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SURVEYING*

Surveying is the science and art of making the measurements necessary to determine the relative positions of points above, on, or beneath the surface of the earth or to establish such points. Surveying continues to undergo important changes.

12.1 Types of Surveys

Plane surveying neglects curvature of the earth and is suitable for small areas.

Geodetic surveying takes into account curvature of the earth. It is applicable for large areas, long lines, and precisely locating basic points suitable for controlling other surveys.

Land, boundary, and cadastral surveys usually are closed surveys that establish property lines and corners. The term *cadastral* is now generally reserved for surveys of the public lands. There are two major categories: **retracement surveys** and **subdivision surveys.**

Topographic surveys provide the location of natural and artificial features and elevations used in map making.

Route surveys normally start at a control point and progress to another control point in the most direct manner permitted by field conditions. They are used for surveys for railroads, highways, pipelines, etc.

Construction surveys are made while construction is in progress to control elevations, horizontal positions and dimensions, and configuration. Such surveys also are made to obtain essential data for computing construction pay quantities.

*Revised and updated from Sec. 12, Surveying, by Roy Minnick in the 3rd edition.

As-built surveys are postconstruction surveys that show the exact final location and layout of civil engineering works, to provide positional verification and records that include design changes.

Hydrographic surveys determine the shoreline and depths of lakes, streams, oceans, reservoirs, and other bodies of water.

Sea surveying covers surveys for port and offshore industries and the marine environment, including measurement and marine investigations by ship-borne personnel.

Solar surveying includes surveying and mapping of property boundaries, solar access easements, positions of obstructions and collectors, determination of minimum vertical sun angles, and other requirements of zoning boards and title insurance companies.

Satellite surveying provides positioning data and imagery, which is received by equipment, stored, and automatically verified in selected data coordinates with each satellite pass. Doppler and global positioning are used as standard practice in remote regions and on subdivided lands.

Global positioning system (GPS) utilizes a constellation of 24 high-altitude navigational satellites positioned in six orbital planes and spaced so that an operator of specialized equipment can receive signals from between five to eight satellites at all times.

Inertial surveying systems acquire coordinate data obtained by use of a helicopter or ground vehicle. Inertial equipment now coming into use has a dramatic impact on the installation of geodetic and cadastral control.

Photogrammetric surveys utilize terrestrial and aerial photographs or other sensors that provide data and can be a part of all the types of surveys listed in the preceding.

12.2 Surveying Sources and Organizations

Land and boundary surveying is a regulated activity; each state licenses those who practice land surveying. Boards are established to test prospective land surveyors and to ensure compliance with state laws. Rosters of licensees are usually maintained. There is no national licensing of land surveyors. Each state defines land surveying, who must be licensed, and the activities that are subject to regulation, and those that are exempt. Information about licensing and regulations may be obtained from the American Congress of Surveying and Mapping (ACSM), 6 Montgomery Village Avenue, Suite 403, Gaithersburg, MD 20879 (www.acsm.org). ACSM is also the national membership organization for all branches of surveying. It can provide information about survey education and licensing, state societies, and state registration boards.

The National Geodetic Survey (NGS), formerly called the U.S. Coast and Geodetic Survey, coordinates activities of the Federal Geodetic Control Committee, which develops standards and specifications for conducting Federal geodetic surveys. NGS is the source for geodetic control data, both historic and current. Information on products, programs, and services may be obtained from the National Geodetic Information Branch, 1315 East-West Highway, Silver Spring, MD 20910 (www.ngs.noaa.gov).

The Geological Survey's (USGS) National Mapping Program is responsible for commonly used 7.5-min quadrangle maps and other multipurpose maps. The Earth Science Information Office, in USGS, informs the public of sources of maps, aerial photographs, digital products, and other cartographic and earth science products. The U.S. Geological Survey and the Earth Science Information Center are located at National Center, John W. Powers Federal Building, 12201 Sunrise Valley Drive, Reston, VA 20192 (www.usgs.gov).

The Bureau of Land Management, Cadastral Survey (BLM) is the agency responsible for survey and resurvey of the public lands. The agency is the source for information about the public lands surveys. A starting place for seeking survey information is the Division of Cadastral Surveys, 1849 C Street NW, MS L302, Washington, DC 20240.

Surveying equipment, using the computer, satellites, and a wide array of other technologies, is evolving rapidly. Two magazines, sent free on request, contain topical articles and useful information about all types of surveying and surveying equipment: Professional Surveyor Magazine, 100 Tuscany Drive, Suite B1, Frederick, MD 21702 (www.profsurv.com), and P.O.B. Magazine, 755 West Big Beaver Rd, Ste 1000, Troy, MI 48084.

See also Art. 12.19.

12.3 Units of Measurement

Units of measurement used in past and present surveys are:

For construction work: feet, inches, fractions of inches

For most surveys: feet, tenths, hundredths, thousandths

For National Geodetic Survey control surveys: metres, 0.1, 0.01, 0.001 m

The most-used equivalents are:

1 metre = 39.37 in (exactly) = 3.2808 ft

1 rod = 1 pole = 1 perch = $16\frac{1}{2}$ ft

1 engineer's chain = 100 ft = 100 links

1 Gunter's chain = 66 ft = 100 Gunter's links (lk) = 4 rods = $\frac{1}{80}$ mi

1 acre = 100,000 sq (Gunter's) links = 43,560 ft^2 = 160 $rods^2$ = 10 sq (Gunter's) chains = 4046.87 m^2 = 0.4047 hectare

1 rood = $\frac{1}{4}$ acre = 40 $rods^2$ (also local unit = $5\frac{1}{2}$ to 8 yd)

1 hectare = 10,000 m^2 = 107,639.10 ft^2 = 2.471 acres

1 arpent = about 0.85 acre, or length of side of 1 square arpent (varies)

1 statute mile = 5280 ft = 1609.35 m

1 mi^2 = 640 acres

1 nautical mile (U.S.) = 6080.27 ft = 1853.248 m

1 fathom = 6 ft

1 cubit = 18 in

1 vara = 33 in (Calif.), $33\frac{1}{3}$ in (Texas), varies

1 degree = $\frac{1}{360}$ circle = 60 min = 3600 s = 0.01745 rad

sin $1^\circ = 0.01745241$

1 rad $= 57^\circ 17' 44.8''$ or about 57.30°

1 grad (grade) $= \frac{1}{400}$ circle $= \frac{1}{100}$ quadrant $= 100$ centesimal min $= 10^4$ centesimals (French)

1 mil $= \frac{1}{6400}$ circle $= 0.05625^\circ$

1 military pace (milpace) $= 2\frac{1}{2}$ ft

12.4 Theory of Errors

When a number of measurements of the same quantity have been made, they must be analyzed on the basis of probability and the theory of errors. After all systematic (cumulative) errors and mistakes have been eliminated, random (compensating) errors are investigated to determine the most probable value (**mean**) and other critical values. Formulas determined from statistical theory and the normal, or Gaussian, bell-shaped probability distribution curve, for the most common of these values follow:

Standard deviation of a series of observations is

$$\sigma_s = \pm\sqrt{\frac{\Sigma d^2}{n-1}} \tag{12.1}$$

where d = residual (difference from mean) of single observation

n = number of observations

The **probable error** of a single observation is

$$PE_s = \pm 0.6745\sigma_s \tag{12.2}$$

(The probability that an error within this range will occur is 0.50.)

The probability that an error will lie between two values is given by the ratio of the area of the probability curve included between the values to the total area. Inasmuch as the area under the entire probability curve is unity, there is a 100% probability that all measurements will lie within the range of the curve.

The area of the curve between $\pm\sigma_s$ is 0.683; that is, there is a 68.3% probability of an error between $\pm\sigma_s$ in a single measurement. This error range is also called the one-sigma or 68.3% confidence level. The area of the curve between $\pm 2\sigma_s$ is 0.955. Thus there is a 95.5% probability of an error between $\pm 2\sigma_s$, and $\pm 2\sigma_s$ represents the 95.5% error (two-sigma or 95.5% confidence level). Similarly, $\pm 3\sigma_s$ is referred to as the 99.7% error (three-sigma or 99.7% confidence level). For practical purposes, a maximum tolerable level often is assumed to be the 99.9% error. Table 12.1 indicates the probability of occurrence of larger errors in a single measurement.

The probable error of the combined effects of accidental errors from different causes is

$$E_{\text{sum}} = \sqrt{E_1^2 + E_2^2 + E_3^2 + \cdots} \tag{12.3}$$

where $E_1, E_2, E_3, \ldots$ are probable errors of the separate measurements.

Error of the mean is

$$E_m = \frac{E_{\text{sum}}}{n} = \frac{E_s\sqrt{n}}{n} = \frac{E_s}{\sqrt{n}} \tag{12.4}$$

where E_s = specified error of a single measurement.

Probable error of the mean is

$$PE_m = \frac{PE_s}{\sqrt{n}} = \pm 0.6745\sqrt{\frac{\Sigma d^2}{n(n-1)}} \tag{12.5}$$

12.5 Significant Figures

These are the digits read directly from a measuring device plus one digit that must be estimated and therefore is questionable. For example, a reading of 654.32 ft from a steel tape graduated in tenths of a foot has five significant figures. In multiplying 798.16 by 37.1, the answer cannot have more significant figures than either number used; i.e., three in this case. The same rule applies in division. In addition or subtraction, for example, 73.148 + 6.93 + 482, the answer will have three significant figures, all on the left side of the decimal point.

Table 12.1 Probability of Error in a Single Measurement

Error	Confidence level, percent	Probability of larger error
Probable ($0.6745\sigma_s$)	50	1 in 2
Standard deviation (σ_s)	68.3	1 in 3
90% ($1.6449\sigma_s$)	90	1 in 10
$2\sigma_s$ or 95.5%	95.5	1 in 20
$3\sigma_s$ or 97.7%	99.7	1 in 370
Maximum ($3.29\sigma_s$)	99.9+	1 in 1000

Small, hand-held and large computers now available provide 10 or more digits, but carrying computation results beyond justifiable significant figures leads to false impressions of precision.

12.6 Measurement of Distance with Tapes

Reasonable precisions for different methods of measuring distances are

Pacing (ordinary terrain): $1/50$ to $1/100$.

Taping (ordinary steel tape): $1/1000$ to $1/10{,}000$. (Results can be improved by use of tension apparatus, transit alignment, leveling.)

Base line (invar tape): $1/50{,}000$ to $1/1{,}000{,}000$.

Stadia: $1/300$ to $1/500$ (with special procedures).

Subtense bar: $1/1000$ to $1/700$ (for short distances, with a 1-s theodolite, averaging angles taken at both ends).

Electronic distance measurement (EDM) devices have been in use since the middle of the twentieth century and have now largely replaced steel tape measurements on large projects. The continued development, and the resulting drop in prices, are making their use widespread. A knowledge of steel taping errors and corrections remains important, however, because use of earlier survey data requires a knowledge of how the measurements were made, common sources for errors, and corrections that were typically required.

Slope Corrections ▪ In slope measurements, the horizontal distance $H = L \cos x$, where $L =$ slope distance and $x =$ vertical angle, measured from the horizontal—a simple hand calculator operation. For slopes of 10% or less, the correction to be applied to L for a difference d in elevation between tape ends, or for a horizontal offset d between tape ends, may be computed from

$$C_s = \frac{d^2}{2L} \tag{12.6}$$

For a slope greater than 10%, C_s may be determined from

$$C_s = \frac{d^2}{2L} + \frac{d^4}{8L^3} \tag{12.7}$$

Temperature Corrections ▪ Table 12.2 lists temperature corrections for steel tapes. Formulas for other tape corrections, ft, with L as the measured distance ft, are as follows: **For incorrect tape length,**

$$C_t = \frac{(\text{actual tape length} - \text{nominal tape length})L}{\text{nominal tape length}} \tag{12.8}$$

For nonstandard tension,

$$C_t = \frac{(\text{applied pull} - \text{standard tension})L}{AE} \tag{12.9}$$

where $A =$ cross-sectional area of tape, in^2

$E =$ modulus of elasticity $= 29{,}000{,}000$ psi for steel

For sag correction between points of support, ft,

$$C = -\frac{w^2 L_s^3}{24P^2} \tag{12.10}$$

where $w =$ weight of tape per foot, lb

$L_s =$ unsupported length of tape, ft

$P =$ pull on tape, lb

Sources and Types of Error ▪ There are three sources of error in taping—instrumental, natural, and personal—and nine general types of errors. Table 12.3 lists the types of errors and their sources and classifies them as systematic or accidental.

All errors in Table 12.3 produce, in effect, an incorrect tape length. Therefore, only four basic tape problems exist: *measuring* a line between fixed points with a tape too long or too short, and *laying out* a line from one fixed point with a tape too long or too short. A simple oneline sketch (Fig. 12.1) with tick marks for nominal and actual tape lengths is a foolproof method for deciding whether to add or subtract the correction in any case.

In base-line measurements with steel or invar tapes (three or more tapes should be used on different sections of the line), corrections are applied for inclination; temperature; nonstandard length of tape, for both full and partial tape lengths; and reduction to sea level.

12.7 Leveling

A few definitions introduce the subject:

Vertical Line ▪ A line to the center of the earth from any point. Commonly considered to coincide with a plumb line.

Table 12.2 Temperature Corrections for Steel Tapes*

Subtract corrections for these temperatures, °F	Length of line, ft				*Add* corrections for these temperatures, °F
	5000	1000	500	100	
68	0.00	0.00	0.00	0.00	68
66	0.06	0.01	0.01	0.00	70
64	0.13	0.03	0.01	0.00	72
62	0.20	0.04	0.02	0.00	74
60	0.26	0.05	0.03	0.01	76
58	0.32	0.06	0.03	0.01	78
56	0.39	0.08	0.04	0.01	80
54	0.46	0.09	0.04	0.01	82
52	0.52	0.10	0.05	0.01	84
50	0.58	0.12	0.06	0.01	86
48	0.65	0.13	0.06	0.01	88
46	0.72	0.14	0.07	0.01	90
44	0.78	0.16	0.08	0.02	92
42	0.84	0.17	0.08	0.02	94
40	0.91	0.18	0.09	0.02	96
38	0.98	0.20	0.10	0.02	98
36	1.04	0.21	0.10	0.02	100
34	1.10	0.22	0.11	0.02	102
32	1.17	0.23	0.12	0.02	104
30	1.24	0.25	0.12	0.02	106
28	1.30	0.26	0.13	0.03	108
26	1.36	0.27	0.14	0.03	110

Example: Given a recorded distance of 8785.32 ft for a line measured when the average temperature is 80°F. Correction to be added is $0.39 + 3(0.08) + 0.04 + 2(0.01) + 0.01 = 0.70$ ft. Because of rounding off in the table, the total correction of 0.70 is 0.01 ft larger than the value computed directly by formula, $C = 0.0000065(T - 68F)L$.

* By permission of Marvin C. May, University of New Mexico.

Table 12.3 Types, Sources, and Classification of Taping Errors

Type of Error	Source*	Classification†	Departure from standard to produce 0.01-ft error for a 100-ft tape
Tape length	I	S	0.01 ft
Temperature	N	S or A	15°F
Tension	P	S or A	15 lb
Sag	N, P	S	$7\frac{3}{8}$ in at center as compared with support throughout
Alignment	P	S	1.4 ft at one end, or $8\frac{1}{2}$ in at midpoint
Tape not level	P	S	1.4 ft
Interpolation	P	A	0.01 ft
Marking	P	A	0.01 ft
Plumbing	P	A	0.01 ft

* I = instrumental, N = natural, P = personal.

† S = systematic, A = accidental.

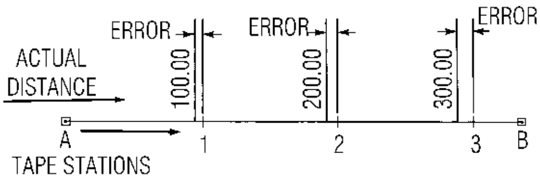

Fig. 12.1 Cumulative error from measuring with a tape that is too long.

Level Surface ▪ A curved surface that, at every point, is perpendicular to a plumb line through the point.

Level Line ▪ A line in a level surface, therefore a curved line.

Horizontal Plane ▪ A plane perpendicular to the plumb line.

Horizontal Line ▪ A straight line perpendicular to the vertical.

Datum ▪ Any level surface to which elevations are referred, such as mean sea level, which is most commonly used; also called datum plane, although not actually a plane.

Mean Sea Level (MSL) ▪ The average height of the surface of the sea. MSL was established originally over a 19-year period, for all tidal stages, at United States and Canadian coastal stations. The basic National Geodetic Vertical Datum net is being connected to all accessible primary tide and water-level stations.

Orthometric Correction ▪ This is a correction applied to preliminary elevations due to flattening of the earth in the polar direction. Its value is a function of the latitude and elevation of the level circuit.

Curvature of the earth causes a horizontal line to depart from a level surface. The departure, C_f, ft; or C_m, m, may be computed from

$$C_f = 0.667.4M^2 = 0.0239F^2 \qquad (12.11a)$$

$$C_m = 0.0785K^2 \qquad (12.11b)$$

where M, F, and K are distances in miles, thousands of feet, and kilometres, respectively, from the point of tangency to the earth.

Refraction causes light rays that pass through the earth's atmosphere to bend toward the earth's surface. For horizontal sights, the average angular displacement (like the sun's diameter) is about 32 min. The displacement, R_f, ft, or R_m, m, is given approximately by

$$R_f = 0.093M^2 = 0.0033F^2 \qquad (12.12a)$$

$$R_m = 0.011K^2 \qquad (12.12b)$$

To obtain the combined effect of refraction and curvature of the earth, subtract R_f from C_f or R_m from C_m.

Differential leveling is the process of determining the difference in elevation of two points. The procedure involves sighting with a level on a ruled rod set on a point of known elevation (backsight or plus sight), then on the rod set on points (or intermediate points) whose elevations are to be determined (foresights). These elevations equal the height of instrument minus the foresight reading. The height of instrument equals the known elevation plus the backsight reading. For accuracy, the sum of backsight and foresight distances should be kept nearly equal.

Elevations commonly are taken to 0.01 ft in engineering surveys and to 0.001 m in precise National Geodetic Survey work.

Table 12.4 shows a typical left-hand page of open-style notes. In closed-style (condensed) notes, B.S., H.I., F.S., and elevation values are placed on the same line, thereby saving space (which is cheap in a field book) but reducing the clarity of steps for beginners. The right-hand page contains benchmark descriptions, sketches, date of survey, names of survey-party members, and information on the weather, equipment used, and other necessary remarks.

As noted in Brinker, Austin, and Minnick, "Note Forms for Surveying Measurements," Landmark Enterprises, Rancho Cordova, Cal.: The critical importance of field notes is sometimes neglected. If any of the five main features used in evaluating notes—accuracy, integrity, legibility, arrangement and clarity—is absent, delays, mistakes and increased costs in completing field work, computations, and mapping result.

Table 12.4 Differential Leveling Notes

Differential Leveling—BM Civil to BM Dorm					
Station	B.S.	H.I.*	F.S.	Elev.†	Dist.
BM Civil				100.00	
	4.08	104.08			175
TP 1			0.20	103.88	180
	6.09	109.97			160
BM Dorm			4.32	105.65	155
	10.17		4.52		670
	4.52				
	5.65				
BM Dorm				105.65	
	4.37	110.02			165
TP 2			6.14	103.88	165
	0.93	104.81			170
BM Civil			4.80	100.01	175
	5.30		10.94		675
			5.30		
			5.64		
			Elev Diff = 5.64 ft		
			Loop Closure = 0.01 ft		

* Height of instrument (H.I.) = elevation + backsight (B.S.).
† Elevation = H.I. − foresight (F.S.).

Digital (electronic) notes for field measurements of angles and distances, as well as reduction of slope distances and computation of coordinates, are now being recorded in various types of Data Collectors. They are displayed and the data automatically recorded. Reading and transcribing errors are thus eliminated, both in the field and office where the data collector automatically transfers the field notes to a computer for processing. The results then go to a printer, which makes working plots and convenient page-width printouts.

Data collectors should not completely replace the field book, which still is used to record backup information, including sketches and notes to show station identification for the permanent project. Actually, since only a small part of total field time is occupied in recording measurements in a field book, the important time-saving advantage of a data collector is gained in the office and plan production.

A usable tool for notekeepers is photography. With a reasonably priced, digital camera a photographic record of monuments set or found, and other field evidence to the survey can be prepared.

Profile leveling determines the elevations of points at known distances along a line. When these points are plotted, a vertical section through the earth's surface is shown. Elevations are taken at full stations (100 ft) or closer in irregular terrain, at breaks in the ground surface, and at critical points such as bridge abutments and road crossings. Profiles are generally plotted on special paper with an exaggeration of from 5:1 up to 20:1, or even more, so that elevation differentials will show up better. Profiles are needed for route surveys, to select grades and find earthwork quantities. Elevations are usually taken to 0.01 ft on bench marks and 0.1 ft on the ground.

Reciprocal leveling is used to cross streams, lakes, canyons, and other topographic barriers that prevent balancing of backsights and foresights. On each side of the obstruction to be crossed, a plus sight is taken on the near rod and several minus sights on the far rod. The resulting differences in elevation are averaged to eliminate the effects of curvature and refraction, and inadjustment of the instrument. Even though a number of minus sights are taken for averaging, their length may reduce the accuracy of results.

Borrow-pit or cross-section leveling produces elevations at the corners of squares or rectangles whose sides are dependent on the area to be covered, type of terrain, and accuracy desired. For example, sides may be 10, 20, 40, 50, or 100 ft. Contours can be located readily, topographic features not so well. Quantities of material to be excavated or filled are computed, in cubic yards, by selecting a grade elevation, or final ground elevation, computing elevation differences for the corners, and substituting in

$$Q = \frac{nxA}{108} \tag{12.13}$$

where n = number of times a particular corner enters as part of a division block

x = difference in ground and grade elevation for each corner, ft

A = area of each block, ft^2

Cross-section leveling also is the term applied to the procedure for locating contours or taking elevations on lines at right angles to the center line in a route survey.

Three-wire leveling is a type of differential leveling with three horizontal sighting wires in the level. Upper, middle, and lower wires are read to obtain an average value for the sight, check the precision of reading the individual wires, and secure stadia distances for checking lengths of backsights and foresights. The height of instrument is not needed or computed. The National Geodetic Survey has long used three-wire leveling for its control work, but more general use is now being made of the method.

Grade designates the elevation of the finished surface of an engineering project and also the rise or fall in 100 ft of horizontal distance, for example, a 4% grade (also called gradient). Note that since the common stadia interval factor is 100, the difference in readings between the middle and upper (or lower) wire represents ½ ft in 100 ft, or a ½% grade.

Types of levels in general use are listed in Table 12.5.

Special construction levels include the Blout & George Laser Tracking Level (which can search a 360° horizontal plane and lock on a pocket-size target), the Dietzgen Laser Swinger, Spectra-Physics Rotolite Building Laser, and AGL Construction Laser. Some laser instruments are available for shaft plumbing and setups inside large pipe lines.

12.8 Vertical Control

The National Geodetic Survey provides vertical control for all types of surveys. NGS furnishes descriptions and elevations of bench marks on request. As given in "Standards and Specifications for Geodetic Control Networks," Federal Geodetic Control Committee, the relative accuracy C, mm,

Table 12.5 Types of Levels

Type	Use
Hand level	Rough work. Sights on ordinary level rod limited to about 50 ft because of zero- to 2-power magnification
Engineer's level, Wye or Dumpy	Suitable for ordinary work (third- or fourth-order). Elevations to 0.01 ft without target
Tilting level	Faster, more accurate sighting. Good for third-, second-, or first order work depending upon refinement
Self-leveling, automatic levels	Fast, suitable for second-order and third-order work
Precise level	Very sensitive level vials, high magnification, tilting, other features

Note: Instruments are arranged in ascending order of cost.

required between directly connected bench marks for the three orders of leveling is:

First-order: $C = 0.5\sqrt{K}$ for Class I and $0.7\sqrt{K}$ for Class II

Second-order: $C = 1.0\sqrt{K}$ for Class I and $1.3\sqrt{K}$ for Class II

Third-order: $C = 2.0\sqrt{K}$

where K is the distance between bench marks, km.

12.9 Magnetic Compass

A magnetic compass consists of a magnetized needle mounted on a pivot at the center of a graduated circle. The compass is now used primarily for retracement purposes and checking, although some surveys not requiring precision are made with a compass, for example, in forestry and geology. American transits have traditionally come with a long compass needle, whereas on optical instruments the compass is merely an accessory, and therefore the instruments can be smaller and lighter.

A small weight is placed on the south end of the needle in the northern hemisphere to counteract the dip caused by magnetic lines of force. Since the magnetic poles are not located at the geographic poles, a horizontal angle (declination) results between the axis of the needle and a true meridian. East declination occurs if the needle points east of true (due) north, west declination if the needle points west of true north.

The National Geodetic Survey publishes a world chart every fifth year showing the positions of the agonic line, isogonic lines for each degree, and values for annual variation of the needle. The **agonic line** is a line of zero declination; i.e., a magnetic compass set up on points along this line would point to true north as well as magnetic north. For points along an isogonic line, declination should be constant, barring local attraction.

Table 12.6 lists the periodic variations in the declination of the needle that make it unreliable. In addition, local attraction resulting from power sources, metal objects, etc., may produce considerable error in bearings taken with a compass. If the source of local attraction is fixed and constant, however, angles between bearings are correct, even though the bearings are uniformly distorted.

The Brunton compass or pocket transit has some of the features of a sighting compass, a prismatic compass, a hand level, and a clinometer. It is suitable for some forest, geological, topographical, and preliminary surveys of various kinds.

A common problem today is the conversion of past magnetic bearings based upon the declination of a given date to present bearings with today's declination, or to true bearings. A sketch, such as Fig. 12.2, showing all data with pencils of different colors, will make the answer evident.

12.10 Bearings and Azimuths

The direction of a line is the angle measured from any reference line, such as a magnetic or true meridian. Bearings are angles measured from the north and the south, toward the east or the west. They can never be greater than 90° (Fig. 12.3).

Bearings read in the advancing direction are forward bearings; those in the opposite direction are back bearings. Computed bearings are obtained by using a bearing and applying a direct, deflection, or other angle. Bearings, either magnetic or true, are used in rerunning old surveys, in computations, on maps, and in deed descriptions.

An azimuth is a clockwise angle measured from some reference line, usually a meridian. Government surveys use geodetic south as the base of azimuths. Other surveys in the northern hemisphere may employ north. Azimuths are advantageous in topographic surveys, plotting, direction

Table 12.6 Periodic Variations in Declination of Magnetic Needle

Variation	Remarks
Secular	Largest and most important. Produces wide unpredictable swings over a period of years, but records permit comparison of past and present declinations
Daily (diurnal)	Swings about 8 min per day in the U.S. Relatively unimportant
Annual	Periodic swing amounting to less than 1 min of arc; it is unimportant
Irregular	From magnetic storms and other sources. Can pull needle off more than a degree

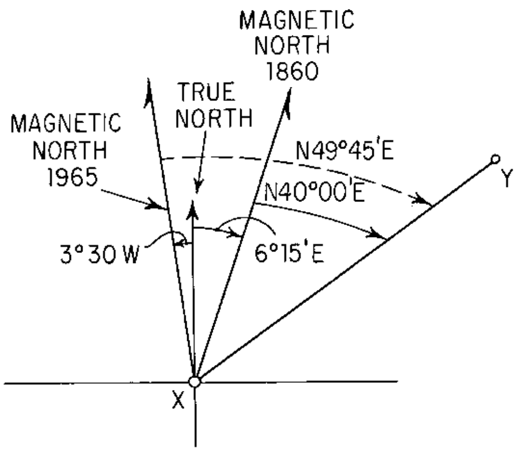

Fig. 12.2 Magnetic bearing of a line *XY* in a past year is found by plotting magnetic north for that year with respect to true north.

problems, and other work where omission of the quadrant letters and a range of angular values from 0 to 360° simplify the work.

12.11 Horizontal Control

All surveys require some kind of control, be it a base line or bench mark, or both. Horizontal control consists of points whose positions are established by traverse, triangulation, or trilateration. The National Geodetic Survey has established control monuments throughout the country and tabulated azimuths, latitude and longitude, statewide coordinates, and other data for them. Surveys on the statewide coordinate system have increased the number of control points available to all surveyors.

12.11.1 Traverses

For a traverse, the survey follows a line from point to point in succession. The lengths of lines between points and their directions are measured. If the traverse returns to the point of origin, it is called a closed traverse. The United States–Canada boundary, for example, was run by traverse. In contrast, the boundary of a construction site would be surveyed by a closed traverse. Permissible closures for traverses that make a closed loop or connect adjusted positions of equal-order or higher-order control surveys are given in Table 12.7.

Transit-tape traverses provide control for areas of limited size as well as for the final results on property surveys, route surveys, and other work. Stadia traverses are good enough for small-area topographic surveys when tied to higher control. Faster and more accurate traversing may be accomplished with electronic distance-measuring devices and with theodolites with direct readings to seconds and much lighter than the older-type, bulky transits.

As a result of developing technologies, the acceptable ratio of error to distance measured for

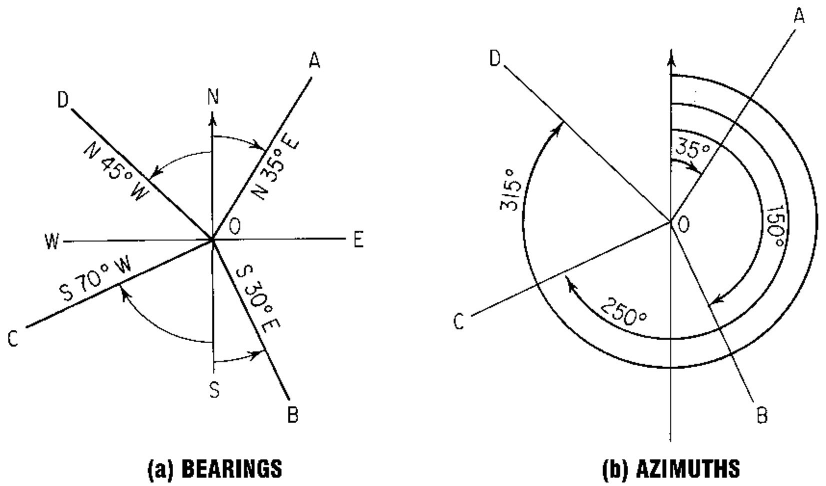

Fig. 12.3 Direction of lines may be specified by (*a*) bearings or (*b*) azimuth.

Table 12.7 Periodic Traverse Closures*

Traverse order	Max permissible closure after azimuth adjustment	Max azimuth closure at azimuth checkpoint: Sec per station	Max azimuth closure at azimuth checkpoint: Sec†
First order	1:100,000	1.0	$2\sqrt{N}$
Second order			
Class I	1:50,000	1.5	$3\sqrt{N}$
Class II	1:20,000	2.0	$6\sqrt{N}$
Third order			
Class I	1:10,000	3.0	$10\sqrt{N}$
Class II	1:5,000	8.0	$30\sqrt{N}$

* National Geodetic Survey.
† N = number of stations.

various types of survey is being reviewed and is subject to change. To obtain the latest recommendations, contact the organizations mentioned in Art. 12.2. See also Art. 12.19.

12.11.2 Triangulation

In triangulation, points are located at the apexes of triangles, and all angles and one base line are measured. Additional base lines are used when a chain of triangles, quadrilaterals, or central-point figures is required (Fig. 12.4). All other sides are computed and adjustments carried from the fixed base lines forward and backward to minimize the corrections. Angles used in computation should exceed 15°, and preferably 30°, to avoid the rapid change in sines for small angles.

Chains of triangles are unsuitable for high-precision work since they do not permit the rigid adjustments available in quadrilaterals and more complicated figures. Quadrilaterals are advantageous for long, relatively narrow chains; polygons and central-point figures for wide systems and perhaps for large cities, where stations can be set on tops of buildings.

Strength of figure in triangulation is an expression of relative precision possible in the system based on the route of computation of a triangle side. It is independent of the accuracy of observations and utilizes the number of directions observed, conditions to be satisfied, and rates of changes for the sines of distance angles. Triangulation stations that cannot be occupied require additional computation for reduction to center in obtaining coordinates and other data.

Permissible triangulation closures for the three orders of triangulation specified by the National Geodetic Survey are given in Table 12.8 and specifications for base-line measurements in Table 12.9.

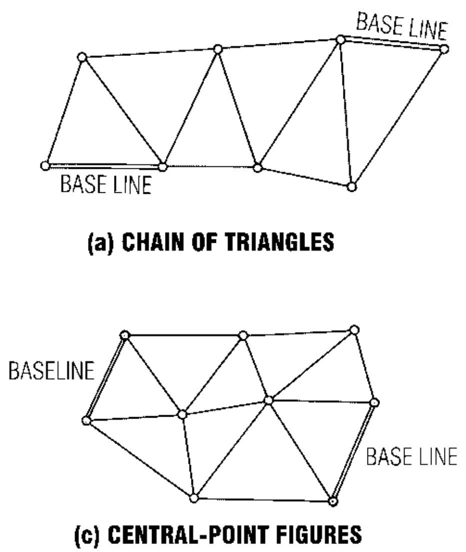

Fig. 12.4 Triangular chains.

Table 12.8 Triangulation Closures

Specification, item	First order	Second order Class I	Second order Class II	Third order Class I	Third order Class II
Avg. triangle closure, s	1.0	1.2	2.0	3.0	2.0
Max triangle closure, s	3.0	3.0	5.0	5.0	10.0

12.11.3 Trilateration

This method has replaced triangulation for establishment of control in many cases, such as photogrammetry, since the development of electronic measuring devices. All distances are measured and the angles computed as needed.

12.11.4 Trilateration vs. Triangulation

In triangulation, one or more base lines and all angles are measured. Astronomical observations made at some monuments control directions. In trilateration, lengths of all lines to be used are measured, slope and atmospheric corrections applied, and astronomical observations taken at intervals. Reading some directions contributes additional strengthening.

Various field and office studies show that time and cost for triangulation or trilateration are about the same for some networks. A combination of observed directions and distances determined with electronic distance-measuring instruments may be best. Trilateration networks covering basically square blocks provide a better strength of figure than long narrow chains (where a number of angles should be read also).

Table 12.9 Specifications for Base-Line Measurements

Order	Max standard error of base
First	1/1,000,000
Second	
Class I	1/900,000
Class II	1/800,000
Third	
Class I	1/500,000
Class II	1/250,000

If one monument is fixed and one azimuth known, both trilateration and triangulation surveys can be extended through other points.

Usefulness of the trilateration method is not confined to high-order-precision large geodetic control networks. Field-proven (using only simple mathematics) satisfactory closures are obtained for small jobs with reasonably strong triangles.

12.12 Stadia

Stadia is a method of measuring distances by noting the length of a stadia or level rod intercepted between the upper and lower sighting wires of a transit, theodolite, or level. Most transits and levels have an interval between stadia wires that gives a vertical intercept of 1 ft on a rod 100 ft away. A stadia constant varying from about $\frac{3}{4}$ to $1\frac{1}{4}$ ft (usually assumed to be 1 ft) must be added for older-type external-focusing telescopes. The internal-focusing short-length telescopes common today have a stadia constant of only a few tenths of a foot, and so it can be neglected for normal readings taken to the nearest foot.

Figure 12.5 shows stadia relationships for a horizontal sight with the older-type external-focusing telescope. Relationships are comparable for the internal-focusing type.

For horizontal sights, the stadia distance, ft (from instrument spindle to rod), is

$$D = R\frac{f}{i} + C \qquad (12.14)$$

where R = intercept on rod between two sighting wires, ft

f = focal length of telescope, ft (constant for specific instrument)

i = distance between stadia wires, ft

$C = f + c$

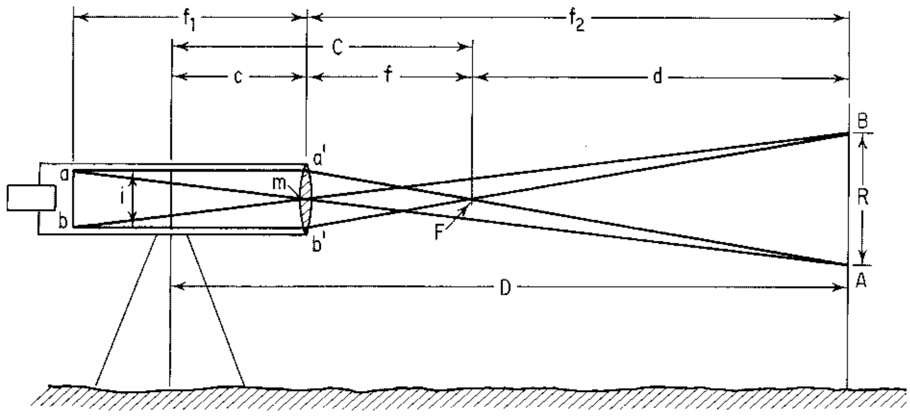

Fig. 12.5 Distance D is measured with an external-focusing telescope by determining interval R intercepted on a rod AB by two horizontal sighting wires a and b.

c = distance from center of spindle to center of objective lens, ft

C is called the stadia constant, although c and C vary slightly.

The value of f/i, the stadia factor, is set by the manufacturer to be about 100, but it is not necessarily 100.00. The value should be checked before use on important work, or when the wires or reticle are damaged and replaced.

For inclined sights (Fig. 12.6) the rod is held vertical, as indicated by a rod level or other means because it is difficult to assure perpendicularity to the sight line on sloping shots. Reduction to horizontal and vertical distances is made according to formulas, such as

$$H = 100R - 100R \sin^2 \alpha + C \tag{12.15}$$

$$V = 100R(\tfrac{1}{2} \sin 2\alpha) \tag{12.16}$$

where H = horizontal distance from instrument to rod, ft

V = vertical distance from instrument to rod, ft

α = vertical angle above or below level sight

A Beaman arc on transits and alidades simplifies reduction of slope sights. It consists of an H scale and a V scale, both graduated in percent, with spacing based on the stadia formulas. The H scale gives the correction per 100 ft of slope distance, which is subtracted from $100R + C$ to get the horizontal distance. A V-scale index of 50 for level sights eliminates minus values in determining vertical distance. Readings above 50 are angles of elevation; below 50, angles of depression. Each unit above or below 50 represents 1-ft difference in elevation per 100 ft of sight. By setting the V scale to a whole number, even though the middle wire does not fall on the height of the instrument, you need only mental arithmetic to compute vertical distance. The H scale is read by interpolation since the value generally is small and falls in the area of wide spaces.

As an illustration, to determine the elevation of a point X from a setup at point Y, compute elevation X = elevation Y + height of instrument + (arc reading − 50)(rod intercept) − reading of middle wire.

Some self-reducing tachymeters have curved stadia lines engraved on a glass plate, which turns and appears to make the lines move closer or farther apart. A fixed stadia factor of 100 is used for horizontal reduction, but several factors are required for elevation differences, depending on the slope.

Stadia traverses can be run with direct or azimuth angles. Distances and elevation differences should be averaged for the foresights and backsights. Elevation checks on bench marks are necessary at frequent intervals to maintain reasonable precision.

Poor closures in stadia work usually result from incorrect rod readings rather than errors in angles.

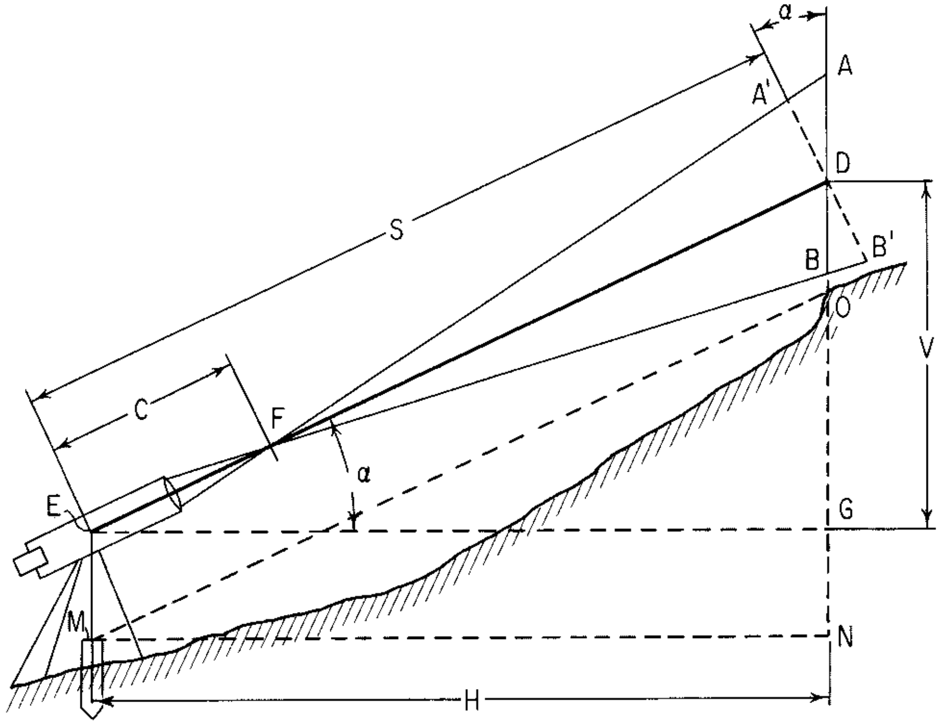

Fig. 12.6 Stadia measurement of vertical distance V and horizontal distance H by reading with a telescope the rod intercept AB and vertical angle α.

A difference of 1 min in vertical angle has little effect on horizontal distances and produces a correction smaller than 0.1 ft for sights up to 300 ft.

Stadia distances, normally read to the nearest foot, are assumed to be valid within about ½ ft. For the same line and lateral error on a 300-ft shot, $\sin \alpha = \frac{1}{2}/300 = 0.00167$ and $\alpha = 5.7$ min. Thus for stadia sights up to 300 ft, comparable distance-angle precision is obtained by reading horizontal angles to the nearest 5 or 6 min. This can be done by estimation on the scale without using the vernier graduations. (See Fig. 12.7.)

Closely approximate answers to many problems in surveying, engineering, mechanics, and other

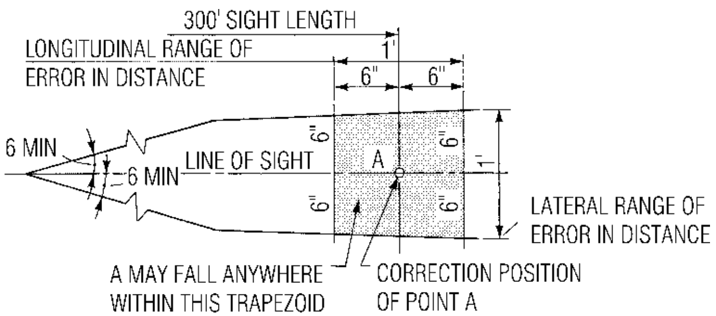

Fig. 12.7 Comparable precision of angles and stadia distances.

fields can be computed mentally by memorizing the sin of $1' = 0.00029$ (or roughly 0.0003), and $\sin 1^\circ = 0.01745$ (about $0.01\frac{3}{4}$). For sines of angles from 0° to 10°, the numerical values increase almost linearly. The divergence from true value at 10° is only $\frac{1}{2}\%$; at 30° just $4\frac{1}{2}\%$—high for surveying but within, say, some design load estimates. Values of tangents found by multiplying the tangent of 1° by other angular sizes diverge more rapidly but still are off only 1% at 10°.

12.13 Topographic Surveys

Topographic surveys are made to locate natural and constructed features for mapping purposes. By means of conventional symbols, culture (bridges, buildings, boundary lines, etc.), relief, hydrography, vegetation, soil types, and other topographic details are shown for a portion of the earth's surface.

Planimetric (line) maps define natural and cultural features in plane only. **Hypsometric maps** give elevations by contours, or less definitely by means of hachures, shading, and tinting.

Horizontal and vertical control of a high order is necessary for accurate topographic work. Triangulation, trilateration, traversing, and photogrammetry furnish the skeleton on which the topographic details are hung. A level net must provide elevations with closures smaller than expected of the topographic traverse and side shots. For surveys near lake shores or slow-moving streams, the water surface on calm days is a continuous bench mark.

Seven methods are used to locate points in the field, as listed in Table 12.10. The first four require a *base line* of known length. An experienced instrument person selects the simplest method considering both fieldwork and office work involved.

A **contour** is a line connecting points of equal elevation. The shoreline of a lake not disturbed by wind, inlet, or outlet water forms a contour. The vertical distance (elevation) between successive contours is the contour interval. Intervals commonly used are 1, 2, 5, 10, 20, 25, 40, 50, 80, and 100 ft, depending on the map scale, type of terrain, purpose of the map, and other factors.

Methods of taking topography and pertinent points on the suitability of each for given conditions are given in Table 12.11.

12.14 Satellite Doppler Positioning

Satellite Doppler positioning is a three-dimensional measurement system based on the radio signals of the U.S. Navy Navigational Satellite System (NNSS), commonly referred to as the TRANSIT system. Satellite Doppler positioning is used primarily to establish horizontal control.

The Doppler observations are processed to determine station positions in Cartesian coordinates, which can be transformed to geodetic coordinates (geodetic latitude and longitude and height above reference ellipsoid). There are two methods by which station positions can be derived: point positioning and relative positioning.

Point positioning, for geodetic applications, requires that the processing of the Doppler data be performed with the precise ephemerides that are supplied by the Defense Mapping Agency. In this method, data from a single station are processed to yield the station coordinates.

Table 12.10 Methods for Locating Points in the Field

Method	Principal use
1. Two distances	Short taping, details close together, trilateration
2. Two angles	Graphical triangulation, phone table
3. One angle, adjacent distance	Transit and stadia
4. One angle, opposite distance	Special cases
5. One distance, right-angle offset	Route surveys, curved shorelines, or boundaries
6. String lines from straddle hubs	Referencing hubs for relocation
7. Two angles at point to be located	Three-point location for planetable, navigation

Table 12.11 Methods of Obtaining Topography

Method	Suitability
Transit and tape	Accurate, but slow and costly. Used where accuracy beyond plotting precision is desired
Transit and stadia	Fast, reasonably accurate for plotting purposes. Contours by direct (trace contour) method in gently rolling country, or by indirect (controlling-point) system where high, low, and break points are found in rugged terrain, or on uniform slopes, and contours interpolated
Planetable	Plotting and checking in field. Good in cluttered areas of many details. Contours by direct or indirect method. Now generally replaced by photogrammetry for large areas. Used to check photogrammetric maps
Coordinate squares	Better for contours than culture. Elevations at corners and slope changes interpolated for contours. Size of squares dependent on area covered, accuracy desired, and terrain. Best in level to gently rolling country
Offsets from center line or cross sectioning	On route surveys, right-angle offsets taken by eye or prism at full stations and critical points, along with elevations, to get a cross profile and topographic details. Contours by direct or indirect method. Elevations or contours recorded as numerator, and distance out as denominator
Photogrammetry	Fast, cheap, and now commonly used for large areas covering any terrain, where ground can be seen. Basic control by ground methods, some additional control from photographs

Relative positioning is possible when two or more receivers are operated together in the survey area. The processing of the Doppler data can be performed in four modes: simultaneous point positioning, translocation, semishort arc, and short arc. The specifications for relative positioning are valid only for data reduced by the semishort- or short-arc methods. The semishort-arc mode allows up to 5 degrees of freedom in the ephemerides; the short arc mode allows 6 or more degrees of freedom. These modes allow the use of the broadcast ephemerides in place of the precise ephemerides.

See also Arts. 12.2 and 12.18.

12.15 Global Positioning System (GPS)

This system utilizes radio signals from a worldwide set of navigational satellites, which broadcast continuously on two L-band carrier frequencies. These provide coded information, such as predicted satellite ephemeris, satellite identification, and time data. Each satellite provides strong radio signals that can be compared with the same signals arriving at other positions on earth for determination of relative earth positions (Fig. 12.8). For the most precise measurements, the surveyor should have three or more receivers simultaneously observing GPS satellites. When four satellites are observed simultaneously, it is possible to determine the timing and three-dimensional positioning of a ground receiver. In effect, the satellites serve as control points and ground positions are determined by distance–distance intersection. Compared with satellite Doppler positioning, GPS offers an order-of-magnitude increase in accuracy and shorter occupation time.

A variation of this system known as real-time kinematic (RTK) GPS offers advantages over other systems for boundary surveys. It enables a surveyor to determine the position of a corner and establish a corner without having to make

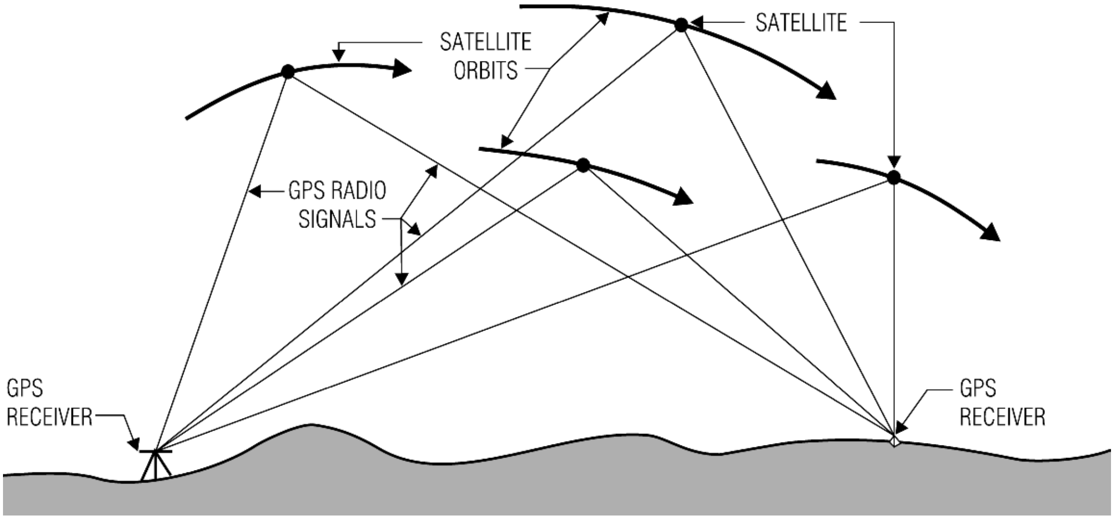

Fig. 12.8 Radio signals from orbiting GPS satellites determine the relative position of receivers on the earth's surface.

traditional corner moves with conventional surveying instruments and practices and without having to postprocess the data. An RTK GPS system generally comprises two or more GPS receivers, three or more radio modems, a fixed-plate initializer, and a hand-held data collector and portable computer. One receiver occupies a control point and broadcasts a correction message, or compact measurement record, to one or more roving receivers. These process the information to produce an accurate position relative to the control point. (C. W. Sumpter and G. W. Asher, "GPS Goes Real Time," *Civil Engineering*, September 1994, p. 64.)

See also Arts. 12.2 and 12.18.

12.16 Inertial Surveying

The inertial surveying system (ISS) is a relative positioning system, in which changes in position are determined from measurements of acceleration and time and by sensing the earth's rotation and the local vertical direction. Distance components are measured from an initial known reference location, used as a control point, and new positions are located relative to that point. Equipment required, which may be mounted on a light-duty truck or a helicopter, consists of accelerometers, stabilized by gyroscopes and mounted on an inertial platform, and control and data-handling components, including a computer. The system is self-contained and has no line-of-sight limitations. The equipment can be moved rapidly and produces three-dimensional geodetic positions with an accuracy acceptable for many purposes.

Inertial surveying is a measurement system composed of lines or a grid of ISS observations (Fig. 12.9). Specifications given in Table 12.12 cover use of ISS only for horizontal control.

Each inertial survey line is required to tie into a minimum of four horizontal network control points spaced well apart and should begin and end at network control points. These network control points must have horizontal datum values better than the intended order (and class) of the new survey. Whenever the shortest distance between two new unconnected survey points is less than 20% of the distance between those points traced along existing or new connections, then a direct connection should be made between those two survey points. In addition, the survey should connect to any sufficiently accurate network control points within the distance specified by the station spacing. The connections may be measured by electronic distance measurement or tape traverse, or by another ISS line. If an ISS line is used, then these lines should follow the same specifications as all other ISS lines in the survey.

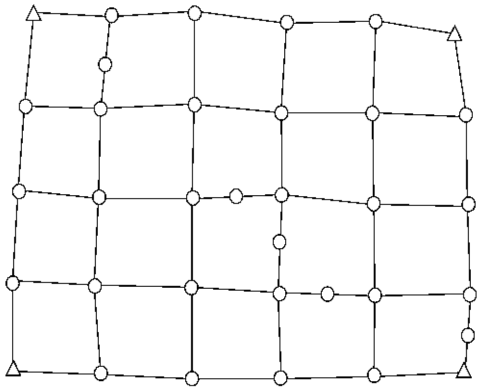

Fig. 12.9 Inertial grid configuration for an inertial traverse survey.

For extended area surveys by ISS, a grid of intersecting lines that satisfies the 20% rule stated above can be designed.

A grid of intersecting lines should contain a minimum of eight network points, and should have a network control point at each corner. The remaining network control points may be distributed about the interior or the periphery of the grid. However, there should be at least one network control point at an intersection of the grid lines near the center of the grid. If the required network points are not available, then they should be established by some other measurement system.

See also Arts. 12.2 and 12.18.

12.17 Photogrammetry

Photogrammetry is the art and science of obtaining reliable measurements by photography (metric photogrammetry) and qualitative evaluation of image data (photo interpretation). It includes use of terrestrial, close-range, aerial, vertical, oblique, strip, and space photographs along with their interpretation. Remote sensing and side-looking radar are also used. Some advantages of mapping by aerial photographs are rapid coverage of large accessible or inaccessible areas and assurance of getting all visible detail. Note that an aerial photo is not a map, i.e., an orthographic projection; it is a perspective projection and may contain unnecessary details that overshadow the critical ones. However, orthophotos may be prepared from a pair of overlapping photos to eliminate the perspective factor and can serve as topographic maps.

Four of the five cameras most commonly used for topographic mapping have f5.6 lenses. They may have narrow-, normal-, wide-, or ultrawide-angle lenses. Most use roll film. Four fiducial marks printed on each photograph locate the geometric axes and principal point. Photographs are taken in strips with a side lap (strip overlap) of 25% and a forward overlap (advance) averaging 60% to ensure that images of ground points appear in at least two and preferably three or more pictures.

Because vertical photographs are perspective views, the scale is not uniform. Equal-length ground lines at higher elevations and near the edges of photographs will be longer than those at lower elevations and near the center. An average scale can be selected as an approximate value.

The basic photogrammetric formulas presented in the following paragraphs are used by the equipment and operators to make measurements and draw maps. Three types available use direct-optical, mechanical, or optical-mechanical projection systems. Among the various models are the Multiplex, Balplex, Kelsh, Zeiss Double Projection, Planimat and Stereoplanigraph, Wild Aviograph and Autograph A10, and Kern PG2.

Scale formulas are as follows (refer to Fig. 12.10)

$$\frac{\text{Photo scale}}{\text{Map scale}} = \frac{\text{photo distance}}{\text{map distance}} \tag{12.17}$$

$$\text{Photo scale} = \frac{ab}{AB} = \frac{f}{H - h_1} \tag{12.18}$$

Table 12.12 Network Geometry

Order class	Second I	Second II	Third I	Third II
Minimum station spacing, km	10	4	2	1
Maximum deviation from straight line connecting endpoints, deg	20	25	30	35

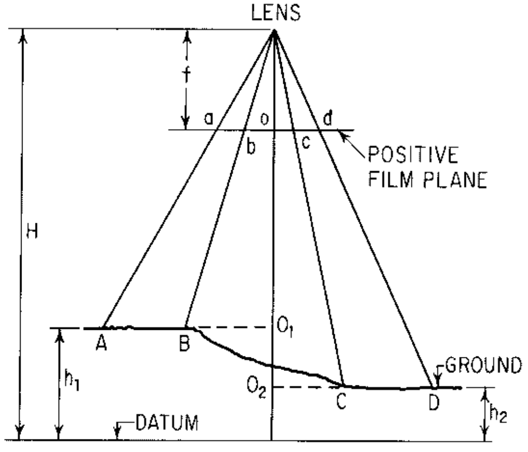

Fig. 12.10 Photographic scale depends on focal length of lens f and height H of airplane. *(Reprinted with permission from R. C. Brinker, "Elementary Surveying," Harper & Row, Publishers, New York.)*

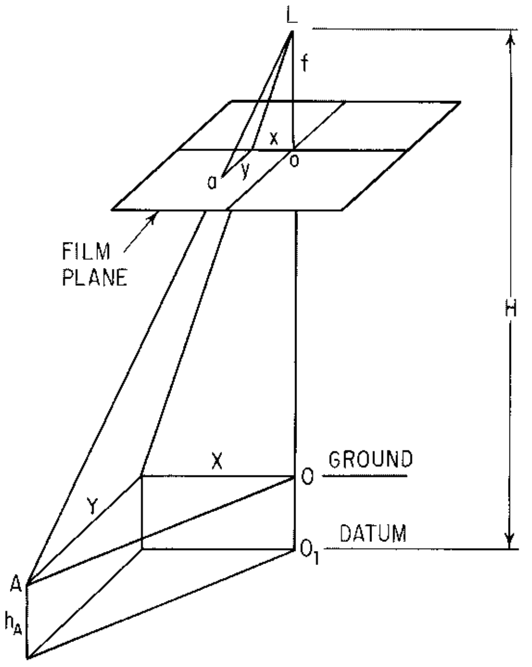

Fig. 12.11 Photograph coordinates x, y are proportional to ground coordinates X, Y when the optical axis is vertical. *(Reprinted with permission from R. C. Brinker, "Elementary Surveying," Harper & Row, Publishers, New York.)*

where f = focal length of lens, in

H = flying height of airplane above datum (usually mean sea level), ft

h_1 = elevation of point, line, or area with respect to datum, ft

Ground distances can be found from measurements on a photograph by using photograph coordinates x, y and ground coordinates X, Y (Fig. 12.10 and 12.11). For a line AB with unequal elevations at A and B, length is determined by

$$AB = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2} \tag{12.19}$$

where $X_A = x_a(H - h_A)/f$

$Y_A = y_a(H - h_A)/f$

$X_B = x_b(H - h_B)/f$

$Y_B = y_b(H - h_B)/f$

Average displacements caused by topographic relief on vertical aerial photographs always radiate from the principal point o (Fig. 12.12), which is directly above the nadir point O on the ground when the optical axis is vertical. The displacement d, in, is the distance on a photograph from the image of a ground point to its fictitious image projected on a datum plane (Fig. 12.12). Then,

$$d = r - r_1 \qquad r = \frac{Rf}{H - h_1} \qquad r_1 = \frac{Rf}{H} \tag{12.20}$$

Substituting for r and r_1 in the first equation yields

$$d = \frac{Rf}{H - h_1} - \frac{Rf}{H} = \frac{Rfh_1}{H(H - h_1)} = \frac{rh_1}{H}$$

$$= \frac{r_1 h_1}{H - h_1} \tag{12.21}$$

where r = radial distance on photograph from principal point to ground image of point P in (or mm)

r_1 = radial distance on photograph from principal point to P_1, the fictitious image position of point P projected to datum, in (or mm)

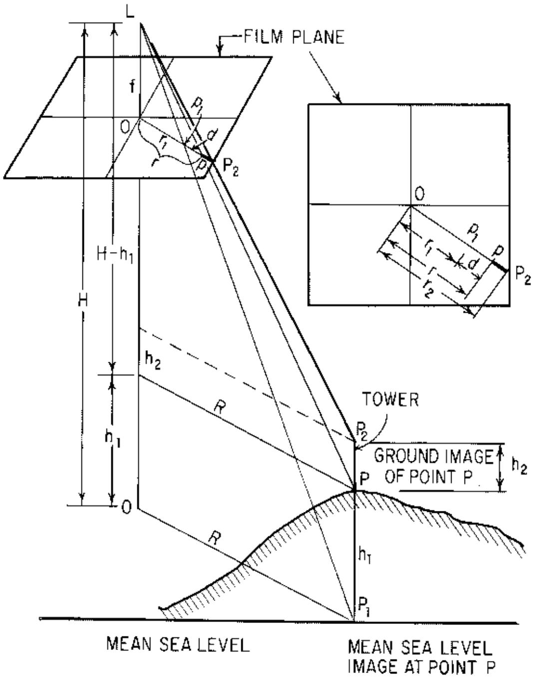

Fig. 12.12 Elevation differences cause topographic-relief displacements. *(Reprinted with permission from R. C. Brinker, "Elementary Surveying," Harper & Row, Publishers, New York.)*

$$h_1 = \text{height of point P above datum, ft}$$

$$H = \text{height of airplane above datum, ft}$$

As an example, find the height of a tower on an aerial photograph where the flight altitude above mean sea level is 5000 ft. ground elevation is 1000 ft. and measurements give $r_2 = 8.65$ mm and $r = 8.52$ mm (Fig. 12.12).

$$d' = r_2 - r = 0.13\,\text{mm}$$

$$h_2 = \frac{d'(H - h_1)}{r_2} = \frac{0.13(5000 - 1000)}{8.65} = 60\,\text{ft}$$

Stereoscopic vision is that particular application of binocular vision (simultaneous vision with both eyes) that enables an observer to view two different perspective photographs of an object (such as two photographs taken from different camera stations) and get the mental impression of three dimensions. Thus, a stereoscope permits each eye to see as one a pair of photographs that shows an area from different exposure points and thereby produces a three-dimensional (stereoscopic) image (model).

Parallax is the apparent displacement of the position of a body with respect to a reference point or system caused by a shift in the point of observation. As a result of the forward movement of a camera in flight, positions of all images travel across the focal plane from one exposure to the next, with images of higher elevations moving farther than those at lower levels.

Absolute parallax of a point is the total movement of the image of a point in the focal plane between exposures and is found as follows: (1) Locating the principal points of adjacent photographs containing the images of the point (Fig. 12.13); (2) transferring each principal point to the other photograph; (3) connecting each principal and transferred principal point to define the flight line; (4) drawing a line on each photograph through the principal point perpendicular to the flight line; and (5) measuring the x coordinate (parallel to flight line) of the point under study on each photograph.

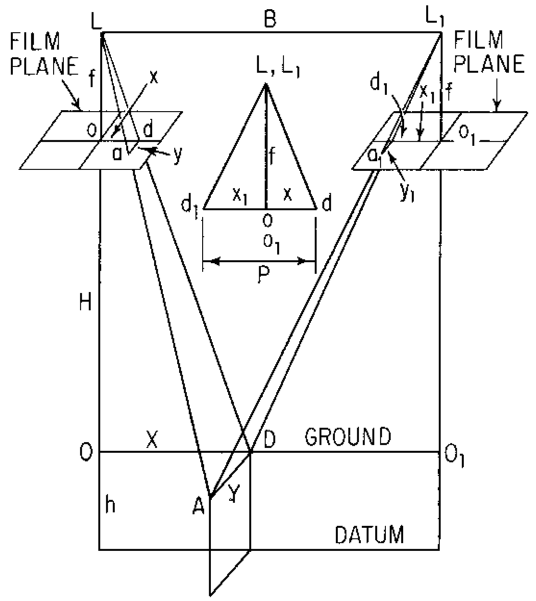

Fig. 12.13 Parallax shifts image of line AD on successive photographs. *(Reprinted with permission from R. C. Brinker, "Elementary Surveying," Harper & Row, Publishers, New York.)*

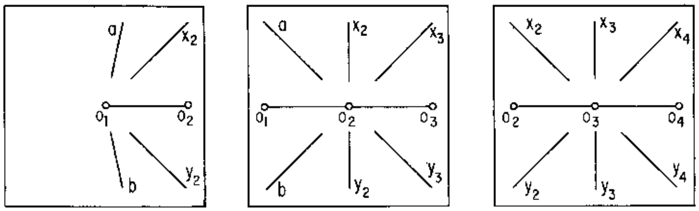

Fig. 12.14 Radial-line plotting of horizontal control points on aerial photographs.

Absolute parallax of a point, in (or mm) (observing algebraic signs), is

$$p = x - x_1 \tag{12.22}$$

Also,

$$X = \frac{xB}{p} \quad Y = \frac{yB}{p} \quad H - h = \frac{fB}{p} \tag{12.23}$$

For untilted photographs,

$$Y = \frac{y_1 B}{p}$$

where X, Y = ground coordinates measured from plumb point, ft

B = air base = distance between exposure stations, ft

x, y = photograph coordinates, in (or mm)

H = altitude of airplane above datum, ft

h = elevation of object above datum, ft

f = focal length of lens, in (or mm)

Measuring stereoscopes, such as the stereocomparator and contour finder, are satisfactory for small areas. The multiplex, Kelsh plotter, Wild Autograph, and other large plotters usually are preferred for extensive projects. The latter instruments measure differences in parallax by means of a floating dot—actually two dots superimposed on the photographs and mentally fused by the operator to produce the floating dot. The operator places it at apparent ground level in the photograph for contouring or finding spot elevations.

The accuracy of photogrammetric contouring depends on camera precision, type of terrain and ground cover, type of stereoscopic plotter, and experience of the operator.

$$C \text{ factor} = \frac{\text{flying height}}{\text{control interval}} \tag{12.24}$$

is an empirical ratio used to express the efficiency of stereoscopic plotters. Photogrammetrists get C factors of 750 to 2500, and elevations to the nearest foot and half foot with present equipment.

Radial-line plotting is a graphical method of extending horizontal control between fixed ground points on aerial photographs. In Fig. 12.14, points o_1, o_2, and o_3, the principal points in photographs 1, 2, and 3, are located on adjacent photographs. Control points a and b are identified in photograph 1. Additional control points called pass points (x_2 and y_2 in photograph 1, a and b in photograph 2, x_3 and y_3 in photograph 3) are established and transferred to the other photographs. On a sheet of tracing paper or template

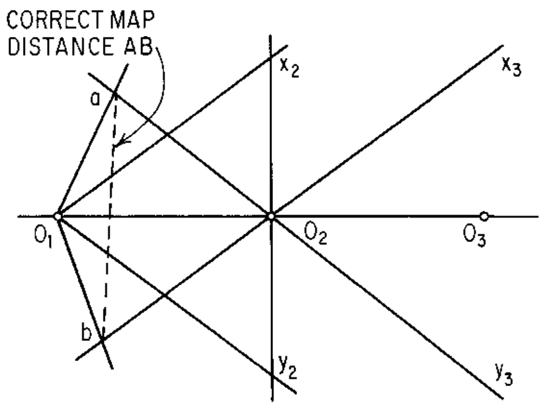

Fig. 12.15 Correct map scale and location of points are obtained with the aid of radial lines of Fig. 12.14.

placed over each photograph, a set of rays is drawn from the principal point, through each conjugate principal point, control point, and pass point. The templates are superimposed, as shown in Fig. 12.15, until all rays to each point, such as *a* or *b*, provide a single intersection. The map positions of the points then have been fixed.

The method is based upon two fundamental photogrammetric principles:

On truly vertical photographs, image displacements caused by topographic relief radiate from the principal point.

Angles between rays passing through the principal point are equal to the horizontal angles formed by the corresponding lines on the ground.

12.18 Bibliography

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See also Art. 12.2.