method of transportation or equipment used, the concrete should be deposited as nearly as possible in its final position. Concrete should not be allowed to flow into position but should be placed in horizontal layers because then less durable mortar concentrates in ends and corners where durability is most important.

### 8.5.2 Vibration of Concrete in Forms

This is desirable because it eliminates voids. The resulting consolidation also ensures close contact of the concrete with the forms, reinforcement, and other embedded items. It usually is accomplished with electric or pneumatic vibrators.

For consolidation of structural concrete and tunnel-invert concrete, immersion vibrators are recommended. Oscillation should be at least 7000 vibrations per minute when the vibrator head is immersed in concrete. Precast concrete of relatively small dimensions and concrete in tunnel arch and sidewalls may be vibrated with vibrators rigidly attached to the forms and operating at 8000 vibrations per minute or more. Concrete in canal and lateral linings should be vibrated at more than 4000 vibrations per minute, with the immersion type, though external vibration may be used for linings less than 3 in thick. For mass concrete, with 3- and 6-in coarse aggregate, vibrating heads should be at least 4 in in diameter and operate at frequencies of at least 6000 vibrations per minute when immersed. Each cubic yard should be vibrated for at least 1 min. A good small vibrator can handle from 5 to 10 $yd^3/h$ and a large two-person, heavy-duty type, about 50 $yd^3/h$ in uncramped areas. Over vibration can be detrimental as it can cause segregation of the aggregate and bleeding of the concrete.

### 8.5.3 Construction Joints

A construction joint is formed when unhardened concrete is placed against concrete that has become so rigid that the new concrete cannot be incorporated into the old by vibration. Generally, steps must be taken to ensure bond between the two.

Method of preparation of surfaces at construction joints vary depending on the orientation of the surface.

("Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, DC, 20402 (www.gpo.gov); ACI 311 "Recommended Practice for Concrete Inspection"; ACI 304, "Recommended Practice for Measuring, Mixing, Transporting, and Placing Concrete"; and ACI 506 "Recommended Practice for Shotcreting"; also, ACI 304.2R, "Placing Concrete by Pumping Methods," ACI 304.1R, "Preplaced Aggregate Concrete for Structural and Mass Concrete," and "ACI Manual of Concrete Inspection," SP-2, American Concrete Institute (www.aci-int.org).)

## 8.6 Finishing of Unformed Concrete Surfaces

After concrete has been consolidated, screeding, floating, and the first troweling should be performed with as little working and manipulation of the surface as possible. Excessive manipulation draws inferior fines and water to the top and can cause checking, crazing, and dusting.

To avoid bringing fines and water to the top in the rest of the finishing operations, each step should be delayed as long as possible. If water accumulates, it should be removed by blotting with mats or draining, or it should be pulled off with a loop of hose, and the next finishing operation should be delayed until the water sheen disappears. Do not work neat cement into wet areas to dry them.

**Screeds** are guides for a straightedge to bring a concrete surface to a desired elevation or for a template to produce a desired curved shape. The screeds must be sufficiently rigid to resist distortion as the concrete is spread. They may be made of lumber or steel pipe.

For floors, screeding is followed by hand floating with wood floats or power floating. Permitting a stiffer mix with a higher percentage of large-size aggregate, power-driven floats with revolving disks and vibrators produce a sounder, more durable surface than wood floats. Floating may begin as soon as the concrete surface has hardened sufficiently to bear a person's weight without leaving an indentation. The operation continues until hollows and humps are removed or, if the surface is to be troweled, until a small amount of mortar is brought to the top.

If a finer finish is desired, the surface may be steel-troweled, by hand or by powered equipment. This is done as soon as the floated surface has hardened enough so that excess fine material will not be drawn to the top. Heavy pressure during troweling will produce a dense, smooth, watertight surface. Do not permit sprinkling of cement or cement and sand on the surface to absorb excess water or facilitate troweling. If an extra hard finish is desired, the floor should be troweled again when it has nearly hardened.

Concrete surfaces dust to some extent and may benefit from treatment with certain chemicals. They penetrate the pores to form crystalline or gummy deposits. Thus, they make the surface less pervious and reduce dusting by acting as plastic binders or by making the surface harder. Poor-quality concrete floors may be improved more by such treatments than high-quality concrete, but the improvement is likely to be temporary and the treatment will have to be repeated periodically.

("Concrete Manual," U.S. Bureau of Reclamation, U.S. Government Printing Office, Washington, DC 20402 (www.gpo.gov).)

## 8.7 Forms for Concrete

Formwork retains concrete until it has set and produces the desired shapes and, sometimes, desired surface finishes. Forms must be supported on falsework of adequate strength and sufficient rigidity to keep deflections within acceptable limits. The forms too must be strong and rigid, to meet dimensional tolerances. But they also must be tight, or mortar will leak out during vibration and cause unsightly sand streaks and rock pockets. Yet they must be low-cost and often easily demountable to permit reuse. These requirements are met by steel, reinforced plastic, and plain or coated lumber and plywood.

Unsightly bulges and offsets at horizontal joints should be avoided. This can be done by resetting forms with only 1 in of form lining overlapping the existing concrete below the line made by a grade strip. Also, the forms should be tied and bolted close to the joint to keep the lining snug against existing concrete (Fig. 8.2). If a groove along a joint will not be esthetically objectionable, forming of a groove along the joint will obscure unsightliness often associated with construction joints (Art. 8.5.3).

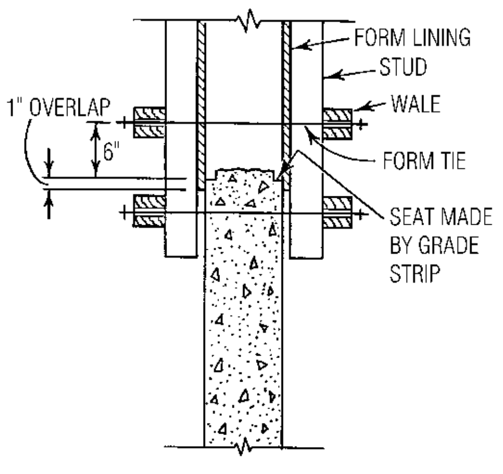



**Fig. 8.2** Form set to avoid bulges at a horizontal joint in a concrete wall.

Where form ties have to pass through the concrete, they should be as small in cross section as possible. (The holes they form sometimes have to be plugged to stop leaks.) Ends of form ties should be removed without spalling adjacent concrete.

Plastic coatings, proper oiling, or effective wetting can protect forms from deterioration, weather, and shrinkage before concreting. Form surfaces should be clean. They should be treated with a suitable form-release oil or other coating that will prevent the concrete from sticking to them. A straight, refined, pale, paraffin-base mineral oil usually is acceptable for wood forms. Synthetic castor oil and some marine-engine oils are examples of compounded oils that give good results on steel forms. The oil or coating should be brushed or sprayed evenly over the forms. It should not be permitted to get on construction joint surfaces or reinforcing bars because it will interfere with bond.

Forms should provide ready access for placement and vibration of concrete for inspection. Formed areas should be clean of debris prior to concrete placement.

Generally, forms are stationary. But for some applications, such as highway pavements, precast-concrete slabs, silos, and service cores of buildings, use of continuous moving forms—sliding forms or slip forms—is advantageous.

### 8.7.1 Slip Forms

A slip form for vertical structures consists principally of a form lining or sheathing about 4 ft high, wales or ribs, yokes, working platforms, suspended scaffolds, jacks, climbing rods, and control equipment (Fig. 8.3). Spacing of the sheathing is slightly larger at the top to permit easy upward movement. The wales hold the sheathing in alignment, support the working platforms and scaffolds, and transmit lifting forces from yokes to sheathing. Each yoke has a horizontal cross member perpendicular to the wall and connected to a jack. From each end of the member, vertical legs extend downward on opposite sides of and outside the wall. The lower end of each leg is attached to a bottom wale. The jack pulls the slip form upward by climbing a vertical steel rod, usually about 1 in in diameter, embedded in the concrete. The suspended scaffolds provide access for finishers to the wall. Slip-form climbing rates range upward from about 2 to about 12 in/h.

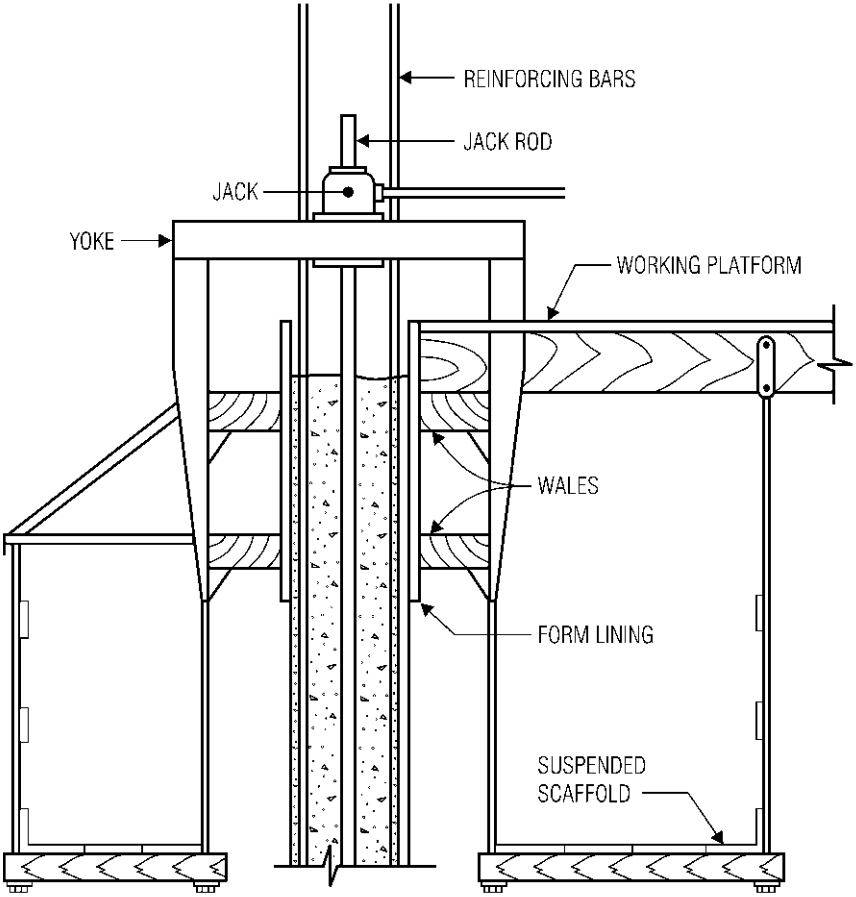



**Fig. 8.3** Slip form for a concrete wall.

### 8.7.2 Form Removal

Stationary forms should be removed only after the concrete has attained sufficient strength so that there will be no noticeable deformation or damage to the concrete. If supports are removed before beams or floors are capable of carrying superimposed loads, they should be reshored until they have gained sufficient strength.

Early removal of forms generally is desirable to permit quick reuse, start curing as soon as possible, and allow repairs and surface treatment while the concrete is still green and conditions are favorable for good bond. In cold weather, however, forms should not be removed while the concrete is still warm. Rapid cooling of the surface will cause checking and surface cracks. For this reason also, curing water applied to newly stripped surfaces should not be much cooler than the concrete.

(R. L. Peurifoy, "Formwork for Concrete Structures," 2nd ed., McGraw-Hill Book Company, New York (books.mcgraw-hill.com); "Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, DC, 20402 (www.gpo.gov); ACI 347 "Recommended Practice for Concrete Formwork," "ACI Manual of Concrete Inspection," SP-2, and "Formwork for Concrete," SP-4, American Concrete Institute (www.aci-int.org).)

## 8.8 Curing Concrete

While more than enough mixing water for hydration is incorporated into normal concrete mixes, drying of the concrete after initial set may delay or prevent complete hydration. Curing includes all operations after concrete has set that improve hydration. Properly done for a sufficiently long period, curing produces stronger, more watertight concrete.

Methods may be classified as one of the following: maintenance of a moist environment by addition of water, sealing in the water in the concrete, and those hastening hydration.

### 8.8.1 Curing by Surface Moistening

Maintenance of a moist environment by addition of water is the most common field procedure. Generally, exposed concrete surfaces are kept continuously moist by spraying or ponding or by a covering of earth, sand, or burlap kept moist. Concrete made with ordinary and sulfate-resistant cements (Types I, II, and V) should be cured this way for 7 to 14 days, that made with low-heat cement (Type IV) for at least 21 days. Concrete made with high-early-strength cement should be kept moist until sufficient strength has been attained, as indicated by test cylinders.

### 8.8.2 Steam Curing

Precast concrete and concrete placed in cold weather often are steam-cured in enclosures. Although this is a form of moist curing, hydration is speeded by the higher-than-normal temperature, and the concrete attains a high early strength. Temperatures maintained usually range between 100 and 165 °F. Higher temperatures produce greater strengths shortly after steam curing commences, but there are severe losses in strength after 2 days. A delay of 1 to 6 h before steam curing will produce concrete with higher 24-h strength than if the curing starts immediately after the concrete is cast. This "preset" period allows early cement reactions to occur and development of sufficient hardness to withstand the more rapid temperature curing to follow. Length of the preset period depends on the type of aggregate and temperature. The period should be longer for ordinary aggregate than for lightweight and for higher temperatures. Duration of steam curing depends on the concrete mix, temperature, and desired results.

Autoclaving, or high-pressure steam curing, maintains concrete in a saturated atmosphere at temperatures above the boiling point of water. Generally, temperatures range from 325 to 375 °F at pressures from 80 to 170 psig. Main application is for concrete masonry. Advantages claimed are high early strength, reduced volume change in drying, better chemical resistance, and lower susceptibility to efflorescence. For steam curing, a preset period of 1 to 6 h is desirable, followed by single- or two-stage curing. Single-curing consists of a pressure buildup of at least 3 h, 8 h at maximum pressure, and rapid pressure release (20 to 30 min). The rapid release vaporizes moisture from the block. In two-stage curing, the concrete products are placed in kilns for the duration of the preset period. Saturated steam then is introduced into the kiln. After the concrete has developed sufficient strength to permit handling, the products are removed from the kiln, set in a compact arrangement, and placed in the autoclave.

### 8.8.3 Curing by Surface Sealing

Curing concrete by sealing the water in can be accomplished by either covering the concrete or coating it with a waterproof membrane. When coverings, such as heavy building paper or plastic sheets, are used, care must be taken that the sheets are sealed airtight and corners and edges are adequately protected against loss of moisture. Coverings can be placed as soon as the concrete has been finished.

Coating concrete with a sealing compound generally is done by spraying to ensure a continuous membrane. Brushing may damage the concrete surface. Sealing compound may be applied after the surface has stiffened so that it will no longer respond to float finishing. But in hot climates, it may be desirable, before spraying, to moist cure for 1 day surfaces exposed to the sun. Surfaces from which forms have been removed should be saturated with water before spraying with compound. But the compound should not be applied to either formed or unformed surfaces until the moisture film on them has disappeared. Spraying should be started as soon as the surfaces assume a dull appearance. The coating should be protected against damage. Continuity must be maintained for at least 28 days.

White or gray pigmented compound often is used for sealing because it facilitates inspection and reflects heat from the sun. Temperatures with white pigments may be decreased as much as 40 °F, reducing cracking caused by thermal changes.

Surfaces of ceilings and walls inside buildings require no curing other than that provided by forms left in place at least 4 days. But wood forms are not acceptable for moist curing outdoor concrete. Water should be applied at the top, for example, by a soil-soaker hose and allowed to drip down between the forms and the concrete.

("Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, DC, 20402 (www.gpo.gov); ACI 517, "Recommended Practice for Atmospheric Pressure Steam Curing of Concrete," ACI 517.1R, "Low-Pressure Steam Curing," and ACI 516R, "High-Pressure Steam Curing: Modern Practice, and Properties of Autoclaved Products," American Concrete Institute (www.aci-int.org).)

## 8.9 Cold-Weather Concreting

Hydration of cement takes place in the presence of moisture at temperatures above 50 °F. Methods used during cold weather should prevent damage to concrete from freezing and thawing at an early age. (Concrete that is protected from freezing until it has attained a compressive strength of at least 500 psi will not be damaged by exposure to a single freezing cycle.) Neglect of protection against freezing can cause immediate destruction or permanent weakening of concrete. Therefore, if concreting is performed in cold weather, protection from low temperatures and proper curing are essential. Except within heated protective enclosures, little or no external supply of moisture is required for curing during cold weather. Under such conditions, the temperature of concrete placed in the forms should not be lower than the values listed in Table 8.5. Protection against freezing should be provided until concrete has gained sufficient strength to withstand exposure to low temperatures, anticipated environment, and construction and service loads.

The time needed for concrete to attain the strength required for safe removal of shores is influenced by the initial concrete temperature at placement, temperatures after placement, type of cement, type and amount of accelerating admixture, and the conditions of protection and curing. The use of high-early-strength cement or the addition of accelerating admixtures may be an economic solution when schedule considerations are critical. The use of such admixtures does not justify a reduction in the amount of protective cover, heat, or other winter protection.

Although freezing is a danger to concrete, so is overheating the concrete to prevent it. By accelerating chemical action, overheating can cause excessive loss of slump, raise the water requirement for a given slump, and increase thermal shrinkage. Rarely will mass concrete leaving the mixer have to be at more than 55 °F and thin-section concrete at more than 75 °F.

**Table 8.5** Recommended Concrete Temperatures for Cold-Weather Construction—Air Entrained Concrete

| | Minimum Cross-Sectional Dimension, in | | | |
|---|---|---|---|---|
| | less than 12 | 12 to 36 | 36 to 72 | 72 or more |
| | (*a*) Minimum Temperature of Concrete as Placed or Maintained, °F | | | |
| | 55 | 50 | 45 | 40 |
| | (*b*) Maximum Allowable Gradual Temperature Drop of Concrete in First 24 h after Protection Is Discounted, °F | | | |
| | 50 | 40 | 30 | 20 |
| Temperature of air, °F | (*c*) Minimum Temperature of Concrete as Mixed, °F | | | |
| 30 or higher | 60 | 55 | 50 | 45 |
| 0 to 30 | 65 | 60 | 55 | 50 |
| 0 or lower | 70 | 65 | 60 | 55 |

To obtain the minimum temperatures for concrete mixes in cold weather, the water and, if necessary, the aggregates should be heated. The proper mixing water temperature for the required concrete temperature is based upon the temperature and weight of the materials in the concrete and the free moisture on aggregates. To avoid flash set of cement and loss of entrained air due to the heated water, aggregates and water should be placed in the mixer before the cement and air-entraining agent so that the colder aggregates will reduce the water temperature to below 80 °F.

When heating of aggregates is necessary, it is best done with steam or hot water in pipes. Use of steam jets is objectionable because of resulting variations in moisture content of the aggregates. For small jobs, aggregates may be heated over culvert pipe in which fires are maintained, but care must be taken not to overheat.

Before concrete is placed in the forms, the interior should be cleared of ice, snow, and frost. This may be done with steam under canvas or plastic covers.

Concrete should not be placed on frozen earth. It would lower the concrete temperature below the minimum and may cause settlement on thawing. The subgrade may be protected from freezing by a covering of straw and tarpaulins or other insulating blankets. If it does freeze, the subgrade must be thawed deep enough so that it will not freeze back up to the concrete during the required protection period.

The usual method of protecting concrete after it has been cast is to enclose the structure with tarpaulins or plastic and heat the interior. Since corners and edges are especially vulnerable to low temperatures, the enclosure should enclose corners and edges, not rest on them. The enclosure must be not only strong but windproof. If wind can penetrate it, required concrete temperatures may not be maintained despite high fuel consumption. Heat may be supplied by live or piped steam, salamanders, stoves, or warm air blown in through ducts from heaters outside the enclosure. But strict fire-prevention measures should be enforced. When dry heat is used, the concrete should be kept moist to prevent it from drying.

Concrete also may be protected with insulation. For example, pavements may be covered with layers of straw, shavings, or dry earth. For structures, forms may be insulated.

When protection is discontinued or when forms are removed, precautions should be taken that the drop in temperature of the concrete will be gradual. Otherwise, the concrete may crack and deteriorate. Table 8.5 lists recommended limitations on temperature drop in the first 24 hours. Special care shall be taken with concrete test specimens used for acceptance of concrete. Cylinders shall be properly stored and protected in insulated boxes with a thermometer to maintain temperature records.

("Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, DC 20402 (www.gpo.gov); ACI 306R "Cold-Weather Concreting," American Concrete Institute (www.aci-int.org).)

## 8.10 Hot-Weather Concreting

Hot weather is defined as any combination of the following: high ambient air temperature, high concrete temperature, low relative humidity, high wind velocity, and intense solar radiation. Such weather may lead to conditions in mixing, placing, and curing concrete that can adversely affect the properties and serviceability of the concrete.

The higher the temperature, the more rapid the hydration of cement, the faster the evaporation of mixing water, the lower the concrete strength and the larger the volume change. Unless precautions are taken, setting and rate of hardening will accelerate, shortening the available time for placing and finishing the concrete. Quick stiffening encourages undesirable additions of mixing water, or retempering, and may also result in inadequate consolidation and cold joints. The tendency to crack is increased because of rapid evaporation of water, increased drying shrinkage, or rapid cooling of the concrete from its high initial temperature. If an air-entrained concrete is specified, control of the air content is more difficult. And curing becomes more critical. Precautionary measures required on a calm, humid day will be less restrictive than those required on a dry, windy, sunny day, even if the air temperatures are identical.

Placement of concrete in hot weather is too complex to be dealt with adequately by simply setting a maximum temperature at which concrete may be placed. A rule of thumb, however, has been that concrete temperature during placement should be maintained as much below 90 °F as is economically feasible.

The following measures are advisable in hot weather: The concrete should have ingredients and proportions with satisfactory records in field use in hot weather. To keep the concrete temperature within a safe range, the concrete should be cooled with iced water or cooled aggregate, or both. Also, to minimize slump loss and other temperature effects, the concrete should be transported, placed, consolidated, and finished as speedily as possible. Materials and facilities not otherwise protected from the heat should be shaded. Mixing drums should be insulated or cooled with water sprays or wet burlap coverings. Also, water-supply lines and tanks should be insulated or at least painted white. Cement with a temperature exceeding 170 °F should not be used. Forms, reinforcing steel, and the subgrade should be sprinklered with cool water. If necessary, work should be done only at night. Futhermore, the concrete should be protected against moisture loss at all times during placing and curing.

Self-retarding admixtures counteract the accelerating effects of high temperature and lessen the need for increase in mixing water. Their use should be considered when the weather is so hot that the temperature of concrete being placed is consistently above 75 °F.

Continuous water curing gives best results in hot weather. Curing should be started as soon as the concrete has hardened sufficiently to withstand surface damage. Water should be applied to formed surfaces while forms are still in place. Surfaces without forms should be kept moist by wet curing for at least 24 h. Moist coverings are effective in eliminating evaporation loss from concrete, by protecting it from sun and wind. If moist curing is discontinued after the first day, the surface should be protected with a curing compound (Art. 8.8).

(ACI 305R, "Hot-Weather Concreting," American Concrete Institute (www.aci-int.org).)

## 8.11 Contraction and Expansion Joints

Contraction joints are used mainly to control locations of cracks caused by shrinkage of concrete after it has hardened. If the concrete, while shrinking, is restrained from moving, by friction or attachment to more rigid construction, cracks are likely to occur at points of weakness. Contraction joints, in effect, are deliberately made weakness planes. They are formed in the expectation that if a crack occurs it will be along the neat geometric pattern of a joint, and thus irregular, unsightly cracking will be prevented. Such joints are used principally in floors, roofs, pavements, and walls.

A contraction joint is an indentation in the concrete. Width may be ¼ or ⅜ in and depth one-fourth the thickness of the slab. The indentation may be made with a saw cut while the concrete still is green but before appreciable shrinkage stress develops. Or the joint may be formed by insertion of a strip of joint material before the concrete sets or by grooving the surface during finishing. Spacing of joints depends on the mix, strength and thickness of the concrete, and the restraint to shrinkage. The indentation in highway and airport pavements usually is filled with a sealing compound.

Construction joints occur where two successive placements of concrete meet. They may be designed to permit movement and/or to transfer load.

**Expansion or isolation** joints are used to help prevent cracking due to thermal dimension changes in concrete. They usually are placed where there are abrupt changes in thickness, offsets, or changes in types of construction, for example, between a bridge pavement and a highway pavement. Expansion joints provide a complete separation between two parts of a slab. The opening must be large enough to prevent buckling or other undesirable deformation due to expansion of the concrete.

To prevent the joint from being jammed with dirt and becoming ineffective, the opening is sealed with a compressible material. For watertightness, a flexible water stop should be placed across the joint. And if load transfer is desired, dowels should be embedded between the parts separated by the joint. The sliding ends of the dowels should be enclosed in a close-fitting metal cap or thimble, to provide space for movement of the dowel during expansion of the concrete. This space should be at least ¼ in longer than the width of the joint.

(ACI 504R, "Guide to Joint Sealants for Concrete Structures," American Concrete Institute (www.aci-int.org).)

## 8.12 Steel Reinforcement in Concrete

Because of the low tensile strength of concrete, steel reinforcement is embedded in it to resist tensile stresses. Steel, however, also is used to take compression, in beams and columns, to permit use of smaller members. It serves other purposes too: It controls strains due to temperature and shrinkage and distributes load to the concrete and other reinforcing steel; it can be used to prestress the concrete; and it ties other reinforcing together for easy placement or to resist lateral stresses.

Most reinforcing is in the form of bars or wires whose surfaces may be smooth or deformed. The latter type is generally used because it produces better bond with the concrete because of the raised pattern on the steel.

Bars range in diameter from $\frac{1}{4}$ to $2\frac{1}{4}$ in (Table 8.11, p 8.36). Sizes are designated by numbers, which are approximately eight times the nominal diameters. (See the latest edition of ASTM "Specifications for Steel Bars for Concrete Reinforcement." These also list the minimum yield points and tensile strengths for each type of steel.) Use of bars with yield points over 60 ksi for flexural reinforcement is limited because special measures are required to control cracking and deflection.

Wires usually are used for reinforcing concrete pipe and, in the form of welded-wire fabric, for slab reinforcement. The latter consists of a rectangular grid of uniformly spaced wires, welded at all intersections, and meeting the minimum requirements of ASTM A185 and A497. Fabric offers the advantages of easy, fast placement of both longitudinal and transverse reinforcement and excellent crack control because of high mechanical bond with the concrete. (Deformed wires are designated by D followed by a number equal to the nominal area, in$^2$, times 100.) Bars and rods also may be prefabricated into grids, by clipping or welding (ASTM A184).

Sometimes, metal lath is used for reinforcing concrete, for example, in thin shells. It may serve as both form and reinforcing when concrete is applied by spray (gunite or shotcrete.)

### 8.12.1 Bending and Placing Reinforcing Steel

Bars are shipped by a mill to a fabricator in uniform long lengths and in bundles of 5 or more tons. The fabricator transports them to the job straight and cut to length or cut and bent.

Bends may be required for beam-and-girder reinforcing, longitudinal reinforcing of columns where they change size, stirrups, column ties and spirals, and slab reinforcing. Dimensions of standard hooks and typical bends and tolerances for cutting and bending are given in ACI 315, "Manual of Standard Practice for Detailing Reinforced Concrete Structures," American Concrete Institute (www.aci-int.org).

Some preassembling of reinforcing steel is done in the fabricating shop or on the job. Beam, girder, and column steel often is wired into frames before placement in the forms. Slab reinforcing may be clipped or welded into grids, or mats, if not supplied as welded-wire fabric.

Some rust is permissible on reinforcing if it is not loose and there is no appreciable loss of cross-sectional area. In fact, rust, by creating a rough surface, will improve bond between the steel and concrete. But the bars should be free of loose rust, scale, grease, oil, or other coatings that would impair bond.

Bars should not be bent or straightened in any way that will damage them. All reinforcement shall be bent cold unless permitted by the engineer. If heat is necessary for bending, the temperature should not be higher than that indicated by a cherry-red color (1200 °F), and the steel should be allowed to cool slowly, not quenched, to 600 °F.

Reinforcing should be supported and tied in the locations and positions called for in the plans. The steel should be inspected before concrete is placed. Neither the reinforcing nor other parts to be embedded should be moved out of position before or during the casting of the concrete.

Bars and wire fabric should not be kinked or have unspecified curvatures when positioned. Kinked and curved bars, including those misshaped by workers walking on them, may cause the hardened concrete to crack when the bars are tensioned by service loads.

Usually, reinforcing is set on wire bar supports, preferably galvanized for exposed surfaces. Lower-layer bars in slabs usually are supported on bolsters consisting of a horizontal wire welded to two legs about 5 in apart. The upper layer generally is supported on bolsters with runner wires on the bottom so that they can rest on bars already in place. Or individual or continuous high chairs can be used to hold up a support bar, often a No. 5, at appropriate intervals, usually 5 ft. An individual high chair is a bar seat that looks roughly like an inverted U braced transversely by another inverted U in a perpendicular plane. A continuous high chair consists of a horizontal wire welded to two inverted-U legs 8 or 12 in apart. Beam and joist chairs have notches to receive the reinforcing. These chairs usually are placed at 5-ft intervals.

Although it is essential that reinforcement be placed exactly where called for in the plans, some tolerances are necessary. Reinforcement in beams and slabs, walls and compression members should be within $\pm\frac{3}{8}''$ for members where $d \leq 8''$, $\pm\frac{1}{2}''$ for members where $d > 8''$ of the specified distance from the tension or compression face. Lengthwise, a cutting tolerance of $\pm 1$ in and a placement tolerance of $\pm 2$ in are normally acceptable. If length of embedment is critical, the designer should specify bars 3 in longer than the computed minimum to allow for accumulation of tolerances. Spacing of reinforcing in wide slabs and tall walls may be permitted to vary $\pm\frac{1}{2}$ in or slightly more if necessary to clear obstructions, so long as the required number of bars are present.

Lateral spacing of bars in beams and columns, spacing between multiple reinforcement layers, and concrete cover over stirrups, ties, and spirals in beams and columns should never be less than that specified but may exceed it by $\frac{1}{4}$ in. A variation in setting of an individual stirrup or column hoop of 1 in may be acceptable, but the error should not be permitted to accumulate.

("CRSI Recommended Practice for Placing Reinforcing Bars," and "Manual of Standard Practice," Concrete Reinforcing Steel Institute, 180 North La Salle St., Chicago, IL 60601 (www.crsi.org).)

### 8.12.2 Minimum Spacing of Reinforcement

In buildings, the minimum clear distance between parallel bars should be 1 in for bars up to No. 8 and the nominal bar diameter for larger bars. For columns, however, the clear distance between longitudinal bars should be at least 1.5 in for bars up to No. 8 and 1.5 times the nominal bar diameter for larger bars. And the clear distance between multiple layers of reinforcement in building beams and girders should be at least 1 in. Upper-layer bars should be directly above corresponding bars below. These minimum-distance requirements also apply to the clear distance between a contact splice and adjacent splices or bars.

A common requirement for minimum clear distance between parallel bars in highway bridges is 1.5 times the diameter of the bars, and spacing center to center should be at least 1.5 times the maximum size of coarse aggregate.

Many codes and specifications relate the minimum bar spacing to maximum size of coarse aggregate. This is done with the intention of providing enough space for all of the concrete mix to pass between the reinforcing. But if there is a space to place concrete between layers of steel and between the layers and the forms, and the concrete is effectively vibrated, experience has shown that bar spacing or form clearance does not have to exceed the maximum size of coarse aggregate to ensure good filling and consolidation. That portion of the mix which is molded by vibration around bars, and between bars and forms, is not inferior to that which would have filled those parts had a larger bar spacing been used. The remainder of the mix in the interior, if consolidated layer after layer, is superior because of its reduced mortar and water content ("Concrete Manual," U.S. Bureau of Reclamation, Government Printing Office, Washington, D.C. 20402 (www.gpo.gov)).

**Bundled Bars ▪** Groups of parallel reinforcing bars bundled in contact to act as a unit may be used only when they are enclosed by ties or stirrups. Four bars are the maximum permitted in a bundle, and all must be deformed bars. If full-length bars cannot be used between supports, then there should be a stagger of at least 40 bar diameters between any discontinuities. Also, the length of lap should be increased 20% for a three-bar bundle and 33% for a four-bar bundle. In determining minimum clear distance between a bundle and parallel reinforcing, the bundle should be treated as a single bar of equivalent area.

### 8.12.3 Maximum Spacing

In walls and slabs in buildings, except for concrete-joist construction, maximum spacing, center to center, of principal reinforcement should be 18 in, or three times the wall or slab thickness, whichever is smaller.

### 8.12.4 Concept of Development Length

Bond of steel reinforcement to the concrete in a reinforced concrete member must be sufficient so that the steel will yield before it is freed from the concrete. Furthermore, the length of embedment must be adequate to prevent highly stressed reinforcement from splitting relatively thin sections of restraining concrete. Hence, design codes specify a required length of embedment, called development length, for reinforcing steel. The concept of development length is based on the attainable average bond stress over the embedment length of the reinforcement.

Each reinforcing bar at a section of a member must develop on each side of the section the calculated tension or compression in the bar through development length $l_d$ or end anchorage, or both. Development of tension bars can be assisted by hooks.

### 8.12.5 Tension Development Lengths

For bars and deformed wire in tension, basic development length is defined by Eqs. (8.4). For No. 11 and smaller bars,

$$l_d = \left[ \frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha\beta\gamma\lambda}{(c + k_{tr})/d_b} \right] d_b \qquad (8.4)$$

Where $\alpha$ = traditional reinforcement location factor

$\beta$ = coating factor

$\gamma$ = reinforcement size factor

$\lambda$ = lightweight aggregate factor

$c$ = spacing or cover dimension

$k_{tr}$ = transverse reinforcement index

$d_b$ = bar diameter

### 8.12.6 Compression Development Lengths

For bars in compression, the basic development length $l_d$ is defined as

$$l_d = \frac{0.02 f_y d_b}{\sqrt{f_c'}} \geq 0.0003 d_b f_y \qquad (8.5)$$

but $l_d$ not be less than 8 in. See Table 8.6.

For $f_y$ greater than 60 ksi or concrete strengths less than 3000 psi, the required development length in Table 8.6 should be increased as indicated by Eq. (8.5). The values in Table 8.6 may be multiplied by the applicable factors:

a) reinforcement in excess of that required by analyses: $\frac{As\ required}{As\ provided}$

b) reinforcement enclosed within spiral reinforcement not less than ¼″ diameter and not more than 4″ pitch or within #4 ties spaced not more than 4″ on center.

### 8.12.7 Bar Lap Splices

Because of the difficulty of transporting very long bars, reinforcement cannot always be continuous. When splices are necessary, it is advisable that they

**Table 8.6** Compression Development in Normal-Weight Concrete for Grade 60 Bars

| | $f_c'$ (Normal-Weight Concrete) | | | |
|---|---|---|---|---|
| Bar Size No. | 3000 psi | 3750 psi | 4000 psi | Over 4444 psi* |
| 3 | 8 | 8 | 8 | 8 |
| 4 | 11 | 10 | 10 | 9 |
| 5 | 14 | 12 | 12 | 11 |
| 6 | 17 | 15 | 15 | 14 |
| 7 | 19 | 17 | 17 | 16 |
| 8 | 22 | 20 | 19 | 18 |
| 9 | 25 | 22 | 22 | 20 |
| 10 | 28 | 25 | 24 | 23 |
| 11 | 31 | 27 | 27 | 25 |
| 14 | 38 | 34 | 34 | 32 |
| 18 | 50 | 44 | 43 | 41 |

* For $f_c' > 4444$ psi, minimum embedment = $18d_b$.

should be made where the tensile stress is less than half the permissible stress.

Bars up to No. 11 in size may be spliced by overlapping them and wiring them together.

Bars spliced by noncontact lap splices in flexural members should not be spaced transversely farther apart than one-fifth the required lap length or 6 in.

### 8.12.8 Welded or Mechanical Splices

These other positive connections should be used for bars larger than No. 11 and are an acceptable alternative for smaller bars. Welding should conform to AWS D12.1, "Reinforcing Steel Welding Code," American Welding Society, 550 N.W. LeJeune Road, Miami, FL 33126 (www.aws.org). Bars to be spliced by welding should be butted and welded so that the splice develops in tension at least 125% of their specified yield strength. Mechanical coupling devices should be equivalent in strength.

### 8.12.9 Tension Lap Splices

The length of lap for bars in tension should conform to the following, with $l_d$ taken as the tensile development length for the full yield strength $f_y$ of the reinforcing steel [Eq. (8.4)]:

*Class A splices* (lap of $l_d$) are permitted where both conditions 1 and 2 occur.

1. The area of reinforcement provided is at least twice that required by analysis over the entire lengths of splices.
2. No more than one-half of the total reinforcement is spliced within the required lap length.

*Class B splices* (lap of 1.3 $l_d$) are required where either 1 or 2 does not apply.

Bars in tension splices should lap at least 12 in.

Splices for **tension tie members** should be fully welded or made with full mechanical connections and should be staggered at least 30 in. Where feasible, splices in regions of high stress also should be staggered.7

### 8.12.10 Compression Lap Splices

For a bar in compression, the minimum length of a lap splice should be the largest of 12 in, or $0.0005 f_y d_b$, for $f'_c$ of 3000 psi or larger and steel yield strength $f_y$ of 60 ksi or less, where $d_b$ is the bar diameter.

For tied compression members where the ties have an area, in$^2$, of at least $0.0015hs$ in the vicinity of the lap, the lap length may be reduced to 83% of the preceding requirements but not to less than 12 in ($h$ is the overall thickness of the member, in, and $s$ is the tie spacing, in).

For spirally reinforced compression members, the lap length may be reduced to 75% of the basic required lap but not to less than 12 in.

In columns where reinforcing bars are offset and one bar of a splice has to be bent to lap and contact the other one, the slope of the bent bar should not exceed 1 in 6. Portions of the bent bar above and below the offset should be parallel to the column axis. The design should account for a horizontal thrust at the bend taken equal to at least 1.5 times the horizontal component of the nominal stress in the inclined part of the bar. This thrust should be resisted by steel ties, or spirals, or members framing into the column. This resistance should be provided within a distance of 6 in of the point of the bend.

Where column faces are offset 3 in or more, vertical bars should be lapped by separate dowels.

In columns, a minimum tensile strength at each face equal to one-fourth the area of vertical reinforcement multiplied by $f_y$ should be provided at horizontal cross sections where splices are located. In columns with substantial bending, full tensile splices equal to double the factored tensile stress in the bar are required.

### 8.12.11 Splices of Welded-Wire Fabric

Wire reinforcing normally is spliced by lapping. For plainwire fabric in tension, when the area of reinforcing provided is more than twice that required, the overlap measured between outermost cross wires should be at least 2 in or $1.5 l_d$. Otherwise, the overlap should equal the spacing of the cross wires but not less than $1.5 l_d$ nor 6 in. For deformed wire fabric, the overlap measured between outermost cross wires should be at least 2 in. The overlap should be at least 8″ or $1.3 l_d$.

### 8.12.12 Slab Reinforcement

Structural floor and roof slabs with principal reinforcement in only one direction should be reinforced for shrinkage and temperature stresses in a perpendicular direction. The crossbars may be spaced at a maximum of 18 in or five times the slab thickness. The ratio of reinforcement area of these bars to gross concrete area should be at least 0.0020 for deformed bars with less than 60 ksi yield strength, 0.0018 for deformed bars with 60 ksi yield strength and welded-wire fabric with welded intersections in the direction of stress not more than 12 in apart, and 0.0018 $(60/f_y)$ for bars with $f_y$ greater than 60 ksi.

### 8.12.13 Concrete Cover

To protect reinforcement against fire and corrosion, thickness of concrete cover over the outermost steel should be at least that given in Table 8.7.

(ACI 318, "Building Code Requirements for Reinforced Concrete," American Concrete Institute; "Standard Specifications for Highway Bridges," American Association of State Highway and Transportation Officials, 444 N. Capitol St., N.W., Washington, DC 20001 (www.aashto.org).)

## 8.13 Tendons

High-strength steel is required for prestressing concrete to make the stress loss due to creep and shrinkage of concrete and to other factors a small percentage of the applied stress (Art. 8.37). This type of loss does not increase as fast as increase in strength in the prestressing steel, or tendons.

Tendons should have specific characteristics in addition to high strength to meet the requirements of prestressed concrete. They should elongate uniformly up to initial tension for accuracy in applying the prestressing force. After the yield strength has been reached, the steel should continue to stretch as stress increases, before failure occurs. ASTM Specifications for prestressing wire and strands, A421 and A416, set the yield strength at 80 to 85% of the tensile strength. Furthermore, the tendons should exhibit little or no creep, or relaxation, at the high stresses used.

ASTM A421 covers two types of uncoated, stress-relieved, high-carbon-steel wire commonly used for linear prestressed-concrete construction. Type BA wire is used for applications in which cold-end deformation is used for end anchorages, such as buttonheads. Type WA wire is intended for end anchorages by wedges and where no cold-end deformation of the wire is involved. The wire is required to be stress-relieved by a continuous-strand heat treatment after it has been cold-drawn to size. Type BA usually is furnished 0.196 and 0.250 in in diameter, with an ultimate strength of 240 ksi and yield strength (at 1% extension) of 192 ksi. Type WA is available in those sizes and also 0.192 and 0.276 in in diameter, with ultimate strengths ranging from 250 for the smaller diameters to 235 ksi for the largest. Yield strengths range from 200 for the smallest to 188 ksi for the largest (Table 8.8).

For pretensioning, where the steel is tensioned before the concrete is cast, wires usually are used individually, as is common for reinforced concrete. For posttensioning, where the tendons are tensioned and anchored to the concrete after it has attained sufficient strength, the wires generally are placed parallel to each other in groups, or cables, sheathed or ducted to prevent bond with the concrete.

A seven-wire strand consists of a straight center wire and six wires of slightly smaller diameter winding helically around and gripping it. High friction between the center and outer wires is important where stress is transferred between the strand and concrete through bond. ASTM A416 covers strand with ultimate strengths of 250 and 270 ksi (Table 8.8).

Galvanized strands sometimes are used for posttensioning, particularly when the tendons may not be embedded in grout. Sizes normally available range from a 0.5-in-diameter seven-wire strand, with 41.3-kip breaking strength, to $1\frac{11}{16}$-in-diameter strand, with 352-kip breaking strength. The cold-drawn wire comprising the strand is stress-relieved when galvanized, and stresses due to stranding are offset by prestretching the strand to about 70% of

**Table 8.7** Cast-in-Place Concrete Cover for Steel Reinforcement (Non-prestressed)

1. Concrete deposited against and permanently exposed to the ground, 3 in.
2. Concrete exposed to seawater, 4 in; except precast-concrete piles, 3 in.
3. Concrete exposed to the weather or in contact with the ground after form removal, 2 in for bars larger than No. 5 and $1\frac{1}{2}$ in for No. 5 or smaller.
4. Unexposed concrete slabs, walls, or joists, $\frac{3}{4}$ in for No. 11 and smaller, $1\frac{1}{2}$ in for No. 14 and No. 18 bars. Beams, girders, and columns, $1\frac{1}{2}$ in. Shells and folded-plate members, $\frac{3}{4}$ in for bars larger than No. 5, and $\frac{1}{2}$ inch for No. 5 and smaller.

**Table 8.8** Properties of Tendons

| Diameter, in | Area, $in^2$ | Weight per ft-kip | Ultimate Strength |
|---|---|---|---|
| Uncoated Type WA Wire | | | |
| 0.276 | 0.05983 | 203.2 | 235 ksi |
| 0.250 | 0.04909 | 166.7 | 240 ksi |
| 0.196 | 0.03017 | 102.5 | 250 ksi |
| 0.192 | 0.02895 | 98.3 | 250 ksi |
| Uncoated Type BA Wire | | | |
| 0.250 | 0.04909 | 166.7 | 240 ksi |
| 0.196 | 0.03017 | 102.5 | 240 ksi |
| Uncoated Seven-Wire Strands, 250 Grade | | | |
| 1/4 | 0.04 | 122 | 9 kips |
| 5/16 | 0.058 | 197 | 14.5 kips |
| 3/8 | 0.080 | 272 | 20 kips |
| 7/16 | 0.108 | 367 | 27 kips |
| 1/2 | 0.144 | 490 | 36 kips |
| 270 Grade | | | |
| 3/8 | 0.085 | 290 | 23 kips |
| 7/16 | 0.115 | 390 | 31 kips |
| 1/2 | 0.153 | 520 | 41.3 kips |

its ultimate strength. Tendons 0.5 and 0.6 in in diameter are typically used sheathed and unbonded.

Hot-rolled alloy-steel bars used for prestressing concrete generally are not so strong as wire or strands. The bars usually are stress-relieved, then cold-stretched to at least 90% of ultimate strength to raise the yield point. The cold stretching also serves as proof stressing, eliminating bars with defects.

(H. K. Preston and N. J. Sollenberger, "Modern Prestressed Concrete," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); J. R. Libby, "Modern Prestressed Concrete," Van Nostrand Reinhold Company, New York.)

## 8.14 Fabrication of Prestressed-Concrete Members

Prestressed concrete may be produced much like high-strength reinforced concrete, either cast in place or precast. Prestressing offers several advantages for precast members, which have to be transported from casting bed to final position and handled several times. Prestressed members are lighter than reinforced members of the same capacity, both because higher-strength concrete generally is used and because the full cross section is effective. In addition, prestressing of precast members normally counteracts handling stresses. And, if a prestressed, precast member survives the full prestress and handling, the probability of its failing under service loads is very small.

Two general methods of prestressing are commonly used—pretensioning and posttensioning—and both may be used for the same member. See also Art. 8.37.

**Pretensioning**, where the tendons are tensioned before embedment in the concrete and stress transfer from steel to concrete usually is by bond, is especially useful for mass production of precast elements. Often, elements may be fabricated in long lines, by stretching the tendons (Art. 8.13) between abutments at the ends of the lines. By use of tiedowns and struts, the tendons may be draped in a vertical plane to develop upward and downward components on release. After the tendons have been jacked to their full stress, they are anchored to the abutments.

The casting bed over which the tendons are stretched usually is made of a smooth-surface concrete slab with easily stripped side forms of steel. (Forms for pretensioned members must permit them to move on release of the tendons.) Separators are placed in the forms to divide the long line into members of required length and provide space for cutting the tendons. After the concrete has been cast and has attained its specified strength, generally after a preset period and steam curing, side forms are removed. Then, the tendons are detached from the anchorages at the ends of the line and relieved of their stress. Restrained from shortening by bond with the concrete, the tendons compress it. At this time, it is safe to cut the tendons between the members and remove the members from the forms.

In pretensioning, the tendons may be tensioned one at a time to permit the use of relatively light jacks, in groups, or all simultaneously. A typical stressing arrangement consists of a stationary anchor post, against which jacks act, and a moving crosshead, which is pushed by the jacks and to which the tendons are attached. Usually, the tendons are anchored to a thick steel plate that serves as a combination anchor plate and template. It has holes through which the tendons pass to place them in the desired pattern. Various patented grips are available for anchoring the tendons to the plate. Generally, they are a wedge or chuck type capable of developing the full strength of the tendons.

**Posttensioning** frequently is used for cast-in-place members and long-span flexural members. Cables or bars (Art. 8.13) are placed in the forms in flexible ducts to prevent bond with the concrete. They may be draped in a vertical plane to develop upward and downward forces when tensioned. After the concrete has been placed and has attained sufficient strength, the tendons are tensioned by jacking against the member and then are anchored to it. Grout may be pumped into the duct to establish bond with the concrete and protect the tendons against corrosion. Applied at pressures of 75 to 100 psi, a typical grout consists of 1 part portland cement, 0.75 parts sand (capable of passing through a No. 30 sieve), and 0.75 parts water, by volume.

Concrete with higher strengths than ordinarily used for reinforced concrete offers economic advantages for prestressed concrete. In reinforced concrete, much of the concrete in a slab or beam is assumed to be ineffective because it is in tension and likely to crack under service loads. In prestressed concrete, the full section is effective because it is always under either compression or very low tension. Furthermore, high-strength concrete develops higher bond stresses with the tendons, greater bearing strength to withstand the pressure of anchorages, and a higher modulus of elasticity. The last indicates reductions in initial strain and camber when prestress is applied initially and in creep strain. The reduction in creep strain reduces the loss of prestress with time. Generally, concrete with a 28-day strength of 5000 psi or more is advantageous for prestressed concrete.

**Concrete cover** over prestressing steel, ducts, and nonprestressed steel should be at least 3 in for concrete surfaces in contact with the ground; $1\frac{1}{2}$ in for prestressing steel and main reinforcing bars, and 1 in for stirrups and ties in beams and girders, 1 in in slabs and joists exposed to the weather; and $\frac{3}{4}$ in for unexposed slabs and joists. In extremely corrosive atmospheres or other severe exposures, the amount of protective cover should be increased.

**Minimum clear spacing** between pretensioning steel at the ends of a member should be four times the diameter of individual wires and three times the diameter of strands. Some codes also require that the spacing be at least $1\frac{1}{3}$ times the maximum size of aggregate. (See also Art. 8.12.2.) Away from the ends of a member, prestressing steel or ducts may be bundled. Concentrations of steel or ducts, however, should be reinforced to control cracking.

**Prestressing force** may be determined by measuring tendon elongation, by checking jack pressure on a recently calibrated gage, or by using a recently calibrated dynamometer. If several wires or strands are stretched simultaneously, the method used should be such as to induce approximately equal stress in each.

**Splices** should not be used in parallel-wire cables, especially if a splice has to be made by welding, which would weaken the wire. Failure is likely to occur during tensioning of the tendon.

Strands may be spliced, if necessary, when the coupling will develop the full strength of the tendon, not cause it to fail under fatigue loading, and does not displace sufficient concrete to weaken the member.

High-strength bars are generally spliced mechanically. The couplers should be capable of developing the full strength of the bars without decreasing resistance to fatigue and without replacing an excessive amount of concrete.

**Posttensioning End Anchorages ▪** Anchor fittings are different for pretensioned and posttensioned members. For pretensioned members, the fittings hold the tendons temporarily against anchors outside the members and therefore can be reused. In posttensioning, the fittings usually anchor the tendons permanently to the members. In unbonded tendons, the sheathing is typically plastic or impregnated paper.

A variety of patented fittings are available for anchoring in posttensioned members. Such fittings should be capable of developing the full strength of the tendons under static and fatigue loadings. The fittings also should spread the prestressing force over the concrete or transmit it to a bearing plate. Sufficient space must be provided for the fittings in the anchor zone.

Generally, all the wires of a parallel-wire cable are anchored with a single fitting (Figs. 8.4 and 8.5). The type shown in Fig. 8.5 requires that the wires

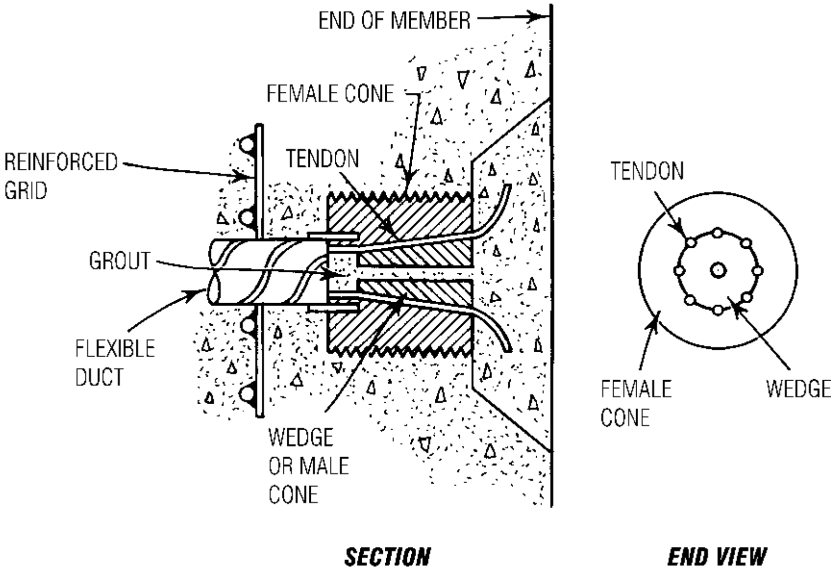



**Fig. 8.4** Conical wedge anchorage for prestressing wires.

be cut to exact length and a buttonhead be cold-formed on the ends for anchoring.

The wedge type in Fig. 8.4 requires a double-acting jack. One piston, with the wires wedged to it, stresses them, and a second piston forces the male cone into the female cone to grip the tendons. Normally, a hole is provided in the male cone for grouting the wires. After final stress is applied, the anchorage may be embedded in concrete to prevent corrosion and improve appearance.

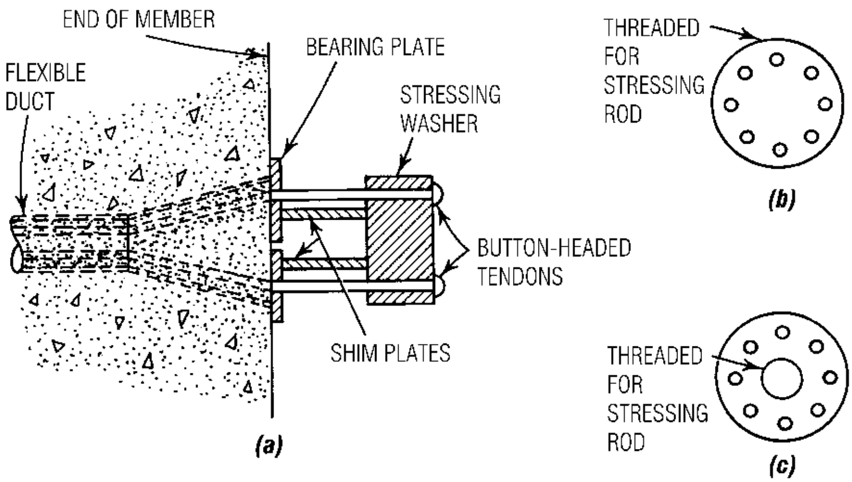



**Fig. 8.5** Detail at end of prestressed concrete member. (*a*) End anchorage for button-headed wires. (*b*) Externally threaded stressing head. (*c*) Internally threaded stressing head. Heads are used for attachment to stressing jack.

With the buttonhead type, a stressing rod may be screwed over threads on the circumference of a thick, steel stressing washer (Fig. 8.5*b*) or into a center hole in the washer (Fig. 8.5*c*). The rod then is bolted to a jack. When the tendons have been stressed, the washer is held in position by steel shims inserted between it and a bearing plate embedded in the member. The jack pressure then can be released and the jack and stressing rod removed. Finally, the anchorage is embedded in concrete.

Posttensioning bars may be anchored individually with steel wedges (Fig. 8.6*a*) or by tightening a nut against a bearing plate (Fig. 8.6*b*). The former has the advantage that the bars do not have to be threaded.

Posttensioning strands normally are shop-fabricated in complete assemblies, cut to length, anchor fittings attached, and sheathed in flexible duct. Swaged to the strands, the anchor fittings have a threaded steel stud projecting from the end. The threaded stud is used for jacking the stress into the strand and for anchoring by tightening a nut against a bearing plate in the member (Fig. 8.7).

To avoid overstressing and failure in the anchorage zone, the anchorage assembly must be placed with care. Bearing plates should be placed perpendicular to the tendons to prevent eccentric loading. Jacks should be centered for the same reason and so as not to scrape the tendons against the plates. The entire area of the plates should bear against the concrete.

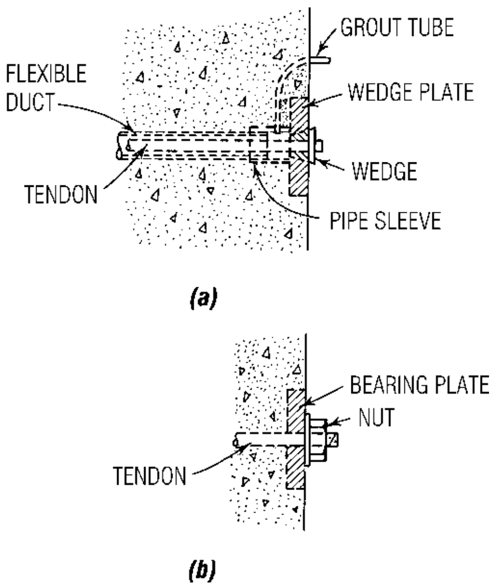



**Fig. 8.6** End anchorages for bars. (*a*) Conical wedge. (*b*) Nut and washer acting against a bearing plate at a threaded end of tendon.

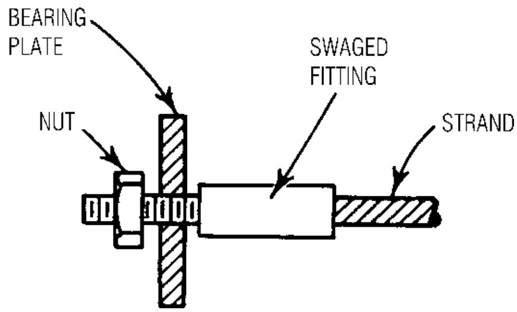



**Fig. 8.7** Swaged fitting for strands. Prestress is maintained by tightening the nut against the bearing plate.

Prestress normally is applied with hydraulic jacks. The amount of prestressing force is determined by measuring tendon elongation and comparing with an average load-elongation curve for the steel used. In addition, the force thus determined should be checked against the jack pressure registered on a recently calibrated gage or by use of a recently calibrated dynamometer. Discrepancies of less than 5% may be ignored.

When prestressed-concrete beams do not have a solid rectangular cross section in the anchorage zone, an enlarged end section, called an end block, may be necessary to transmit the prestress from the tendons to the full concrete cross section a short distance from the anchor zone. End blocks also are desirable for transmitting vertical and lateral forces to supports and to provide adequate space for the anchor fittings for the tendons.

The transition from end block to main cross section should be gradual (Fig. 8.8). Length of end block, from beginning of anchorage area to the start of the main cross section, should be at least 24 in. The length normally ranges from three-fourths the depth of the member for deep beams to the full depth for shallow beams. The end block should be reinforced vertically and horizontally to resist tensile bursting and spalling forces induced by the concentrated loads of the tendons. In particular, a grid of reinforcing should be placed directly behind the anchorages to resist spalling.

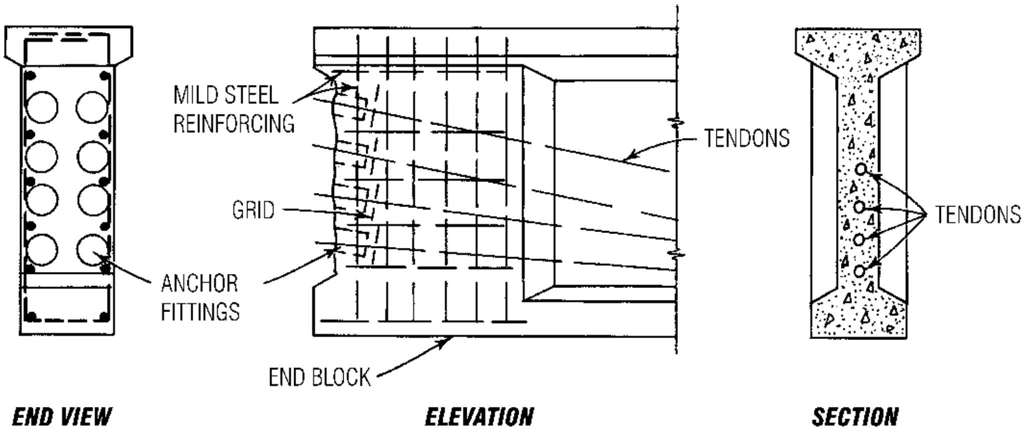



**Fig. 8.8** Transition from cross section of the end block of a prestressed concrete beam to the main cross section.

Ends of pretensioned beams should be reinforced with vertical stirrups over a distance equal to one-fourth the beam depth. The stirrups should be capable of resisting in tension a force equal to at least 4% of the prestressing force.

**Camber ▪** Control of camber is important for prestressed members. Camber tends to increase with time because of creep. If a prestressed beam or slab has an upward camber under prestress and long-time loading, the camber will tend to increase upward. Excessive camber should be avoided, and for deck-type structures, such as highway bridges and building floors and roofs, the camber of all beams and girders of the same span should be the same.

Computation of camber with great accuracy is difficult, mainly because of the difficulty of ascertaining with accuracy the modulus of elasticity of the concrete, which varies with time. Other difficult-to-evaluate factors also influence camber: departure of the actual prestressing force from that calculated, effects of long-time loading, influence of length of time between prestressing and application of full service loads, methods of supporting members after removal from the forms, and influence of composite construction.

When camber is excessive, it may be necessary to use concrete with higher strength and modulus of elasticity, for example, change from lightweight to ordinary concrete; increase the moment of inertia of the section; use partial prestressing, that is, decrease the prestressing force and add reinforcing steel to resist the tensile stresses; or use a larger prestressing force with less eccentricity.

To ensure uniformity of camber, a combination of pretensioning and posttensioning can be provided for precast members. Sufficient prestress may be applied initially to permit removal of the member from the forms and transportation to a storage yard. After the member has increased in strength but before erection, additional prestress is applied by posttensioning to bring the camber to the desired value. During storage, the member should be supported in the same manner as it will be in the structure.

(H. K. Preston and N. J. Sollenberger, "Modern Prestressed Concrete," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); J. B. Libby, "Modern Prestressed Concrete," Van Nostrand Reinhold Company, New York.)

## 8.15 Precast Concrete

When concrete products are made in other than their final position, they are considered precast. They may be unreinforced, reinforced, or prestressed. They include in their number a wide range of products: block, brick, pipe, plank, slabs, conduit, joists, beams and girders, trusses and truss components, curbs, lintels, sills, piles, pile caps, and walls.

Precasting often is chosen because it permits efficient mass production of concrete units. With precasting, it usually is easier to maintain quality control and produce higher-strength concrete than with field concreting. Formwork is simpler, and a good deal of falsework can be eliminated. Also, since precasting normally is done at ground level, workers can move about more freely. But sometimes these advantages are more than offset by the cost of handling, transporting, and erecting the precast units. Also, joints may be troublesome and costly.

Design of precast products follows the same rules, in general, as for cast-in-place units. However, ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute (www.ACI-int.org)), permits the concrete cover over reinforcing steel to be as low as $\frac{5}{8}$ in for slabs, walls, or joists not exposed to weather. Also, ACI Standard 525, "Minimum Requirements for Thin-Section Precast Concrete Construction," permits the cover for units not exposed to weather to be only $\frac{3}{8}$ in for bars smaller than #6.

Precast units must be designed for handling and erection stresses, which may be more severe than those they will be subjected to in service. Normally, inserts are embedded in the concrete for picking up the units. They should be picked up by these inserts, and when set down, they should be supported right side up, in such a manner as not to induce stresses higher than the units would have to resist in service.

For precast beams, girders, joists, columns, slabs, and walls, joints usually are made with cast-in-place concrete. Often, in addition, steel reinforcing projecting from the units to be joined is welded together. (ACI 512.1R, "Suggested Design of Joints and Connections in Precast Structural Concrete," American Concrete Institute (www.aci-int.org).)

## 8.16 Lift-Slab Construction

A type of precasting used in building construction involves casting floor and roof slabs at or near ground level and lifting them to their final position, hence the name lift-slab construction. It offers many of the advantages of precasting (Art. 8.15) and eliminates many of the storing, handling, and transporting disadvantages. It normally requires fewer joints than other types of precast building systems.

Typically, columns are erected first, but not necessarily for the full height of the building. Near the base of the columns, floor slabs are cast in succession, one atop another, with a parting compound between them to prevent bond. The roof slab is cast last, on top. Usually, the construction is flat plate, and the slabs have uniform thickness; waffle slabs or other types also can be used. Openings are left around the columns, and a steel collar is slid down each column for embedment in every slab. The collar is used for lifting the slab, connecting it to the column, and reinforcing the slab against shear.

To raise the slabs, jacks are set atop the columns and turn threaded rods that pass through the collars and do the lifting. As each slab reaches its final position, it is wedged in place and the collars are welded to the columns.

## Design of Concrete Flexural Members

ACI 318, "Building Code Requirements for Reinforced Concrete," specifies that the span of members not integral with supports should be taken as the clear span plus the depth of the member but not greater than the distance center to center of supports. For analysis of continuous frames, spans should be taken center-to-center of supports for determination of bending moments in beams and girders, but moments at the faces of supports may be used in the design of the members. Solid or ribbed slabs integral with supports and with clear spans up to 10 ft may be designed for the clear span.

"Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) has the same requirements as the ACI Code for spans of simply supported beams and slabs. For slabs continuous over more than two supports, the effective span is the clear span for slabs monolithic with beams or walls (without haunches); the distance between stringer-flange edges plus half the stringer-flange width for slabs supported on steel stringers; clear span plus half the stringer thickness for slabs supported on timber stringers. For rigid frames, the span should be taken as the distance between centers of bearings at the top of the footings. The span of continuous beams should be the clear distance between faces of supports.

Where fillets or haunches make an angle of 45° or more with the axis of a continuous or restrained slab and are built integral with the slab and support, AASHTO requires that the span be measured from the section where the combined depth of the slab and fillet is at least 1.5 times the thickness of slab. The moments at the ends of this span should be used in the slab design, but no portion of the fillet should be considered as adding to the effective depth of the slab.

## 8.17 Ultimate-Strength Theory for Reinforced-Concrete Beams

For consistent, safe, economical design of beams, their actual load-carrying capacity should be known. The safe load then can be determined by dividing this capacity by a safety factor. Or the design load can be multiplied by the safety factor to indicate what the capacity of the beams should be. It should be noted, however, that under service loads, stresses and deflections may be computed with good approximation on the assumption of a linear stress-strain diagram and a cracked cross section.

ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), provides for design by ultimate-strength theory. Bending moments in members are determined as if the structure were elastic. Ultimate-strength theory is used to design critical sections, those with the largest bending moments, shear, torsion, etc. The ultimate strength of each section is computed, and the section is designed for this capacity.

### 8.17.1 Stress Redistribution

The ACI Code recognizes that, below ultimate load, a redistribution of stress occurs in continuous beams, frames, and arches. This allows the structure to carry loads higher than those indicated by elastic analysis. The code permits an increase or decrease of up to 10% in the negative moments calculated by elastic theory at the supports of continuous flexural members. But these modified moments must also be used for determining the moments at other sections for the same loading conditions. [The modifications, however, are permissible only for relatively small steel ratios at each support. The steel ratios $\rho$ or $\rho - \rho'$ (see Arts. 8.20, 8.21, and 8.24 to 8.27) should be less than half $\rho_b$, the steel ratio for balanced conditions (concrete strength equal to steel strength) at ultimate load.] For example, suppose elastic analysis of a continuous beam indicates a maximum negative moment at a support of $wL^2/12$ and maximum positive moment at midspan of $wL^2/8 - wL^2/12$, or $wL^2/24$. Then, the code permits the negative moment to be decreased to $0.9wL^2/12$, if the positive moment is increased to $wL^2/8 - 0.9wL^2/12$, or $1.2wL^2/24$.

### 8.17.2 Design Assumptions for Ultimate-Strength Design

Ultimate strength of any section of a reinforced-concrete beam may computed assuming the following:

1. Strain in the concrete is directly proportional to the distance from the neutral axis (Fig. 8.9*b*).
2. Except in anchorage zones, strain in reinforcing steel equals strain in adjoining concrete.
3. At ultimate strength, maximum strain at the extreme compression surface equals 0.003 in/in.
4. When the reinforcing steel is not stressed to its yield strength $f_y$, the steel stress is 29,000 ksi times the steel strain, in/in. After the yield strength has been reached, the stress remains constant at $f_y$, though the strain increases.
5. Tensile strength of the concrete is negligible.

At ultimate strength, concrete stress is not proportional to strain. The actual stress distribution may be represented by an equivalent rectangle, known as the Whitney rectangular stress block, that yields ultimate strengths in agreement with numerous, comprehensive tests (Fig. 8.9*c*).

The ACI Code recommends that the compressive stress for the equivalent rectangle be taken as $0.85f'_c$, where $f'_c$ is the 28-day compressive strength of the concrete. The stress is assumed constant from the surface of maximum compressive strain over a depth $a = \beta_1 c$, where $c$ is the distance to the neutral axis (Fig. 8.9*c*). For $f'_c \leq 4000$ psi, $\beta_1 = 0.85$; for greater concrete strengths, $\beta_1$ is reduced 0.05 for each 1000 psi in excess of 4000.

Formulas in the ACI Code based on these assumptions usually contain a factor $\phi$ which is applied to the theoretical ultimate strength of a section, to provide for the possibility that small adverse variations in materials, quality of work, and dimensions, while individually within acceptable tolerances, occasionally may combine, and actual

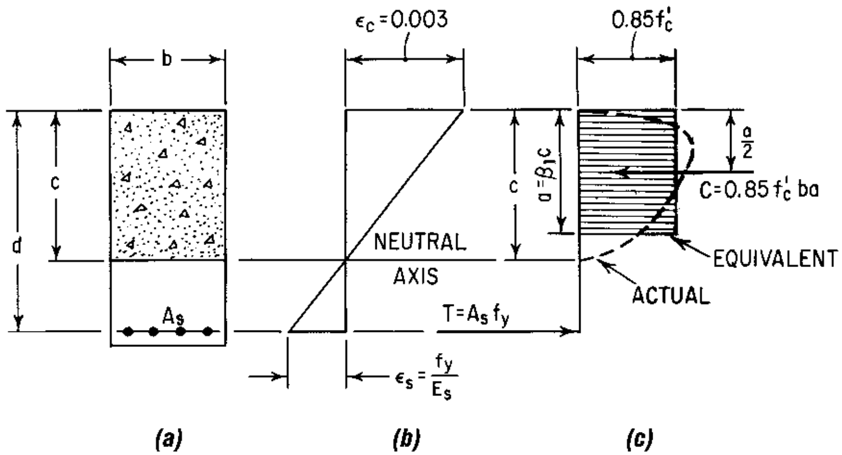



**Fig. 8.9** Stresses and strains on a reinforced-concrete beam section: (*a*) At ultimate load, after the section has cracked and only the steel carries tension. (*b*) Strain diagram. (*c*) Actual and assumed compression-stress block.

capacity may be less than that computed. The coefficient $\phi$ is taken as 0.90 for flexure, 0.85 for shear and torsion, 0.75 for spirally reinforced compression members, and 0.70 for tied compression members. Under certain conditions of load (as the value of the axial load approaches zero) and geometry, the $\phi$ value for compression members may increase linearly to a maximum value of 0.90.

### 8.17.3 Crack Control of Flexural Members

Because of the risk of large cracks opening up when reinforcement is subjected to high stresses, the ACI Code recommends specific provisions on crack control through reinforcement distribution limits on spacings:

$$s = \frac{540}{f_s} - 2.5C_c \tag{8.6}$$

where $s$ = center to center spacing of flexural tension reinforcement (in),

$f_s = 0.6 f_y$(ksi),

$C_c$ = clear cover from nearest surface in tension to flexural tension reinforcement (in). These provisions apply to reinforced concrete beams and one-way slabs subject to normal environmental condition.

### 8.17.4 Required Strength

For combinations of loads, the ACI Code requires that a structure and its members should have the following ultimate strengths (capacities to resist design loads and their related internal moments and forces):

$$U = 1.4D + 1.7L \tag{8.7a}$$
$$U = 1.4(D + F) \tag{8.7b}$$
$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \tag{8.7c}$$
$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W) \tag{8.7d}$$
$$U = 1.2D + 1.6W + 0.5L + 1.0(L_r \text{ or } S \text{ or } R) \tag{8.7e}$$
$$U = 1.2D + 1.0E + 1.0L + 0.2S \tag{8.7f}$$
$$U = 0.9D + 1.6W + 1.6H \tag{8.7g}$$
$$U = 0.9D + 1.0E + 1.6H \tag{8.7h}$$

where $D$ = dead load; $E$ = earthquake load; $F$ = lateral fluid pressure load and maximum height;

$H$ = load due to the weight and lateral pressure of soil and water in soil;

$L$ = live load; $L_r$ = roof load; $R$ = rain load; $S$ = snow load;

$T$ = self-straining force such as creep, shrinkage, and temperature effects;

$W$ = wind load

For ultimate-strength loads (load-factor method) for bridges, see Art. 17.4.

Although structures may be designed by ultimate-strength theory, it is not anticipated that service loads will be substantially exceeded. Hence, deflections that will be of concern to the designer are those that occur under service loads. These deflections may be computed by working-stress theory. (See Art. 8.18.)

### 8.17.5 Deep Members

Due to the nonlinearity of strain distribution and the possibility of lateral buckling, deep flexural members must be given special consideration. The ACI Code considers members with clear span, ln, equal to or less than 4 times the overall member depth as deep members. The ACI Code provides special shear design requirements and minimum requirements for both horizontal and vertical reinforcement for such members.

## 8.18 Working-Stress Theory for Reinforced-Concrete Beams

Stress distribution in a reinforced-concrete beam under service loads is different from that at ultimate strength (Art. 8.17). Knowledge of this stress distribution is desirable for many reasons, including the requirements of some design codes that specified working stresses in steel and concrete not be exceeded.

Working stresses in reinforced-concrete beams are computed from the following assumptions:

1. Longitudinal stresses and strains vary with distance from the neutral axis (Fig. 8.10*c* and *d*); that is, plane sections remain plane after bending. (Strains in longitudinal reinforcing steel and adjoining concrete are equal.)
2. The concrete does not develop any tension. (Concrete cracks under tension.)
3. Except in anchorage zones, strain in reinforcing steel equals strain in adjoining concrete. But because of creep, strain in compressive steel in beams may be taken as half that in the adjoining concrete.
4. The modular ratio $n = E_s/E_c$ is constant. $E_s$ is the modulus of elasticity of the reinforcing steel and $E_c$ of the concrete.

Table 8.9 lists allowable stresses that may be used for flexure. For other than the flexural stresses in Table 8.9*a*, allowable or maximum stresses to be used in design are stated as a percentage of the values given for ultimate-strength design. See, for example, service loads in Table 8.9*b*.

Allowable stresses may be increased one-third when wind or earthquake forces are combined with other loads, but the capacity of the resulting

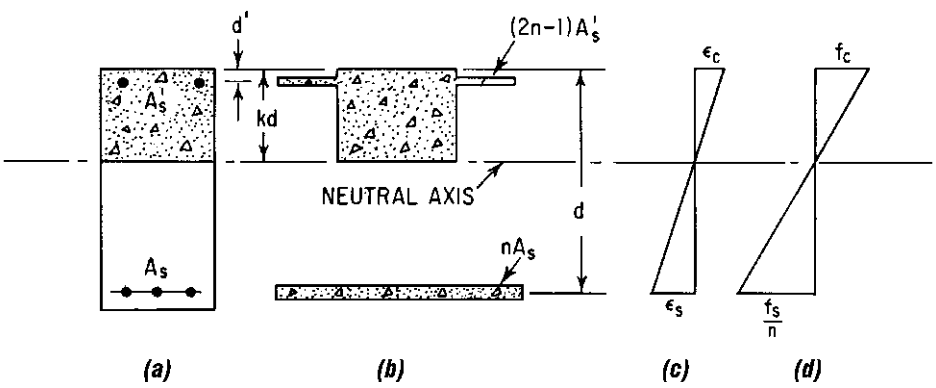



**Fig. 8.10** Typical cracked cross section of a reinforced concrete beam: (*a*) Only the reinforcing steel is effective in tension. (*b*) Section treated as an all-concrete transformed section. In working-stress design, linear distribution is assumed for (*c*) strains and (*d*) stresses.

**Table 8.9** Allowable Stresses for Concrete Flexural Members

(*a*)

| Type of Stress | Buildings | Bridges |
|---|---|---|
| Compression in extreme compression surface | $0.45 f_c'^*$ | $0.4 f_c'^*$ |
| Tension in reinforcement | | |
| Grade 40 or 50 steel | 20 ksi | 20 ksi |
| Grade 60 or higher yield strength | 24 ksi | 24 ksi |

(*b*)

| Type of Member and Stress | Allowable Stresses or Capacity, %, of Ultimate (Nominal) |
|---|---|
| Compression members, walls | 40 |
| Shear or tension in beams, joists, walls, one-way slabs | 55 |
| Shear or tension in two-way slabs, footings | 50 |
| Bearing in concrete | 35 |

* $f_c'$ is the 28-day compressive strength of the concrete.

section should not be less than that required for dead plus live loads.

Other equivalency factors are also given in terms of ultimate-strength values. Thus, the predominant design procedure is the ultimate-strength method, but for reasons of background and historical significance and because the working-stress design method is sometimes preferred for bridges and certain foundation and retaining-wall design, examples of working-stress design procedure are presented in Arts. 8.21, 8.25, and 8.27.

**Transformed Section ▪** According to the working-stress theory for reinforced-concrete beams, strains in reinforcing steel and adjoining concrete are equal. Hence $f_s$, the stress in the steel, is $n$ times $f_c$, the stress in the concrete, where $n$ is the ratio of modulus of elasticity of the steel $E_s$ to that of the concrete $E_c$. The total force acting on the steel then equals $(nA_s)f_c$. This indicates that the steel area can be replaced in stress calculations by a concrete area $n$ times as large.

The transformed section of a concrete beam is one in which the reinforcing has been replaced by an equivalent area of concrete (Fig. 8.10*b*). (In doubly reinforced beams and slabs, an effective modular ratio of $2n$ should be used to transform the compression reinforcement, to account for the effects of creep and nonlinearity of the stress-strain diagram for concrete. But the computed stress should not exceed the allowable tensile stress.) Since stresses and strains are assumed to vary with distance from the neutral axis, conventional elastic theory for homogeneous beams holds for the transformed section. Section properties, such as location of neutral axis, moment of inertia, and section modulus $S$, can be computed in the usual way, and stresses can be found from the flexure formula $f = M/S$, where $M$ is the bending moment.

## 8.19 Deflection Computations and Criteria for Concrete Beams

The assumptions of working-stress theory (Art. 8.18) may also be used for computing deflections under service loads; that is, elastic-theory deflection formulas may be used for reinforced-concrete beams (Art. 6.32). In these formulas, the **effective moment of inertia** $I_e$ is given by Eq. (8.8).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \qquad (8.8)$$

where $I_g$ = moment of inertia of the gross concrete section

$M_{cr}$ = cracking moment

$M_a$ = moment for which deflection is being computed

$I_{cr}$ = cracked concrete (transformed) section

If $y_t$ is taken as the distance from the centroidal axis of the gross section, neglecting the reinforcement, to the extreme surface in tension, the cracking moment may be computed from

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad (8.9)$$

with the modulus of rupture of the concrete $f_r = 7.5\sqrt{f_c'}$ Eq. (8.8) takes into account the variation of the moment of inertia of a concrete section based on whether the section is cracked or uncracked. The modulus of elasticity of the concrete $E_c$ may be computed from Eq. (8.3) in Art. 8.1.

The deflections thus calculated are those assumed to occur immediately on application of load. The total long-term deflection is

$$\Delta_{LT} = \Delta_L + \lambda_\infty \Delta_D + \lambda_t \Delta_{LS} \tag{8.10}$$

where $\Delta_L$ = initial live load deflection,

$\Delta_D$ = initial dead load deflection,

$\Delta_{LS}$ = initial sustained live-load deflection,

$\lambda_\infty$ = time dependent multiplier for infinite duration of sustained load,

$\lambda_t$ = time dependent multiplier for limited load duration.

**Deflection Limitations** ▪ The ACI Code recommends the following limits on deflections in buildings:

For roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections, maximum immediate deflection under live load should not exceed $L/180$, where $L$ is the span of beam or slab.

For floors not supporting partitions and not attached to nonstructural elements, the maximum immediate deflection under live load should not exceed $L/360$.

For a floor or roof construction intended to support or to be attached to partitions or other construction likely to be damaged by large deflections of the support, the allowable limit for the sum of immediate deflection due to live loads and the additional deflection due to shrinkage and creep under all sustained loads should not exceed $L/480$. If the construction is not likely to be damaged by large deflections, the deflection limitation may be increased to $L/240$. But tolerances should be established and adequate measures should be taken to prevent damage to supported or nonstructural elements resulting from the deflections of structural members.

## 8.20 Ultimate-Strength Design of Rectangular Beams with Tension Reinforcement Only

Generally, the area $A_s$ of tension reinforcement in a reinforced-concrete beam is represented by the ratio $\rho = A_s/bd$, where $b$ is the beam width and $d$ the distance from extreme compression surface to the centroid of tension reinforcement (Fig. 8.11*a*). At ultimate strength, the steel at a critical section of the beam will be at its yield strength $f_y$ if the concrete does not fail in compression first (Art. 8.17). Total tension in the steel then will be $A_s f_y = \rho f_y bd$. It will be opposed, according to Fig. 8.11*c*, by an equal compressive force, $0.85 f'_c ba = 0.85 f'_c b \beta_1 c$, where $f'_c$ is the 28-day strength of the concrete, ksi, $\alpha$ the depth of the equivalent rectangular stress distribution, $c$ the distance from the extreme compression surface to the neutral axis, and $\beta_1$ a constant (see Art. 8.17). Equating the compression and tension at the critical section yields

$$c = \frac{\rho f_y}{0.85 \beta_1 f'_c} d \tag{8.11}$$

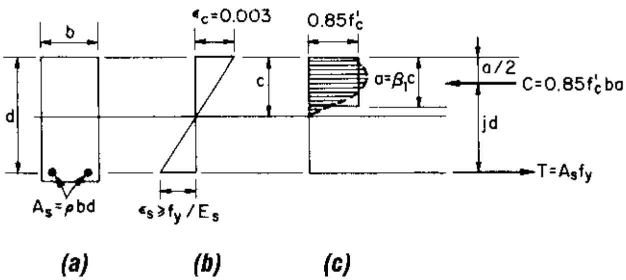



**Fig. 8.11** Rectangular concrete beam reinforced for tension only: (*a*) Beam cross section. (*b*) Linear distribution assumed for strains at ultimate load. (*c*) Equivalent rectangular stress block assumed for compression stresses at ultimate load.

The criterion for compression failure is that the maximum strain in the concrete equals 0.003 in/in. In that case

$$c = \frac{0.003}{f_s/E_s + 0.003} d \tag{8.12}$$

where $f_s$ = steel stress, ksi
$E_s$ = modulus of elasticity of steel
= 29,000 ksi

Table 8.10 lists the nominal diameters, weights, and cross-sectional areas of standard steel reinforcing bars.

### 8.20.1 Balanced Reinforcing

Under balanced conditions, the concrete will reach its maximum strain of 0.003 when the steel reaches its yield strength $f_y$. Then, $c$ as given by Eq. (8.11) will equal $c$ as given by Eq. (8.12) since $c$ determines the location of the neutral axis. This determines the steel ratio for balanced conditions:

$$\rho_b = \frac{0.85\beta_1 f'_c}{f_y} \frac{87{,}000}{87{,}000 + f_y} \tag{8.13}$$

### 8.20.2 Reinforcing Limitations

All structures are designed to collapse not suddenly but by gradual deformation when overloaded. This condition is referred to as a ductile mode of failure. To achieve this end in concrete, the reinforcement should yield before the concrete crushes. This will occur if the quantity of tensile reinforcement is less than the critical percentage determined by ultimate-strength theory [Eq. 8.13]. The ACI Code, to avoid compression failures, limits the steel ratio $\rho$ to a maximum of $0.75\rho_b$. The Code also requires that $\rho$ for positive-moment reinforcement be at least $200/f_y$.

### 8.20.3 Moment Capacity

For such underreinforced beams, the nominal moment strength is

$$M_n = [bd^2 f'_c w(1 - 0.59w)]$$
$$= \left[A_s f_y \left(d - \frac{a}{2}\right)\right] \tag{8.14}$$

where $w = \rho f_y / f'_c$
$a = A_s f_y / 0.85 f'_c b$

The design moment strength, $\phi M_n$, must be equal to or greater than the external factored moment, $M_u$.

### 8.20.4 Shear Reinforcement

The nominal shear strength, $V_n$, of a section of a beam equals the sum of the nominal shear strength provided by the concrete, $V_c$, and the nominal shear strength provided by the reinforcement, $V_s$;

**Table 8.10** Areas of Groups of Standard Bars, in$^2$

| Bar No. | Diam, in | Weight, lb per ft | Number of Bars: 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.250 | 0.167 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 |
| 3 | 0.375 | 0.376 | 0.11 | 0.22 | 0.33 | 0.44 | 0.55 | 0.66 | 0.77 | 0.88 | 0.99 |
| 4 | 0.500 | 0.668 | 0.20 | 0.39 | 0.58 | 0.78 | 0.98 | 1.18 | 1.37 | 1.57 | 1.77 |
| 5 | 0.625 | 1.043 | 0.31 | 0.61 | 0.91 | 1.23 | 1.53 | 1.84 | 2.15 | 2.45 | 2.76 |
| 6 | 0.750 | 1.502 | 0.44 | 0.88 | 1.32 | 1.77 | 2.21 | 2.65 | 3.09 | 3.53 | 3.98 |
| 7 | 0.875 | 2.044 | 0.60 | 1.20 | 1.80 | 2.41 | 3.01 | 3.61 | 4.21 | 4.81 | 5.41 |
| 8 | 1.000 | 2.670 | 0.79 | 1.57 | 2.35 | 3.14 | 3.93 | 4.71 | 5.50 | 6.28 | 7.07 |
| 9 | 1.128 | 3.400 | 1.00 | 2.00 | 3.00 | 4.00 | 5.00 | 6.00 | 7.00 | 8.00 | 9.00 |
| 10 | 1.270 | 4.303 | 1.27 | 2.53 | 3.79 | 5.06 | 6.33 | 7.59 | 8.86 | 10.12 | 11.39 |
| 11 | 1.410 | 5.313 | 1.56 | 3.12 | 4.68 | 6.25 | 7.81 | 9.37 | 10.94 | 12.50 | 14.06 |
| 14 | 1.693 | 7.650 | 2.25 | 4.50 | 6.75 | 9.00 | 11.25 | 13.50 | 15.75 | 18.00 | 20.25 |
| 18 | 2.257 | 13.600 | 4.00 | 8.00 | 12.00 | 16.00 | 20.00 | 24.00 | 28.00 | 32.00 | 36.00 |

that is, $V_n = V_c + V_s$. The factored shear force, $V_n$, on a section should not exceed

$$\phi V_n = \phi(V_c + V_s) \tag{8.15}$$

where $\phi$ = strength reduction factor (0.75 for shear and torsion). Except for brackets and other short cantilevers, the section for maximum shear may be taken at a distance equal to $d$ from the face of the support.

The shear $V_c$ carried by the concrete alone should not exceed $2\sqrt{f'_c}b_w d$ where $b_w$ is the width of the beam web and $d$ the depth from the extreme compression fiber to centroid of longitudinal tension reinforcement. (For members subject to shear and flexure only, the maximum for $V_c$ may be taken as

$$V_c = \left(1.9\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \tag{8.16}$$

$$\leq 3.5\sqrt{f'_c} b_w d$$

where $\rho_w = A_s/b_w d$ and $V_u$ and $M_u$ are the shear and bending moment, respectively, at the section considered, but $M_u$ should not be less than $V_u d$.)

When $V_u$ is larger than $\phi V_c$, the excess shear will have to be resisted by web reinforcement. In general, this reinforcement should be stirrups perpendicular to the axis of the member (Fig. 8.12). Shear or torsion reinforcement should extend the full depth $d$ of the member and should be adequately anchored at both ends to develop the design yield strength of the reinforcement. An alternative is to incorporate welded-wire fabric with wires perpendicular to the axis of the member. In members without prestressing, however, the stirrups may be inclined, as long as the angle is at least 45° with the axis of the member. As an alternative, longitudinal reinforcing bars may be bent up at an angle of 30° or more with the axis, or spirals may be used. Spacing should be such that every 45° line, representing a potential crack and extending from middepth $d/2$ to the longitudinal tension bars, should be crossed by at least one line of reinforcing.

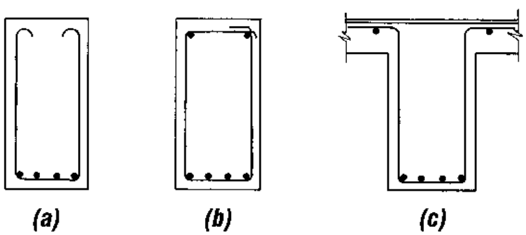



**Fig. 8.12** Typical stirrups in a concrete beam.

The area of steel required in vertical stirrups, in$^2$ per stirrup, with a spacing $s$, in, is

$$A_v = \frac{V_s s}{f_y d} \tag{8.17}$$

where $f_y$ = yield strength of the shear reinforcement. $A_v$ is the area of the stirrups cut by a horizontal plane. $V_s$ should not exceed $8\sqrt{f'_c}b_w d$ in sections with web reinforcement, nor should $f_y$ exceed 60 ksi. Where shear reinforcement is required and is placed perpendicular to the axis of the member, it should not be spaced farther apart than $0.5d$, nor more than 24 in c to c. When $V_s$ exceeds $4\sqrt{f'_c}b_w d$, however, the maximum spacing should be limited to $0.25d$.

Alternatively, for practical design, Eq. (8.17) can be transformed into Eq. (8.18) to indicate the stirrup spacing $s$ for the design shear $V_u$, stirrup area $A_v$, and geometry of the member $b_w$ and $d$:

$$s = \frac{A_v \phi f_y d}{V_u - 2\phi\sqrt{f'_c} b_w d} \tag{8.18}$$

The area required when a single bar or a single group of parallel bars are all bent up at the same distance from the support at $\alpha$ angle with the longitudinal axis of the member is

$$A_v = \frac{V_s}{f_y \sin\alpha} \tag{8.19}$$

in which $V_s$ should not exceed $3\sqrt{f'_c}b_w d$. $A_v$ is the area cut by a plane normal to the axis of the bars. The area required when a series of such bars are bent up at different distances from the support or when inclined stirrups are used is

$$A_v = \frac{V_s s}{(\sin\alpha + \cos\alpha) f_y d} \tag{8.20}$$

A minimum area of shear reinforcement is required in all members, except slabs, footings, and joists or where $V_u$ exceeds $0.5V_c$.

### 8.20.5 Torsion Reinforcement

Types of stresses induced by torsion and reinforcement requirements for members subjected to torsion are discussed in Art. 8.28.

### 8.20.6 Development of Tensile Reinforcement

To prevent bond failure or splitting, the calculated stress in any bar at any section must be developed on each side of the section by adequate embedment length, end anchorage, or hooks. The critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates. See Art. 8.22.

At least one-third of the positive-moment reinforcement in simple beams and one-fourth of the positive-moment reinforcement in continuous beams should extend along the same face of the member into the support, in both cases, at least 6 in into the support. At simple supports and at points of inflection, the diameter of the reinforcement should be limited to a diameter such that the development length $l_d$ defined in Art. 8.12.5 satisfies

$$l_d \le \frac{M_n}{V_u} + l_a \tag{8.21}$$

where $M_n$ = nominal moment strength with all reinforcing steel at section stressed to $f_y$

$V_u$ = factored shear at section

$l_a$ = additional embedment length beyond inflection point or center of support

At an inflection point, $l_a$ is limited to a maximum of $d$, the depth of the centroid of the reinforcement, or 12 times the reinforcement diameter.

Negative-moment reinforcement should have an embedment length into the span to develop the calculated tension in the bar, or a length equal to the effective depth of the member, or 12 bar diameters, whichever is greatest. At least one-third of the total negative reinforcement should have an embedment length beyond the point of inflection not less than the effective depth of the member, or 12 bar diameters, or one-sixteenth of the clear span, whichever is greatest.

### 8.20.7 Hooks on Bars

When straight embedment of reinforcing bars in tension is inadequate to provide the required development lengths of the bars as specified in Art. 8.12.5, the bar ends may be bent into standard 90° and 180° hooks (Table 8.11) to provide additional development. The basic development length for a hooked bar with $f_y = 60$ ksi is defined as

$$l_{dh} = \left(\frac{0.02\beta\lambda f_y}{f'_c}\right) d_b \tag{8.22}$$

where $d_b$ is the bar diameter, in, and $f'_c$ is the 28-day compressive strength of the concrete, psi. $\beta = 1.2$ for epoxy coated reinforcement and $\lambda = 1.3$ for lightweight aggregate concrete. For all other cases, $\beta$ and $\lambda$ shall be taken as 1.0. Figure 8.13 illustrates embedment lengths for standard hooks.

A footnote to Table 8.12 indicates some of the factors by which basic development length should be multiplied for values of $f_y$ other than 60 ksi and for excess reinforcement. For bars sizes up to No. 11, side cover (normal to the plane of the hook) of at least 2½ in, and for a 90° hook, cover on the bar extension of 2 in or more, the modification may be taken as 0.7. Also, for bars sizes up to No. 11 with the hook enclosed vertically or horizontally and enclosed within ties or stirrup-ties spaced along the full development length at $3d_b$ or less, the modification factor may be taken as 0.8.

Hooks should not be considered effective in adding to the compressive resistance of reinforcement. Thus, hooks should not be used on footing dowels. Instead, when depth of footing is less than that required by large-size bars, the designer should substitute smaller-diameter bars with equivalent area and lesser embedment length. It may be possible sometimes to increase the footing depth where large-diameter dowel reinforcement is used so that footing dowels can have the proper embedment length. Footing dowels need only transfer the excess load above that transmitted in bearing and therefore may be bars with areas different from those required for compression design for the first column lift.

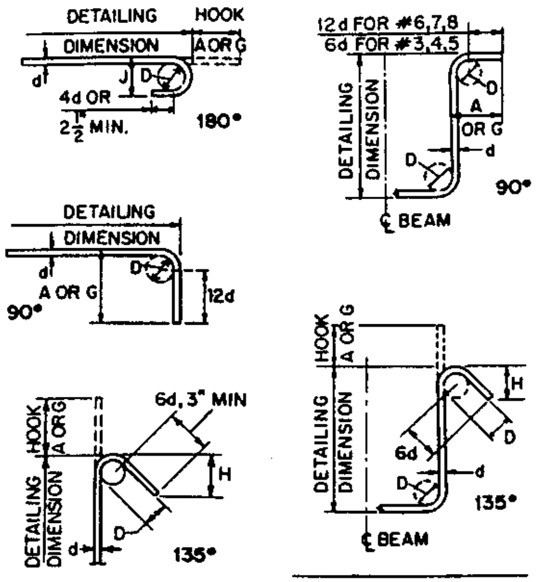
(P. F. Rice and E. S. Hoffman, "Structural Design Guide to the ACI Building Code," Van Nostrand Reinhold Company, New York; "CRSI Handbook," Concrete Reinforcing Steel Institute, Chicago, III.; ACI SP-17, "Design Handbook in Accordance with the Strength Design Method of ACI 318-77 (www.aci-int.org)," American Concrete Institute; G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com).)

**Table 8.11** Standard Hooks*

Recommended End Hooks—All Grades, in or ft-in

| Bar Size No. | 180° Hooks | | | 90° Hooks |
|---|---|---|---|---|
| | *D*† | *A* or *G* | *J* | *A* or *G* |
| 3 | 2¼ | 5 | 3 | 6 |
| 4 | 3 | 6 | 4 | 8 |
| 5 | 3¾ | 7 | 5 | 10 |
| 6 | 4½ | 8 | 6 | 1–0 |
| 7 | 5¼ | 10 | 7 | 1–2 |
| 8 | 6 | 11 | 8 | 1–4 |
| 9 | 9½ | 1–3 | 11¾ | 1–7 |
| 10 | 10¾ | 1–5 | 1–1¼ | 1–10 |
| 11 | 12 | 1–7 | 1–2¾ | 2–0 |
| 14 | 18¼ | 2–3 | 1–9¾ | 2–7 |
| 18 | 24 | 3–0 | 2–4¼ | 3–5 |

†*D* = finished bend diameter, in.




Stirrups (ties similar)

Stirrup and Tie Hook Dimensions, in—Grades 40–50–60 ksi

| Bar Size No. | *D*, in | 90° Hook | 135° Hook | |
|---|---|---|---|---|
| | | Hook *A* or *G* | Hook *A* or *G* | *H*, Approx. |
| 3 | 1½ | 4 | 4 | 2½ |
| 4 | 2 | 4½ | 4½ | 3 |
| 5 | 2½ | 6 | 5½ | 3¾ |
| 6 | 4½ | 1–0 | 8 | 4½ |
| 7 | 5¼ | 1–2 | 9 | 5¼ |
| 8 | 6 | 1–4 | 10½ | 6 |

135° Seismic Stirrup/Tie Hook Dimensions (Ties Similar) in—Grades 40–50–60 ksi

| Bar Size No. | *D*, in | 135° Hook | |
|---|---|---|---|
| | | Hook *A* or *G* | *H*, Approx. |
| 3 | 1½ | 4¼ | 3 |
| 4 | 2 | 4½ | 3 |
| 5 | 2½ | 5½ | 3¾ |
| 6 | 4½ | 8 | 4½ |
| 7 | 5¼ | 9 | 5¼ |
| 8 | 6 | 10½ | 6 |

* Notes:

1. All specific sizes recommended by CRSI in this table meet minimum requirements of ACI 318.
2. 180° hook *J* dimension (sizes 10, 11, 14, and 18) and A or G dimension (Nos. 14 and 18) have been revised to reflect recent test research using ASTM/ACI bend-test criteria as a minimum.
3. Tables for stirrup and tie hook dimensions have been expanded to include sizes 6, 7, and 8, to reflect current design practices.

Courtesy of the Concrete Reinforcing Steel Institute.

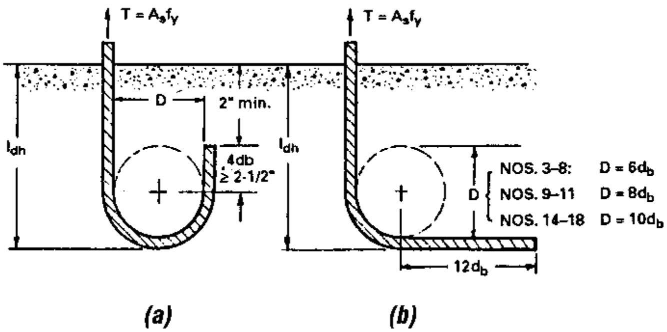



**Fig. 8.13** Embedment lengths for 90° and 180° hooks.

**Table 8.12** Minimum Embedment Lengths for Hooks on Steel Reinforcement in Tension

*a*. Embedment Lengths $l_{dh}$, in, for Standard End Hooks on Grade 60 Bars in Normal-Weight Concrete*

| Bar Size No. | Concrete Compressive Strength $f'_c$, psi 3000 | 4000 | 5000 | 6000 | 7000 | 8000 |
|---|---|---|---|---|---|---|
| 3 | 6 | 6 | 6 | 6 | 6 | 6 |
| 4 | 8 | 7 | 6[†] | 6[†] | 6[†] | 6[†] |
| 5 | 10 | 9 | 8 | 7 | 7 | 6[†] |
| 6 | 12 | 10 | 9 | 8 | 8 | 7[†] |
| 7 | 14 | 12 | 11 | 10 | 9 | 9 |
| 8 | 16 | 14 | 12 | 11 | 10 | 10 |
| 9 | 18 | 15 | 14 | 13 | 12 | 11 |
| 10 | 20 | 17 | 15 | 14 | 13 | 12[†] |
| 11 | 22 | 29 | 17 | 16 | 14 | 14[†] |
| 14 | 37 | 32 | 29 | 27 | 25 | 23 |
| 18 | 50 | 43 | 39 | 35 | 33 | 31 |

*b*. Embedment Lengths, in, to Provide 2-in Concrete Cover over Tail of Standard 180° End Hooks

| No. 3 | No. 4 | No. 5 | No. 6 | No. 7 | No. 8 | No. 9 | No. 10 | No. 11 | No. 14 | No. 18 |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 7 | 8 | 9 | 10 | 12 | 14 | 15 | 20 | 25 |

* Embedment length for 90° and 180° standard hooks is illustrated in Fig. 8.13. Details of standard hooks are given in Table 8.11. Side cover required is a minimum of 2½ in. End cover required for 90° hooks is a minimum of 2 in. To obtain embedment lengths for grades of steel different from Grade 60, multiply $l_{dh}$ given in Table 8.12 by $f_y/60$. If reinforcement exceeds that required, multiply $l_{dh}$ by the ratio of area required to that provided.

† For 180° hooks at right angles to exposed surfaces, obtain $l_{dh}$ from Table 8.12*b* to provide 2-in minimum cover to tail (Fig. 8.13*a*).

## 8.21 Alternate Design of Rectangular Beams with Tension Reinforcement Only

From the assumption that stress varies across a beam section with the distance from the neutral axis (Art. 8.18), it follows that (see Fig. 8.14)

$$\frac{nf_c}{f_s} = \frac{k}{1-k} \tag{8.23}$$

where $n$ = modular ratio $E_s/E_c$

$E_s$ = modulus of elasticity of steel reinforcement, ksi

$E_c$ = modulus of elasticity of concrete, ksi

$f_c$ = compressive stress in extreme surface of concrete, ksi

$f_s$ = stress in steel, ksi

$kd$ = distance from extreme compression surface to neutral axis, in

$d$ = distance from extreme compression to centroid of reinforcement, in

When the steel ratio $\rho = A_s/bd$, where $A_s$ = area of tension reinforcement, in$^2$, and $b$ = beam width, in, is known, $k$ can be computed from

$$k = \sqrt{2n\rho + (n\rho)^2} - n\rho \tag{8.24}$$

Wherever positive-moment steel is required, $\rho$ should be at least $200/f_y$, where $f_y$ is the steel yield stress. The distance $jd$ between the centroid of compression and the centroid of tension, in, can be obtained from Fig. 8.14:

$$j = 1 - \frac{k}{3} \tag{8.25}$$

### 8.21.1 Allowable Bending Moment

The moment resistance of the concrete, in-kips, is

$$M_c = \tfrac{1}{2} f_c kjbd^2 = K_c bd^2 \tag{8.26}$$

where $K_c = \tfrac{1}{2} f_c kj$. The moment resistance of the steel reinforcement is

$$M_s = f_s A_s jd = f_s \rho jbd^2 = K_s bd^2 \tag{8.27}$$

where $K_s = f_s \rho j$. Allowable stresses are given in Art. 8.18. Table 8.10 lists nominal diameters, weights, and cross-sectional areas of standard steel reinforcing bars.

### 8.21.2 Allowable Shear

The nominal unit shear stress acting on a section with shear $V$ is

$$v = \frac{V}{bd} \tag{8.28}$$

Allowable shear stresses are 55% of those for ultimate-strength design (Art. 8.20.4). Otherwise, designs for shear by the working-stress and ultimate-strength methods are the same. Except for brackets and other short cantilevers, the section for maximum shear may be taken at a distance $d$ from the face of the support. In working-stress design, the shear stress $v_c$ carried by the concrete

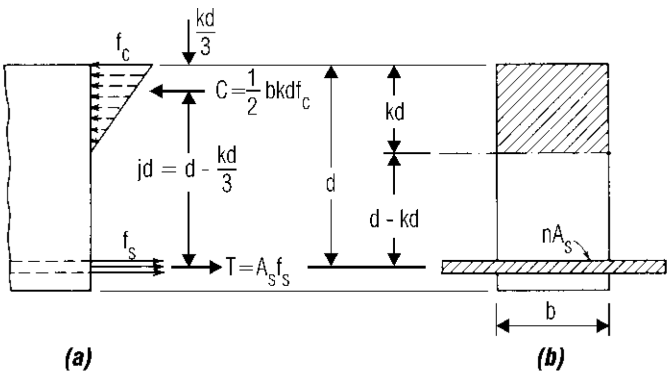



**Fig. 8.14** Rectangular concrete beam reinforced for tension only: (*a*) In working-stress design, a linear distribution is assumed for compression stresses. (*b*) Transformed all-concrete section.

alone should not exceed $1.1\sqrt{f_c'}$. (As an alternative, the maximum for $v_c$ may be taken as $\sqrt{f_c'} + 1300\rho Vd/M$, with a maximum of $1.9\sqrt{f_c'}$, $f_c'$ is the 28-day compressive strength of the concrete, psi, and $M$ is the bending moment at the section but should not be less than $Vd$.)

At cross sections where the torsional stress $v_t$ exceeds $0.825\sqrt{f_c'}$, $v_c$ should not exceed

$$v_c = \frac{1.1\sqrt{f_c'}}{\sqrt{1 + (v_t/1.2v)^2}} \qquad (8.29)$$

The excess shear $v - v_c$ should not exceed $4.4\sqrt{f_c'}$ in sections with web reinforcement. Stirrups and bent bars should be capable of resisting the excess shear $V' = V - v_c bd$.

The area required in the legs of a vertical stirrup, in$^2$, is

$$A_v = \frac{V's}{f_v d} \qquad (8.30)$$

where $s$ = spacing of stirrups, in

$f_v$ = allowable stress in stirrup steel, psi (see Art. 8.21)

For a single bent bar or a single group of parallel bars all bent at an angle $\alpha$ with the longitudinal axis at the same distance from the support, the required area is

$$A_v = \frac{V'}{f_v \sin\alpha} \qquad (8.31)$$

For inclined stirrups and groups of bars bent up at different distances from the support, the required area is

$$A_v = \frac{V's}{f_v d(\sin\alpha + \cos\alpha)} \qquad (8.32)$$

Where shear reinforcing is required and the torsional moment $T$ exceeds the value calculated from Eq. (8.64), the minimum area of shear reinforcement provided should be that given by Eq. (8.60).

### 8.21.3 Allowable Torsion

Torsion effects should be considered whenever the torsion $T$ due to service loads exceeds the torsion capacity of the concrete $T_c$ given by Eq. (8.63). For working-stress design for torsion, see Art. 8.28.2.

### 8.21.4 Development of Reinforcement

To prevent bond failure or splitting, the calculated stress in reinforcement at any section should be developed on each side of that section by adequate embedment length, end anchorage, or, for tension only, hooks. Requirements are the same as those given for ultimate-strength design in Art. 8.20.6. Embedment length required at simple supports and inflection points can be computed from Eq. (8.25) by substituting double the computed shears for $V_u$. In computation of $M_t$, the moment arm, $d - a/2$ may be taken as $0.85d$ (Fig. 8.12). See also Art. 8.22.

## 8.22 Bar Cutoffs and Bend Points

It is common practice to stop or bend main reinforcement in beams and slabs where it is no longer required. But tensile steel should never be discontinued exactly at the theoretical cutoff or bend points. It is necessary to resist tensile forces in the reinforcement through embedment beyond those points.

All reinforcement should extend beyond the point at which it is no longer needed to resist flexure for a distance equal to the effective depth of the member or 12 bar diameters, whichever is greater except at supports of simple spans and at free end of a cantilever. Lesser extensions, however, may be used at supports of a simple span and at the free end of a cantilever. See Art. 8.20.6 for embedment requirements at simple supports and inflection points and for termination of negative-moment bars. Continuing reinforcement should have an embedment length beyond the point where bent or terminated reinforcement is no longer required to resist flexure. The embedment should be at least as long as the development length $l_d$ defined in Art. 8.12.5.

Flexural reinforcement should not be terminated in a tension zone unless one of the following conditions is satisfied:

1. Shear is less than two-thirds that normally permitted, including allowance for shear reinforcement, if any.
2. Continuing bars provide double the area required for flexure at the cutoff, and the shear

does not exceed three-quarters of that permitted (No. 11 bar or smaller).

3. Stirrups in excess of those normally required are provided each way from the cutoff for a distance equal to 75% of the effective depth of the member. Area and spacing of the excess stirrups should be such that

$$A_v \geq 60 \frac{b_w s}{f_y} \tag{8.33}$$

where $A_v$ = stirrup cross-sectional area, in$^2$

$b_w$ = web width, in

$s$ = stirrup spacing, in

$f_y$ = yield strength of stirrup steel, psi

Stirrup spacing $s$ should not exceed $d/8\beta_b$, where $\beta_b$ is the ratio of the area of bars cut off to the total area of tension bars at the section and $d$ is the effective depth of the member.

The location of theoretical cutoffs or bend points may usually be determined from bending moments since the steel stresses are approximately proportional to them. The bars generally are discontinued in groups or pairs. So, for example, if one-third the bars are to be bent up, the theoretical bend-up point lies at the section where the bending moment is two-thirds the maximum moment. The point may be found analytically or graphically.

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); P. F. Rice and E. S. Hoffman, "Structural Design Guide to the ACI Building Code," Van Nostrand Reinhold Company, New York, ACI 315, "Manual of Standard Practice for Detailing Reinforced Concrete Structures," American Concrete Institute (www.aci-int.org).)

## 8.23 One-Way Slabs

If a slab supported on beams or walls spans a distance in one direction more than twice that in the perpendicular direction, so much of the load is carried on the short span that the slab may reasonably be assumed to be carrying all the load in that direction. Such a slab is called a one-way slab.

Generally, a one-way slab is designed by selecting a 12-in-wide strip parallel to the short direction and treating it as a rectangular beam. Reinforcing steel usually is spaced uniformly in both directions (Table 8.13). In addition to the main reinforcing in the short span, steel should be provided in the long direction to distribute

**Table 8.13** Areas of Bars in Slabs, in$^2$/ft of Slab

| | Bar No. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Spacing, in | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 0.44 | 0.78 | 1.23 | 1.77 | 2.40 | 3.14 | 4.00 | 5.06 | 6.25 |
| 3½ | 0.38 | 0.67 | 1.05 | 1.51 | 2.06 | 2.69 | 3.43 | 4.34 | 5.36 |
| 4 | 0.33 | 0.59 | 0.92 | 1.32 | 1.80 | 2.36 | 3.00 | 3.80 | 4.68 |
| 4½ | 0.29 | 0.52 | 0.82 | 1.18 | 1.60 | 2.09 | 2.67 | 3.37 | 4.17 |
| 5 | 0.26 | 0.47 | 0.74 | 1.06 | 1.44 | 1.88 | 2.40 | 3.04 | 3.75 |
| 5½ | 0.24 | 0.43 | 0.67 | 0.96 | 1.31 | 1.71 | 2.18 | 2.76 | 3.41 |
| 6 | 0.22 | 0.39 | 0.61 | 0.88 | 1.20 | 1.57 | 2.00 | 2.53 | 3.12 |
| 6½ | 0.20 | 0.36 | 0.57 | 0.82 | 1.11 | 1.45 | 1.85 | 2.34 | 2.89 |
| 7 | 0.19 | 0.34 | 0.53 | 0.76 | 1.03 | 1.35 | 1.71 | 2.17 | 2.68 |
| 7½ | 0.18 | 0.31 | 0.49 | 0.71 | 0.96 | 1.26 | 1.60 | 2.02 | 2.50 |
| 8 | 0.17 | 0.29 | 0.46 | 0.66 | 0.90 | 1.18 | 1.50 | 1.89 | 2.34 |
| 9 | 0.15 | 0.26 | 0.41 | 0.59 | 0.80 | 1.05 | 1.33 | 1.69 | 2.08 |
| 10 | 0.13 | 0.24 | 0.37 | 0.53 | 0.72 | 0.94 | 1.20 | 1.52 | 1.87 |
| 12 | 0.11 | 0.20 | 0.31 | 0.44 | 0.60 | 0.79 | 1.00 | 1.27 | 1.56 |

concentrated loads and resist shrinkage and thermal stresses. The bars or wires should not be spaced farther apart than five times the slab thickness for shrinkage and temperature steel and three times the slab thickness for main reinforcing. Spacing in either direction should not exceed 18 in.

For shrinkage and temperature stresses, ACI 318, "Building Code Requirements for Reinforced Concrete," requires the following ratio of reinforcement to gross concrete areas, in$^2$/ft: deformed bars with yield strength less than 60 ksi, 0.0020; deformed bars with 60 ksi yield strength or welded-wire fabric with wires not more than 12 in apart, 0.0018. For highway bridge slabs, "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) requires reinforcing steel in the bottoms of all slabs transverse to the main reinforcement for lateral distribution of wheel loads. The area of the distribution steel should be at least the following percentages of the main steel required for positive moment, where $S$ is the effective span, ft. When main steel is parallel to traffic, $100/\sqrt{S}$ with a maximum of 50%; when the main steel is perpendicular to traffic, $200/\sqrt{S}$, with a maximum of 67%.

To control deflections, the ACI Code sets limitations on slab thickness unless deflections are computed and determined to be acceptable (Art. 8.19). Otherwise, thickness of one-way slabs must be at least $L/20$ for simply supported slabs; $L/24$ for slabs with one end continuous; $L/28$ for slabs with both ends continuous; and $L/10$ for cantilevers; where $L$ is the span, in.

## 8.24 Ultimate-Strength Design of Rectangular Beams with Compression Bars

The steel ratio $\rho_b$ for balanced conditions at ultimate strength of a rectangular beam is given by Eq. (8.13) in Art. 8.20.1. When the tensile steel ratio $\rho$ exceeds $0.75\rho_b$, compression reinforcement should be used. When $\rho$ is equal to or less than $0.75\rho_b$, the strength of the beam may be approximated by Eq. (8.14), disregarding any compression bars that may be present, since the strength of the beam will usually be controlled by yielding of the tensile steel.

The bending-moment capacity of a rectangular beam with both tension and compression steel is

$$M_u = 0.90\left[(A_s - A_s')f_y\left(d - \frac{a}{2}\right) + A_s' f_y (d - d')\right] \tag{8.34}$$

where $a$ = depth of equivalent rectangular compressive stress distribution
$= (A_s - A_s')f_y/f_c'b$
$b$ = width of beam, in
$d$ = distance from extreme compression surface to centroid of tensile steel, in
$d'$ = distance from extreme compression surface to centroid of compressive steel, in
$A_s$ = area of tensile steel, in$^2$
$A_s'$ = area of compressive steel, in$^2$
$f_y$ = yield strength of steel, ksi
$f_c'$ = 28-day strength of concrete, ksi

Equation (8.35) is valid only when the compressive steel reaches $f_y$. This occurs when

$$(\rho - \rho') \geq 0.85\beta_1 \frac{f_c' d'}{f_y d} \frac{87{,}000}{87{,}000 - f_y} \tag{8.35}$$

where $\rho = A_s/bd$, $\rho' = A_s'/bd$, and $\beta_1$ is a constant defined in Art. 8.17. When $\rho - \rho'$ is less than the right-hand side of Eq. (8.36), calculate the moment capacity from Eq. (8.15) or from an analysis based on the assumptions of Art. 8.17. ACI 318, "Building Code Requirements for Reinforced Concrete," requires also that $\rho - \rho'$ not exceed $0.75\rho_b$ to avoid brittle failure of the concrete.

Compressive steel should be anchored by ties or stirrups at least ⅜ in in diameter and spaced no more than 16 bar diameters or 48 tie diameters apart. Tie reinforcement requirements are the same as those for columns.

Design for shear and development lengths of reinforcement is the same as for beams with tension reinforcement only (Art. 8.20.4 and 8.20.6).

## 8.25 Alternate Design of Rectangular Beams with Compression Bars

The following formulas, based on the linear variation of stress and strain with distance from the neutral axis (Fig. 8.15) may be used in design:

$$k = \frac{1}{1 + f_s/nf_c} \tag{8.36}$$

where $f_s$ = stress in tensile steel, ksi

$f_c$ = stress in extreme compression surface, ksi

$n$ = modular ratio, $E_s/E_c$

$$f'_s = \frac{kd - d'}{d - kd} 2f_s \tag{8.37}$$

where $f'_s$ = stress in compressive steel, ksi

$d$ = distance from extreme compression surface to centroid of tensile steel, in

$d'$ = distance from extreme compression surface to centroid of compressive steel, in

The factor 2 is incorporated into Eq. (8.37) in accordance with ACI 318, "Building Code Requirements for Reinforced Concrete," to account for the effects of creep and nonlinearity of the stress-strain diagram for concrete. But $f'_s$ should not exceed the allowable tensile stress for the steel.

Since total compressive force equals total tensile force on a section,

$$C = C_c + C'_s = T \tag{8.38}$$

where $C$ = total compression on beam cross section, kips

$C_c$ = total compression on concrete, kips, at section

$C'_s$ = force acting on compressive steel, kips

$T$ = force acting on tensile steel, kips

$$\frac{f_s}{f_c} = \frac{k}{2[\rho - \rho'(kd - d')/(d - kd)]} \tag{8.39}$$

where $\rho = A_s/bd$ and $\rho' = A'_s/bd$.

For reviewing a design, the following formulas may be used:

$$k = \sqrt{2n\left(\rho + \rho'\frac{d'}{d}\right) + n^2(\rho + \rho')^2} - n(\rho + \rho') \tag{8.40}$$

$$\bar{z} = \frac{(k^3 d/3) + 4n\rho' d'[k - (d'/d)]}{k^2 + 4n\rho'[k - (d'/d)]} \tag{8.41}$$

$$jd = d - \bar{z} \tag{8.42}$$

where $jd$ is the distance between the centroid of compression and the centroid of the tensile steel. The moment resistance of the tensile steel is

$$M_s = Tjd = A_s f_s jd \tag{8.43}$$

$$f_s = \frac{M}{A_s jd} \tag{8.44}$$

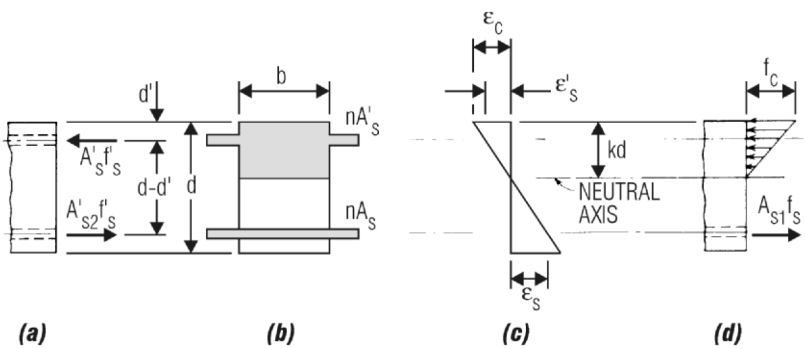



**Fig. 8.15** Rectangular concrete beam: (*a*) Reinforced for both tension and compression. (*b*) Transformed all-concrete section. (*c*) Strain distribution. (*d*) Stresses.

where $M$ is the bending moment at the section of beam under consideration. The moment resistance in compression is

$$M_c = \frac{1}{2} f_c jbd^2 \left[ k + 2n\rho' \left( 1 - \frac{d'}{kd} \right) \right] \tag{8.45}$$

$$f_c = \frac{2M}{jbd^2\{k + 2n\rho'[1 - (d'/kd)]\}} \tag{8.46}$$

Computer software is available for the preceding calculations. Many designers, however, prefer the following approximate formulas:

$$M_1 = \frac{1}{2} f_c bkd \left( d - \frac{kd}{3} \right) \tag{8.47}$$

$$M'_s = M - M_1 = 2f'_s A'_s (d - d') \tag{8.48}$$

where $M$ = bending moment

$M'_s$ = moment-resisting capacity of compressive steel

$M_1$ = moment-resisting capacity of concrete

For determination of shear, see Art. 8.21. Compressive steel should be anchored by ties or stirrups at least No. 3 in size and spaced not more than 16 bar diameters or 48 tie diameters apart. At least one tie within the required spacing, throughout the length of the beam where compressive reinforcement is required, should extend completely around all longitudinal bars.

## 8.26 Ultimate-Strength Design of I and T Beams

A reinforced-concrete beam may be shaped in cross section like a T, or it may be composed of a slab and integral rectangular beam that, in effect, act as a T beam. According to ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), and "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials), when the slab forms the compression flange, its effective width $b$ may be assumed to be not larger than one-fourth the beam span and not greater than the distance center to center of beams. In addition, the ACI Code requires that the overhanging width on either side of the beam web should not be assumed to be larger than eight times the slab thickness nor one-half the clear distance to the next web. The AASHTO Specifications more conservatively limit the effective width to 12 times the slab thickness plus the beam width. For beams with a flange on only one side, the effective overhanging flange width should not exceed one-twelfth the beam span, or six times the slab thickness, or half the clear distance to the next beam.

Two cases may occur in the design of T and I beams: The neutral axis lies in the compression flange (Fig. 8.16*a* and *b*) or in the web (Fig. 8.16*c* and *d*). For negative moment, a T beam should be designed as a rectangular beam with width $b$ equal to that of the stem. (See Arts. 8.17 and 8.20.)

When the neutral axis lies in the flange, the member may be designed as a rectangular beam, with effective width $b$ and depth $d$, by Eq. (8.14). For that condition, the flange thickness $t$ will be greater than the distance $c$ from the extreme compression surface to the neutral axis.

$$c = \frac{1.18\omega d}{\beta_1} \tag{8.49}$$

where $\beta_1$ = constant defined in Art. 8.17

$\omega = A_s f_y / bdf'_c$

$A_s$ = area of tensile steel, in$^2$

$f_y$ = yield strength of steel, ksi

$f'_c$ = 28-day strength of concrete, ksi

When the neutral axis lies in the web, the ultimate moment should not exceed

$$M_u = 0.90\left[ (A_s - A_{sf}) f_y \left( d - \frac{a}{2} \right) + A_{sf} f_y \left( d - \frac{t}{2} \right) \right] \tag{8.50}$$

where $A_{sf}$ = area of tensile steel required to develop compressive strength of overhanging flange, in$^2$ $= 0.85(b - b_w)tf'_c/f_y$

$b_w$ = width of beam web or stem, in

$a$ = depth of equivalent rectangular compressive stress distribution, in $= (A_s - A_{sf})f_y/0.85f'_c b_w$

The quantity $\rho_w - \rho_f$ should not exceed $0.75\rho_b$, where $\rho_b$ is the steel ratio for balanced conditions [Eq. (8.13)], $\rho_w = A_s/b_w d$, and $\rho_f = A_{sf}/b_w d$.

For determination of ultimate shear, see Art. 8.20.4. Note, however, that the web or stem

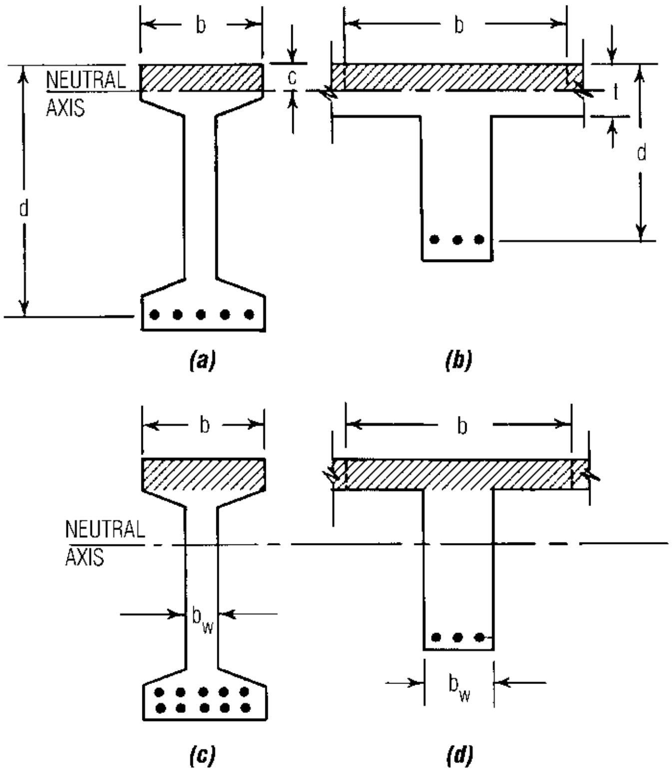



**Fig. 8.16** I and T beams: (*a*) and (*b*) Neutral axis in the flange. (*c*) and (*d*) Neutral axis in the web.

width $b_w$ should be used instead of $b$ in these calculations.

## 8.27 Working-Stress Design of I and T Beams

For T beams, effective width of compression flange is determined by the same rules as for ultimate-strength design (Art. 8.26). Also, for working-stress design, two cases may occur: The neutral axis may lie in the flange (Fig. 8.16*a* and *b*) or in the web (Fig. 8.16*c* and *d*). (For negative moment, a T beam should be designed as a rectangular beam with width $b$ equal to that of the stem.) See Art. 8.21.

If the neutral axis lies in the flange, a T or I beam may be designed as a rectangular beam with effective width $b$. If the neutral axis lies in the web or stem, an I or T beam may be designed by the following formulas, which ignore the compression in the stem, as is customary:

$$k = \frac{I}{1 + f_s/nf_c} \tag{8.51}$$

where $kd$ = distance from extreme compression surface to neutral axis, in

$d$ = distance from extreme compression surface to centroid of tensile steel, in

$f_s$ = stress in tensile steel, ksi

$f_c$ = stress in concrete at extreme compression surface, ksi

$n$ = modular ratio = $E_s/E_c$

Since the total compressive force $C$ equals the total tension $T$,

$$C = \frac{1}{2} f_c(2kd - t)\frac{bt}{kd} = T = A_s f_s \tag{8.52}$$

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \tag{8.53}$$

where $A_s$ = area of tensile steel, in$^2$

$t$ = flange thickness, in

The distance between the centroid of the area in compression and the centroid of the tensile steel is

$$jd = d - \bar{z} \tag{8.54}$$

$$\bar{z} = \frac{t(3kd - 2t)}{3(2kd - t)} \tag{8.55}$$

The moment resistance of the steel is

$$M_s = Tjd = A_s f_s jd \tag{8.56}$$

The moment resistance of the concrete is

$$M_c = Cjd = \frac{f_c btjd}{2kd}(2kd - t) \tag{8.57}$$

In design, $M_s$ and $M_c$ can be approximated by

$$M_s = A_s f_s \left(d - \frac{t}{2}\right) \tag{8.58}$$

$$M_c = \frac{1}{2} f_c bt \left(d - \frac{t}{2}\right) \tag{8.59}$$

derived by substituting $d - t/2$ for $jd$ and $f_c/2$ for $f_c(1 - t/2kd)$, the average compressive stress on the section.

For determination of shear, see Art. 8.21. Note, however, that the web or stem width $b_w$ should be used instead of $b$ in these calculations.

## 8.28 Torsion in Reinforced-Concrete Members

Under twisting or torsional loads, a member develops normal (warping) and shear stresses. The warping, normal stresses help greatly in resisting torsion. But there are no accurate ways of computing this added resistance.

The maximum shears at any point are accompanied by equal tensile stresses on planes bisecting the angles between the planes of maximum shears.

As for ordinary shear, reinforcement should be incorporated to resist the diagonal tension in excess of the tensile capacity of the concrete. If web reinforcement is required for vertical shear in a horizontal beam subjected to both flexure and torsion, additional web reinforcement should be included to take care of the full torsional shear.

### 8.28.1 Ultimate-Strength Design for Torsion

When the factored torsion $T_u$ is less than the value calculated from Eq. (8.63), the area $A_v$ of shear reinforcement should be at least

$$A_v = 50\frac{b_w s}{f_y} \tag{8.60}$$

But when the ultimate torsion exceeds $T_u$ calculated from Eq. (8.63) and where web reinforcement is required, either nominally or by calculation, the minimum area of closed stirrups required is

$$A_v + 2A_t = \frac{50 b_w s}{f_y} \tag{8.61}$$

where $A_t$ is the area of one leg of a closed stirrup resisting torsion within a distance $s$.

While shear reinforcement may consist of stirrups (Fig. 8.12), bent-up longitudinal bars, spirals, or welded-wire fabric (Art. 8.20.4), torsion reinforcement should consist of closed ties, closed stirrups, or spirals—all combined with longitudinal bars. Closed ties or stirrups may be formed either in one piece by overlapping standard tie end hooks around a longitudinal bar (Fig. 8.12*b*), or in two pieces spliced as a Class B splice or adequately embedded. Pairs of U stirrups placed so as to form a closed unit should be lapped at least $1.3l_d$, where $l_d$ is the tensile development length (Art. 8.12.5).

Torsion effects should be considered whenever the ultimate torsion exceeds

$$T_u = \phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) \tag{8.62}$$

where $A_{cp}$ = area enclosed by the outside perimeter of concrete cross section

$p_{cp}$ = outside perimeter of the concrete cross section

The design torsional strength should be equal to or greater than the required torsional strength:

$$\phi T_n \geq T_u \tag{8.63}$$

The nominal torsional moment strength in terms of stirrup yield strength was derived above.

$$T_n = \frac{2A_o A_t f_{yv}}{s}\cot\theta \tag{8.64}$$

where $A_o = 0.85A_{oh}$ (this is an assumption for simplicity)

$A_{oh}$ = area enclosed by centerline of the outermost closed transverse torsional reinforcement

$\theta$ = angle of compression diagonal, ranges between 30° and 60°. It is suggested in 11.6.3.6 to use 45° for nonprestressed members and 37.5° for prestressed members with prestress force greater than 40 percent of tensile strength of the longitudinal reinforcement.

The spacing of closed stirrups, however, should not exceed $p_h/8$ nor 12 in, where $p_h$ is the perimeter of centerline of outermost closed transverse torsional reinforcement, in.

At least one longitudinal bar should be placed in each corner of the stirrups. Size of longitudinal bars should be at least No. 3, and their spacing around the perimeters of the stirrups should not exceed 12 in. Longitudinal bars larger than No. 3 are required if indicated by the larger of the values of $Al$ computed from Eq. (8.65).

$$Al = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta \qquad (8.65)$$

## 8.29 Two-Way Slabs

When a rectangular reinforced-concrete slab is supported on all four sides, reinforcement placed perpendicular to the sides may be assumed to be effective in the two directions if the ratio of the long sides to the short sides is less than about 2 : 1. "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) requires that the slab be designed as a one-way slab if the ratio is more than 1.5 : 1. In effect, a two-way slab distributes part of the load on it in the long direction and usually a much larger part in the short direction. For a symmetrically supported square slab, however, distribution is the same in the two directions for symmetrical loading.

Because precise determination of reactions and moments for two-way slabs with various edge conditions is complex and tedious, most codes offer empirical formulas to simplify the calculation.

The reactions of the slab on supporting beams and walls are not constant along the sides, which should be taken into account in the design of the supports. (One method is to use a triangular distribution on the short sides and a trapezoidal distribution on the long sides. The legs of the triangles and the trapezoids usually are assumed to make a 45° angle with the slab edges.)

### 8.29.1 Flat-Slab Construction

Two-way slab systems supported directly on columns, without beams or girders, and thickened locally around the columns creating drop panels, are classified as flat slabs. Generally, the columns flare out at the top in capitals (Fig. 8.17*a*). But only the portion of the inverted truncated cone thus formed that lies inside a 90° vertex angle is considered effective in resisting stress. Sometimes, the capital for an exterior column is a bracket on the inner face.

To reduce the shear stresses in the region of the columns and the amount of steel needed for negative bending moments, especially when the live load exceeds 150 psf, a rectangular drop panel, or thicker slab, is formed over the columns (Fig. 8.17*a*). For similar spans and loads, use of a drop panel permits a reduced slab thickness between panels. For the full effective depth of the drop to be used in determination of negative-moment reinforcement, ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), specifies that a drop panel should extend in each direction from the center of support a distance equal to at least one-sixth the span in that direction. The difference in thickness between the drop panel and slab should be at least one-fourth the slab thickness but, for determining reinforcement, should not be taken as more than one-fourth the distance from the edge of the drop panel to the edge of the column or capital.

To control deflection, the ACI Code establishes minimum thicknesses for slabs, without interior beams as a ratio of the length of the clear space in the long direction. The minimum slab thickness for slabs with drop panels is 4 inches.

In general, flat slabs are more economical than beam-and-girder construction. They yield a lower building for the same number of stories. Formwork is simpler. Fire resistance is greater because of fewer sharp corners where spalling may occur. And there is less obstruction to light with flat slabs. The design procedure is similar to that for flat plates and is described in Art. 8.29.2.

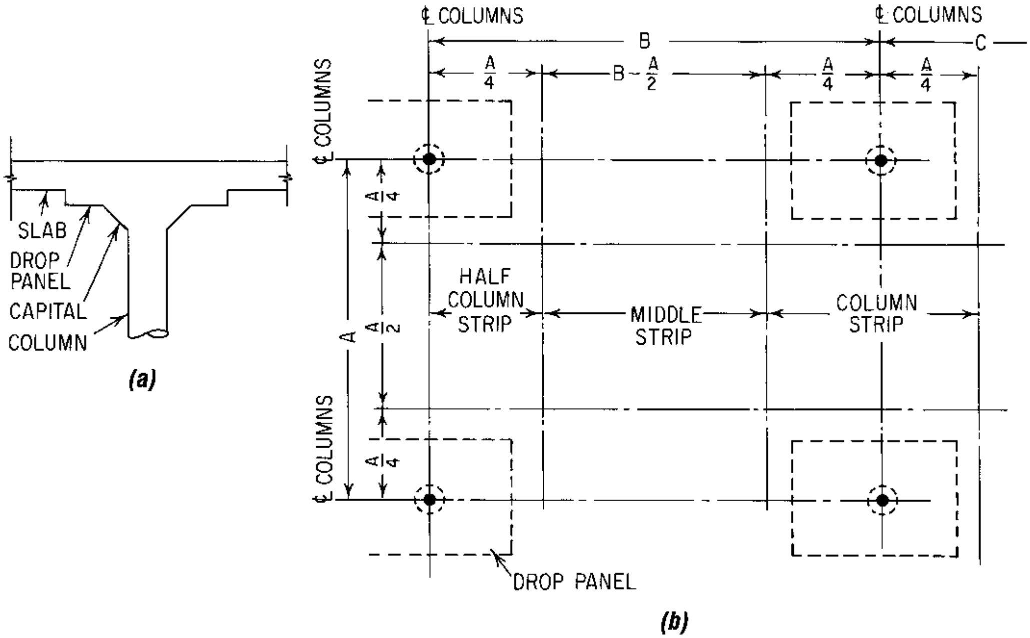



**Fig. 8.17** Concrete flat slab: (*a*) Vertical section through drop panel and column at a support. (*b*) Plan view indicates division of slab into column and middle strips.

## 8.29.2 Flat-Plate Construction

Flat slabs with constant thickness between supports are called flat plates. Generally, capitals are omitted from the columns.

Exact analysis or design of flat slabs or flat plates is very complex. It is common practice to use approximate methods. The ACI Code presents two such methods: direct design and equivalent-frame.

In both methods, a flat slab is considered to consist of strips parallel to column lines in two perpendicular directions. In each direction, a **column strip** spans between columns and has a width of one-fourth the shorter of the two perpendicular spans on each side of the column centerline. The portion of a slab between parallel column strips in each panel is called the **middle strip** (Fig. 8.17).

**Direct Design Method ▪** This may be used when all the following conditions exist:

The slab has three or more bays in each direction.

Ratio of length to width of panel is 2 or less.

Loads are uniformly distributed over the panel.

Ratio of live to dead load is 2 or less.

Columns form an approximately rectangular grid (10% maximum offset).

Successive spans in each direction do not differ by more than one-third of the longer span.

Moment redistribution shall not be applied.

When a panel is supported by beams on all sides, the relative stiffness of the beams satisfies

$$0.2 \le \frac{\alpha_1}{\alpha_2}\left(\frac{l_2}{l_1}\right)^2 \le 5 \qquad (8.66)$$

where $\alpha_1 = \alpha$ in direction of $l_1$

$\alpha_2 = \alpha$ in direction of $l_2$

$\alpha =$ ratio of flexural stiffness $E_{cb}I_b$ of the beam section to flexural stiffness $E_{cs}I_s$ of width of slab bounded laterally by centerline of adjacent panel, if any, on each side of beam.

$l_1$ = span in the direction in which moments are being determined, c to c of supports

$l_2$ = span perpendicular to $l_1$, c to c of supports

The basic equation used in direct design is the total static design moment in a strip bounded laterally by the centerline of the panel on each side of the centerline of the supports:

$$M_o = \frac{wl_2 l_n^2}{8} \tag{8.67}$$

where $w$ = uniform design load per unit of slab area

$l_n$ = clear span in direction moments are being determined

The strip, with width $l_2$, should be designed for bending moments for which the sum in each span of the absolute values of the positive and average negative moments equals or exceeds $M_o$.

**Interior Panels.** Following is the procedure for direct design of an interior panel of a flat slab (or flat plate or two-way beam-and-slab construction):

**Step 1.** Determine the minimum allowable and practical slab thickness to control deflections and verify adequacy for shear strength.

**Step 2.** Determine the ultimate design load from Eq. (8.7*a*), $U = 1.4D + 1.7L$, where $D$ represents the moments and shears caused by dead load and $L$ those caused by live load. (This assumes that horizontal loads are taken by shear walls or other vertical elements.)

**Step 3.** Determine $M_o$ from Eq. (8.67).

**Step 4.** For an interior span, distribute $M_o$ as follows:

$$\text{Negative design moment} = 0.65M_o$$
$$\text{Positive design moment} = 0.35M_o$$

The negative-moment section should be designed to resist the larger of the two interior negative design moments determined for the spans framing into a common support.

**Step 5.** Proportion design moments and shears in column and middle strips as follows:

**1. Column Strip.** The interior negative moment should be determined in accordance with Table 8.14. Values not given may be obtained by linear interpolation.

The positive design moment should be determined in accordance with Table 8.15. Values not given may be obtained by linear interpolation.

When there is a beam between columns in the direction of the span in which moments are being considered, the beam should be proportioned to resist 85% of the column strip moment if $\alpha_1 l_2/l_1$ is greater than 1.0. For values of $\alpha_1 l_2/l_1$ between 1.0 and zero, the proportion of moment resisted by the beam may be obtained by linear interpolation between 85 and 0%. The slab in the column strip should be proportioned to resist that portion of the design moment not resisted by the beam.

**2. Middle Strip.** The interior negative or positive design moment assigned to a middle strip is that portion of the design moments not resisted by the column strips bounding it. Thus, each middle strip should be proportioned to resist the sum of the negative moment not taken by the column strip along one side and the negative moment not resisted by the column strip on the other side and, similarly, the sum of the positive moments.

**3. Moment Modification.** A design moment may be modified by 10% if the total static design moment for the panel in the direction considered is not less than that required by Eq. (8.67).

**Step 6.** Walls and columns built integrally with the slab should be designed to resist the moments due to loads on the slab system.

**Exterior Panels.** The ACI Code lists design criteria for exterior panels for a wide range of support conditions. These criteria require determination of the relative flexural stiffness of supports at edges, including torsional resistance.

**Equivalent-Frame Method ▪** The equivalent-frame method typically is used when all

**Table 8.14** Percent of Interior Negative Design Moment in Column Strips

| | Span Ratio $l_2/l_1$ | | |
|---|---|---|---|
| $\alpha_1 l_2/l_1$ | 0.5 | 1.0 | 2.0 |
| 0 | 75 | 75 | 75 |
| 1 or more | 90 | 75 | 45 |

**Table 8.15** Percent of Positive Design Moment in Column Strips

| | Span Ratio $l_2/l_1$ | | |
|---|---|---|---|
| $\alpha_1 l_2/l_1$ | 0.5 | 1.0 | 2.0 |
| 0 | 60 | 60 | 60 |
| 1.0 or more | 90 | 75 | 45 |

the conditions required for the direct design method are not satisfied. The slab is initially divided into a series of bents, or equivalent frames, on column lines taken longitudinally and transversely through the building. Each frame consists of a row of equivalent columns and slab-beam strips, bounded laterally by the centerline of the panel on each side of the column line under investigation. Each such frame may be analyzed in its entirety. Or for vertical loads, each floor may be analyzed, with columns, above and below, assumed fixed at floors above and below. For purposes of computation, the slab-beam may be assumed fixed at any support two panels away from the support where the bending moment is being determined. The moments thus determined may be distributed to the column strips, middle strips, and beams as previously described for the direct design method if Eq. (8.66) is satisfied.

The critical section for negative moment in both the column and middle strips should be taken at the face of supports, but for interior supports not farther than $0.175 l_1$ from the center of the column, where $l_1$ is the span center to center of supports.

Note that where slabs designed by the equivalent-frame method meet the criteria of the direct design method, the computed moments in any span may be reduced in a proportion such that the sum of the absolute values of the positive and average negative bending moments used in design does not exceed $M_o$ given by Eq. (8.67).

Determination of reinforcement, based on the bending moments at critical sections, is the same as described for rectangular beams (Art. 8.20 or 8.21). Requirements for minimum reinforcement should be respected.

The equivalent-frame method attempts to represent the effects of torsional stiffness of the three-dimensional slab system by defining and using the flexural stiffness of the slab-beam-column system in geometric terms applicable to a two-dimensional analysis. The ACI Code assigns a finite moment of inertia to the slab-beam from center to face of column equal to the moment of inertia of the slab-beam at the face of the column divided by $(1 - c_2/l_2)^2$, where $c_2$ is the dimension of column, capital, or bracket in the direction of $l_2$. This assigned $I$ represents the flexibility of the slab on the sides of the column. This simulates additional stiffness in the area of the slab-column and is reflected by the change in the coefficients used to determine fixed-end moments, stiffness factors, and carry-over factors for slabs. The ACI Code also modifies the column flexural stiffness to account for the torsional flexibility of the slab. The part of the slab providing the torsional restraint is transverse to the direction in which moments are being determined for the width of the column and extends to the bounding lateral panel centerlines on each side of the column.

### 8.29.3 Shear in Slabs

Slabs should also be investigated for shear, both beam-type and punching shear. For beam-type shear, the slab is considered as a thin, wide rectangular beam. The critical section for diagonal tension should be taken at a distance from the face of the column or capital equal to the effective depth $d$ of the slab. The critical section extends across the full width $b$ of the slab. Across this section, the nominal shear stress $v_u$ on the unreinforced concrete should not exceed the ultimate capacity $2\sqrt{f'_c}$.

Punching shear may occur along several sections extending completely around the support, for example, around the face of the column or column capital or around the drop panel. These critical sections occur at a distance $d/2$ from the faces of the supports, where $d$ is the effective depth of the slab or drop panel. Design for punching shear should be based on Eq. (8.16), with shear strength $V_n$ taken not larger than the concrete strength $V_c$. $V_c$ shall be the smallest of (a), (b) and (c).

(a) $$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\, b_o d \qquad (8.68)$$

where $b_o$ = perimeter of critical section

$\beta_c$ = ratio of long side to short side of critical section

(b)
$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'}b_o d \quad (8.69)$$

where $\alpha_s$ = 40 for interior columns, 30 for edge columns, 20 for corner columns

(c)
$$V_{c2} 4\sqrt{f_c'}b_o d \quad (8.70)$$

Shear reinforcement for slabs generally consists of bent bars and is designed in accordance with the provisions for beams (Art. 8.20.4), with the shear strength of the concrete at critical sections taken as $2\sqrt{f_c'}b_o d$ at ultimate strength and $V_n \leq 6\sqrt{f_c'}b_o d$. Extreme care should be taken to ensure that shear reinforcement is accurately placed and properly anchored, especially in thin slabs.

The ACI Code also includes instructions for design of steel shear heads. Because of the cost of steel shear-head reinforcement, however, it is preferable to either thicken the slab or design concrete beams to support heavy loads.

### 8.29.4 Column Moments

Another important consideration in design of two-way slab systems is the transfer of moments to columns. This is generally a critical condition at edge columns, where the unbalanced slab moment is very high due to the one-sided panel.

The unbalanced slab moment is considered to be transferred to the column partly by flexure across a critical section, which is $d/2$ from the periphery of the column, and partly by eccentric shear forces acting about the centroid of the critical section.

That portion of unbalanced slab moment $M_u$ transferred by the eccentricity of the shear is given by $\gamma_v M_u$.

$$\gamma_v = 1 - \gamma_f \quad (8.71)$$

$$= 1 - \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \quad (8.72)$$

where $b_1$ = width, in, of critical section in the span direction for which moments are being computed

$b_2$ = width, in, of critical section in the span direction perpendicular to $b_1$

For that portion of the unbalanced moment transferred to the column by flexure, it is accepted practice to concentrate or add reinforcement across the critical slab width, determined as the sum of the column width plus the thickness of the slab.

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); P. F. Rice and E. S. Hoffman, "Structural Design Guide to the ACI Building Code," Van Nostrand Reinhold Company, New York: "CRSI Handbook," and "Two-Way Slab Design Supplements," Concrete Reinforcing Steel Institute, Chicago, Ill (www.crsi.org).)

## 8.30 Brackets and Corbels

Brackets and corbels are members having a ratio of shear span to depth $a/d$ of 1 or less. The shear span $a$ is the distance from the point of load to the face of support (Fig. 8.18).

The depth of a bracket or corbel at its outer edge should not be less than one-half of the required depth $d$ at the support. Reinforcement should consist of main tension bars with area $A_s$ and shear reinforcement with area $A_h$. The shear reinforcement should consist of closed ties parallel to the main tension reinforcement (Fig. 8.18). The area of shear reinforcing should not be less than $0.5(A_s - A_n)$ where $A_n$ is the area of reinforcement to resist the tensile force and should be uniformly distributed within two-thirds of the depth of the bracket adjacent to the main tension bars. Also, the ratio $\rho = A_s/bd$ should not be less than $0.04 f_c'/f_y$, where $f_c'$ is the 28-day concrete strength and $f_y$ the steel yield point.

It is good practice to anchor main tension reinforcement bars as close as possible to the outer edge by welding a crossbar or steel angle to them. Also, the bearing area should be kept at least 2 in from the outer edge, and the bearing plate should be welded to the main tension reinforcement if horizontal forces are present.

**Tension Reinforcement ▪** $A_s$ should be adequate at the face of the support to resist the moments due to the vertical load and any horizontal forces. This reinforcement must be properly developed to prevent pull-out, by proper anchorage within the support and by a crossbar welded to the bars at the end of the bracket.

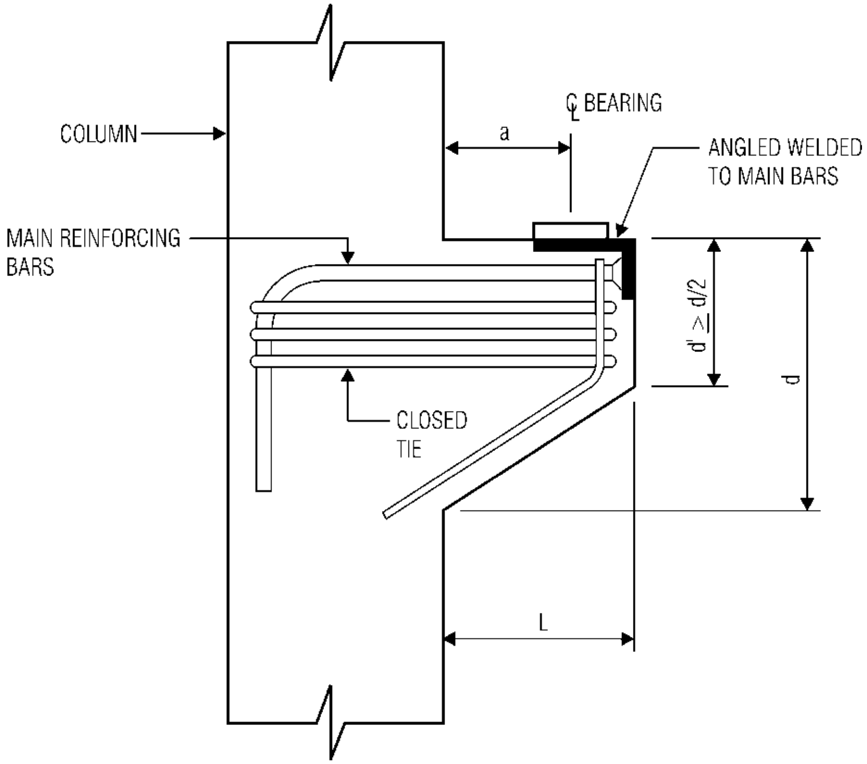



**Fig. 8.18** Steel reinforcement of concrete corbel.

## Concrete Compression Members

ACI 318, "Building Code Requirements for Reinforced Concrete," American Concrete Institute, sets limitations on column geometry and reinforcement. Following are some of the more important.

## 8.31 Column Reinforcement

In reinforced concrete columns, longitudinal steel bars help the concrete carry the load. Steel ties or spiral wrapping around those bars prevent the bars from buckling outward and spalling the outer concrete shell. Since spirals are more effective, columns with closely spaced spirals are allowed to carry greater loads than comparable columns with ties.

**Reinforcement Cover ▪** For cast-in-place columns, spirals and ties should be protected with a monolithic concrete cover of at least $1\frac{1}{2}$ in. But for severe exposures, the amount of cover should be increased.

**Minimum Reinforcement ▪** Columns should be reinforced with at least six longitudinal bars in a circular arrangement or with four longitudinal bars in a rectangular arrangement, of at least No. 5 bar size. Area of column reinforcement should not be less than 1% or more than 8% of the gross cross-sectional area of a column.

**Excess Concrete ▪** In a column that has a larger cross section than that required by load, the effective area $A_g$ used to determine minimum reinforcement area and load capacity may be reduced proportionately, but not to less than half the total area.

### 8.31.1 Spirals

This type of transverse reinforcement should be at least $\frac{3}{8}$ in in diameter. A spiral may be anchored at each of its ends by $1\frac{1}{2}$ extra turns of the spiral. Splices may be made by welding or by a lap of 48 bar diameters (but at least 12 in). Spacing (pitch) of spirals should not exceed 3 in or be less than 1 in. Clear spacing should be at least $1\frac{1}{3}$ times the maximum size of coarse aggregate.

A spiral should extend to the level of the lowest horizontal reinforcement in the slab, beam, or drop panel above. Where beams are of different depth or are not present on all sides of a column, ties should extend above the termination of the spiral to the bottom of the shallowest member. In a column with a capital, the spiral should extend to a plane at which the diameter or width of the capital is twice that of the column.

The ratio of the volume of spiral reinforcement to volume of concrete core (out to out of spiral) should be at least

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_y} \qquad (8.73)$$

where $A_g$ = gross area of column

$A_c$ = area of core of column measured to outside of spiral

$f_y$ = yield strength of spiral reinforcement

$f'_c$ = 28-day compressive strength of concrete

### 8.31.2 Column Ties

Lateral ties should be at least $\frac{3}{8}$ in in diameter for No. 10 or smaller bars and $\frac{1}{2}$ in diameter for No. 11 and larger bars. Spacing should not exceed 16 bar diameters, 48 tie diameters, or the least dimension of the column. The ties should be so arranged that every corner bar and alternate longitudinal bars will have lateral support provided by the corner of a tie having an included angle of not more than 135° (Fig. 8.19). No bar should be more than 6 in from such a laterally supported bar. Where bars are located around a circle, a complete circular tie may be used. (For more details, see ACI 315, "Manual of Standard Practice for Detailing Reinforced Concrete Structures," American Concrete Institute (www.aci-int.org).)

## 8.32 Effects of Column Slenderness

Building columns generally are relatively short. Thus, an approximate evaluation of slenderness effects can usually be used in design. Slenderness, which is a function of column geometry and bracing, can reduce the load-carrying capacity of compression members by introducing bending stresses and can lead to a buckling failure.

Load-carrying capacity of a column decreases with increase in unsupported length $l_u$, beyond a certain length. In buildings, $l_u$ should be taken as the clear distance between floor slabs, girders, or other members capable of providing lateral support to the column or as the distance from a floor to a column capital or a haunch, if one is present.

In contrast, load-carrying capacity increases with increase in radius of gyration $r$ of the column cross section. For rectangular columns, $r$ may be taken as 30% of the overall dimension in the direction in which stability is being considered and for circular members as 25% of the diameter.

### 8.32.1 Effective Column Length

Also, the greater the resistance offered by a column to sidesway, or drift, because of lateral bracing or

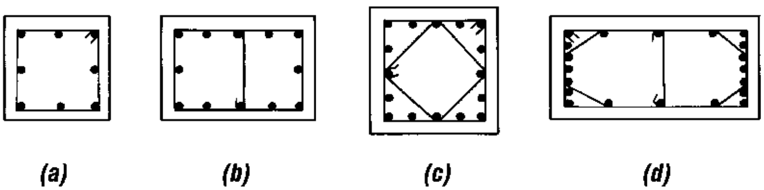



**Fig. 8.19** Column ties provide lateral support at corners and to alternate reinforcing bars at a horizontal section. (*a*) Square column with single tie. (*b*) Rectangular column with a pair of ties. (*c*) Square column with a pair of ties. (*d*) Rectangular column with inclined ties.

restraint against end rotations, the higher the load-carrying capacity. This resistance is represented by application of a factor $k$ to the unsupported length of the column, and $kl_u$ is referred to as the **effective length** of the column.

The combination of these factors, which is a measure of the slenderness of a column, $kl_u/r$, is called the **slenderness ratio** of the column.

The effective-length factor $k$ can be determined by analysis. If an analysis is not made, for compression members in nonsway frames, $k$ should be taken as unity. For columns not braced against sidesway, $k$ will be greater than unity; analysis should take into account the effects of cracking and reinforcement on relative stiffness. See also Art. 8.32.3.

ACI Committee 441 has proposed that $k$ should be obtained from the Jackson and Moreland alignment chart, reproduced as Fig. 8.20. For determination of $k$ with this chart, a parameter $\psi_A$ must be computed for end $A$ of column $AB$, and a similar parameter $\psi_B$ must be computed for end $B$. Each parameter equals the ratio at that end of the column of the sum of $EI/l_u$ for the compression members meeting there to the sum of $EI/l$ for the flexural members meeting there, where $EI$ is the flexural stiffness of a member.

### 8.32.2 Non-Sway and Sway Frames

As a guide in judging whether a frame is non-sway or sway, ACI 318 indicates that a column in a structure can be considered non-sway if the column end moments due to second-order effects do not exceed 5% of the first-order end moments. It is also permitted to assume a story within a structure is non-sway if:

$$Q = \frac{\sum P_u \Delta_o}{V_u \ell_c} \leq 0.05 \tag{8.74}$$

where $Q$ = stability index for a story

$\sum P_u$ = total factored vertical load in the story corresponding to the lateral loading case for which $\sum P_u$ is greatest

$V_u$ = total story shear

$\Delta_o$ = first-order relative deflection between the top and bottom of the story due to $V_u$

$\ell_c$ = column length, measured from center-to-center of the joints in the frame

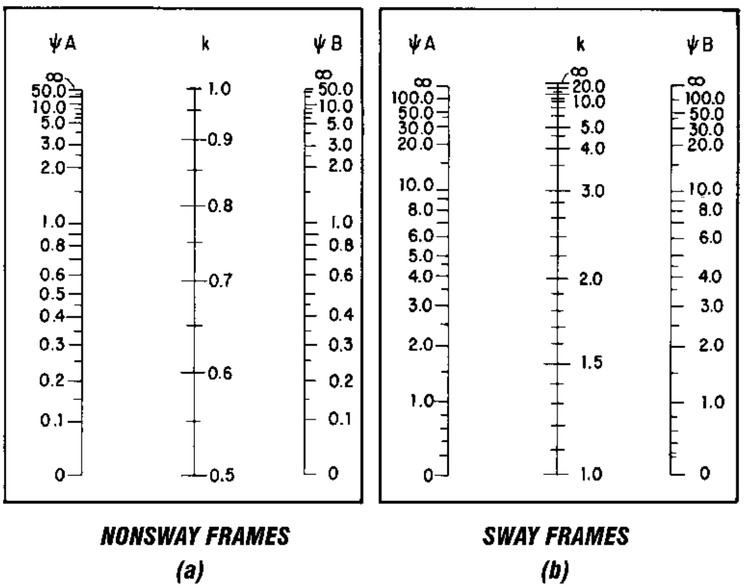



**Fig. 8.20** Alignment charts for determination of effective-length factor $k$ for columns. $\psi$ is the ratio for each end of a column of $\Sigma EI/l_u$ for the compression members to $\Sigma EI/l$ for the girders.

For compression members in non-sway frames, the slenderness effect may be neglected under the following conditions:

For columns braced against sideway, when

$$\frac{kl_u}{r} < 34 - 12\frac{M_1}{M_2} \tag{8.75}$$

where $M_1$ = smaller of two end moments on column as determined by conventional elastic frame analysis, with positive sign if column is bent in single curvature and negative sign if column is bent in double curvature

$M_2$ = absolute value of larger of the two end moments on column as determined by conventional elastic frame analysis

For columns not braced against sidesway, when

$$\frac{kl_u}{r} < 22 \tag{8.76}$$

### 8.32.3 Column Design Loads

Analysis taking into account the influence of axial loads and variable moment of inertia on member stiffness and fixed-end moments, the effects of deflections on moments and forces, and the effects of duration of loads is required for all columns when

$$\frac{kl_u}{r} > 100 \tag{8.77}$$

For columns for which the slenderness ratio lies between 22 and 100, and therefore the slenderness effect on load-carrying capacity must be taken into account, either an elastic analysis can be performed to evaluate the effects of lateral deflections and other effects producing secondary stresses, or an approximate method based on moment magnification may be used. In the approximate method, the compression member in a non-sway frame is designed for the factored axial load $P_u$ and the moment amplified for the effects of member curvature $M_c$ defined by

$$M_c = \delta_{ns} M_2 \tag{8.78}$$

where $\delta_{ns}$ is the moment magnification factor for non-sway frames and may be determined from:

$$\delta_{ns} = \frac{C_m}{1 - P_u/0.75P_c} \geq 1 \tag{8.79}$$

where $C_m$ = factor relating actual moment diagram to that for equivalent uniform moment

$P_c$ = critical load for column

$$= \frac{\pi^2 EI}{(kl_u)^2} \tag{8.80}$$

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d} \tag{8.81}$$

Or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \tag{8.82}$$

For members without transverse loads between supports,

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{8.83}$$

For members with transverse loads between supports $C_m = 1$.

The critical load is given by

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{8.84}$$

where $EI$ is the flexural stiffness of the column.

The flexural stiffness $EI$ may be computed approximately from

$$EI = \frac{E_cI_g/2.5}{1 + \beta_d} \tag{8.85}$$

where $E_c$ = modulus of elasticity of concrete, psi

$I_g$ = moment of inertia about centroidal axis of gross concrete section, neglecting load reinforcement, $\text{in}^4$

$E_s$ = modulus of elasticity of reinforcement, psi

$I_{se}$ = moment of inertia of reinforcement, $\text{in}^4$

$\beta_d$ = ratio of maximum design dead load to total load moment (always taken positive)

Because a column has different properties, such as stiffness, slenderness ratio, and $\delta$, in different directions, it is necessary to check the strength of a column in each of its two principal directions.

For design of compression members in sway frames for slenderness, the magnified sway moment may be computed using a second-order elastic analysis, or an approximate method in the ACI 318 code.

## 8.33 Unified Design Provisions of ACI 318-02

The Unified Design Provisions, which were introduced in Appendix B of the 1995 edition of ACI 318 "Building Code Requirements for Structural Concrete (American Concrete Institute), are incorporated in the body of the 2002 edition. A version of this design method was initially introduced in a paper by Robert Mast in the *ACI Structural Journal*. (Robert Mast, "Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members," *ACI Structural Journal*, vol. 89, no. 2, March–April 1992, pp. 185–199)

Before describing the Unified Design Provisions, a brief review of the Strength Design Method that has been utilized for many years to design reinforced concrete members may be helpful. According to this method, the design strength of a member at any section must be greater than or equal to the required strength that is calculated by the load combinations specified in Chapter 9 (see also Art. 8.17.4) of the code:

$$\text{Design Strength} \geq \text{Required Strength}$$

where

$$\text{Design Strength} = \text{Strength Reduction Factor } (\phi) \times \text{Nominal Strength}$$

$$\text{Required Strength} = \text{Load Factors} \times \text{Service Load Effects}$$

Strength reduction factors ($\phi$-factors) account for the probability of understrength of a member due to variations in material strengths and member dimensions, inaccuracies in design equations, the degree of ductility (range of deformations beyond the stage of elastic response, over which full gravity loads can be sustained), the probable quality control achievable, and the importance of a member in a structure.

The nominal strength of a member or cross-section is determined using the assumptions given in Chapter 10 of the code and the design equations given in various chapters throughout the code.

The Unified Design Provisions modify the Strength Design Method for nonprestressed and prestressed members subjected to flexure and axial loads. Affected are strength reduction factors, reinforcement limits, and moment redistribution.

Like the Strength Design Method, members are proportioned by the Unified Design Provisions using factored loads and strength reduction factors. It is important to recognize that these provisions do not alter nominal strength calculations; the nominal strength of a section is computed in the same way as before. What is modified is the design strength of a section via the strength reduction factors. According to the Unified Design Provisions, $\phi$-factors are determined based on the strain conditions in the reinforcement farthest from the extreme compression face. Prior to this, $\phi$-factors depended only on the type of loading (axial load, flexure, or both) on the section. The Unified Design Provisions provide a rational means for designing nonprestressed and prestressed concrete members subjected to flexural and axial loads, and eliminate many of the inconsistencies in the previous design requirements. This method produces results similar to those from the Strength Design Method. The Unified Design Provisions apply to:

- Flexural and compression members
- Nonprestressed members, prestressed members, and members with a combination of nonprestressed and prestressed reinforcement
- Sections with reinforcement at various depths
- Sections of any shape
- Composite (precast and cast-in-place) concrete sections

These provisions, as they appear in the body of the 2002 ACI code, are described below.

The following definitions are relevant to the Unified Design Provisions. They can be found in Chapter 2 of the code.

- **Net tensile strain, $\varepsilon_t$:** the tensile strain at nominal strength, exclusive of strains due to effective prestress, creep, shrinkage, and temperature.

  The net tensile strain is caused by external axial loads and/or bending moments at a section due to the loads applied on the member at the time when the concrete strain at the extreme compression fiber reaches its assumed limit of 0.003. Generally speaking, the net tensile strain can be used as a measure of excessive cracking or excessive deflection.

- **Extreme tension steel:** the reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber.

  Figure 8.21 depicts the location of the extreme tension steel for two sections with different reinforcement arrangements where the top fiber of the section is the extreme compression fiber. The distance from the extreme compression fiber to the centroid of the extreme tension steel is denoted in the figure as $d_t$. The net tensile strain $\varepsilon_t$ in the extreme tension steel due to the external loads can be determined from a strain compatibility analysis for sections with multiple layers of reinforcement. For sections with one layer of reinforcement, it can easily be determined from the strain diagram by similar triangles.

- **Compression-controlled strain limit:** The net tensile strain at balanced conditions.

  The definition of a balanced strain condition, which is given in ACI Section 10.3.2, is unchanged from previous editions of the code: a balanced strain condition exists at a cross-section when tension reinforcement reaches the strain corresponding to its specified yield strength just as the concrete strain in the extreme compression fiber reaches its assumed limit of 0.003.

  For Grade 60 reinforcement and all prestressed reinforcement, ACI Section 10.3.3 permits the compression-controlled strain limit to be taken equal to 0.002. For Grade 60 bars, this limit is actually equal to $f_y/E_s = 60{,}000/29{,}000{,}000 = 0.00207$ where $f_y$ and $E_s$ are the specified yield strength and modulus of elasticity of the nonprestressed reinforcement, respectively. For other grades of nonprestressed steel, this limit is computed from the ratio $f_y/E_s$.

- **Compression-controlled section:** a cross-section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal the compression-controlled strain limit.

  When the net tensile strain in the extreme tension steel is small, a brittle failure condition is expected. In such cases, there is little warning of impending failure. Cross-sections of compression members such as columns, subject to significant axial compression, are usually compression-controlled.

- **Tension-controlled section:** a cross-section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005.

  The net tensile strain limit of 0.005 applies to both nonprestressed and prestressed reinforcement and provides ductile behavior for most designs. When the net tensile strain in the extreme tension steel is greater than or equal to 0.005, the section is expected to have sufficient ductility so that ample warning of failure in the form of visible cracking and deflection should be available. Cross-sections of flexural members such as beams, if not heavily reinforced, are usually tension-controlled.

Some sections have a net tensile strain in the extreme tension steel between the limits for compression-controlled and tension-controlled sec-

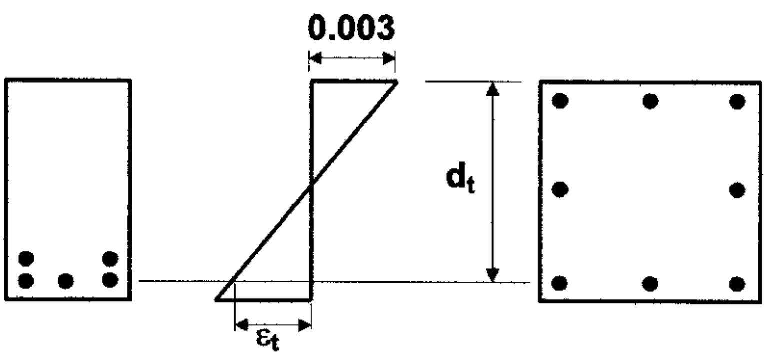



**Fig. 8.21** Location of extreme tension steel and net tensile strain at nominal strength.

tions. An example of this is a section subjected to a small axial load and a large bending moment. These members are in a transition region, which is described below.

All of these definitions are utilized when determining strength reduction factors and design strengths.

### 8.33.1 Strength Reduction Factors, $\phi$—Unified Design

In previous editions of the code, the appropriate $\phi$-factor to use in design depended on the type of loading that the member was subjected to. For example, for members subjected to flexure without axial load, $\phi$ was equal to 0.90.

According to the Unified Design Provisions, strength reduction factors are a function of the net tensile strain $\varepsilon_t$ in the extreme tension steel. ACI Section 9.3.2 contains $\phi$-factors for tension-controlled sections, compression-controlled sections, and sections in which the net tensile strain in the extreme tension steel is between the limits for tension-controlled and compression-controlled sections. Variation of $\phi$ with respect to $\varepsilon_t$ is depicted in ACI Fig. R9.3.2, which is reproduced here as Fig. 8.22.

For compression-controlled sections, $\phi$ is equal to 0.70 for members with spiral reinforcement conforming to ACI Section 10.9.3 and is equal to 0.65 for other members. For tension-controlled sections, $\phi$ is equal to 0.90. For sections that fall between these two limits, it is permitted to linearly increase $\phi$ from the applicable value for compression-controlled sections to 0.90.

The following equations can be used to determine $\phi$ in the transition region:

- For sections with spiral reinforcement:

$$\phi = 0.57 + 67\varepsilon_t \tag{8.86}$$

- For other sections:

$$\phi = 0.48 + 83\varepsilon_t \tag{8.87}$$

ACI Fig. R9.3.2 also contains equations to determine $\phi$ as a function of the ratio $c/d_t$ where $c$ is the distance from the extreme compression fiber to the neutral axis at nominal strength.

Once $\varepsilon_t$ has been computed, Fig. 8.22 can be used to determine the appropriate $\phi$-factor.

### 8.33.2 Nominal Flexural Strength—Unified Design

As noted previously, nominal strength calculations for members subjected to flexure and/or axial loads have not been changed in the Unified Design Provisions. Nominal strength of any cross-section with any amount and arrangement of reinforcement is determined by satisfying force

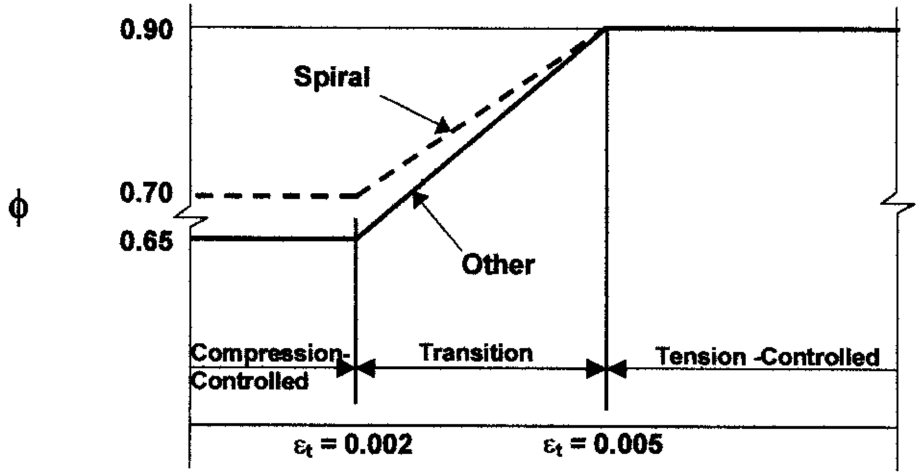



**Fig. 8.22** Variation of $\phi$ with net tensile strain $\varepsilon_t$.

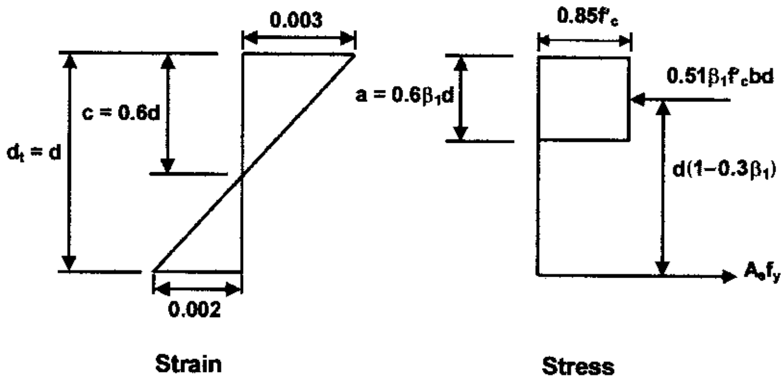



**Fig. 8.23** Strains and stresses at compression-controlled limit.

and moment equilibrium, based on the design assumptions given in Section 10.2 of the code, which include compatibility of strain.

The following equations can be used to determine the nominal flexural strength of compression-controlled sections and tension-controlled sections.

**Compression-Controlled Sections.** For a rectangular section with one layer of Grade 60 reinforcement or prestressed reinforcement, when it is at the compression-controlled strain limit of $\varepsilon_t = 0.002$, the nominal flexural strength is obtained by summing moments about any point on the stress diagram shown in Fig. 8.23:

$$M_{nc} = f_c' bd^2(0.51\beta_1 - 0.153\beta_1^2) \tag{8.88}$$

Since the compression-controlled strain limit is not equal to 0.002 for other grades of nonprestressed reinforcement, similar equations to determine the nominal flexural strength at the compression-controlled strain limit for those grades of steel can easily be derived.

**Tension-Controlled Sections.** The nominal flexural strength of a rectangular section with one layer of Grade 60 or prestressed reinforcement, when it is at the tension-controlled strain limit of $\varepsilon_t = 0.005$, is determined in a similar fashion (see Fig. 8.24):

$$M_{nt} = f_c' bd^2(0.319\beta_1 - 0.06\beta_1^2) \tag{8.89}$$

By equating the tension force in the reinforcing steel to the compression force in the concrete,

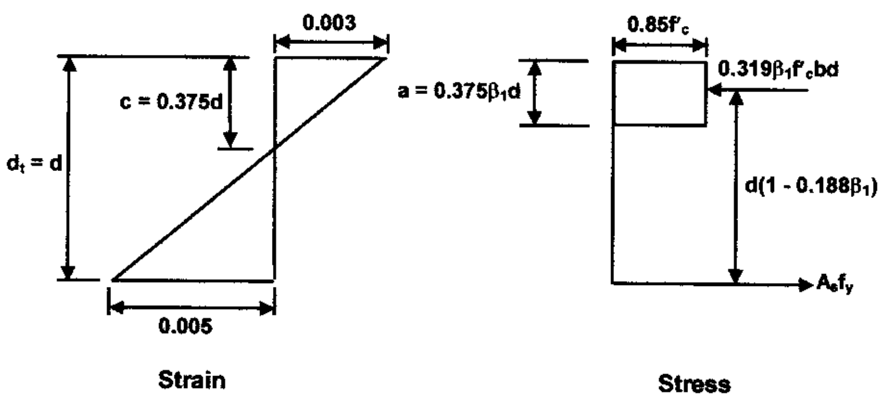



**Fig. 8.24** Strains and stresses at tension-controlled limit.

the area of steel at the tension-controlled limit is:

$$A_s = \frac{0.319\beta_1 f_c' bd}{f_y} \tag{8.90}$$

or

$$\rho_t = \frac{A_s}{bd} = \frac{0.319\beta_1 f_c'}{f_y} \tag{8.91}$$

Define the reinforcement index $\omega_t$ at the tension-controlled strain limit as $\rho_t f_y/f_c'$ and substitute this into the above equation for $M_{nt}$:

$$M_{nt} = \omega_t(1 - 0.59\omega_t)f_c' bd^2 \tag{8.92}$$

or

$$R_{nt} = \frac{M_{nt}}{bd^2} = \omega_t(1 - 0.59\omega_t)f_c' \tag{8.93}$$

where $R_{nt}$ is the nominal strength coefficient of resistance at the tension-controlled strain limit. Values of $R_{nt}$ can readily be determined for any concrete strength and reinforcement. For example, for a section with 4000 psi concrete and Grade 60 reinforcing bars:

$$\rho_t = \frac{0.319 \times 0.85 \times 4}{60} = 0.0181 \tag{8.94}$$

$$\omega_t = \frac{0.1081 \times 60}{4} = 0.2715 \tag{8.95}$$

$$R_{nt} = \frac{M_{nt}}{bd^2} = 0.2715[1 - (0.59 \times 0.2715)] \times 4000$$

$$= 912\ \text{psi} \tag{8.96}$$

The nominal strength of tension-controlled sections is controlled by the strength of the reinforcement, which is less variable than that of the concrete.

**General Case.** The following equation can be used to determine the nominal flexural strength of a rectangular section with tension reinforcement only:

$$M_n = \omega(1 - 0.59\omega)f_c' bd^2 \tag{8.97}$$

where

$$\omega = \frac{\rho f_y}{f_c'} \tag{8.98}$$

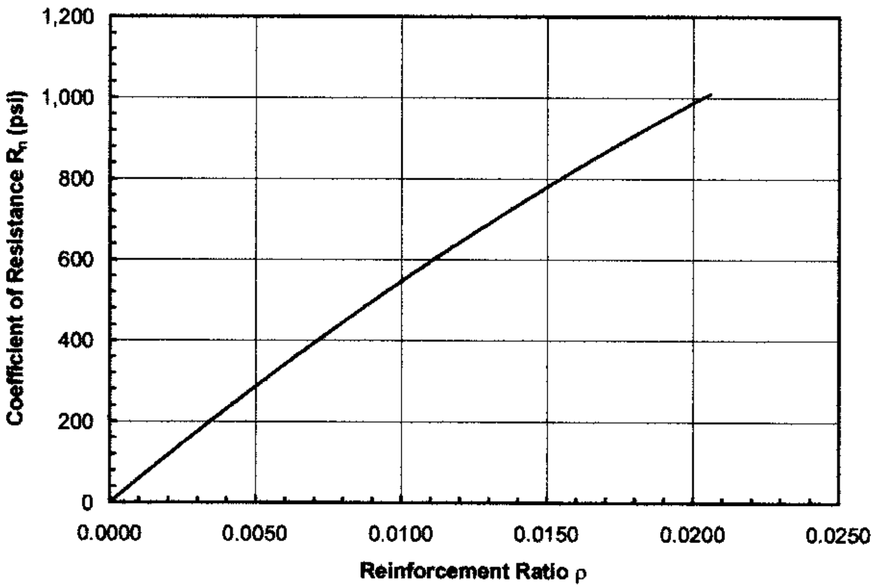



**Fig. 8.25** Strength curve for 4000 psi concrete and Grade 60 reinforcement.

and

$$\rho = \frac{A_s}{bd}. \tag{8.99}$$

The nominal strength coefficient of resistance $R_n$ is:

$$R_n = \frac{M_n}{bd^2} = \omega(1 - 0.59\omega)f_c' \tag{8.100}$$

$$= \rho f_y\left(1 - 0.59\rho\frac{f_y}{f_c'}\right) \tag{8.101}$$

Figure 8.25 shows the strength curve for 4000 psi concrete and Grade 60 reinforcement.

It is important to note that ACI Section 10.3.5 limits the amount of flexural reinforcement in nonprestressed flexural members that are subjected to an axial load less than $0.10f_c'A_g$ by requiring that the net tensile strain $\varepsilon_t$ must be greater than or equal to 0.004. This limit is slightly more conservative than the reinforcement limit of $0.75\rho_b$ that was imposed in previous editions of the code, since the net tensile strain at nominal strength is 0.00376 when $\rho = 0.75\rho_b$.

### 8.33.3 Design Flexural Strength—Unified Design

Once the nominal flexural strength has been determined, the design flexural strength is computed by multiplying the nominal flexural strength by the strength reduction factor $\phi$, which, as shown above, is determined based on the magnitude of the net tensile strain in the extreme tension steel. The design flexural strength must be greater than or equal to the required strength $M_u$ due to the factored loads:

$$\phi M_n \geq M_u \tag{8.102}$$

The following equation can be used to determine the required area of reinforcement for a factored bending moment $M_u$. Substitute $M_n = M_u/\phi$ into the equation for $R_n$:

$$R_n = \frac{M_u}{\phi bd^2} = \rho f_y\left(1 - 0.59\rho\frac{f_y}{f_c'}\right) \tag{8.103}$$

$$\phi R_n = \frac{M_u}{bd^2} = \phi\rho f_y\left(1 - 0.59\rho\frac{f_y}{f_c'}\right) \tag{8.104}$$

The design strength curve for 4000 psi concrete and Grade 60 reinforcement is depicted in Fig. 8.26.

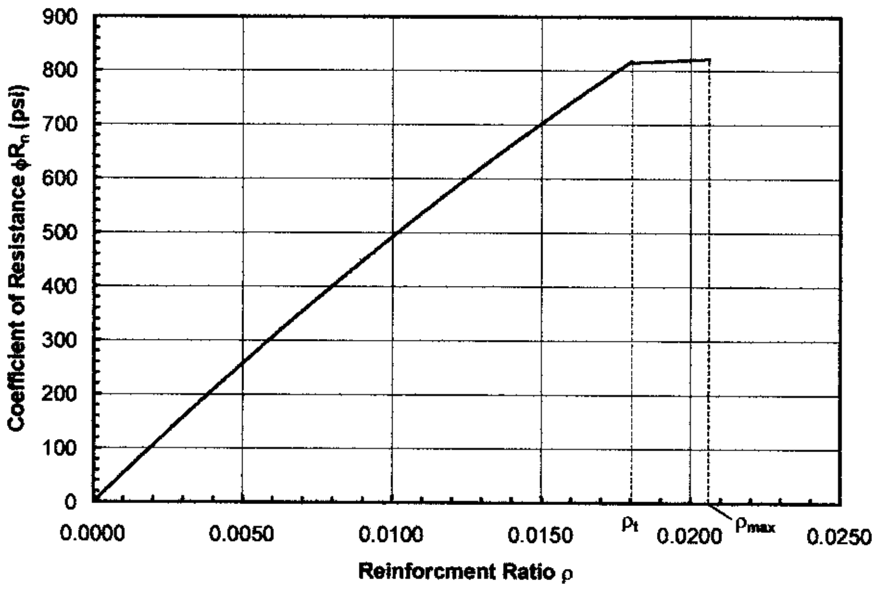



**Fig. 8.26** Design strength curve for 4000 psi concrete and Grade 60 reinforcement.

The required reinforcement ratio $\rho$ can be obtained from the figure for values of $\phi R_n = M_u/bd^2$.

Figure 8.26 shows the effect that $\phi$ has on the design strength. The relationship between the design strength and the reinforcement ratio is approximately linear up to $\rho = \rho_t$, which is the reinforcement ratio corresponding to the net tensile strain $\varepsilon_t$ at the tension-controlled limit of 0.005. For the portion of the design strength curve up to $\rho = \rho_t$, $\phi = 0.90$. For reinforcement ratios greater than $\rho_t$, the net tensile strain is less than 0.005, and $\phi$ is less than 0.90. The maximum reinforcement ratio $\rho_{max}$ corresponds to a net tensile strain of 0.004.

It is clear from the figure that there is no benefit in designing a flexural member with a reinforcement ratio greater than $\rho_t$, since any gain in strength with greater amounts of tension reinforcement is cancelled by the reduction in $\phi$. Therefore, whenever possible, flexural members should be designed as tension-controlled sections. In cases where member size is limited, it is advisable to increase member strength by adding compression reinforcement instead of additional tension reinforcement so that the section remains tension-controlled.

### 8.33.4 Nominal Strength for Combined Flexure and Axial Load—Unified Design

The nominal strength of a member subjected to combined flexure and axial load must satisfy equilibrium and compatibility of strains, which are the same two conditions required for members subjected to flexure only. Fig. 8.27 depicts the general condition of strain and stress at nominal strength for a member subjected to combined flexure and axial compression.

The nominal axial load strength is computed by summing forces on the section, while the nominal flexure strength is obtained by summing moments about any point on the stress diagram. The forces in the reinforcement depend on the corresponding magnitude of the strains, which are determined from a strain compatibility analysis.

Once the net tensile strain is determined in the extreme tension steel for a given combination of axial load and bending moment, $\phi$ is determined as described above. The design axial load strength and design flexural strength of the section are obtained by multiplying the nominal axial load strength and nominal flexural strength by the strength reduction factor, respectively.

A design axial load-bending moment interaction diagram can be constructed for a section in the same manner as before. The only difference in using the Unified Design Provisions is that the $\phi$-factors are computed at the various points along the strength curve as a function of the net tensile strain in the extreme tensile steel, as shown above.

### 8.33.5 Redistribution of Negative Moments— Unified Design

Prior to the 2002 code, the permissible percentage of redistribution of negative moments in continuous

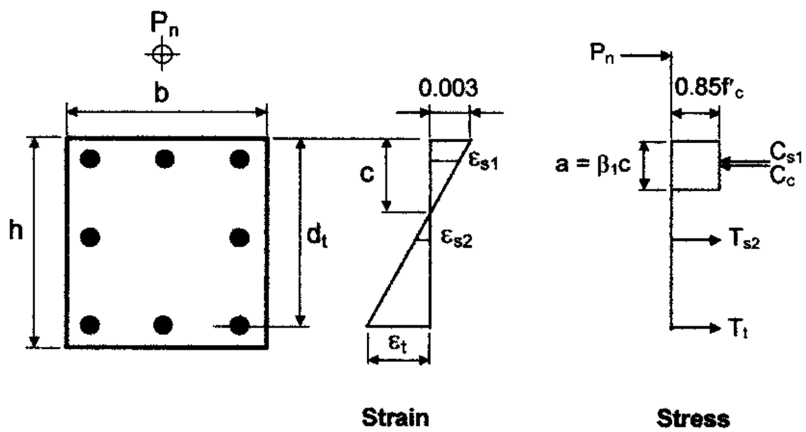



**Fig. 8.27** Strains and stresses for section subjected to combined flexure and axial load.

nonprestressed flexural members was determined as a function of the negative and positive reinforcement in the section and the balanced reinforcement ratio. Similar provisions for continuous prestressed flexural members were also given in a different section of the code. Moment redistribution requirements based on the Unified Design Provisions are given in Section 8.4 of the 2002 code, and are applicable to both continuous nonprestressed and prestressed flexural members. They may not be applied to cases where approximate values for bending moments are used, as in the Direct Design Method of two-way slab design.

According to ACI 318-02 requirements, negative moments may be increased or decreased by not more than $1000\varepsilon_t$ percent, with a maximum of 20 percent, at sections where $\varepsilon_t$ is greater than or equal to 0.0075. The lower limit on $\varepsilon_t$ is required since moment redistribution depends on adequate ductility. The lower limit in prior codes used to be $\rho \le 0.5\rho_b$.

These new requirements for moment redistribution are much simpler to apply than those in previous editions of the code. Figure R8.4 in the commentary of the 2002 ACI code shows a comparison between the permissible moment redistribution in the 2002 and earlier codes.

## 8.34 Planar Walls

These are vertical or near vertical members with length exceeding three times the thickness. Concrete walls may be classified as non-load bearing, load-bearing, or shear walls, which may be either load-bearing or non-load-bearing. Retaining walls are discussed in Arts. 8.41 to 8.43.

### 8.34.1 Non-Load-Bearing Walls

These are generally basement, retaining, or facade-type walls that support only their own weight and also resist lateral loads. Such walls are principally designed for flexure. By the ACI Code, design requirements include:

1. Ratio of vertical reinforcement to gross concrete area should be at least 0.0012 for deformed bars No. 5 or smaller, 0.0015 for deformed bars No. 6 and larger, and 0.0012 for welded-wire fabric not larger than ⅝ in in diameter.
2. Spacing of vertical bars should not exceed three times the wall thickness or 18 in.
3. Lateral or cross ties are not required if the vertical reinforcement is 1% or less of the concrete area, or where the vertical reinforcement is not required as compression reinforcement.
4. Ratio of horizontal reinforcement to gross concrete area should be at least 0.0020 for deformed bars No. 5 or smaller, 0.0025 for deformed bars No. 6 and larger, and 0.0020 for welded-wire fabric not larger than ⅝ in in diameter.
5. Spacing of horizontal bars should not exceed 3 times the wall thickness or 18 in.

Note that walls more than 10″ thick shall have nominal reinforcement for each direction place within two layers parallel with faces of wall.

### 8.34.2 Load-Bearing Walls

These are subject to axial compression loads in addition to their own weight and, where there is eccentricity of load or lateral loads, to flexure. Load-bearing walls may be designed in a manner similar to that for columns, but including the preceding design requirements for non-load-bearing walls.

As an alternative, load-bearing walls may be designed by an empirical procedure given in the ACI Code when the resultant of all factored loads is located within the middle third of the overall wall thickness.

Load-bearing walls designed by either method should meet the minimum reinforcing requirements for non-load-bearing walls.

In the empirical method the axial capacity, kips, of the wall is

$$\phi P_{nw} = 0.55\phi f'_c A_g \left[1 - \left(\frac{kl_c}{32h}\right)^2\right] \tag{8.105}$$

where $f'_c$ = 28-day compressive strength of concrete, ksi

$A_g$ = gross area of wall section, in$^2$

$\phi$ = strength reduction factor = 0.70

$l_c$ = vertical distance between supports, in

$h$ = overall thickness of wall, in

$k$ = effective-length factor

For a wall supporting a concentrated load, the length of wall effective for the support of that concentrated load should be taken as the smaller of the distance center to center between loads and the bearing width plus $4h$.

Reinforced bearing walls designed using Eq. (8.86) should have a thickness of at least $\frac{1}{25}$ of the unsupported height or width, whichever is shorter, but not less than 4 in. Thickness of exterior basement walls and foundation walls must be $7\frac{1}{2}$ in or greater. Also, walls more than 10 in thick, except for basement walls, should have two layers of reinforcement in each direction, with between one-half and two-thirds of the total steel area in the layer near the exterior face of the wall. This layer should be placed at least 2 in but not more than one-third the wall thickness from the face. Walls should be anchored to the floors, or to the columns, pilasters, or intersecting walls.

Walls designed as grade beams should have top and bottom reinforcement as required by the ACI Code for beam design.

### 8.34.3 Shear Walls

Walls subject to horizontal shear forces in the plane of the wall should, in addition to satisfying flexural requirements, be capable of resisting the shear. The nominal shear stress can be computed from

$$v_u = \frac{V_u}{\phi h d} \tag{8.106}$$

where $V_u$ = total design shear force

$\phi$ = capacity reduction factor = 0.75

$d = 0.8 l_w$

$h$ = overall thickness of wall

$l_w$ = horizontal length of wall

The shear $V_c$ carried by the concrete depends on whether $N_u$, the design axial load, lb, normal to the wall horizontal cross section and occurring simultaneously with $V_u$ at the section, is a compression or tension force. When $N_u$ is a compression force, $V_c$ may be taken as $2\sqrt{f'_c}hd$, where $f'_c$ is the 28-day strength of concrete, psi. When $N_u$ is a tension force, $V_c$ may be taken as:

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f'_c}b_w d > 0 \tag{8.107}$$

where $N_u$ = negative for tension

$A_g$ = gross area of section

$N_u/A_g$ = expressed in psi

Alternatively, more detailed calculations may be made for $V_c$ when $V_c$ is the smallest of:

$$V_c = 3.3\sqrt{f'_c}hd + \frac{N_u d}{4l_w} \tag{8.108}$$

$$V_c = hd\left[0.6\sqrt{f'_c} + \frac{l_w(1.25\sqrt{f'_c} + 0.2N_u/l_w h)}{M_u/V_u - l_w/2}\right] \tag{8.108a}$$

where $N_u$ is negative for tension.

Equation (8.90) does not apply, however, when $M_u/V_u - l_w/2$ is negative.

When the factored shear $V_u$ is less than $0.5\phi V_c$, reinforcement should be provided as required by the empirical method for bearing walls.

When $V_u$ exceeds $0.5\phi V_c$, horizontal reinforcement should be provided in accordance with Eq. (8.18), with $V_s = A_v f_y d/s_2$, where $s_2$ = spacing of horizontal reinforcement and $A_v$ = reinforcement area. Also, the ratio $\rho_h$ of horizontal shear reinforcement to the gross concrete area of the vertical section of the wall should be at least 0.0025. Spacing of horizontal shear bars should not exceed $l_w/5$, $3h$, or 18 in. In addition, the ratio of vertical shear reinforcement area to gross concrete area of the horizontal section of wall need not be greater than that required for horizontal reinforcement but should not be less than

$$\rho_h = 0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right) \tag{8.109}$$

$$(\rho_h - 0.0025) \le 0.0025$$

where $h_w$ = total height of wall. Spacing of vertical shear reinforcement should not exceed $l_w/3$, $3h$, or 18 in.

In no case should the shear strength $V_n$ be taken greater than $10\sqrt{f'_c}hd$ at any section.

## 8.35 Composite Columns

A composite column consists of a structural-steel shape, pipe, or tube compression member completely encased in concrete, with or without longitudinal reinforcement.

Composite compression members should be designed in accordance with the provisions applicable to ordinary reinforced concrete columns. Loads assigned to the concrete portion of a member must be transferred by direct bearing on the concrete through brackets, plates, reinforcing bars, or other structural shapes that have been welded to the central structural-steel compression members prior to placement of the perimeter concrete. The balance of the load should be assigned to the structural-steel shape and should be developed by direct connection to the structural shape.

### 8.35.1 Concrete-Filled Steel Columns

When the composite member consists of a steel-encased concrete core, the required thickness of metal face of width $b$ of a rectangular section is not less than

$$t = b\sqrt{\frac{f_y}{3E_s}} \tag{8.110}$$

for each face of width, $b$ for circular sections of diameter $h$,

$$t = h\sqrt{\frac{f_y}{8E_s}} \tag{8.111}$$

where $f_y$ is the yield strength and $E_s$ the modulus of elasticity of the steel.

### 8.35.2 Steel-Core Columns

When the composite member consists of a spiral-bound concrete encasement around a structural-steel core, the concrete should have a minimum strength of 2500 psi, and spiral reinforcement should conform to the requirements of Art. 8.31.

When the composite member consists of a laterally tied concrete encasement around a steel core, the concrete should have a minimum strength of 2500 psi. The lateral ties should completely encase the core. Ties should be No. 3 bars or larger but should have a diameter of at least 1/50 the longest side of the cross section. Ties need not be larger than No. 5. Vertical spacing should not exceed one-half of the least width of the cross section, or 48 tie bar diameters, or 16 longitudinal bar diameters. The area of vertical reinforcing bars within the ties should not be less than 1% or more than 8% of the net concrete section. In rectangular sections, one longitudinal bar should be placed in each corner and other bars, if needed, spaced no farther apart than half the least side dimension of the section.

The design yield strength of the structural core should not be taken greater than 50 ksi, even though a larger yield strength may be specified.

## Prestressed Concrete

Prestressing is the application of permanent forces to a member or structure to counteract the effects of subsequent loading. Applied to concrete, prestressing takes the form of precompression, usually to eliminate disadvantages stemming from the weakness of concrete in tension.

## 8.36 Basic Principles of Prestressed Concrete

The usual prestressing procedure is to stretch high-strength steel (Art. 8.13) and anchor it to the concrete, which resists the tendency of the stretched steel to shorten and thus is compressed. The amount of prestress used generally is sufficient to prevent cracking or sometimes to avoid tension entirely, under service loads. As a result, the whole concrete cross section is available to resist tension and bending, whereas in reinforced-concrete construction, concrete in tension is considered ineffective. Hence, it is particularly advantageous with prestressed concrete to use high-strength concrete. (See also Art. 8.14.)

Prestressed-concrete pipe and tanks are made by wrapping steel wire under high tension around concrete cylinders. Domes are prestressed by wrapping tensioned steel wire around the ring girders. Beams and slabs are prestressed linearly with steel tendons anchored at their ends or bonded to the concrete (Art. 8.14). Piles also are prestressed linearly, usually to counteract handling stresses.

Prestressed concrete may be either pretensioned or posttensioned. For pretensioned concrete, the steel is stretched before the concrete is placed around it and the forces are transferred to the concrete by bond. For posttensioned concrete, bars or tendons are sheathed in ducts within the concrete forms and are tensioned after the concrete attains sufficient strength.

The final precompression of the concrete is not equal to the initial tension applied to the tendons.

There are both immediate and long-time losses, (Art. 8.37), which should be deducted from the initial prestress to determine the effective prestress to be used in design. One reason high-tensioned tendons are used for prestressing is to maintain the sum of these losses at a small percentage of the applied prestress.

In determining stresses in prestressed members, the prestressing forces may be treated the same way as other external loads. If the prestress is large enough to prevent cracking under design loads, elastic theory may be applied to the entire concrete cross section.

For example, consider the simple beam in Fig. 8.28*a*. Prestress $P$ is applied by a straight tendon at a distance $e_1$ below the neutral axis. The resulting prestress in the extreme surfaces throughout equals $P/A \pm Pe_1c/I$, where $P/A$ is average stress on a cross section and $Pe_1c/I$, the bending stress (+ represents compression, − represents tension), as indicated in Fig. 8.28*c*. If, now, stresses $+\, Mc/I$ due to downward-acting loads are superimposed at midspan, the net stresses in the extreme surfaces may become zero at the bottom and compressive at the top (Fig. 8.28*c*). Since the stresses due to loads at the beam ends are zero, however, the prestress is the final stress there. Hence, the top of the beam at the ends will be in tension.

If this is objectionable, the tendons may be draped, or harped, in a vertical curve, as shown in Fig. 8.28*b*. Stresses at midspan will be substantially the same as before (assuming the horizontal component of $P$ approximately equal to $P$), and the stress at the ends will be a compression, $P/A$, since $P$ passes through the centroid of the section there. Between midspan and the ends, the cross sections also are in compression (Fig. 8.28*d*).

## 8.37 Losses in Prestress

As pointed out in Art. 8.36, the prestressing force acting on the concrete differs from the initial tension on the tendons by losses that occur immediately and over a long time.

### 8.37.1 Elastic Shortening of Concrete

In pretensioned members (Art. 8.14), when the tendons are released from fixed abutments and the steel stress is transferred to the concrete by bond, the concrete shortens because of the compressive stress. For axial prestress, the decrease in inches per inch of length may be taken as $P_i/AE_c$, where $P_i$ is the initial prestress, kips; $A$ the concrete area, in$^2$; and $E_c$ the modulus of elasticity of the concrete, ksi. Hence, the decrease in unit stress in the tendons equals $P_iE_s/AE_c = nf_c$, where $E_s$ is the modulus of elasticity of the steel, ksi; $n$ the modular ratio; and $f_c$ the stress in the concrete, ksi.

In posttensioned members, if tendons or cables are stretched individually, the stress loss in each due to compression of the concrete depends on the order of stretching. The loss will be greatest for the first tendon or cable stretched and least for the last one. The total loss may be approximated by assigning half the loss in the first cable to all. As an alternative, the tendons may be brought to the final prestress in steps.

### 8.37.2 Frictional Losses

In posttensioned members, there may be a loss of prestress where curved tendons rub against their enclosure. For harped tendons, the loss may be computed in terms of a curvature-friction coefficient $\mu$. Losses due to unintentional misalignment may be calculated from a wobble-friction coefficient $K$ (per lin ft). Since the coefficients vary considerably with duct material and construction methods, they should, if possible, be determined experimentally or obtained from the tendon manufacturer. Table 8.16 lists values of $K$ and $\mu$ suggested in the Commentary to the 2002 Edition "Building Code Requirements for Structural Concrete" (American Concrete Institute (www.aci-int.org)) for posttensioned tendons.

With $K$ and $\mu$ known or estimated, the friction loss can be computed from

$$P_s = P_x e^{Kl_x + \mu\alpha} \tag{8.112}$$

where $P_s$ = force in tendon at prestressing jack, lb

$P_x$ = force in tendon at any point $x$, lb

$e$ = 2.718

$l_x$ = length of tendon from jacking point to point $x$, ft

$\alpha$ = total angular change of tendon profile from jacking end to point $x$, rad

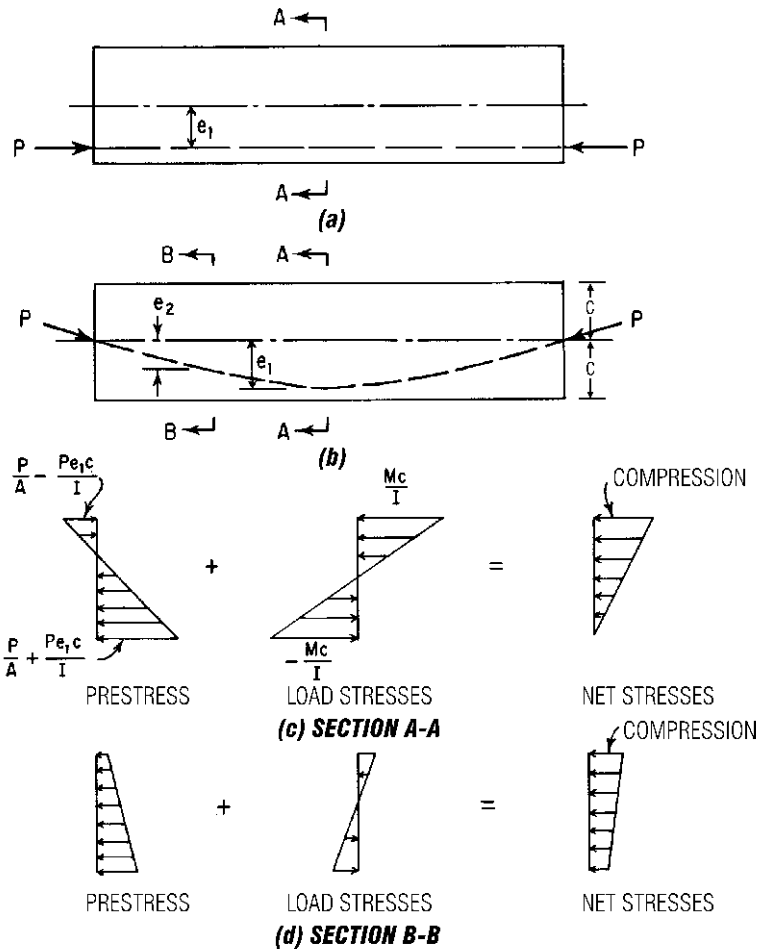



**Fig. 8.28** Concrete beams: (*a*) Prestressed with straight tendons. (*b*) Prestressed with draped tendons. (*c*) Stress distribution at midspan. (*d*) Stress distribution for draped tendons at section between support and midspan. For straight tendons, net stress may be tensile near the supports.

When $Kl_x + \mu\alpha$ does not exceed 0.3, $P_s$ may be obtained from

$$P_s = P_x(1 + Kl_x + \mu\alpha) \qquad (8.113)$$

### 8.37.3 Slip at Anchorages

For posttensioned members, prestress loss may occur at the anchorages during the anchoring. For example, seating of wedges may permit some shortening of the tendons. If tests of a specific anchorage device indicate a shortening $\delta l$, the decrease in unit stress in the steel is $E_s\delta l/l$, where $l$ is the length of the tendon.

### 8.37.4 Shrinkage of Concrete

Change in length of a member due to concrete shrinkage results over time in prestress loss. This should be determined from test or experience. Generally, the loss is greater for pretensioned members than for posttensioned members, which are prestressed after much of the shrinkage has occurred. Assuming a shrinkage of 0.0002 in/in for

**Table 8.16** Friction Coefficients for Post-tensioned Tendons

| | | | Wobble Coefficient, $K$ | Curvature coefficient, $\mu$ |
|---|---|---|---|---|
| Grouted tendons in metal sheathing | | Wire tendons | 0.0010–0.0015 | 0.15–0.25 |
| | | High-strength bars | 0.0001–0.0006 | 0.08–0.30 |
| | | 7-wire strand | 0.0005–0.0020 | 0.15–0.25 |
| Unbonded tendons | Mastic coated | Wire tendons | 0.0010–0.0020 | 0.05–0.15 |
| | | 7-wire strand | 0.0010–0.0020 | 0.05–0.15 |
| | Pre-greased | Wire tendons | 0.0003–0.0020 | 0.05–0.15 |
| | | 7-wire strand | 0.0003–0.0020 | 0.05–0.15 |

a pretensioned member, the loss in tension in the tendons will be

$$0.0002E_s = 0.0002 \times 30{,}000 = 6\,\text{ksi}$$

### 8.37.5 Creep of Concrete

Change in length of concrete under sustained load induces a prestress loss over time. This loss may be several times the elastic shortening. An estimate of the loss may be made with a creep coefficient $C_c$, equal to the ratio of additional long-time deformation to initial elastic deformation, determined by test. Hence, for axial prestress, the loss in tension in the steel is $C_c n f_c$, where $n$ is the modular ratio and $f_c$ is the prestressing force divided by the concrete area. (Values ranging from 1.5 to 2.0 have been recommended for $C_c$.)

### 8.37.6 Relaxation of Steel

Decrease in stress under constant high strain occurs with some steels. For example, for steel tensioned to 60% of ultimate strength, relaxation loss may be 3%. This type of loss may be reduced by temporary overstressing, stabilizing the tendons by artificially accelerating relaxation and thus reducing the loss that will occur later at lower stresses.

Actual losses should be computed based on the actual initial stress level, type of steel (stress-relieved or low relaxation; wire, strand or bar), and prestressing method (pretensioned or posttensioned).

## 8.38 Allowable Stresses in Prestressed Concrete — AASHTO

The "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) require that the design of precast prestressed members ordinarily must be based on $f'_c = 5000$ psi. An increase to 6000 psi is permissible where, in the Engineer's judgment, it is reasonable to expect that this strength will be obtained consistently. Still higher concrete strengths may be considered on an individual area basis. In such cases, the Engineer must satisfy himself completely that the controls over materials and fabrication procedures will provide the required strengths.

In setting allowable stresses for prestressed concrete, design codes recognize two loading stages: application of initial stress and loading under service conditions. The codes permit higher stresses for the temporary loads during the initial stage.

Stresses due to the jacking force and those produced in the concrete and steel immediately after prestress transfer or tendon anchorage, before losses due to creep and shrinkage, are considered temporary. Permissible temporary stresses in the concrete are specified as a percentage of $f'_{ci}$, the compressive strength of the concrete psi, at time of initial prestress, instead of the usual $f'_c$, 28-day strength of the concrete. This is done because prestress usually is applied only a few days after casting the concrete.

**Table 8.17** Allowable Stresses in Prestressed-Concrete Flexural Members—AASHTO Standard Specifications

| | |
|---|---|
| **Prestressing Steel** | |
| Pretensioned members: | |
| Stress immediately prior to transfer– | |
| Low relaxation strands | $0.75f'_s$ |
| Stress-relieved strands | $0.70f'_s$ |
| Post-tensioned members: | |
| Stress immediately after seating– | |
| At anchorage | $0.70f'_s$ |
| At the end of the seating loss zone | $0.83f_y$* |
| Tensioning to $0.90f_y$* for short periods of time prior to seating may be permitted to offset seating and friction losses provided the stress at the anchorage does not exceed the above value. | |
| Stress at service load after losses | $0.80f_y$* |
| **Concrete** | |
| **Temporary Stresses Before Losses Due to Creep and Shrinkage** | |
| Compresion: | |
| Pretensioned members | $0.60f'_{ci}$ |
| Post-tensioned members | $0.55f'_{ci}$ |
| Tension: | |
| Precompressed tensile zone | No temporary allowable stresses are specified |
| Other Areas | |
| In tension areas with no bonded reinforcement | 200 psi or $3\sqrt{f'_{ci}}$ |
| Where the calculated tensile stress exceeds this value, bonded reinforcement shall be provided to resist the total tension force in the concrete computed on the assumption of an uncracked section. The maximum tensile stress shall not exceed | $7.5\sqrt{f'_{ci}}$ |
| **Stress at Service Load After Losses Have Occurred** | |
| Compression: | |
| (a) The compression stresses under all load combinations, except as stated in (b) and (c), shall not exceed $0.60f'_c$. | |
| (b) The compressive stresses due to effective prestress plus permanent (dead) loads shall not exceed $0.40f'_c$ | |
| (c) The compressive stress due to live loads plus one-half of the sum of the compressive stresses due to prestress and permanent (dead) loads shall not exceed $0.40f'_c$. | |
| Tension in the precompressed tensile zone: | |
| (a) For members with bonded reinforcement (including bonded prestressed strands) | $6\sqrt{f'_{ci}}$ |
| (b) For severe corrosive exposure conditions, such as coastal areas | $3\sqrt{f'_{ci}}$ |
| (c) For members without bonded reinforcement | 0 |
| Tension in other areas is limited by allowable temporary stresses specified above. | |
| **Anchorage Bearing Stress** | |
| Post-tensioned anchorage at service load | 3,000 psi (but not to exceed $0.9f'_{ci}$) |

The allowable stresses for prestressed concrete, in accordance with the ASSHTO Standard Specifications, are given in Table 8.17. In the table, $f'_s$ is the ultimate stress (tensile strength) of prestressing steel, and $f^*_y$ is the yield stress (strength) of prestressing steel.

$$
\begin{aligned}
f^*_y &= 0.90\, f'_s \text{ for low-relaxation wire or strand} \\
&= 0.85\, f'_s \text{ for stress-relieved wire or strand} \\
&= 0.85\, f'_s \text{ for Type I (smooth) high-strength bar} \\
&= 0.80\, f'_s \text{ for Type II (deformed) high-strength bar}
\end{aligned}
$$

## 8.39 Allowable Stresses in Prestressed Concrete—ACI 318

The 2002 edition of ACI 318 "Building Code Requirements for Structural Concrete" (American Concrete Institute) requires prestressed flexural members to be classified as Class U, Class T, or Class C based on the computed extreme fiber stress $f_t$ at service loads in the precompressed tensile zone, as follows:

$$\text{Class U:}\quad f_t \le 7.5\sqrt{f'_c}$$

$$\text{Class T:}\quad 7.5\sqrt{f'_c} \le f_t \le 12\sqrt{f'_c}$$

$$\text{Class C:}\quad f_t > 12\sqrt{f'_c}$$

Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in Table R18.3.3, in the Commentary to the 2002 edition of ACI 318. For comparison, the table also shows the corresponding requirements for nonprestressed members.

The classes apply to both bonded and unbonded prestressed flexural members, but prestressed two-way slab systems are required to be designed as Class U.

For Class U and Class T flexural members, stresses at service loads are permited to be calculated using the uncracked section. For Class C flexural members, stresses at service loads must be calculated using the cracked transformed section.

**Table 8.18** Allowable Stresses in Prestressed-Concrete Flexural Members—ACI 318

| | |
|---|---|
| Stresses at transfer or anchoring: | |
| Compression in concrete | $0.60 f'_{ci}$ |
| Tension in concrete without auxiliary reinforcement in the tension zone* | $3\sqrt{f'_{ci}}$ |
| Prestress in tendons due to jacking force† | $0.94 f_{py} \le 0.80 f_{pu}$ |
| Prestress in tendons immediately after transfer or anchoring | $0.82 f_{py} \le 0.74 f_{pu}$ |
| Stresses under service loads: | |
| Compression in concrete‡ | |
| • due to prestress plus sustained load | $0.45 f'_c$ |
| • due to prestress plus total load | $0.60 f'_c$ |

* Where the calculated tension stress exceeds this value, bonded additional reinforcement (prestressed or nonprestressed) should be provided to resist the total tension force on the concrete computed on the assumption of an uncracked section. At ends of simply supported beams, the allowable stress may be taken as $6\sqrt{f'_{ci}}$

† But not greater than the maximum value recommended by the manufacturer of the steel or anchorages. $f_{py}$ = yield strength of tendons.

‡ For Class U and class T prestressed flexural members only.

The allowable stresses for prestressed concrete, in accordance with ACI 318, are listed in Table 8.18. In the table, $f_{pu}$ is the specified tensile strength of prestressing steel, and $f_{py}$ is the specified yield strength of prestressing steel.

Class C prestressed members are, for all practical purposes, treated like nonstressed members. As such, there is no upper limit on the tensile stress that may develop at a section under service loads. There is no explicit limit on the compressive stresses that can develop under service-level loads either. However, crack control and deflection control requirements become applicable for Class C prestressed flexural members. The introduction of the Class C flexural member enables a user of ACI 318 to design a partially prestressed concrete flexural member (combining mild reinforcement with prestressing steel) for the first time under the 2002 edition of the code.

For prestressed concrete members exposed to corrosive environments or other severe exposure conditions, and which are classified as Class T or C, minimum cover to the prestressed reinforcement (see Sections 7.7.2 and 7.7.3 of ACI 318-02) is required to be increased 50 percent. This requirement is permitted to be waived if the precompressed tensile zone is not in tension under sustained loads. The engineer may also consider reducing tensile stresses in the concrete to eliminate possible cracking at service loads.

Comprehensive requirements for post-tensioned anchorage zone design have been included in ACI 318 since its 1999 edition (see Section 18.13).

## 8.40 Design of Prestressed-Concrete Beams

This involves selection of shape and dimensions of the concrete portion, type and positioning of tendons, and amount of prestress. After a concrete shape and dimensions have been assumed, determine geometric properties: cross-sectional area, center of gravity, distances of extreme surfaces from the centroid, section moduli, and dead load of member per unit of length. Treat prestressing forces as a system of external forces acting on the concrete (see Art. 8.37).

Compute bending stresses due to dead and live loads. From these, determine the magnitude and location of the prestressing force at points of maximum moment. This force must provide sufficient compression to offset the tensile stresses caused by the bending moments due to loads (Fig. 8.21). But at the same time, it must not create any stresses exceeding the allowable values listed in Art. 8.38 or Art. 8.39. Investigation of other sections will guide selection of tendons to be used and determine their position in the beam.

After establishing the tendon profile, prestressing forces, and tendon areas, check critical points along the beam under initial and final conditions, on removal from the forms, and during erection. Check ultimate strength in flexure and shear and the percentage of prestressing steel. Design anchorages, if required, and diagonal-tension steel. Finally, check camber.

The design may be based on the following assumptions. Strains vary linearly with depth. At cracked sections, the concrete cannot resist tension. Before cracking, stress is proportional to strain. The transformed area of bonded tendons may be included in pretensioned members and in posttensioned members after the tendons have been grouted. Areas of open ducts should be deducted in calculations of section properties before bonding of tendons. The modulus of rupture should be determined from tests, or the cracking stress may be assumed as $7.5\sqrt{f_c'}$, where $f_c'$ is the 28-day strength of the concrete, psi.

Prestressed beams should be checked by the strength theory (Art. 8.17). Beams for building should be capable of supporting the factored loads given in Chap. 9 of ACI 318-02. For bridge beams, the nominal strength should not be less than

$$\frac{U}{\phi} = \frac{1.30}{\phi}\left[D + \frac{5}{3}(L + I)\right] \tag{8.114}$$

where $D$ = effect of dead load

$L$ = effect of design live load

$I$ = effect of impact

$\phi$ = 1.0 for factory-produced precast, prestressed members

= 0.95 for posttensioned, cast-in-place members

= 0.90 for shear

The "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) recommend that prestressed-concrete flexural members be assumed to act as uncracked members subjected to combined axial and bending stresses under specified service loads. In pretensioned members and in posttensioned members after tendons have been grouted, the transformed area of bonded reinforcement may be taken into account in computations of section properties. For calculations of section properties before bonding of tendons, areas of open ducts should be deducted.

### 8.40.1 Steel Stress—AASHTO

The following definitions will be used for the formulas that follow:

$A_s$ = area of non-prestressed tension reinforcement, in$^2$

$A_s'$ = area of compression reinforcement, in$^2$

$A_s^*$ = area of prestressing steel, in$^2$

$A_{sf}$ = steel area required to develop the compressive strength of the overhanging portions of the flange

$A_{sr}$ = steel area required to develop the compressive strength of the web of a flanged section, in$^2$

$b$ = width of a rectangular member or flange of a flanged member, in

$b'$ = width of a web of a flanged member, in

$d$ = distance, in, from extreme compression fiber to the centroid of the prestressing force

$d_t$ = distance, in, from extreme compression fiber to the centroid of the non-prestressed tension reinforcement

$f'_c$ = 28-day compressive strength of the concrete, psi

$f'_s$ = ultimate strength of prestressing steel, psi

$f^*_{su}$ = average stress in prestressing steel at ultimate load, psi

$f_{sy}$ = yield strength of non-prestressed tension reinforcement, psi

$f'_y$ = yield strength of non-prestressed compression reinforcement, psi

$p = A_s/bd_t$

$p^* = A_s{}^*/bd$

The value of the average stress in prestressing steel at ultimate load $f^*_{su}$ may be determined by analysis. It may be estimated, however, from the following if the effective prestress after losses is at least half the ultimate strength of the prestressing steel:

For prestressed beams with unbonded tendons:

$$f^*_{su} = f_{se} + 900\{(d - y_u)/l_e\} \leq f^*_y \tag{8.115}$$

where $f_{se}$ = effective prestress after losses

$y_u$ = distance from extreme compression fiber to the neutral axis assuming the prestressing steel has yielded

$l_e = l_i/(1 + 0.5N_s)$, effective tendon length

$l_i$ = tendon length between anchorages (in)

$N_s$ = number of support hinges crossed by the tendon between anchorages or discretely bonded points.

$f^*_y$ = average yield stress as defined in Art 8.37.

For prestressed beams with bonded prestressing steel and no non-prestressed tension reinforcement:

$$f^*_{su} = f'_s\left(1 - \frac{\gamma^*}{\beta_1}\frac{p^* f'_s}{f'_c}\right) \tag{8.116}$$

and with non-prestressed tension reinforcement:

$$f^*_{su} = f'_s\left[1 - \frac{\gamma^*}{\beta_1}\left(\frac{p^* f'_s}{f'_c} + \frac{d_t}{d}\frac{p f_{sy}}{f'_c}\right)\right] \tag{8.117}$$

where $f'_s$ = ultimate strength of prestressing steel, psi

$\gamma$ = factor for type of tendon used

= 0.28 for low-relaxation steel

= 0.40 for stress-relieved steel

= 0.55 for high-strength steel bars

$\beta_1$ = factor $a/c$ defined in Art. 8.17.2

The design strength of prestressed beams depends on whether the reinforcement indexes $(p^* f^*_{su})/f'_c$ for rectangular sections and $A_{sr} f^*_{su}/(b' d f'_c)$ for flanged sections are less than $36\beta_1$.

## 8.40.2 Steel Stress—ACI

As an alternative to a more accurate determination of $f_{ps}$ (stress in prestressed reinforcement at nominal strength) based on strain compatibility, the following approximate values of $f_{ps}$ may be used if $f_{se}$ is not less than $0.5 f_{pu}$.

For members with bonded tendons:

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \tag{8.118}$$

If any compression reinforcement is taken into account when calculating $f_{ps}$ by the above equation, the term

$$\left[\rho_p\frac{f_{pu}}{f'_c} + \frac{d}{d_p}(\omega - \omega')\right]$$

is to be taken not less than 0.17 and $d'$ is to be taken no greater than $0.15 d_p$.

For members with unbonded tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p} \tag{8.119}$$

but no greater than $f_{py}$, nor greater than $(f_{se} + 60{,}000)$.

For members with unbonded tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{300\rho_p} \tag{8.120}$$

but no greater than $f_{py}$, nor greater than $(f_{se} + 30{,}000)$

The notation used above is explained below:

$A_{ps}$ = area of prestressed reinforcement in tension zone, in$^2$

$A_s$ = area of nonstressed tension reinforcement, in$^2$

$A'_s$ = area of compression reinforcement, in$^2$

$b$ = width of compression face of member, in

$d$ = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in

$d'$ = distance from extreme compression fiber to centroid of compression reinforcement, in

$d_p$ = distance from extreme compression fiber to centroid of prestressed reinforcement, in

$f'_c$ = specified compressive strength of concrete, psi

$f_{ps}$ = stress in prestressed reinforcement at nominal strength, psi

$f_{pu}$ = specified tensile strength of prestressing steel, psi

$f_{py}$ = specified yield strength of prestressing steel, psi

$f_{se}$ = effective stress in prestressed reinforcement (after allowance for all prestress losses), psi

$f_y$ = specified yield strength of nonstressed reinforcement, psi

$\beta_1$ = stress block depth parameter

= 0.85 for $f'_c \le 4000$ psi

= $0.85 - 0.05(f'_c - 4000/1000) \ge 0.65$ for $f'_c >$ 4000 psi

$\gamma_p$ = factor for type of prestressing steel

= 0.55 for $f_{py}/f_{pu}$ not less than 0.80

= 0.40 for $f_{py}/f_{pu}$ not less than 0.85

= 0.28 for $f_{py}/f_{pu}$ not less than 0.90

$\rho$ = ratio of nonprestressed tension reinforcement = $A_s/bd$

$\rho'$ = ratio of compression reinforcement = $A'_s/bd$

$\rho_p$ = ratio of prestressed reinforcement = $A_{ps}/bd_p$

$\omega = \rho f_y/f'_c$

$\omega' = \rho' f_y/f'_c$

### 8.40.3 Design Strength When Indexes Are 36$\beta_1$ or Less—AASHTO

AASHTO Specifications state that prestressed concrete members should be designed so that the steel is yielding as ultimate strength is approached. This requires, in general, that reinforcement indexes not exceed $36\beta_1$. When this requirement is met, design flexural strength $\phi M_n$, in-kips, with $\phi$ as given for Eq. (8.114), is determined as follows:

For rectangular sections with prestressing steel only and for flanged sections with prestressing steel only, when the depth of the equivalent rectangular stress block, $(A^*_s f^*_{su}/0.85 f'_c b)$, does not exceed the compression flange thickness $t$,

$$\phi M_n = \phi\left[A^*_s f^*_{su} d\left(1 - 0.6\frac{p^* f^*_{su}}{f'_c}\right)\right] \tag{8.121}$$

When non-prestressed tension reinforcement with a yield strength $f_{sy}$ is used and the depth of the equivalent rectangular stress block, $[(A^*_s f^*_{su} + A_s f_y)/0.85 f'_c b]$ does not exceed the compression flange thickness $t$,

$$\phi M_n = \phi\left[A^*_s f^*_{su} d\left(1 - 0.6\frac{p^* f^*_{su}}{f'_c} + \frac{d_t}{d}\frac{p f_{sy}}{f'_c}\right)\right] + A_s f_{sy} d_t\left[1 - 0.6\left(\frac{d}{d_t}\frac{p^* f^*_{su}}{f'_c} + \frac{p f_{sy}}{f'_c}\right)\right] \tag{8.122}$$

For flanged sections with prestressing steel only but with a deeper stress block than that specified for Eq. (8.121),

$$\phi M_n = \phi\left[A_{sr} f_{su}^* d\left(1 - 0.6\frac{A_{sr} f_{su}^*}{b' d f_c'}\right) + 0.85 f_c' t(b - b')(d - 0.5t)\right] \tag{8.123}$$

where $A_{sr} = A_s^* - A_{sf}$

$A_{sf}$ = steel area required to develop the ultimate compressive strength of the overhanging portion of the flange

$= 0.85 f_c'(b - b')t/f_{su}^*$

For flanged sections with non-prestressed tension reinforcement but with a deeper stress block than that specified for Eq. (8.122),

$$\phi M_n = \phi\left[A_{sr} f_{su}^* d\left(1 - 0.6\frac{A_{sr} f_{su}^*}{b' d f_c'}\right) + A_s f_{sy}(d_t - d) + 0.85 f_c' t(b - b')(d - 0.5t)\right] \tag{8.124}$$

where $A_{sr} = A_s^* + A_s f_{sy}/f_{su}^* - A_{sf}$.

### 8.40.4 Design Strength When Indexes Are 36$\beta_1$ or More—AASHTO

The design flexural strength $\phi M_n$, in-kips, for prestressed beams with reinforcement indexes larger than $36\beta_1$ may be determined as follows: For rectangular sections, with $\phi$ given by Eq. (8.114),

$$\phi M_n = \phi[(0.36\beta_1 - 0.08\beta_1^2) f_c' b d^2] \tag{8.125}$$

For flanged sections,

$$\phi M_n = \phi[(0.36\beta_1 - 0.08\beta_1^2) f_c' b' d^2 + 0.85 f_c' t(b - b')(d - 0.5t)] \tag{8.126}$$

### 8.40.5 Design Strength—ACI

Design moment strength of flexural members is to be computed by the strength design method of ACI 318. For prestressing steel, $f_{ps}$ is to be substituted for $f_y$ is strength computations. Irrespective of whether the traditional ACI load combination of Appendix C or the ASCE 7-98 load combinations from Chapter 9 of ACI 318-02 are used, the strength reduction factor, $\phi$ is as follows (see Article 8.33 on Unified Design Procedure)

| | |
|---|---|
| Tension-controlled sections | 0.90 |
| Compression-controlled sections, spirally reinforced | 0.75 |
| Compression-controlled sections, other | 0.70 |

For sections in which the net tensile strain is between the limits for compression-controlled and tension-controlled sections, $\phi$ may be increased linearly from that for compression-controlled sections to 0.90 as the net tensile strain increases from the compression-controlled strain limit (which may be taken as 0.002 for Grade 60 reinforcement and for all prestressed reinforcement) to the tension-controlled strain limit (of 0.005).

If the traditional design approach in Appendix B of ACI 318-02 is used, the indexes $\omega_p$, $[\omega_p + d/d_p(\omega - \omega')]$, or $[\omega_{pw} + d/d_p(\omega_w - \omega_w')]$ are restricted to $0.36\beta_1$,

where $\omega_p = \rho_p f_{ps}/f_c' = A_{ps} f_{ps}/b d_p f_c'$

$\omega_w$, $\omega_{pw}$, $\omega_w'$ = reinforcement indices for flanged sections computed as for $\omega$, $\omega_p$, and $\omega'$ except that $b$ is to be the web width, and reinforcement area must be that required to develop compressive strength of web only.

Design moment strength of over-reinforced sections may be computed using strength equations similar to those for nonprestressed concrete members. The 1983 edition of ACI 318 provided strength equations for over-reinforced rectangular and flanged sections.

If the Unified Design Procedure in Chapter 18 of ACI 318-02 (see Article 8.33) is used, the net tensile strain, $\epsilon_t$ is restricted to an upper limit of 0.004.

### 8.40.6 Minimum Steel Required—AASHTO

The AASHTO Specifications require that the total amount of tendons and non-prestressed reinforcement be adequate to develop an ultimate strength $\phi M_n$ that is at least 20% larger than the cracking moment $M_{cr}^*$. For a composite section,

$$M_{cr}^* = (f_r + f_{pe})S_c - M_{d/nc}\left(\frac{S_c}{S_b} - 1\right) \tag{8.127}$$

where $f_r$ = modulus of rupture of the concrete

$= 7.5\sqrt{f_c'}$ for normal-weight concrete

$f_{pe}$ = compressive stress in the concrete due to effective prestress forces only, after allowing for prestress losses, at the extreme surface of the section where tensile stress is caused by externally applied loads

$S_b$, $S_c$ = noncomposite and composite section modulus, respectively, for the extreme surface of the section where tensile stress is caused by externally applied loads

$M_{d/nc}$ = noncomposite dead-load moment at the section

For a noncomposite section,

$$M^*_{cr} = (f_r + f_{pe})S_b \tag{8.128}$$

The above requirement may be varied if the area of prestressed and non-prestressed reinforcement provided at a section is at least one-third greater than that required by analysis based on the factored load combinations of AASHTO. The minimum amount of non-prestressed longitudinal reinforcement provided in the cast-in-place position of slabs utilizing precast prestressed deck panels must be 0.25 in$^2$ per foot of slab width.

### 8.40.7 Minimum Steel Required—ACI

The total amount of prestressed and nonprestressed reinforcement must be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulars of rupture $f_r$ specified in ACI 318 ($7.5\sqrt{f'_c}$ for normal-weight concrete). This provision may be waived for flexural members with shear and flexural strength at least twice those required by the factored loads.

Part or all of the bonded reinforcement consisting of bars or tendons are to be provided as close as practicable to the extreme tension fiber in all prestressed flexural members. In members prestressed with unbonded tendons, the minimum bonded reinforcement consisting of bars or wires must satisfy specific code requirements (See Article 8.40.8).

### 8.40.8 Shear in Prestressed Beams—AASHTO

"Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) require that prestressed members be designed to resist diagonal tension by the strength theory.

Shear reinforcement should consist of stirrups or welded-wire fabric. The area of shear reinforcement, in$^2$, set perpendicular to the beam axis, should not be less than

$$A_v = \frac{50b's}{f_{sy}} \tag{8.129}$$

where $s$ is the reinforcement spacing, except when the factored shear force $V_u$ is less than one-half $\phi V_c$. The capacity reduction factor $\phi$ should be taken as 0.85.

The yield strength of shear reinforcement, $f_{sy}$, used in design calculations should not exceed 60,000 psi.

Where shear reinforcement is required, it should be placed perpendicular to the axis of the member and should not be spaced farther apart than $0.75h$, where $h$ is the overall depth of the member, or 24 in. Web reinforcement between the face of support and the section at a distance $h/2$ from it should be the same as the reinforcement required at that section.

When $V_u$ exceeds the design shear strength $\phi V_c$ of the concrete, shear reinforcement must be provided. The shear strength provided by concrete, $V_c$, must be taken as the lesser of the value $V_{ci}$ or $V_{cw}$.

The shear strength, $V_{ci}$, is to be computed by

$$V_{ci} = 0.6\sqrt{f'_c}b'd + V_d + \frac{V_i M_{cr}}{M_{\max}} \tag{8.130}$$

$$\geq 1.7\sqrt{f'_c}b'd$$

and $d$ need not be taken less than $0.8h$. $V_d$ = shear force at section due to unfactored dead load.

The moment causing flexural cracking at the section due to externally applied loads, $M_{cr}$, is to be computed by

$$M_{cr} = \frac{1}{Y_t}(6\sqrt{f'_c} + f_{pe} - f_d) \tag{8.131}$$

where $Y_t$ = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension

$f_d$ = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads

$f_{pe}$ = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads.

The shear strength, $V_{cw}$, is to be computed by

$$V_{cw} = (3.5\sqrt{f'_c} + 0.3f_{pc})b'd + V_p \tag{8.132}$$

but $d$ need to be taken less than $0.8h$. $f_{pc}$ = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange. $V_p$ = vertical component of effective prestress force at section.

For a pretensioned member in which the section at a distance $h/2$ from the face of support is closer to the end of the member than the transfer length of the prestressing steel, the reduced prestress should be considered when computing $V_{cw}$. The prestress force may be assumed to vary linearly from zero at the end of the prestressing steel to a maximum at a distance from the end of the prestressing steel equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

When $V_u - \phi V_c$ exceeds $4\sqrt{f'_c}b_w d$, the maximum spacing of stirrups must be reduced to $0.375h$, but not more than 12 in. But $V_u - \phi V_c$ must not exceed $8\sqrt{f'_c}b_w d$.

The shear strength provided by web reinforcement is to be taken as

$$V_s = \frac{A_v f_{sy} d}{s} \tag{8.133}$$

where $A_v$ is the area of web reinforcement within a distance $s$. $V_s$ must not be taken greater than $8\sqrt{f'_c}b'd$ and $d$ need not be taken less than $0.8h$.

## 8.40.9 Shear in Prestressed Beams—ACI

ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute) also requires that prestressed members be designed to resist diagonal tension by the strength theory.

Shear reinforcement should consist of stirrups or welded-wire fabric. The area of shear reinforcement, in$^2$, set perpendicular to the beam axis, should not be less than

$$A_v = 0.75\sqrt{f'_c}\frac{b_w s}{f_y} \geq 50\frac{b_w s}{f_y} \tag{8.134}$$

where $s$ is the reinforcement spacing, in, except when the factored shear force $V_u$ is less than one-half $\phi V_c$; or when the depth of the member $h$ is less than 10 in or 2.5 times the thickness of the compression flange, or one-half the width of the web, whichever is largest. The capacity reduction factor $\phi$ should be taken as 0.85, if the traditional ACI load combinations from Appendix C of ACI 318-02 are used, or 0.75 if the ASCE 7-98 (American Society of Civil Engineers) load combinations adopted in Chapter 9 of ACI 318-02 are used.

Alternatively, a minimum area

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y d}\sqrt{\frac{d}{b_w}} \tag{8.135}$$

may be used if the effective prestress force is at least equal to 40% of the tensile strength of the flexural reinforcement.

The yield strength of shear reinforcement, $f_y$ used in design calculations should not exceed 60,000 psi.

Where shear reinforcement is required, it should be placed perpendicular to the axis of the member and should not be spaced farther apart than $0.75h$, where $h$ is the overall depth of the member, or 24 in. Web reinforcement between the face of support and the section at a distance $h/2$ from it should be the same as the reinforcement required at that section.

When $V_u$ exceeds the nominal shear strength $\phi V_c$ of the concrete, shear reinforcement must be provided. $V_c$ may be computed from Eq. (8.136) when the effective prestress force is 40% or more of the tensile strength of the flexural reinforcement, but this shear stress must not exceed $5\sqrt{f'_c}b_w d$.

$$V_c = \left(0.6\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right)b_w d \geq 2\sqrt{f'_c}b_w d \tag{8.136}$$

where $M_u$ = factored moment at section occurring simultaneously with shear $V_u$ at section

$b_w$ = web width

$d$ = distance from extreme compression surface to centroid of prestressing steel or $0.80h$, whichever is larger

$V_u d/M_u$ should not be taken greater than 1. For some sections, such as medium- and long-span I-shaped members, Eq. (8.136) may be overconservative, and the following more detailed analysis would be preferable.

The ACI Code requires a more detailed analysis when the effective prestress force is less than 40% of the tensile strength of the flexural reinforcement. The governing shear stress is the smaller of the values computed for inclined flexure-shear cracking $V_{ci}$ from Eq. (8.137) and web-shear cracking $V_{cw}$ from Eq. (8.138).

$$V_{ci} = 0.6\sqrt{f_c'}b_w d + V_d + \frac{V_i M_{cr}}{M_{\max}} \tag{8.137}$$

$$\geq 1.7\sqrt{f_c'}b_w d$$

$$V_{cw} = (3.5\sqrt{f_c'}b_w d + 0.3 f_{pc})b_w d + V_p \tag{8.138}$$

where $V_d$ = shear force at section due to unfactored dead load

$V_i$ = factored shear force at section due to externally applied loads occurring simultaneously with $M_{\max}$ and produced by external loads

$M_{cr}$ = moment causing flexural cracking at section due to externally applied loads [see Eq. (8.139)]

$M_{\max}$ = maximum factored moment at section due to externally applied loads

$b_w$ = web width or diameter of circular section

$d$ = distance from extreme compression fiber to centroid of longitudinal tension reinforcement or 80% of overall depth of beam, whichever is larger

$f_{pc}$ = compressive stress in concrete occurring, after all prestress losses have taken place, at centroid of cross section resisting applied loads or at junction of web and flange when centroid lies in flange

$V_p$ = vertical component of effective prestress force at section considered

The cracking moment is given by

$$M_{cr} = \frac{I}{y_t}(6\sqrt{f_c'} + f_{pe} - f_d) \tag{8.139}$$

where $I$ = moment of inertia of section resisting externally applied factored loads, $m^4$

$y_t$ = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in

$f_{pe}$ = compressive stress in concrete due to effective prestress forces only, after all losses, occurring at extreme fiber of section at which tension is produced by externally applied loads, psi

$f_d$ = stress due to unfactored dead load at extreme fiber of section at which tension is produced by externally applied loads, psi

Alternatively, $V_{cw}$ may be taken as the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f_c'}$ at the centroidal axis of the member or, when the centroidal axis is in the flange, induces this tensile stress at the intersection of flange and web.

The values of $M_{\max}$ and $V_i$ used in Eq. (8.137) should be those resulting from the load combination causing maximum moment to occur at the section.

In a pretensioned beam in which the section at a distance of half the overall beam depth $h/2$ from face of support is closer to the end of the beam than the transfer length of the tendon, the reduced prestress in the concrete at sections falling within the transfer length should be considered when calculating $V_{cw}$. The prestress may be assumed to vary linearly along the centroidal axis from zero at the beam end to a maximum at a distance from the beam end equal to the transfer length. This distance may be assumed to be 50 diameters for strand and 100 diameters for single wire.

When $V_u - \phi V_c$ exceeds $4\sqrt{f_c'}b_w d$, the maximum spacing of stirrups must be reduced to $0.375h$ but not to more than 12 in. But $V_u - \phi V_c$ must not exceed $8\sqrt{f_c'}b_w d$.

### 8.40.10 Bonded Reinforcement in Prestressed Beams—ACI

When prestressing steel is not bonded to the concrete, some bonded reinforcement should be provided in the precompressed tension zone of flexural members. The bonded reinforcement should be distributed uniformly over the tension zone near the extreme tension surface in beams and one-way slabs and should have an area of at least

$$A_s = 0.004A \tag{8.140}$$

where $A$ = area, in$^2$, of that part of cross section between flexural tension face and center of gravity of gross section.

In positive-moment regions of two-way slabs where the tensile stress under service loads exceeds $2\sqrt{f'_c}$, the area of bonded reinforcement should be at least

$$A_s = \frac{N_c}{0.5f_y} \tag{8.141}$$

where $N_c$ = tensile force in concrete due to unfactored dead plus live loads, lb

$f_y$ = yield strength, psi, of bonded reinforcement $\leq 60$ ksi

At column supports in negative-moment regions of two-way slabs, at least four bonded reinforcing bars should be placed in each direction and provide a minimum steel area

$$A_s = 0.00075hl \tag{8.142}$$

where $l$ = span of slab in direction parallel to that of reinforcement being determined, in

$h$ = overall thickness of slab, in

The bonded reinforcement should be distributed, with a spacing not exceeding 12 in, over the slab width between lines that are $1.5h$ outside opposite faces of the columns.

## 8.40.11 Prestressed Compression Members

Prestressed concrete members subject to combined flexure and axial load, with or without nonprestressed reinforcement, must be proportioned by the strength design method, including effects of prestress, shrinkage, and creep. Reinforcement in columns with an average prestress less than 225 psi should have an area equal to at least 1% of the gross concrete area $A_c$. For walls subject to an average prestress greater than 225 psi and for which structural analysis shows adequate strength, the minimum reinforcement requirements given in Art. 8.35 may be waived.

Tendons in columns with average prestress $f_{pc}$ equal to or greater than 225 psi should be enclosed in spirals or closed lateral ties. The spiral should comply with the requirements given in Art. 8.31.1. Ties should be at least No. 3 bar size and spacing should not exceed 48 tie diameters or the least dimension of the column.

## 8.40.12 Ducts for Posttensioning

Tendons for posttensioned members generally are sheathed in ducts before prestress is applied so that the tendons are free to move when tensioned. The tendons may be grouted in the ducts after transfer of prestress to the concrete and thus bonded to the concrete.

Ducts for grouting bonded bars or strand should be at least ¼ in larger than the diameter of the posttensioning bars or strand or large enough to produce an internal area at least twice the gross area of the prestressing steel. The temperature of members at time of grouting should be above 50 °F, and members must be maintained at this temperature for at least 48 h.

Unbonded prestressing steel should be completely coated with suitable material to ensure corrosion protection and protect the tendons against infiltration of cement during casting operations.

## 8.40.13 Deflections of Prestressed Beams

The immediate deflection of prestressed members may be computed by the usual formulas for elastic deflections. If cracking may occur, however, the effective moment of inertia (Art. 8.19) should be used. The PCI Design Handbook (Precast/Prestressed Concrete Institute) contains a deflection calculation method using bilinear moment-deflection relationships, which has been widely used in practice. Long-time deflection computations should include effects of the sustained load and effects of creep and shrinkage and relaxation of the steel (Art. 8.19).

("PCI Design Handbook," Precast/Prestressed Concrete Institute, 209 West Jackson Boulevard, Chicago, IL 60606)

# Retaining Walls

## 8.41 Concrete Gravity Walls

Generally economical for walls up to about 15 ft high, gravity walls use their own weight to resist lateral forces from earth or other materials (Fig. 8.29*a*). Such walls usually are sufficiently massive to be unreinforced.

Forces acting on gravity walls include the walls' own weight, the weight of the earth on the sloping back and heel, lateral earth pressure, and resultant soil pressure on the base. It is advisable to include a force at the top of the wall to account for frost action, perhaps 700 lb/lin ft. A wall, consequently, may fail by overturning or sliding, overstressing of the concrete, or settlement due to crushing of the soil.

Design usually starts with selection of a trial shape and dimensions, and this configuration is checked for stability. For convenience, when the wall is of constant height, a 1-ft-long section may be analyzed. Moments are taken about the toe. The

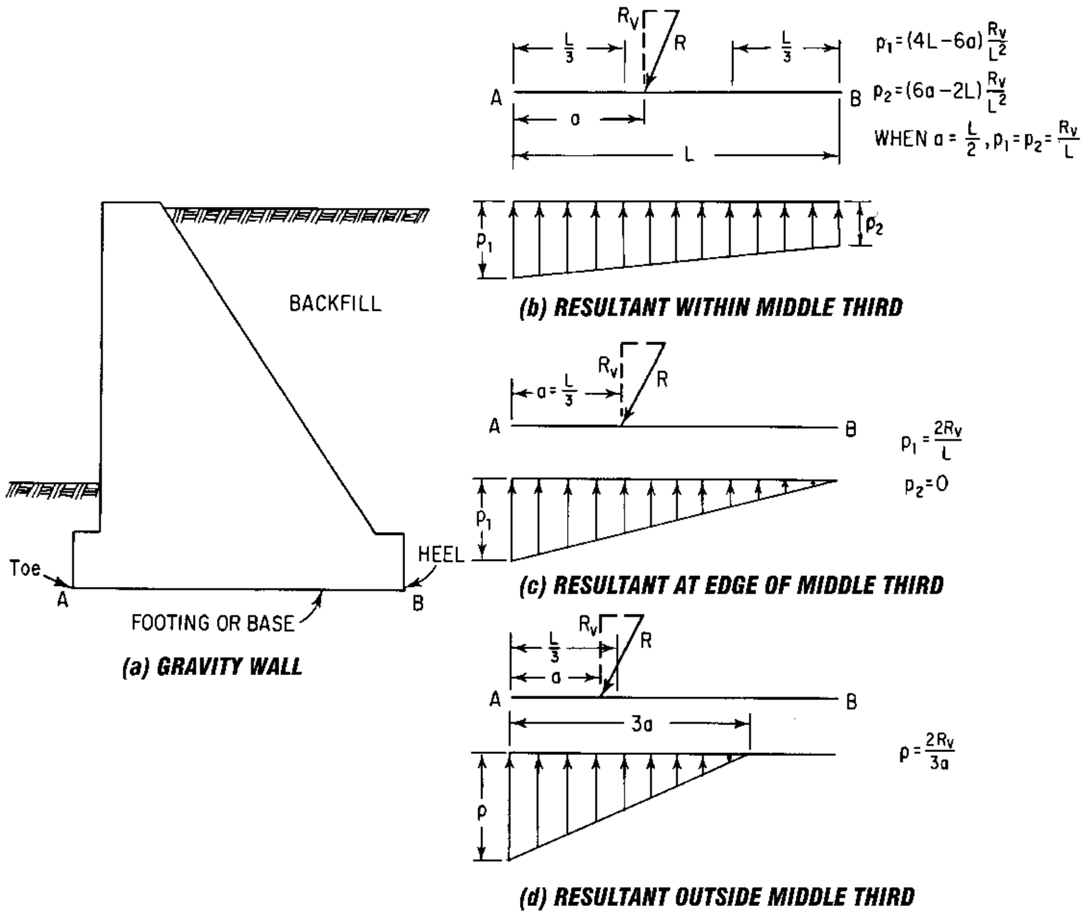



**Fig. 8.29** Diagrams for pressure of the base of a concrete gravity wall on the soil below. (*a*) Vertical section through the wall. (*b*) Significant compression under the entire base. (*c*) No compression along one edge of the base. (*d*) Compression only under part of the base. No support from the soil under the rest of the beam.

sum of the righting moments should be at least 1.5 times the sum of the overturning moments. To prevent sliding

$$\mu R_v \geq 1.5 P_h \tag{8.143}$$

where $\mu$ = coefficient of sliding friction

$R_v$ = total downward force on soil, lb

$P_h$ = horizontal component of earth thrust, lb

Next, the location of the vertical resultant $R_v$ should be found at various sections of the wall by taking moments about the toe and dividing the sum by $R_v$. The resultant should act within the middle third of each section if there is to be no tension in the wall.

Finally, the pressure exerted by the base on the soil should be computed to ensure that the allowable pressure will not be exceeded. When the resultant is within the middle third, the pressures, psf, under the ends of the base are given by

$$p = \frac{R_v}{A} \pm \frac{Mc}{I} = \frac{R_v}{A}\left(1 \pm \frac{6e}{L}\right) \tag{8.144}$$

where $A$ = area of base, ft$^2$

$L$ = width of base, ft

$e$ = distance, parallel to $L$, from centroid of base to $R_v$, ft

Figure 8.29*b* shows the pressure distribution under a 1-ft strip of wall for $e = L/2 - a$, where $a$ is the distance of $R_v$ from the toe. When $R_v$ is exactly $L/3$ from the toe, the pressure at the heel becomes zero (Fig. 8.29*c*). When $R_v$ falls outside the middle third, the pressure vanishes under a zone around the heel, and pressure at the toe is much larger than for the other cases (Fig. 8.29*d*). "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) requires that contraction joints be provided at intervals not exceeding 30 ft. Alternate horizontal bars should be cut at these joints for crack control. Expansion joints should be located at intervals of up to 90 ft.

## 8.42 Cantilever Retaining Walls

This type of wall resists the lateral thrust of earth pressure through cantilever action of a vertical stem and horizontal base (Fig. 8.30*a*). Cantilever walls generally are economical for heights from 10 to 20 ft. For lower walls, gravity walls may be less costly; for taller walls, counterforts may be less expensive.

Usually, the force acting on the stem is the lateral earth pressure, including the effect of frost action, perhaps 700 lb/lin ft. The base is loaded by the moment and shear from the stem, upward soil pressure, its own weight, and that of the earth above. The weight of the soil over the toe, however, may be ignored in computing stresses in the toe since the earth may not be in place when the wall is first loaded or may erode. For walls of constant height, it is convenient to design and analyze a 1-ft-long strip.

The stem is designed to resist the bending moments and shear due to the earth thrust. Then, the size of the base slab is selected to meet requirements for resisting overturning and sliding and to keep the pressure on the soil within the allowable. If the flat bottom of the slab does not provide sufficient friction [Eq. (8.143)], a key, or lengthwise projection, may be added on the bottom for that purpose. The key may be reinforced by extending and bending up the dowels between stem and base.

To provide an adequate safety factor against overturning, the sum of the righting moments about the toe should be at least 1.5 times the sum of the overturning moments. The pressure under the base can be computed, as for gravity walls, from Eq. (8.144). (See also Fig. 8.30*b* to *d*.)

Generally, the stem is made thicker at the bottom than required for shear and balanced design for moment because of the saving in steel. Since the moment decreases from bottom to top, the earth side of the wall usually is tapered, and the top is made as thin as convenient concreting will permit (8 to 12 in). The main reinforcement is set, in vertical planes, parallel to the sloping face and 3 in away. The area of this steel at the bottom can be computed from Eq. (8.27). Some of the steel may be cut off where it no longer is needed. Cutoff points may be determined graphically (Fig. 8.30*b*). The bending-moment diagram is plotted and the resisting moment of steel not cut off is superimposed. The intersection of the two curves determines the theoretical cutoff point. The bars should extend upward beyond this point a distance equal to $d$ or 12 bar diameters.

In addition to the main steel, vertical steel is set in the front face of the wall and horizontal steel in both faces to resist thermal and shrinkage stresses

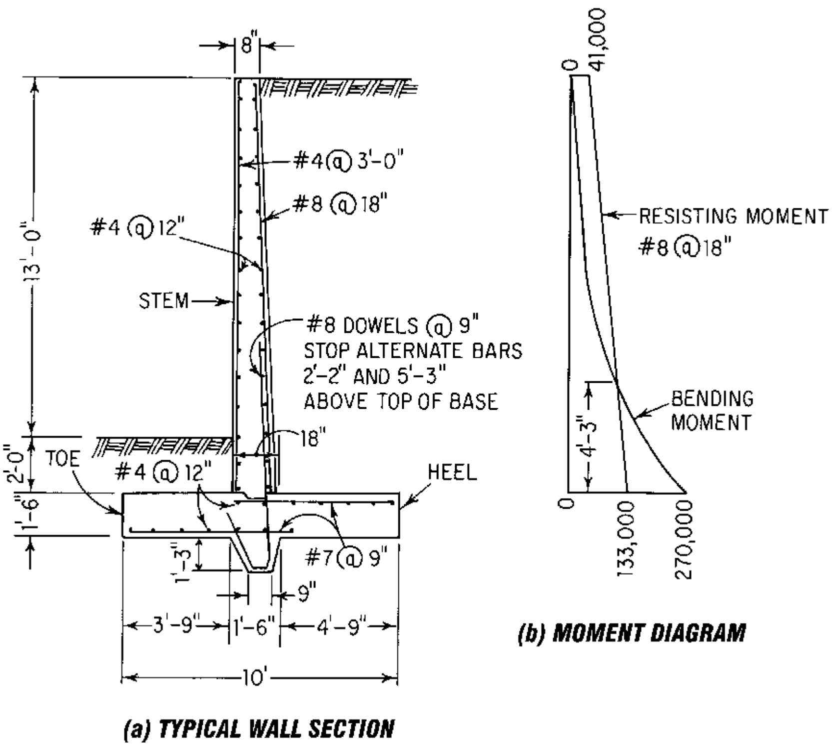



**Fig. 8.30** Cantilever retaining wall. (*a*) Vertical section shows main reinforcing steel placed vertically in the stem. (*b*) Moment diagram.

(Art. 8.23). "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) requires at least $\frac{1}{8}$ in$^2$ of horizontal reinforcement per foot of height.

The heel and toe portions of the base are both designed as cantilevers supported by the stem. The weight of the backfill tends to bend the heel down against relatively small resistance from soil pressure under the base. In contrast, the upward soil pressure tends to bend the toe up. So for the heel, main steel is placed near the top, and for the toe, near the bottom. Also, temperature steel is set lengthwise in the bottom. The area of the main steel may be computed from Eq. (8.27), but the bars should be checked for development length because of the relatively high shear.

To eliminate the need for diagonal-tension reinforcing, the thickness of the base should be sufficient to hold the shear stress, $v_c = V/bd$, below $1.1\sqrt{f'_c}$ where $f'_c$ is the 28 day strength of the concrete, psi, as computed by the working-stress method. The critical section for shear is at a distance *d* from the face of the stem, where *d* is the distance from the extreme compression surface to the tensile steel.

The stem is constructed after the base. A key usually is formed at the top of the base to prevent the stem from sliding. Also, dowels are left projecting from the base to tie the stem to it, one dowel per stem bar. The dowels may be extended to serve also as stem reinforcing (Fig. 8.30*a*).

The AASHTO Specifications require that contraction joints be provided at intervals not exceeding 30 ft. Expansion joints should be located at intervals up to 90 ft.

To relieve the wall of water pressure, weep holes should be formed near the bottom of the stem. Also, porous pipe and backfill may be set behind the wall to conduct water to the weep holes.

(M. Fintel, "Handbook of Concrete Engineering," Van Nostrand Reinhold Company, New York; "CRSI Handbook," Concrete Reinforcing Steel

Institute, 180 North La Salle St., Chicago, IL 60601 (www.crsl.org).)

## 8.43 Counterfort Retaining Walls

Counterforts are ties between the vertical stem of a wall and its base (Fig. 8.31*a*). Placed on the earth side of the stem, they are essentially wedge-shaped cantilevers. (Walls with supports on the opposite side are called buttressed retaining walls.) Counterfort walls are economical for heights for which gravity and cantilever walls are not suitable.

Stability design is the same as for gravity walls (Art. 8.41) and cantilever walls (Art. 8.42). But the design is applied to a section of wall center to center of counterforts.

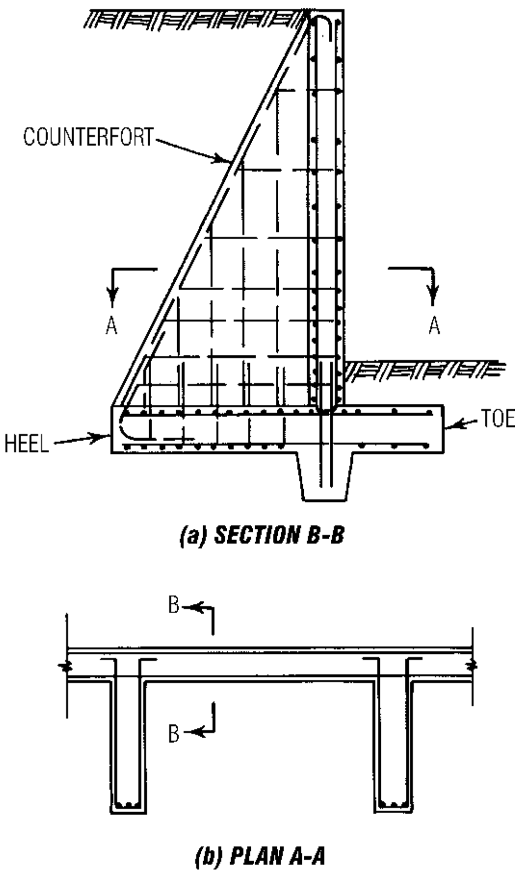



**Fig. 8.31** Counterfort retaining wall. (*a*) Vertical section. (*b*) Horizontal section.

The vertical face resists lateral earth pressure as a continuous slab supported by the counterforts. It also is supported by the base, but an exact analysis of the effects of the three-sided supports would not be worthwhile except for very long walls. Similarly, the heel portion of the base is designed as a continuous slab supported by the counterforts. In turn, the counterforts are subjected to lateral earth pressure on the sloping face and the pull of the vertical stem and base. The toe of the base acts as a cantilever; as in a cantilever wall.

Main reinforcing in the vertical face is horizontal. Since the earth pressure increases with depth, reinforcing area needed also varies with depth. It is customary to design a 1-ft-wide strip of slab spanning between counterforts at the bottom of the wall and at several higher levels. The steel area and spacing for each strip then are held constant between strips. Negative-moment steel should be placed near the backfill face of the wall at the counterforts, and positive-moment steel near the opposite face between counterforts (Fig. 8.31*b*). Concrete cover should be 3 in over reinforcing throughout the wall. Design requirements are substantially the same as for rectangular beams and one-way slabs, except the thickness is made large enough to eliminate the need for shear reinforcing (Arts. 8.20 to 8.23). The vertical face also incorporates vertical steel, equal to about 0.3 to 1% of the concrete area, for placement purposes and to resist temperature and shrinkage stresses.

In the base, main reinforcing in the heel portion extends lengthwise, whereas that in the toe runs across the width. The heel is subjected to the downward weight of the backfill above and its own weight and to the upward pressure of the soil below and the pull of the counterforts. So longitudinal steel should be placed in the top face at the counterforts and near the bottom between counterforts. Main transverse steel should be set near the bottom to resist the cantilever action of the toe.

The counterforts, resisting the lateral earth pressure on the sloping face and the pull of the vertical stem, are designed as T beams. Maximum moment occurs at the bottom. It is resisted by main reinforcing along the sloping face. (The effective depth should be taken as the distance from the outer face of the wall to the steel along a perpendicular to the steel.) At upper levels, main steel not required may be cut off. Some of the steel, however, should be extended and bent down into the vertical face. Also, dowels equal in area to the main steel at the bottom should be hooked into the base to provide anchorage.

Shear unit stress on a horizontal section of a counterfort may be computed from $v_c = V_1/bd$, where $b$ is the thickness of the counterfort and $d$ is the horizontal distance from face of wall to main steel.

$$V_1 = V - \frac{M}{d}(\tan\theta + \tan\phi) \qquad (8.145)$$

where $V$ = shear on section

$M$ = bending moment at section

$\theta$ = angle earth face of counterfort makes with vertical

$\phi$ = angle wall face makes with vertical

For a vertical wall face, $\phi = 0$ and $V_1 = V - (M/d)\tan\theta$. The critical section for shear may be taken conservatively at a distance up from the base equal to $d'\sin\theta\cos\theta$, where $d'$ is the depth of counterfort along the top of the base.

Whether or not horizontal web reinforcing is needed to resist the shear, horizontal bars are required to dowel the counterfort to the vertical face (Fig. 8.25*b*). They should be designed for the full wall reaction. Also, vertical bars are needed in the counterfort to resist the pull of the base. They should be doweled to the base.

The base is concreted first. Vertical bars are left projecting from it to dowel the counterforts and the vertical face. Then, the counterforts and vertical stem are cast together.

# Footings

Footings should be designed to satisfy two objectives: limit total settlement to an acceptable small amount and eliminate differential settlement between parts of a structure as nearly as possible. To limit the amount of settlement, a footing should be constructed on soil with sufficient resistance to deformation, and the load should be spread over a large soil area. The load may be spread horizontally, as is done with spread footings, or vertically, as with friction-pile foundations.

## 8.44 Types of Footings

There are a wide variety of spread footings. The most commonly used ones are illustrated in Fig. 8.32*a* to *g*. A simple pile footing is shown in Fig. 8.32*h*.

For walls, a spread footing is a slab wider than the wall and extending the length of the wall (Fig. 8.32*a*). Square or rectangular slabs are used under single columns (Fig. 8.32*b* to *d*). When two columns are so close that their footings would merge or nearly touch, a combined footing (Fig. 8.32*e*) extending under the two should be constructed. When a column footing cannot project in one direction, perhaps because of the proximity of a property line, the footing may be helped out by an adjacent footing with more space. Either a combined footing or a strap (cantilever) footing (Fig. 8.32*f*) may be used under the two columns.

For structures with heavy loads relative to soil capacity, a mat or raft foundation (Fig. 8.32*g*) may prove economical. A simple form is a thick, two-way-reinforced-concrete slab extending under the entire structure. In effect, it enables the structure to float on the soil, and because of its rigidity, it permits negligible differential settlement. Even greater rigidity can be obtained by building the raft foundation as an inverted beam-and-girder floor, with the girders supporting the columns. Sometimes, also, inverted flat slabs are used as mat foundations.

In general, footings should be so located under walls or columns as to develop uniform pressure below. The pressure under adjacent footings should be as nearly equal as possible, to avoid differential settlement. In the computation of stresses in spread footings, the upward reaction of the soil may be assumed to vary linearly. For pile-cap stresses, the reaction from each pile may be assumed to act at the pile center.

Simple footings act as cantilevers under the downward column or wall loads and upward soil or pile reactions. Therefore, they can be designed as rectangular beams.

## 8.45 Stress Transfer from Columns to Footings

For a footing to serve its purpose, column stresses must be distributed to it and spread over the soil or to piles, with a safety factor against failure of the

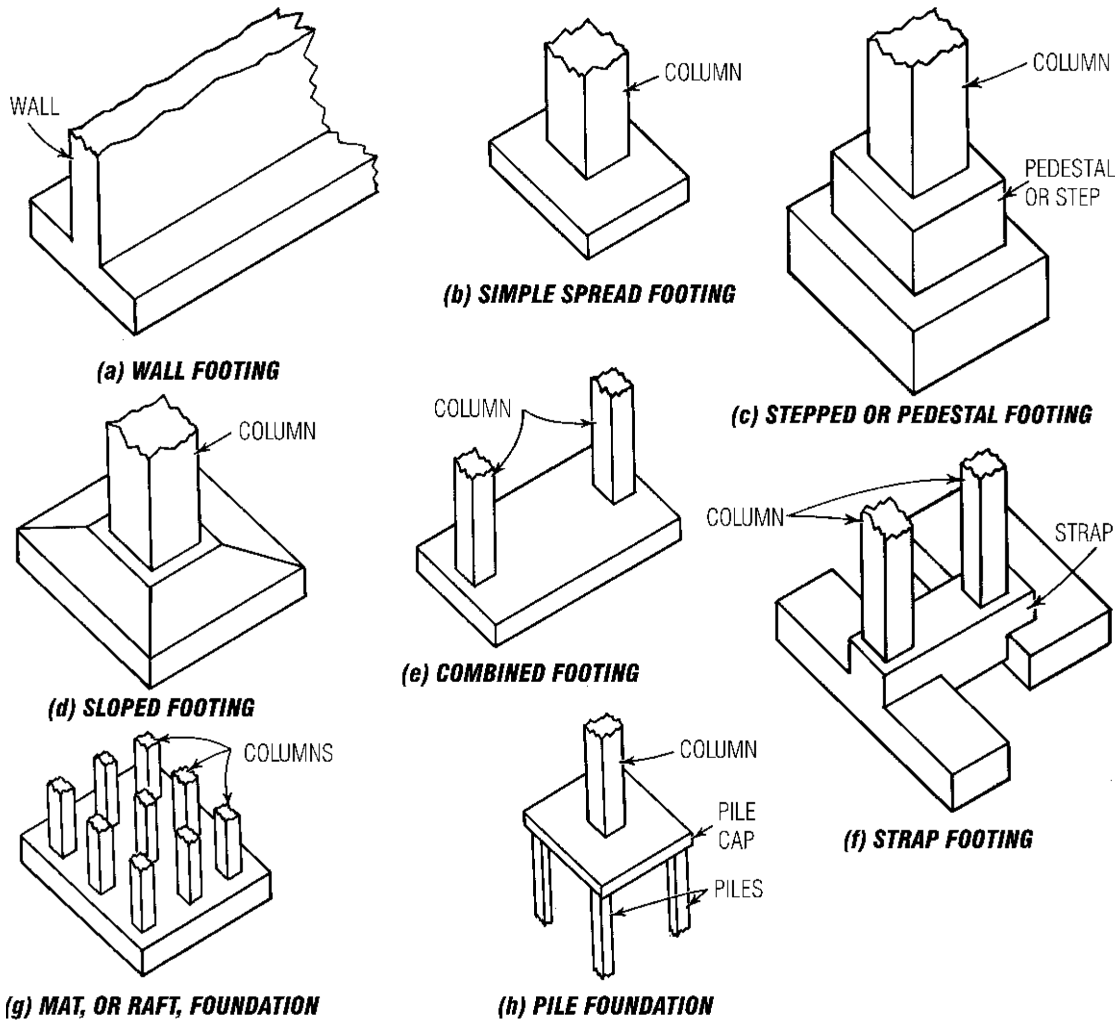



**Fig. 8.32** Common types of concrete footings for walls and columns.

footing. Stress in the longitudinal reinforcement of a column should be transferred to its pedestal or footing either by extending the longitudinal steel into the support or by dowels. At least four bars should be extended or four dowels used. In any case, a minimum steel area of 0.5% of the column area should be supplied for load transfer. The stress-transfer bars should project into the base a sufficient compression-embedment distance to transfer the stress in the column bars to the base concrete. Where dowels are used, their total area should be adequate to transfer the compression in excess of that transmitted by the column concrete to the footing in bearing, and the dowels should not be larger than #11 bars. If the required dowel length is larger than the footing depth less 3 in, either smaller-diameter bars with equivalent area should be used or a monolithic concrete cap should be added to increase the concrete depth. The dowels, in addition, should provide at least one-quarter of the tension capacity of the column bars on each column face. The dowels should extend into the column a distance equal to that required for compression lapping of column bars (Art. 8.12.6).

Stress in the column concrete should be considered transferred to the top of the pedestal or footing by bearing. ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), specifies two bearing stresses:

For a fully loaded area, such as the base of a pedestal, allowable bearing stress is $0.85\phi f_c'$, where $f_c'$ is the strength of the concrete and $\phi = 0.65$.

If the area $A_1$, the loaded portion at the top of a pedestal or footing, is less than the area of the top, the allowable pressure may be multiplied by $\sqrt{A_2/A_1}$ but not more than 2, where $A_2$ is the area of the top that is geometrically similar to and concentric with the loaded area $A_1$.

## 8.46 Wall Footings

The spread footing under a wall (Fig. 8.32*a*) distributes the wall load horizontally to preclude excessive settlement. (For retaining-wall footings, see Arts. 8.41 to 8.43.) The wall should be so located on the footing as to produce uniform bearing pressure on the soil (Fig. 8.33), ignoring the variation due to bending of the footing. The pressure, lb/ft$^2$, is determined by dividing the load per foot by the footing width, ft.

The footing acts as a cantilever on opposite sides of the wall under downward wall loads and upward soil pressure. For footings supporting concrete walls, the critical section for bending moment is at the face of the wall; for footings under masonry walls, halfway between the middle and edge of the wall. Hence, for a 1-ft-long strip of symmetrical concrete-wall footing, symmetrically loaded, the maximum moment, ft-lb, is

$$M = \frac{p}{8}(L - a)^2 \tag{8.146}$$

where $p$ = uniform pressure on soil, psf

$L$ = width of footing, ft

$a$ = wall thickness, ft

If the footing is sufficiently deep that the tensile bending stress at the bottom, $6M/t^2$, where $M$ is the factored moment and $t$ is the footing depth, in, does not exceed $5\phi\sqrt{f_c'}$, where $f_c'$ is the 28-day concrete strength, psi, and $\phi = 0.90$, the footing need not be reinforced. If the tensile stress is larger, the footing should be designed as a 12-in-wide rectangular, reinforced beam. Bars should be placed across the width of the footing, 3 in from the bottom. Bar development length is measured from the point at which the critical section for moment occurs. Wall footings also may be designed by ultimate-strength theory.

ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), requires at least 6 in of cover over the reinforcement at the edges. Hence, allowing about 1 in for the bar diameter, the minimum footing thickness is 10 in.

The critical section for shear is at a distance $d$ from the face of the wall, where $d$ is the distance from the top of the footing to the tensile reinforcement, in. Since diagonal-tension reinforcement is undesirable, $d$ should be large enough to keep the shear unit stress $V/12d$ below $1.1\sqrt{f_c'}$, as computed by the working-stress method, or below $2\sqrt{f_c'}b_w d$ for factored shear loads. $V$ is the shear at the critical section per foot of wall.

In addition to the main steel, some longitudinal steel also should be placed parallel to the wall to resist shrinkage stresses and facilitate placement of the main steel. (See also Art. 8.45.)

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com).)

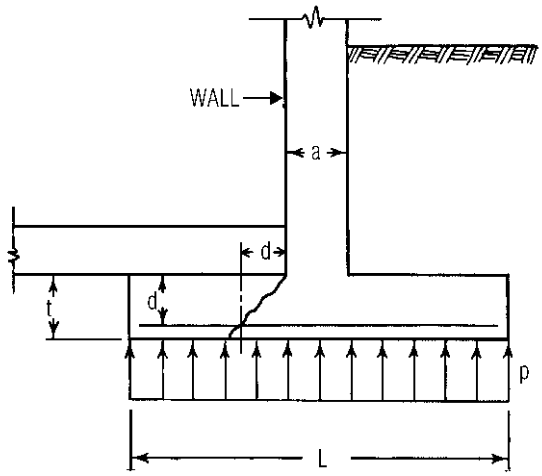



**Fig. 8.33** Reinforced concrete wall footing.

## 8.47 Single-Column Spread Footings

The spread footing under a column (Fig. 8.32*b* to *d*) distributes the column load horizontally to prevent

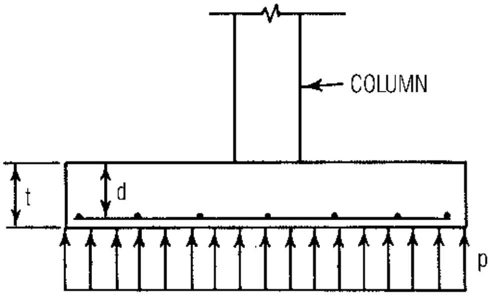



**Fig. 8.34** Spread footing for column.

excessive total and differential settlement. The column should be located on the footing so as to produce uniform bearing pressure on the soil (Fig. 8.34), exclusive of the variation due to bending of the footing. The pressure equals the load divided by the footing area.

Single-column footings usually are square, but they may be made rectangular to satisfy space restrictions or to support elongated columns.

Under the downward load of the column and the upward soil pressure, a footing acts as a cantilever in two perpendicular directions. For rectangular concrete columns and pedestals, the critical section for bending moment is at the face of the loaded member (*ab* in Fig. 8.35*a*). (For round or octagonal columns or pedestals, the face may be taken as the side of a square with the same area.) For steel baseplates, the critical section for moment is halfway between the face of the column and the edge of the plate.

The bending moment on *ab* is produced by the upward pressure of the soil on the area *abcd*. That part of the footing is designed as a rectangular beam to resist the moment. Another critical section lies along a perpendicular column face and should be similarly designed. If the footing is sufficiently deep that the factored tensile bending stress at the bottom does not exceed $5\phi\sqrt{f_c'}$, where $\phi = 0.90$ and $f_c'$ is the 28-day strength of the concrete, psi, the footing need not be reinforced. If the tensile stress is larger, reinforcement should be placed parallel to both sides of the footing, with the lower layer 3 in above the bottom of the footing and the upper layer a bar diameter higher. The critical section for anchorage (or bar embedment length) is the same as for moment.

In square footings, the steel should be uniformly spaced in each layer. Although the effective depth *d* is less for the upper layer, thus requiring more steel, it is general practice to compute the required area and spacing for the upper level and repeat them for the lower layer.

In rectangular footings, reinforcement parallel to the long side, with length *A*, ft, should be uniformly distributed over the width of the footing, *B*, ft. Bars parallel to the short side should be more closely spaced under the column than near the edges. ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), recommends that the short bars should be

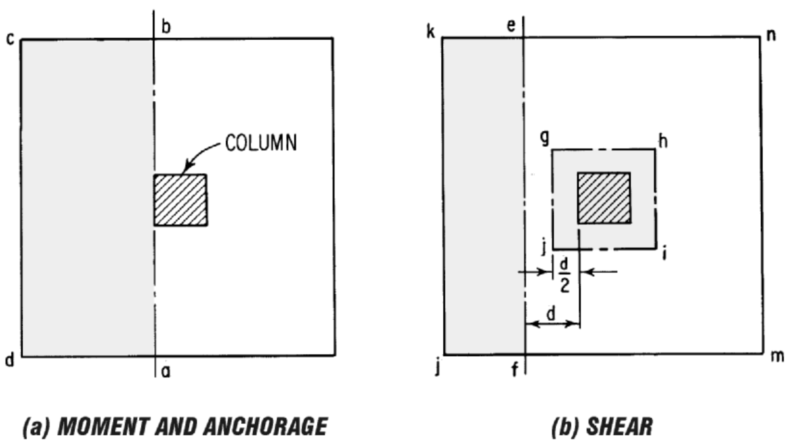



**Fig. 8.35** Critical section in a column footing as viewed in plan.

given a constant but closer spacing over a width B centered under the column. The area of steel in this band should equal twice the total steel area required in the short direction divided by $A/B + 1$. The remainder of the reinforcement should be uniformly distributed on opposite sides of the band. (See also Art. 8.45.)

Two types of shear should be investigated: two-way action (punching shear) and beam-type shear. The critical section for beam-type shear lies at a distance $d$ from the face of column or pedestal (*ef* in Fig. 8.35*b*). The shear equals the total upward pressure on area *efjk*. To eliminate the need for diagonal-tension reinforcing, $d$ should be made large enough that the unit shear stress does not exceed $1.1\sqrt{f_c'}$ ($2\sqrt{f_c'}$ for ultimate-strength design).

The critical section for two-way action is concentric with the column or pedestal. It lies at a distance $d/2$ from the face of the loaded member (*ghij* in Fig. 8.35*b*). The shear equals the column load less the upward soil pressure on area *ghij*. In this case, $d$ should be large enough that the factored shear on the concrete does not exceed

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}b_o d \qquad (8.147)$$

where $\beta_c$ = ratio of long side to short side of critical shear section

$b_o$ = perimeter of critical section, in

$d$ = depth of centroid of reinforcement, in

Shearhead reinforcement (steel shapes), although generally uneconomical, may be used to obtain a shallow footing.

Footings for columns designed to take moment at the base should be designed against overturning and nonuniform soil pressures. When the moments are about only one axis, the footing may be made rectangular with the long direction perpendicular to that axis, for economy. Design for the long direction is similar to that for retaining-wall bases (Art. 8.45 to 8.47).

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); M. Fintel, "Handbook of Concrete Engineering," Van Nostrand Reinhold Company, New York; "CRSI Handbook," Concrete Reinforcing Steel Institute, Chicago, Ill. (www.crsi.org); ACI SP-17, "Design Handbook," American Concrete Institute, Detroit, MI (www.aci-int.org).)

## 8.48 Combined Footings

These are spread footings extended under more than one column (Fig. 8.32*e*). They may be necessary when two or more columns are so closely spaced that individual footings would interfere with each other. Or they may be desirable when space is restricted for a column footing, such as an exterior member so close to a property line that an individual footing would be so short that it would have excessive eccentric loading. In that case, the footing may be extended under a rear column. If the footing can be continued past that column a sufficient distance, and the exterior column has a lighter load, the combined footing may be made rectangular (Fig. 8.36*a*). If not, it may be made trapezoidal.

If possible, the columns should be so placed on the combined footing as to produce a uniform pressure on the soil. Hence, the resultant of the column loads should coincide with the centroid of the footing in plan. This requirement usually determines the length of the footing. The width is computed from the area required to keep the pressure on the soil within the allowable.

In the longitudinal direction, the footing should be designed as a rectangular beam with overhangs. This beam is subjected to the upward pressure of the soil. Hence, the main steel consists of top bars between the columns and bottom bars at the columns where there are overhangs (Fig. 8.36*b*). Depth of footing may be determined by moment or shear (see Art. 8.41).

The column loads may be assumed distributed to the longitudinal beam by beams of the same depth as the footing but extending in the narrow, or transverse, direction. Centered, if possible, under each column, the transverse member should be designed as a rectangular beam subjected to the downward column load and upward soil pressure under the beam. The width of the beam may be estimated by assuming a 60° distribution of the column load, as indicated in Fig. 8.36*c*. Main steel in the transverse beam should be placed near the bottom.

Design procedure for a trapezoidal combined footing is similar. But the reinforcing steel in the longitudinal direction is placed fanwise, and

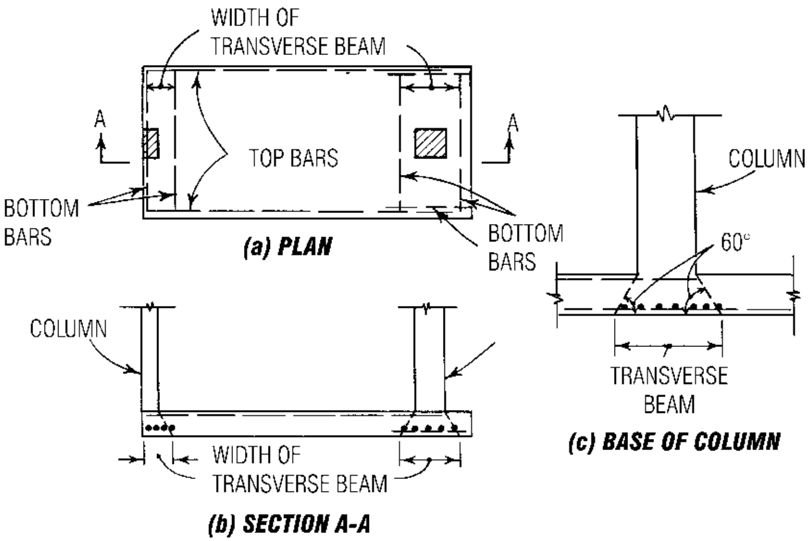



**Fig. 8.36** Combined footing. (*a*) Plan view. (*b*) Vertical section. (*c*) Detail at base of interior column.

alternate bars are cut off as the narrow end is approached. (See also Art. 8.45.)

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); M. Fintel, "Handbook of Concrete Engineering," Van Nostrand Reinhold Company, New York.)

## 8.49 Strap or Cantilever Footings

In Art. 8.48, the design of a combined footing is explained for a column footing in restricted space, such as an exterior column at a property line. As the distance between such a column and a column with adequate space around it increases, the cost of a combined footing rises rapidly. For column spacing more than about 15 ft, a strap footing (Fig. 8.32*f*) may be more economical. It consists of a separate footing under each column connected by a beam or strap to distribute the column loads (Fig. 8.37*a*).

The footings are sized to produce the same, constant pressure under each (Fig. 8.37*c*). This requires that the centroid of their areas coincide with the resultant of the column loads. Usually, the strap is raised above the bottom of the footings so as not to bear on the soil. The sum of the footing areas, therefore, must be large enough for the allowable bearing capacity of the soil not to be exceeded. When these requirements are satisfied, the total net pressure under a footing does not necessarily equal the column design load on the footing.

The strap should be designed as a rectangular beam spanning between the columns. The loads on it include its own weight (when it does not rest on the soil) and the upward pressure from the footings. Width of the strap usually is selected arbitrarily as equal to that of the largest column plus 4 to 8 in so that column forms can be supported on top of the strap. Depth is determined by the maximum bending moment.

The main reinforcing in the strap is placed near the top. Some of the steel can be cut off where not needed. For diagonal tension, stirrups normally will be needed near the columns (Fig. 8.37*b*). In addition, longitudinal placement steel is set near the bottom of the strap, plus reinforcement to guard against settlement stresses.

The footing under the exterior column may be designed as a wall footing (Art. 8.46). The portions on opposite sides of the strap act as cantilevers under the constant upward pressure of the soil.

The interior footing should be designed as a single-column footing (Art. 8.47). The critical section for punching shear, however, differs from that for

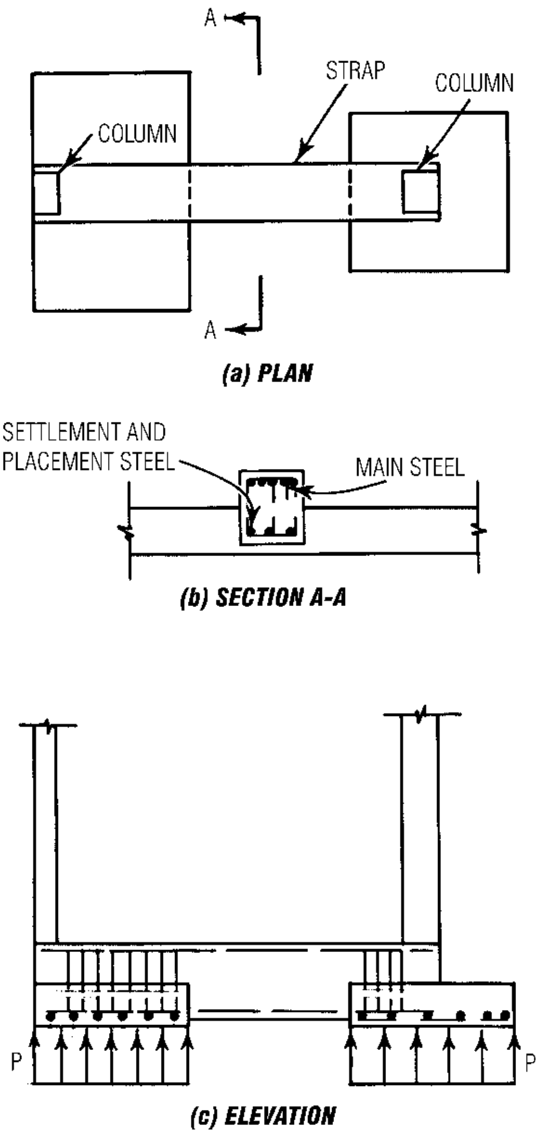



**Fig. 8.37** Strap (cantilever) footing.

a conventional footing. This shear should be computed on a section parallel to the strap and at a distance $d/2$ from the sides and extending around the column at a distance $d/2$ from its faces; $d$ is the effective depth of the footing, the distance from the bottom steel to the top of the footing.

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); M. Fintel, "Handbook of Concrete Engineering," Van Nostrand Reinhold Company, New York.)

## 8.50 Footings on Piles

When piles are required to support a structure, they are capped with a thick concrete slab, on which the structure rests. The pile cap should be reinforced. ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), requires that the thickness above the tops of the piles be at least 12 in. The piles should be embedded from 6 to 9 in, preferably the larger amount, into the footing. They should be cut to required elevation before the footing is cast.

Like spread footings, pile footings for walls are continuous, the piles being driven in line under the wall. For a single column or pier, piles are driven in a cluster. "Standard Specifications for Highway Bridges" (American Association of State Highway and Transportation Officials) requires that piles be spaced at least 2 ft 6 in center to center. And the distance from the side of a pile to the nearest edge of the footing should be 9 in or more.

Whenever possible, the piles should be located so as to place their centroid under the resultant of the column load. If this is done, each pile will carry the same load. If the load is eccentric, then the load on a pile may be assumed to vary linearly with distance from an axis through the centroid.

The critical section for bending moment in the footing and embedment length of the reinforcing should be taken as follows:

At the face of the column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall

Halfway between the middle and edge of the wall, for footings under masonry walls

Halfway between the face of the column or pedestal and the edge of the metallic base, for footings under steel baseplates

The moment is produced at the critical section by the upward forces from all the piles lying between the section and the edge of the footing.

For diagonal tension, two types of shear should be investigated—punching shear and beamlike shear—as for single-column spread footings (Art. 8.47). The ACI Code requires that in computing the external shear on any section through a footing supported on piles, the entire reaction from any pile whose center is located half the pile diameter or more outside the section shall be assumed as producing shear on the section. The reaction from any pile whose center is located half the pile diameter or more inside the section shall be assumed as producing no shear on the section. For intermediate positions of the pile center, the portion of the pile reaction to be assumed as producing shear on the section should be based on straight-line interpolation between the full value at half the pile diameter outside the section and zero value at that distance inside the section.

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York (books.mcgraw-hill.com); M. Fintel, "Handbook of Concrete Engineering," Van Nostrand Reinhold Company, New York).

## Frames and Shells

### 8.51 Structural Analysis of Frames and Shells

Analysis of structural frames yields values of internal forces and moments at various sections. Results include bending moments (about two principal axes of each section), concentric normal forces (axial tension or compression), tangential forces (shear), and torsion (bending moment parallel to the section). In design, critical cross sections are selected and designed to resist the internal forces and moments acting on them.

Geometry of a structural frame and its components has a great bearing on distribution of internal forces and moments and their magnitude. Thus, the geometry affects economy and esthetics of a structural system and its components. Rigid frames, arches, folded plates, and shells are examples of the use of geometry for support of loads at relatively low cost.

Once any of these structures has been analyzed and internal forces and moments on critical cross sections have been determined, design becomes nearly identical with that of cross sections covered in previous articles in this section. Additional consideration, however, should be given to secondary stresses in detailing the reinforcement.

In practice, most structures and their components are analyzed only for the primary stresses caused by external loads. But most structural components, including beams, columns, and slabs discussed previously, are subjected to secondary stresses. They could be due to many causes:

External loads normally not considered during the design, for example, when one side of a building is heated by sun more than the others

Nonhomogeneity of material, such as concrete

Geometry of structural members, for example, deep rather than shallow cross sections

Additional forces and moments due to deformations

Most of the formulas used in everyday structural design are simplified versions of more accurate but complicated mathematical expressions. The simplified formulas give results only for an approximate stress distribution. To provide for the difference between approximate and accurate analyses, design of members, including secondary stresses, should incorporate a margin of safety. Stress concentration, for example, is a secondary stress. In general, there are no set rules or formulas for predicting secondary stresses and designing for them.

In conventional reinforced-concrete structures, secondary stresses are relatively small compared with the primary stresses. But if secondary stresses are not provided for in design, cracks may develop in the structure. Usually, these cracks are not serious and are acceptable. In view of the difficulty, perhaps impossibility, of predicting the location and magnitude of secondary stresses in most cases, normal practice does not include analysis of structures for secondary stresses.

To protect structures against unpredictable stresses, ACI 318, "Building Code Requirements for Reinforced Concrete" (American Concrete Institute), specifies minimum reinforcement for beams, columns, and slabs. Spacing and size of this reinforcement take care of the secondary stresses. These provisions and some additional reinforcement requirements apply to design of rigid frames, arches, folded plates, and shells. But these types of structures often have larger secondary stresses than conventional structures, and these stresses are distributed differently from those in beams and columns. There are no code provisions for designing against these secondary stresses other than the general requirements of elastic behavior, equilibrium checks, and accounting for effects of large deflections, creep, and possible construction defects. But observations of the behavior of rigid frames, arches, folded plates, and shells, along with more accurate mathematical treatment and analysis, do help to design against secondary stresses.

The following articles point out the more salient considerations in designing these reinforced-concrete structures. Engineers, however, should have sufficient experience in design of such structures to take steps to avoid undue cracking of concrete.

One of the most important duties of a structural engineer is to choose an appropriate structural system, for example, to decide whether to span with a simply supported beam, a rigid frame, an arch, a folded plate, or a shell. The engineer must know the advantages of these structural systems to be able to select a proper structure for a project.

In indeterminate structures such as rigid frames, arches, folded plates, and shells, the sizes and thicknesses of the components of these structures affect the magnitude and distribution of the bending moments and, hence, shears and axial forces. For example, if the horizontal member of the rigid frame of Fig. 8.38*a* is made much deeper than the width of the vertical member, that is, the beam is much stiffer than the column, the maximum moment in the beam would be relatively large and that in the column small. Conversely, if the vertical member is made much wider than the depth of the horizontal member, that is, the column is much stiffer than the beam, the maximum bending moment in the column would be relatively large.

Similarly, deepening the haunches in the horizontal member of Fig. 8.38*b* would increase the negative bending moment at the haunches and decrease the positive bending moment at midspan, where the beam is shallow.

Because of the properties described, indeterminate structures are analyzed by first assuming sizes and shapes of components. After internal forces and moments have been determined, the assumed sections are checked for adequacy. If the assumed sizes must be adjusted, another analysis is performed with the adjusted sizes. Then, these are checked for adequacy. If necessary, the cycle is repeated.

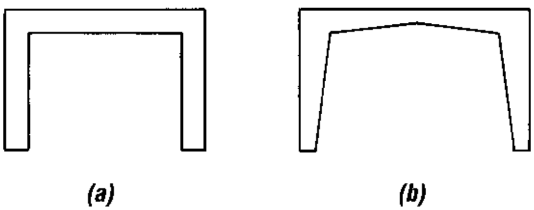


**Fig. 8.38** Rigid frames: (*a*) with prismatic members; (*b*) with haunched beam.

## 8.52 Concrete Rigid Frames

Rigid frame implies a plane structural system consisting of straight members meeting each other at an angle and rigidly connected at the junction. A rigid connection keeps unchanged the angle between members as the entire frame distorts under load.

Rigid frames may be one bay long and one tier high (Fig. 8.38*a* and *b*), or they may have multiple bays and multiple tiers (Fig. 8.39*a* and *b*). They may be built of reinforced concrete or prestressed concrete, cast in place or precast.

Because of continuity between columns and beams, columns in rigid frames participate with the beams in bending and thus in resisting external loads. This participation results in both smaller bending moments and different moment distribution along the beam than in a simply supported beam with the same span and loads. But for these advantages in bending-moment distribution along the beam, the column is penalized. Under vertical loading, for example, it is subjected to bending moments in addition to axial force. (See also Arts. 6.61 to 6.63 and 8.57.)

Since the bases of most rigid frames develop horizontal reactions, the beams usually are subjected to a small axial force. Also, the beams and columns are subjected to shear forces.

It is not advisable, in general, to differentiate between beams and columns in a rigid frame, but to consider each as a member subjected to axial loads and flexure. Find bending moments, shear, and axial forces in each, and design for these.

Because of continuity between members in a rigid frame, this type of structure is particularly advantageous in resisting wind and seismic loads. It does not necessarily have to be subjected to vertical loads only or consist of vertical and horizontal members. Figures 8.40 and 8.41 show examples of rigid frames with sloping members subjected to vertical and lateral loads.

Dimensions of cross sections and the amount of reinforcement in concrete rigid frames are

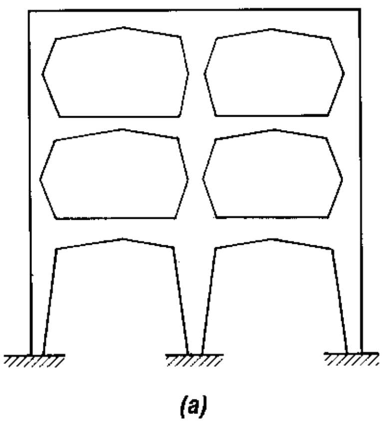



**Fig. 8.39** Multistory rigid frames: (*a*) with haunched members; (*b*) with prismatic members.

determined by primary stresses due to bending moments, axial forces, and shears, as in beams and columns. In addition, the following require special attention:

Rigid joints, where members meet, particularly at reentrant corners

Toes of legs at the foundations

Exceptionally deep members (Art. 8.17.5)

Typical details of rigid joints in a reinforced-concrete frame are shown in Fig. 8.42*a* and *b*. Ample embedment of bars at supports should be provided at all corners, as well as at overlaps (Art. 8.20.6). No interior or exterior face of a rigid joint should be left without reinforcement.

Note that in Fig. 8.42 reinforcing bars extend without bends past the reentrant corners. Reinforcing never should be bent around a reentrant corner. When the reinforcement is in tension, it tends to tear concrete at the corner away from the joint. Furthermore, sufficient stirrups should be provided around all bars that cross a joint. The amount of stirrups may be computed from the

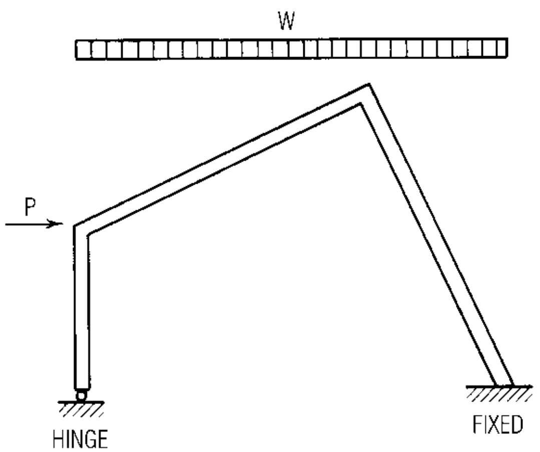



**Fig. 8.40** Rigid frame with sloping beam, one vertical column, and one sloping column.

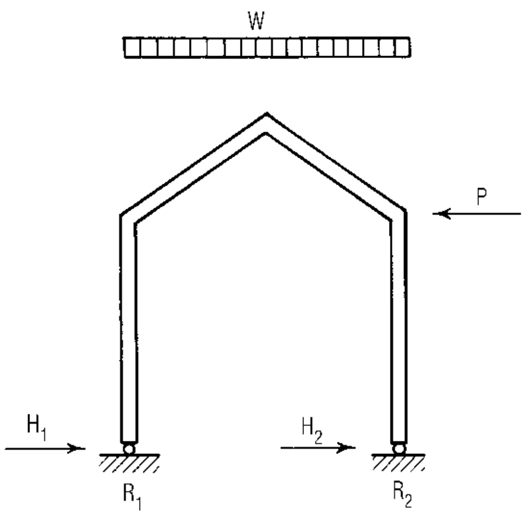



**Fig. 8.41** Gable frame with vertical columns and two sloping beams.

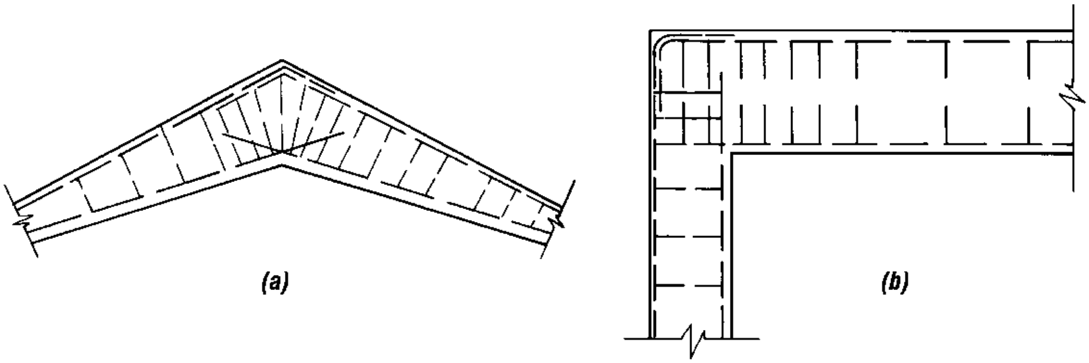



**Fig. 8.42** Reinforcing arrangements at right-frame joints.

component of tensile force in the reinforcement, but preferably a lower limit should be the minimum size and number of ties required for columns.

All toes of rigid frames are subjected to horizontal forces, or thrust. In a hinged rigid frame, an additional axial force (compression or tension) acts on the base, while in a fixed rigid frame, an additional axial force and a bending moment act.

Usually, analysis assumes that the toes of rigid frames do not move relative to each other. The designer should check this assumption in the design. If the toes do spread under load, the horizontal thrust, as well as all the internal forces and moments within the frame, will change. The actual internal forces due to movement of the toes should be computed and the frame designed accordingly. Similarly, if the base is not truly hinged or fixed, but only partly so, the effect of partial fixity on the frame should be taken into account.

The thrust may be resisted by a footing pressing against rock (Fig. 8.43), by friction of the footing against the soil (Fig. 8.44), or by a tie (Fig. 8.45). In the cases illustrated by Figs. 8.44 and 8.45, the likelihood of the toes spreading apart is considerable.

If the toe is hinged, the hinge detail could be provided in the field (Fig. 8.46). Or it could be a prefabricated steel hinge (Fig. 8.47).

In a fixed rigid frame, the connection of the toe to the footing (Fig. 8.48) should be strong enough to develop the computed bending moment. Since this moment is to be transferred to the ground, it is usual to construct a heavy eccentric footing that counterbalances this moment by its weight, as shown in Fig. 8.48.

To obtain an advantageous moment distribution in a frame, a designer might find it desirable to increase the sizes of some members of the frame.

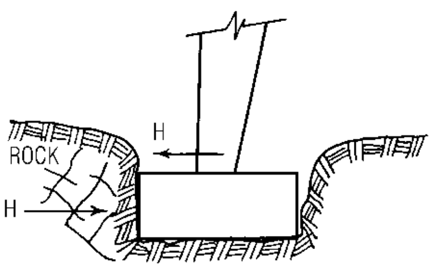



**Fig. 8.43** Footing thrust resisted by side bearing.

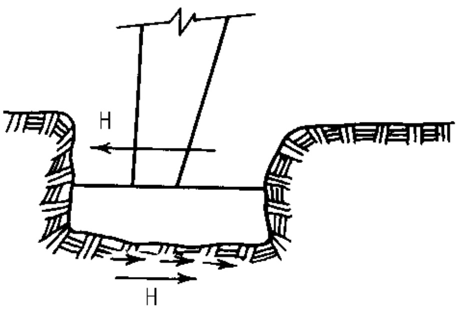



**Fig. 8.44** Footing thrust resisted by base friction.

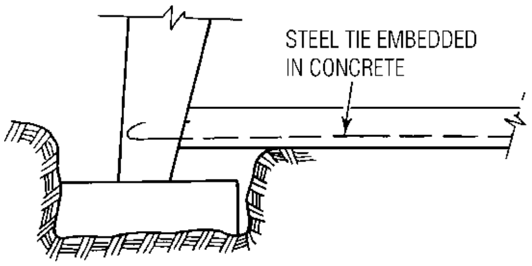



**Fig. 8.45** Ties between footings take thrust at the base of a rigid frame.

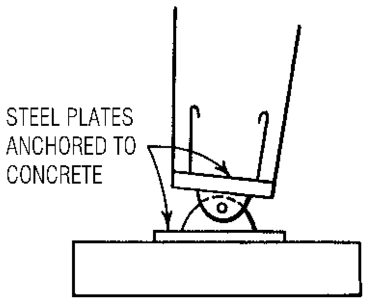



**Fig. 8.47** Base with steel hinge.

For example, for a long-span, low, rigid frame, increasing the width of the vertical legs would reduce positive bending moments in the horizontal members and increase moments in the vertical members. The vertical members could become stubby, as in Fig. 8.49. According to the ACI Code, when the ratio of depth $d$ to length $L$ of a continuous member exceeds 0.4, the member becomes a "deep" beam; the bending stresses and resistance to them do not follow the patterns described previously in this section. The designer should provide more than the usual stirrups and distribute reinforcement along the faces of the deep members, as in Fig. 8.49 (Art 8.17.5).

Design of precast-concrete rigid frames is identical to that of cast-in-place frames, except for connections. It is quite common to precast parts of frames between points of counterflexure, or sections where bending moment is small, as shown in Fig. 8.50*a*. This eliminates the need for a moment connection (often referred to as a continuity connection) at a joint. Only a shear connection is required (Fig. 8.50*b*). Since some bending moment might occur at the joint due to live, wind, seismic, and other loads, moment resistance should be provided by grouting longitudinal bars (Fig. 8.50*b*) or welding steel plates embedded in the precast concrete (Fig. 8.50*c*). When this type of connection is used, however, bending moments in the structure should be determined for continuity at the joint to verify the adequacy of the joint.

Rigid frames also may be prestressed and cast in place or precast. Prestressed, cast-in-place frames are posttensioned. Usually, the prestress is applied to each member with tendons anchored within the

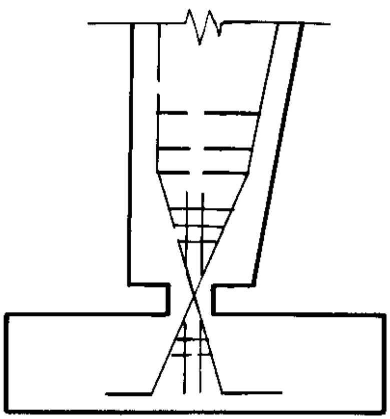


**Fig. 8.46** Hinge built with reinforcing bars at the top of a footing.

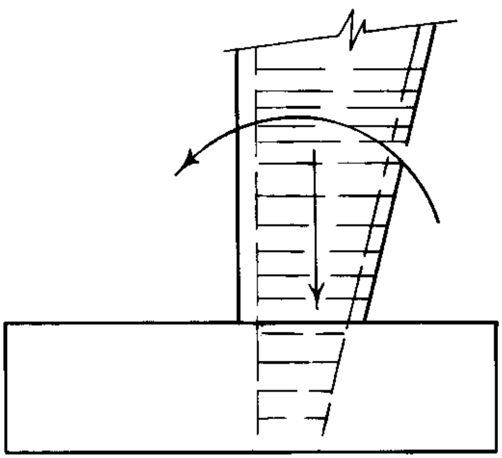


**Fig. 8.48** Base with moment resistance.

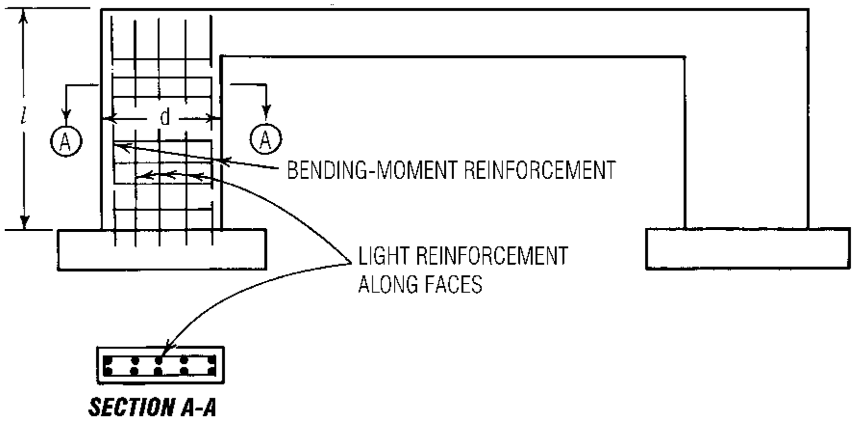



**Fig. 8.49** Rigid frame with stubby columns.

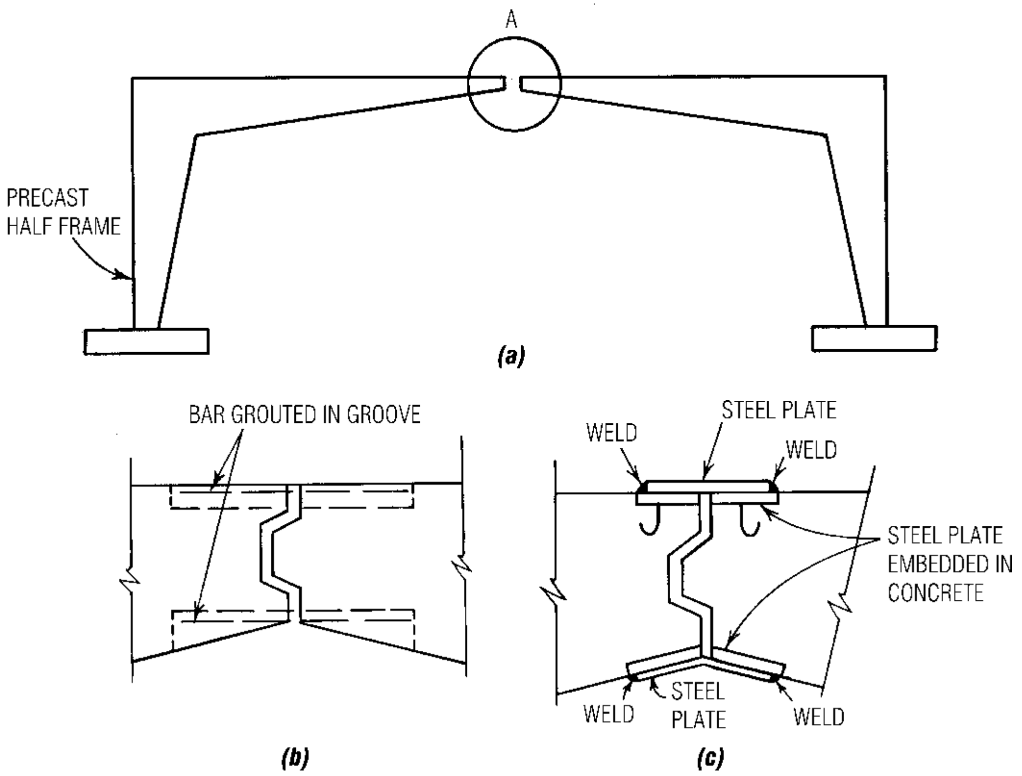



**Fig. 8.50** Precast-concrete rigid frame. (*a*) Halves connected at midspan. (*b*) Midspan joint with grouted longitudinal reinforcing bars. (*c*) Welded connection at midspan.

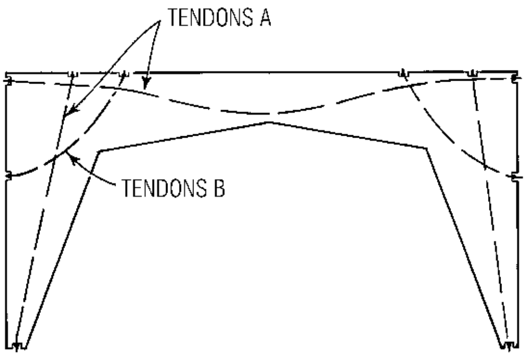



**Fig. 8.51** Prestressed-concrete rigid frame.

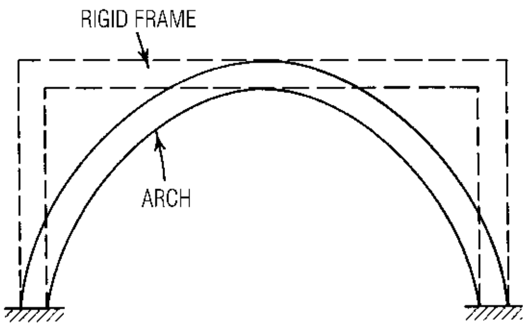



**Fig. 8.52** Arch replacement for rigid frame.

member (Fig. 8.51). Although continuous tendons may be more efficient structurally, friction losses due to bending the tendons make application of prestress in the field as intended by the design difficult. Such losses cannot be estimated. Hence, the magnitude of the prestress imparted is uncertain. The rigid joints, though, may be prestressed by individual straight or slightly bent tendons anchored in adjacent members (tendons B in Fig. 8.51).

When selecting the magnitude of the prestressing force in each member, the designer should ascertain that the bending moments at the ends of members meeting at a joint are in equilibrium and that the end rotation there is the same for each member.

Precast rigid frames may be pretensioned, posttensioned, or both. In prestressed, precast rigid frames, it is common to fabricate the individual members between joints, rather than between points of counterflexure, and connect them rigidly at the joints. The members are connected at the rigid joints by grouting reinforcing bars, welding steel inserts, or posttensioning. In all cases, the designer should make sure that the rotations of the ends of all members meeting at a joint are equal.

## 8.53 Concrete Arches

Structurally, arches are, in many respects, similar to rigid frames (Arts. 8.51 and 8.52). An arch may be considered a rigid frame with one curved member instead of a number of straight members (Fig. 8.52). The internal forces in the two structural systems are of the same nature: bending moments, axial forces, and shears. The difference is that bending moments predominate in rigid frames, while arches may be shaped so that axial (compression) force predominates. Nevertheless, general design procedures for arches and rigid frames are identical.

Design of details, however, differs since arches have no rigid joints above the abutments, and arches, being predominantly subjected to compression, must be provided with more resistance against buckling. Also, because arches are dependent on development of thrust resistance for their strength, all the requirements for rigid frames for thrust resistance are even more critical for arches.

Precasting of arches is not common because the curvature makes stacking for transportation difficult. Some small-span site-precast arches, however, have been successfully erected.

Prestressing of arch ribs is not very common because the arches are subjected to large compressive forces; thus, prestressing rarely offers advantages. But prestressing of abutments and of connections of a fixed-end arch to abutments, where bending moments are large, could be beneficial in resisting these moments.

See also Arts. 6.69 to 6.71.

(G. Winter and A. H. Nilson, "Design of Concrete Structures," McGraw-Hill Book Company, New York.)

## 8.54 Concrete Folded Plates

The basic structural advantage of a folded-plate structure (Fig. 8.53) over beams and slabs for a given span is that more material in a folded plate

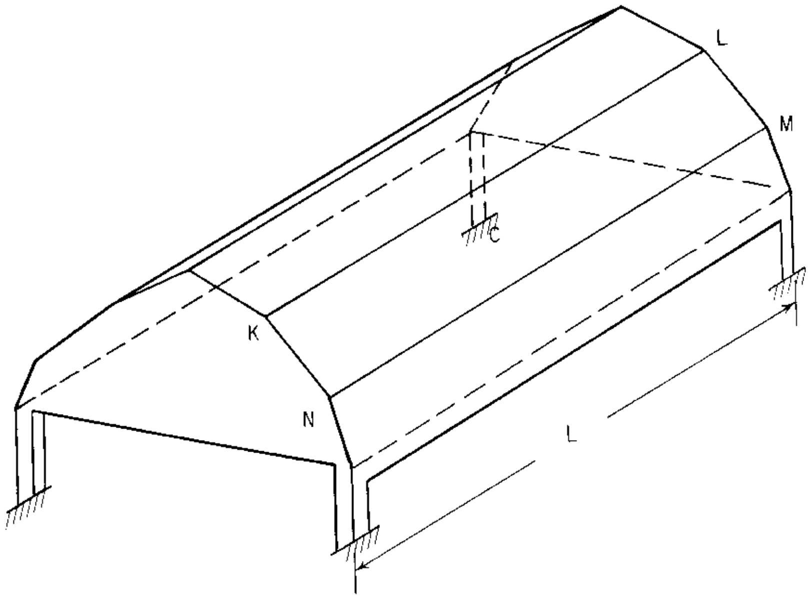



**Fig. 8.53** Folded-plate roof.

carries stresses, and stress distribution may be more uniform. For example, Fig. 8.54*a* shows cross sections of alternative structural systems of the same span and depth superimposed. One section is for a folded plate, the other for a system with two solid beams. The stress distribution in the solid beams is shown in Fig. 8.54*b*. Only the extreme fibers are stressed to the maximum allowable, while the remainder, the largest part of the cross section, is subjected to much smaller stresses. The stresses in the folded plate, as shown in Fig. 8.54*c*, are more uniformly distributed through the depth *D* of the structure. Furthermore, folded plates inherently enclose a space, whereas, for the same function, beams require a deck to span between them. Hence, a folded-plate structure needs less material than solid beams and may therefore be more economical.

It should be noted, however, that longitudinal-stress distribution in a folded-plate structure spanning a distance *L* (Fig. 8.53) is not given accurately by simple-beam theory; that is, the longitudinal normal stresses are not as shown in Fig. 8.54*b*. Under vertical loads, one cannot compute the moment of inertia of the folded-plate section in Fig. 8.54*a* about the centroidal axis and find the stresses from $Mc/I$. The cross section distorts under load, invalidating the elementary bending theory. Hence, the result may be more nearly the stress distribution shown in Fig. 8.54*c*. See also Arts. 6.76 and 6.77.

These normal stresses are perpendicular to the plane of the folded-plate section (Fig. 8.54*a*). They and the shear stresses parallel to the section may be assumed uniformly distributed over the thickness of the plates. The same is true of membrane stresses in shell structures.

Reinforcement in each plate, such as *KLMN* (Fig. 8.53), in the transverse and longitudinal directions, is determined from stresses obtained from analysis. Typical reinforcement is shown in Fig. 8.55. The quantity of longitudinal reinforcement is determined by the tensile stresses in each plate. But reinforcement should not be less than that indicated in Art. 8.23 for minimum quantity in slabs. In addition, a minimum of temperature reinforcement as required for slabs should be distributed uniformly throughout each plate. (See also Art. 8.51.)

Transverse reinforcement is determined by the transverse bending in each plate between support points *A*, *B*, *C*, *D*, . . . (Fig. 8.55). But reinforcement

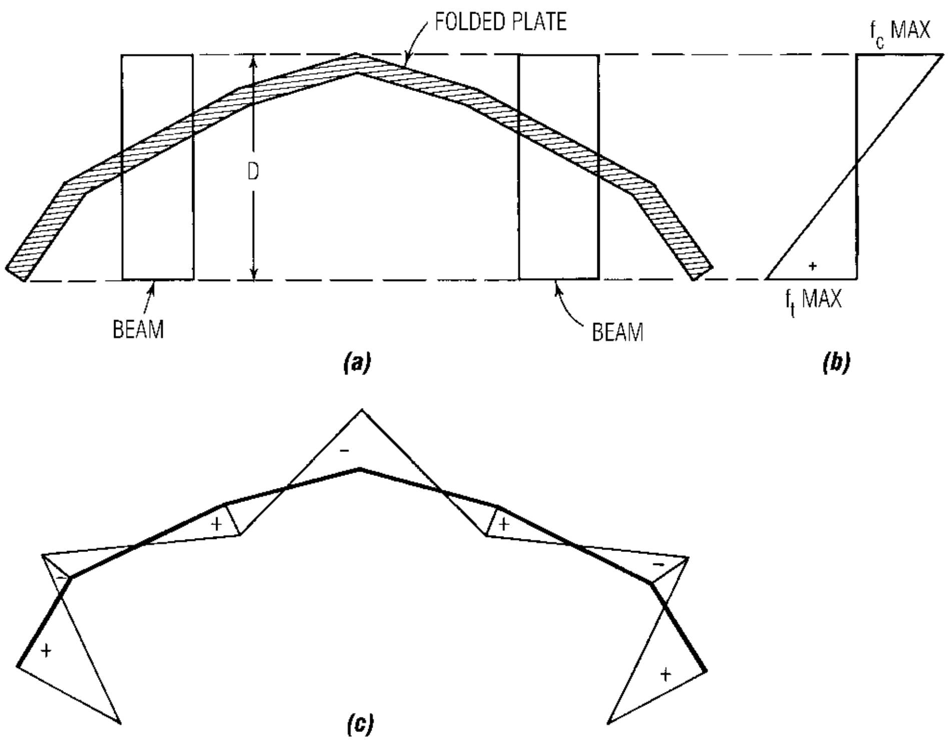



**Fig. 8.54** Comparison of folded-plate with beams. (*a*) Vertical section through a folded-plate roof with superimposition of two solid, rectangular beams that could replace it as roof supports. (*b*) Stress distribution at midspan of a beam. (*c*) Longitudinal stress distribution at midspan of the folded-plate roof.

should not be less than the temperature reinforcement indicated in Art. 8.23. Because the regions around plate intersections, such as *B* and *C*, are subjected to negative transverse bending moments, negative (top) reinforcement is required there. This reinforcement, as well as the bottom bars, should be carried far enough past the corner for proper embedment. Because of the distortions of the section and the uncertainty of the extent of transverse negative moments, it is good practice to carry reinforcement along the top of all plates, as shown for plate *CD* (Fig. 8.55). Such top reinforcement also is efficient in resisting shear.

Essentially, Fig. 8.55 represents a cross section of a rigid frame. The joints between plates have to be maintained rigid to correspond to assumptions made in the analysis. Thus, these joints should be reinforced as in rigid frames. When the angle between two plates is large, it is desirable to tie top and bottom reinforcement with ties, as indicated in Fig. 8.55.

If the concrete alone is not sufficient to resist diagonal tension due to shear, reinforcement should be provided for the excess diagonal tension. Such reinforcement may be inclined, as at *A* in Fig. 8.56, or a grid of longitudinal and transverse bars may be used, as at *B*. In the latter case, the reinforcement will have the pattern indicated in Fig. 8.55. The quantity needed to resist diagonal tension, then, should be added to that required for bending. Both the transverse and longitudinal reinforcement inserted for this purpose preferably should be distributed evenly between the top and bottom faces of the plates.

Elementary analysis of folded plates usually assumes that the cross sections at the supports do not distort. Therefore, it is common practice to provide rigid diaphragms at the ends of folded plates in planes of supports (Fig. 8.57). The diaphragms act as transverse beams, as well as ties, between supports. Hence, they usually have

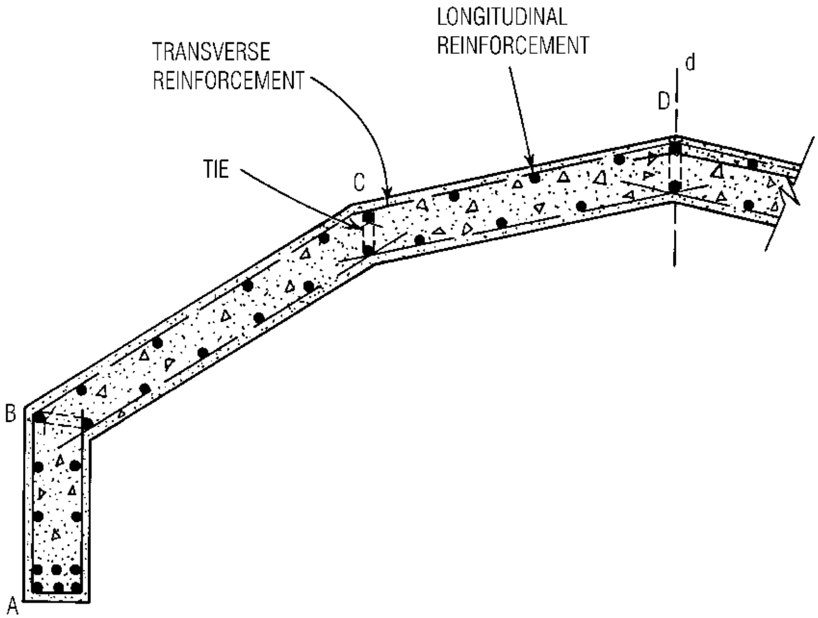



**Fig. 8.55** Typical reinforcement at a section of a folded plate.

relatively heavy bottom reinforcement. The strains in the end diaphragms should be kept small, to keep the end sections of the folded-plate structure from distorting. It is advisable, therefore, that the reinforcement in the diaphragm be evenly distributed throughout each face.

## 8.55 Concrete Shells

Thin shells are curved or folded slabs whose thicknesses are small compared with their other dimensions. In addition, shells are characterized by their three-dimensional load-carrying behavior, which is determined by their geometric shape, their boundary conditions, and the nature of the applied load. Many forms of concrete shells are used. To be amenable to theoretical analysis, these forms have geometrically expressible surfaces.

### 8.55.1 Stress Analysis of Shells

Elastic behavior is usually assumed for shell structural analysis, with suitable assumptions to approximate the three-dimensional behavior of shells. The ACI Building Code includes special provisions for shells. It suggests model studies for complex or unusual shapes, prescribes minimum reinforcement, and specifies design by the ultimate-strength method with the same load factors as for design of other elements.

Stresses usually are determined by membrane theory and are assumed constant across the shell thickness. The membrane theory for shells, however, neglects bending stresses. Yet, every shell is subjected to bending moments, not only under unsymmetrical loads but under uniform and symmetrical loads. Stress analysis of shells, however, by bending theory is more complex than by membrane theory but with the use of computers and finite-element, boundary-element, or numerical integration methods, it can be readily executed. See also Arts. 6.72 to 6.75.

Although unsymmetrical loads cause bending moments throughout a whole shell, symmetrical loads cause moments mainly at edges and supports. These edge and support moments may be very large. Provision should be made to resist them. If they are not properly provided for, not only would unsightly cracks occur, but the shell may distort, progressively increasing the size of

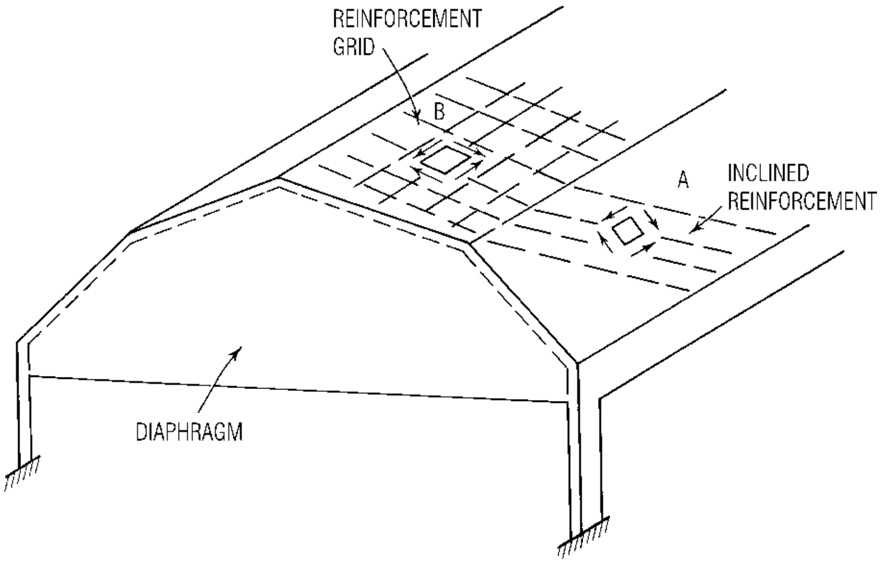



**Fig. 8.56** Reinforcement patterns in the plates of a folded-plate roof.

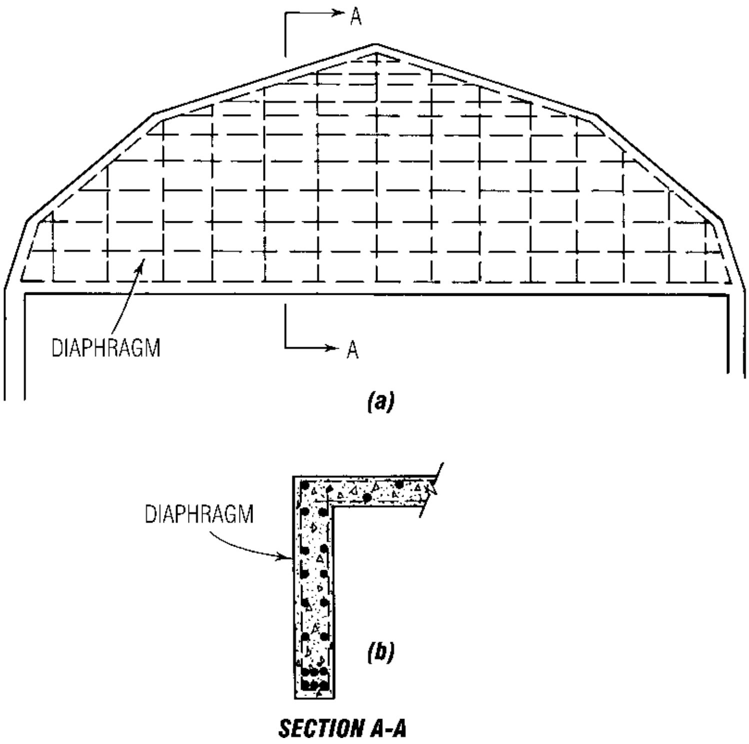



**Fig. 8.57** Reinforcement in the diaphragm of a folded-plate roof.

the cracks and causing large deflections, rendering the shell unusable. Therefore, past experience in design, field observations, and knowledge of results of tests on shells are a necessity for design of shell structures, to insure the proper quantity of reinforcement in critical locations, even though the reinforcement is not predicted by theory. Model testing is a helpful tool for shell design, but small-scale models may not predict all the possible stresses in a prototype.

Because of the difficulties in determining stresses accurately, only those forms of shell that have been successfully constructed and tested in the past are usually undertaken for commercial uses. These forms include barrel arches, domes, and hyperbolic paraboloids (Fig. 8.58).

## 8.55.2 Cylindrical Shells

Also known as **barrel shells**, cylindrical shells may consist of single transverse spans (Fig. 8.58*a*) or multiple spans (Fig. 8.59). Analysis yields a different stress distribution for a single barrel shell from that for a multiple one. But design considerations are the same.

Usually, the design stresses in a shell are quite small, requiring little reinforcement. The reinforcement, both circumferential and longitudinal, however, should not be less than the minimum reinforcement required for slabs (Art. 8.23).

Barrel shells usually are relatively thin. Thickness varies from 4 to 6 in for most parts of shells with spans up to 300 ft transversely and longitudinally. But the shells generally are thickened at

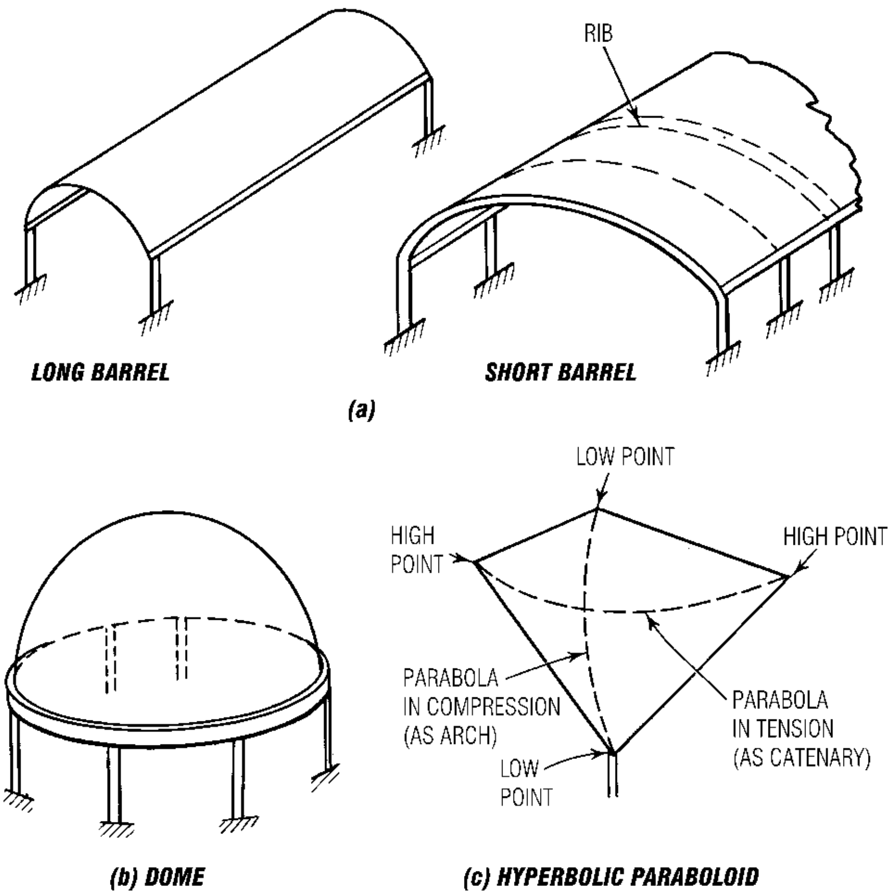



**Fig. 8.58** Common types of concrete shells.

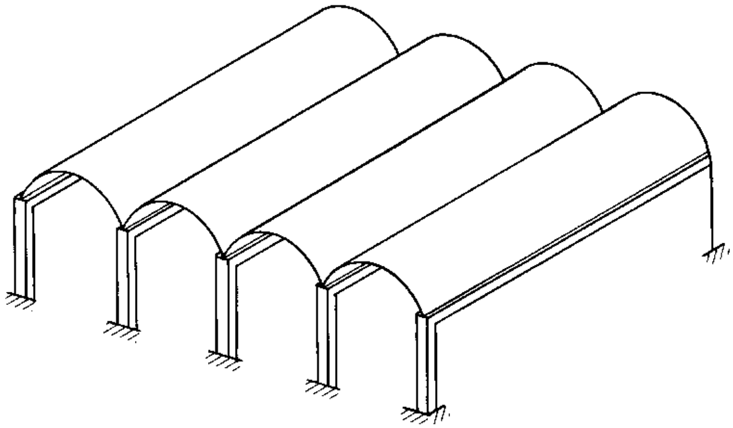


**Fig. 8.59** Multiple barrel-arch roof.

edges and supports and stiffened by edge beams. With analysis, including model testing, it is possible to design barrel shells of uniform thickness throughout, without stiffening edge members. But if the more simplified method of analysis (membrane theory) is employed, which is more usual and practical, stiffening edge members should be provided, as shown in Fig. 8.60. These consist of

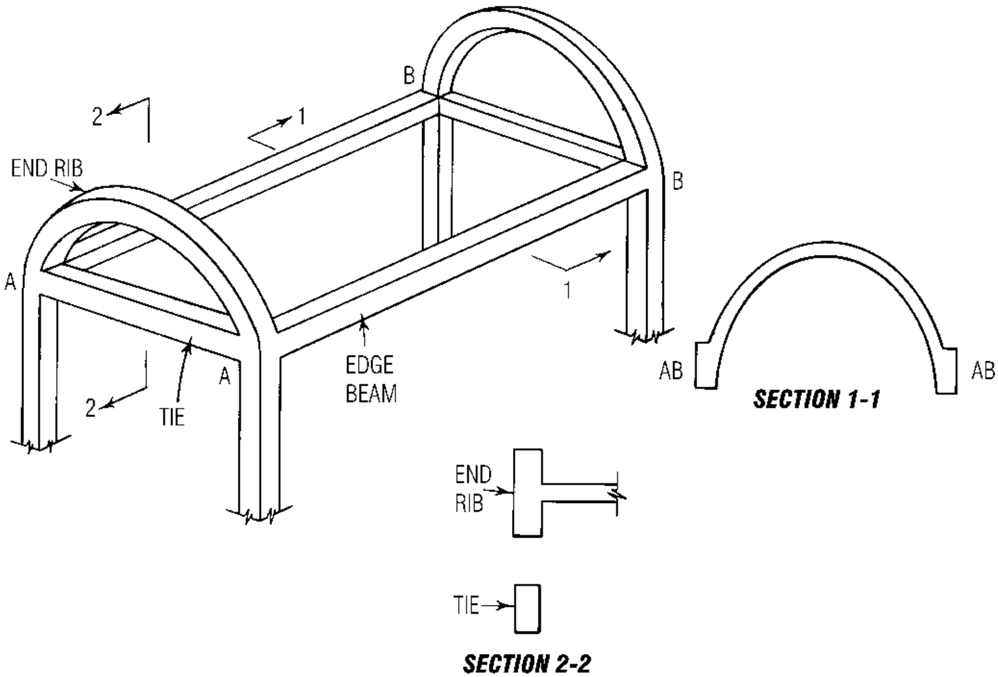


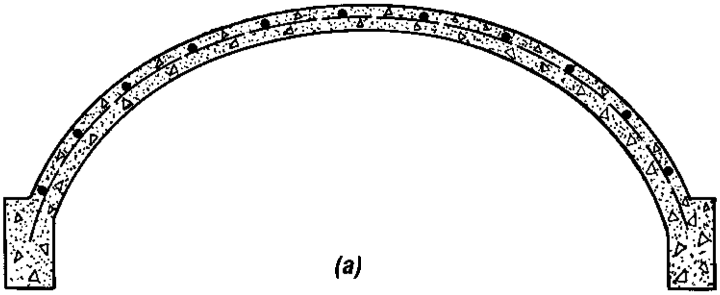

**Fig. 8.60** Stiffening members in thin-shell arch roof.

edge beams *AB* and end arch ribs *AA* and *BB*. Instead of an end arch rib, an end diaphragm may be employed as indicated in Fig. 8.57 for a folded-plate roof.

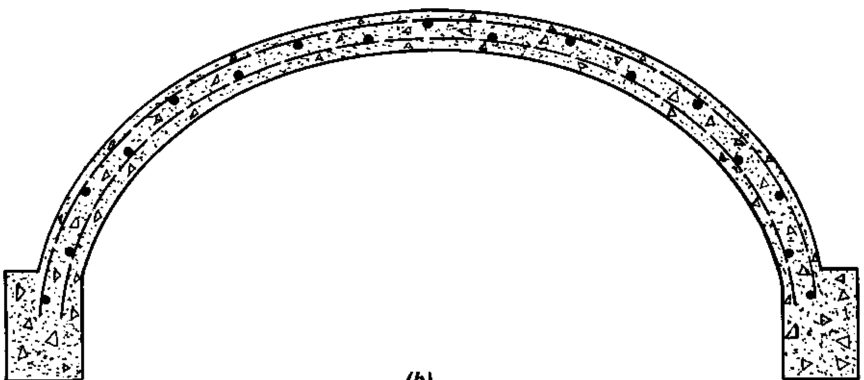
Stresses determined from analysis may be combined to give the principal stresses, or maximum tension and compression, at each point in the shell. If these are plotted on a projection of the shell, the lines of constant stress, or stress trajectories, will be curved. The tensile-stress trajectories generally follow a diagonal pattern near supports and are nearly horizontal around midspan. Reinforcing bars to resist these stresses, therefore, may be draped along the lines of principal stress. This, however, makes fieldwork difficult because large-diameter bars may have to be bent and extra care is needed in placing them. Hence, main steel usually is placed in a grid pattern, with the greatest concentration along longitudinal edges or valleys. To control temperature and shrinkage cracks, minimum reinforcement should be provided.

Reinforcement may be placed in the shell in one layer (Fig. 8.61*a*) or two layers (Fig. 8.61*b*), depending on the stresses; that is, the span and design loads. (Very thin shells, for example, those 3 to $4\frac{1}{2}$ in thick, may offer space for only a single layer.) Shells with one layer of reinforcement are more likely to crack because of local deformations. Although such cracks may not be structurally detrimental, they could permit rainwater leakage. Hence, shells with one layer of reinforcement should have built-up roofing or other waterproofing applied to the outer surface. In reinforcing small-span shells, two-way wire fabric may be used instead of individual bars.

The area of reinforcement, in$^2$/ft width of shell, should not exceed $7.2f'_c/f_y$ or $29{,}000h/f_y$, where $h$ is



(*a*)



(*b*)

**Fig. 8.61** Arch reinforcement: (*a*) single layer; (*b*) double layer.

the overall thickness of the shell, in; $f_y$ the yield strength, psi, of the reinforcement; and $f'_c$ the compressive strength, psi, of the concrete. Reinforcement should not be spaced farther apart than five times the shell thickness or 18 in. Where the computed factored principal tensile stress exceeds $4\sqrt{f'_c}$, the reinforcement should not be spaced farther apart than three times the shell thickness.

Minimum specified compressive strength of concrete $f'_c$ should not be less than 3000 psi, while specified yield strength of reinforcement $f_y$ should not exceed 60,000 psi.

Edge beams of barrel arches behave like ordinary beams under vertical loads, except that additional horizontal shear is applied at the top face at the junction with the shell. (If these shear stresses are high, reinforcement should be provided to resist them.) Also, a portion of the shell equal to the flange width permitted for T beams may be assumed to act with the supporting members. Furthermore, transverse reinforcement from the shell equal to that required for the flange of a T beam should be provided and should be adequately anchored into the edge beam. A typical detail of an edge beam is shown in Fig. 8.62.

Computed stresses in the end arch ribs or diaphragms usually are small. The minimum amount of reinforcement in a rib should be the minimum specified by the ACI Code for a beam and, in a diaphragm, the minimum specified for a slab. Longitudinal reinforcement from the shell should be adequately embedded in the ribs. Because of shear transmission between shell and ribs, the shear stresses should be checked and adequate shear reinforcement provided, if necessary. Typical reinforcement in end ribs and diaphragms is shown in Fig. 8.63.

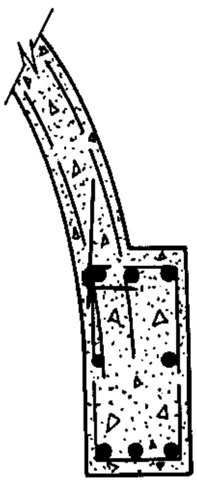


**Fig. 8.62** Edge beam for arch.

High tensile stresses and considerable distortions, particularly in long barrels, usually occur near supports. If the stresses in those areas are not computed accurately, reinforcement should be increased there substantially over that required by simplified analysis. The increased quantity of reinforcement should form a grid. In arches with very long spans and where stresses are computed more accurately, prestressing of critical areas may be efficient and economical. But the ratio of steel to concrete in any portion of the tensile zone should be at least 0.0035.

When barrel shells are subjected to heavy concentrated loads, such as in factory roofs or bridges, economy may be achieved by providing interior ribs (Fig. 8.64), rather than increasing the thickness throughout the whole shell. Such ribs increase both the strength and stiffness of the shell without increasing the weight very much.

In many cases, only part of a barrel shell may be used. This could occur in end bays of multiple barrels or in interior barrels where large openings are to be provided for windows. Stress distribution in such portions of shells is different from that in whole barrels, but design considerations for edge members and reinforcement placement are the same.

### 8.55.3 Domes

These are shells curved in two directions. One of the oldest types of construction, domes were often built of large stone pieces. Having a high ratio of thickness to span, this type of construction is excluded from the family of thin shells.

Concrete domes are built relatively thin. Domes spanning 300 ft have been constructed only 6 in thick. Ratio of rise to span usually is in the range of 0.10 to 0.25.

A dome of revolution is subjected mostly to pure membrane stresses under symmetrical, uniform live load. These stresses are compressive in most of the dome and tensile in some other por-

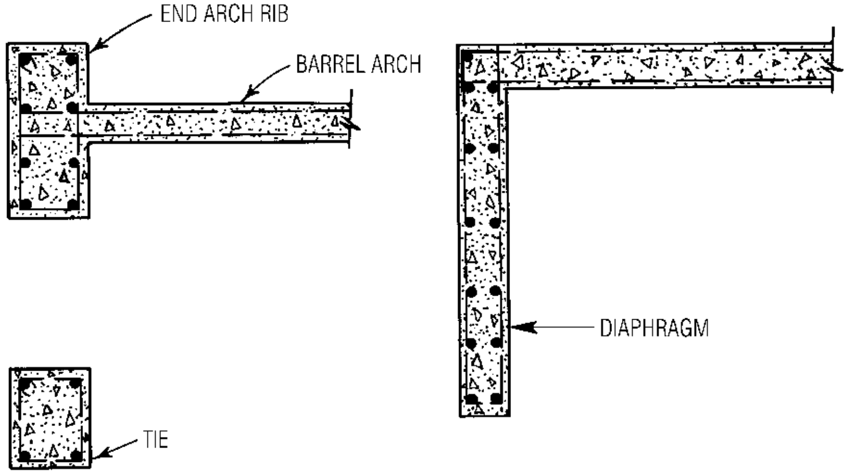



**Fig. 8.63** Reinforcing in end ribs, tie, and diaphragm of an arch.

tions, mainly in the circumferential direction. Under unsymmetrical loading, bending moments may occur. Hence, it is common to place reinforcement both in the circumferential direction and perpendicular to it (Fig. 8.65). The reinforcement may be welded-wire fabric or individual bars. It may be placed in one layer (Fig. 8.65*b*), depending on stresses. Concrete for domes may be cast in

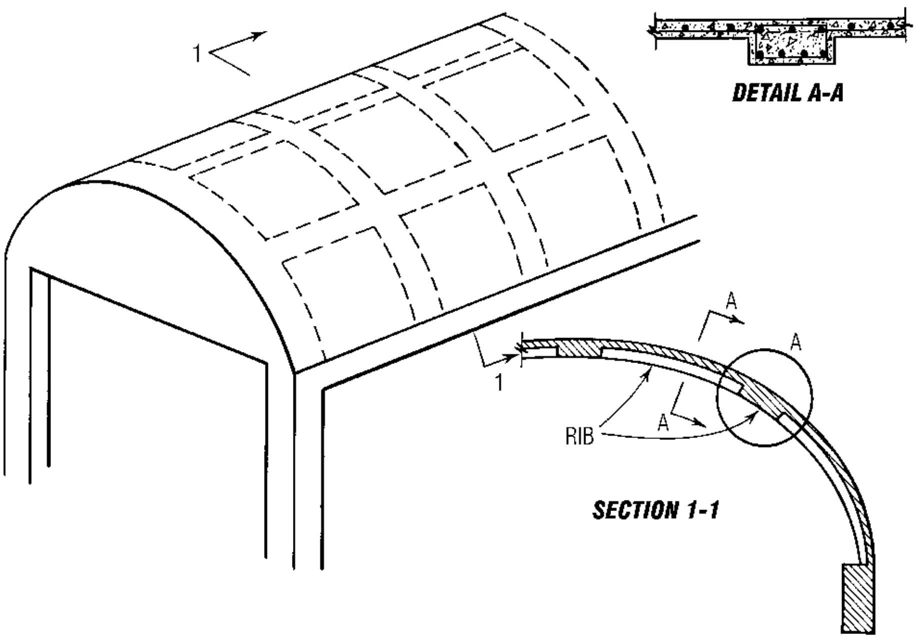



**Fig. 8.64** Arch with ribs in longitudinal and transverse directions.

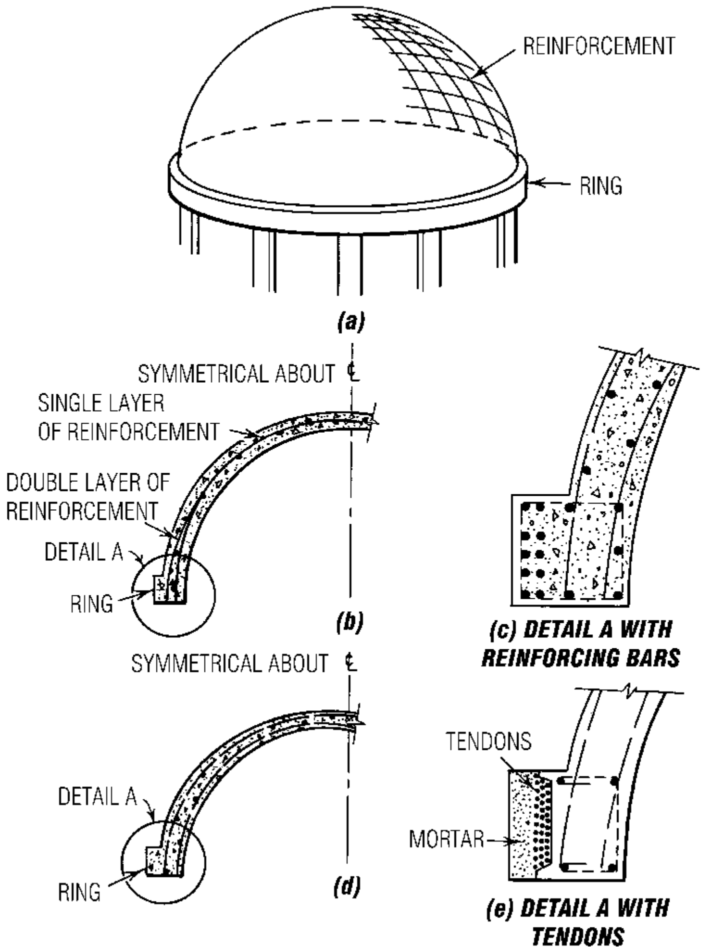



**Fig. 8.65** Reinforcing arrangements for domes.

forms, as are other more conventional structures, or sprayed.

The critical portion of a dome is its base. Whether the dome is supported continuously there, for example, on a continuous footing, or on isolated supports (Fig. 8.65*a*), relatively large bending moments and distortions occur in the shell close to the supports. These regions should be designed to resist the resulting stresses. In domes reinforced with one layer of bars or mesh, it is advisable to provide in the vicinity of the base a double layer of reinforcement (Fig. 8.65*b*). It also is advisable to thicken the dome close to its base.

The base is subjected to a very large outward-acting radial force, causing large circumferential tension. To resist this force, a concrete ring is constructed at the base (Fig. 8.65). The ring and thickening of the concrete shell in the vicinity of the ring help reduce distortions and cracking of the dome at its base.

Reinforcement of the shell should be properly embedded in the ring (detail *A*, Fig. 8.65*c*). The ring should be reinforced or prestressed to resist the circumferential tension. Prestressing is efficient and hence often used. One method of applying prestress is shown in detail *A*, Fig. 8.65*d* and *e*.

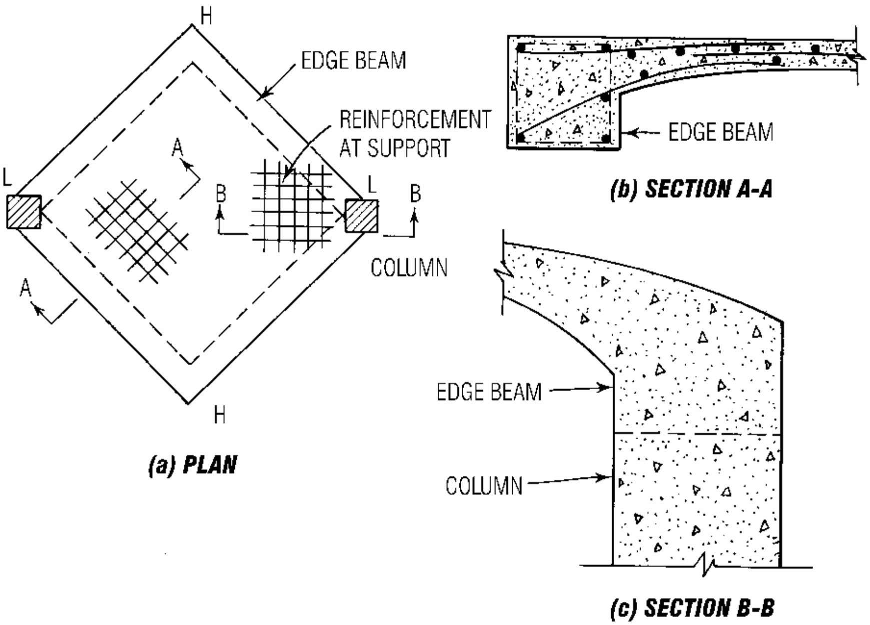



**Fig. 8.66** Hyperbolic-paraboloid shell. *H* indicates high point, *L* low point.

Wires are wrapped under tension around the ring and then covered with mortar, for protection against rust and fire. Stirrups should be provided throughout the ring.

### 8.55.4 Hyperbolic-Paraboloid Shells

Also referred to as a **hypar**, this type of shell, like a dome, is double-curved, but it can be formed with straight boards. Furthermore, since the principal stresses throughout the shell interior consist of equal tension and compression in two perpendicular, constant directions, placement of reinforcement is simple.

Figure 8.66*a* shows a plan of a hypar supported by two columns at the low points *L*. The other corners *H* are the two highest points of the shell. Although strips parallel to *LL* are in compression and strips parallel to *HH* in tension, it is customary to place reinforcement in two perpendicular directions parallel to the generatrices of the shell, as shown at section *A-A*, Fig. 8.66*a*. The reinforcement should be designed for diagonal tension parallel to the generatrices. Since considerable bending moments may occur in the shell at the columns, this region of the shell usually is made thicker than other portions and requires more reinforcement. The added reinforcement may be placed in the *HH* and *LL* directions, as shown at section *B-B*, Fig. 8.66*a*.

Shell reinforcement may be placed in one or two layers, depending on the intensity of stresses and distribution of superimposed load. If the superimposed load is irregular and can cause significant bending moments, it is advisable to place the reinforcement in two layers.

As for other types of shells, edges of a hypar are subjected to larger distortions and bending moments than its interior. Therefore, it is desirable to construct edge beams and thicken the shell in the vicinity of these beams (Fig. 8.66*b*). A double layer of reinforcement at the edge beams helps reduce cracking of the shell in the vicinity of the beams.

The edge beams are designed as compression or tension members, depending on whether the hypar is supported at the low points or high points. Prestress in the shell is most efficient in the vicinity of supports. It also is efficient along the edge beams if supports are at the high points.

### 8.55.5 Shells with Complex Shapes

Curved shells also may be built with more complex shapes. For example, they may be undulating or have elliptical or irregular boundaries. In some cases, they may be derived by inverting structures in pure tension, such as bubbles or fabric hung from posts.

(D. P. Billington "Thin-Shell Concrete Structures," 2nd ed., and A. H. Nilson and G. Winter, "Design of Concrete Structures," 11th ed., McGraw-Hill, Inc., New York.)