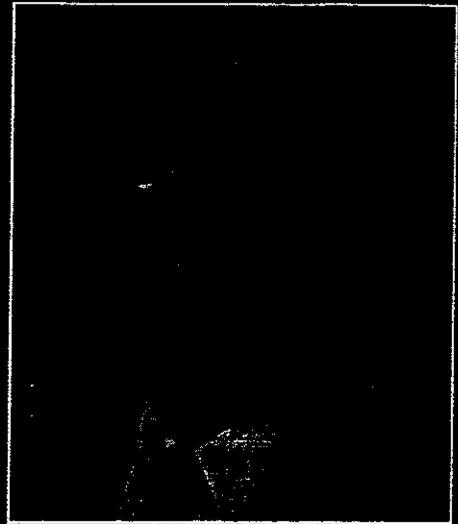

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Smith is author or coauthor of numerous technical papers on natural gas and oil production problems.

GAS ENGINEERING

Second Edition

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Practical Natural Gas Engineering

Second Edition

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Second Edition

R. V. Smith

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Practical Natural Gas Engineering

Second Edition

1 Introduction to Natural Gas Engineering

Natural gas engineering is the art of recovering a maximum amount of gas from a reservoir and bringing the gas to the surface at as high a pressure as practical. Both objectives should be accomplished as efficiently and economically as possible. Natural gas engineering also includes forecasting the future performance of reservoirs and the future production from gas wells. The ability to forecast the volumes and the pressures at which the gas is available from wells and reservoirs with an acceptable degree of accuracy is extremely important to the natural gas industry.

The scope of this book is confined to the flow of natural gas through reservoirs, the flow strings of wells and pipelines. Gas measurement is discussed only for orifice meters and critical flow provers of the Bureau of Mines type. Subjects such as liquid separation, liquid hydrocarbon extraction, dehydration, and compression were considered beyond the scope of this book. Except for compression these subjects are considered a part of process engineering which has an abundance of technical literature.

Natural gas engineering differs very little from conventional reservoir engineering for oil reservoirs. The principal differences are the units used to describe the production process and the effect of pressure-volume-temperature relationships on the behavior of gas as it flows in porous media and in pipelines. Also, natural gas engineering for the most part is concerned with the single-phase flow of an extremely compressible fluid as compared to oil reservoirs. Nevertheless, it is easy to convert calculations for an all-liquid system to a gaseous system. Progress in natural gas engineering has been hindered to some extent by its preoccupation with the so-called back pressure test and the unavoidable conclusion that gas wells showed evidence of non-Darcy flow.

Although papers published in the 1940s have concluded that gas flow in porous media and gas flow into wells could exhibit non-Darcy behavior, many engineers have taken the position that non-Darcy behavior is a result of the changes in viscosity and compressibility of the gas as it moves to the well. This divergence in views, along with the preoccupation of natural gas engineering with well testing, has proved to be a major stumbling block to natural gas engineering progress. The work of Al-Hussainy, Ramey, and Crawford and the work of subsequent investigators have shown that adequately handling changes in viscosity and compressibility still leaves evidence of non-Darcy behavior in the flow of gas to wells.* Further, an understanding of the flow of liquids in porous media has proved that there are no shortcuts to stabilization or magic tests that give the stabilized characteristics of a gas well in a short period of time. Thus, the author believes there is no fundamental difference between the behavior of wells producing liquids and wells producing gas.

The author has attempted to bridge the gap between the results of empirical testing and theory of unsteady-state flow in porous media and thus perhaps to close the gap between conventional reservoir engineering and an understanding of the behavior of gas wells. The latter becomes very important with the recent prices of natural gas and the subsequent push to produce lower-capacity wells at lower-than-ever pressures. The net result has been to increase the importance of natural gas engineering.

Since the author assumes that the reader has some knowledge of reservoir engineering as applied to oil reservoirs and may have had several courses in reservoir engineering, many of the principal subjects in reservoir engineering have not been emphasized. For example, geology has not been mentioned, and the more-technical features of core analysis and results have not been discussed in detail. Instead, the emphasis has been on the behavior of gas wells and gas reservoirs.

RESERVES

The reader will note that the author has avoided the use of the terms *reserves* and *recoverable reserves*. Historically, common practice has been to estimate recoverable gas reserves to be 85% of the gas in place in a dry gas reservoir. Assuming that the West Panhandle and Hugoton fields of Texas had a rock pressure of 435 psig at discovery (shut-in pressure at the

* See Chapter 7

wellhead), the 85% recovery factor indicates an abandonment pressure at a wellhead shut-in pressure of about 65 psig. Many wells in those fields are now approaching the magic pressure of 65 psig, but there has been no movement to abandon the wells. Any sensible observer of practices in the gas industry knows that gas wells are abandoned when current income does not cover current costs of operations.

Historically, recoverable reserves for gas reservoirs have been determined by one or more of the following methods:

1. Volumetric methods with abandonment at an arbitrary shut-in pressure
2. Extrapolation of pressure-cumulative-production curves to an arbitrary abandonment pressure
3. Extrapolation of production rate-time decline curves to an abandonment rate of production
4. Type curve analysis based on theoretical producing rate-time curves

The author contends that recoverable reserves for gas can be determined only by a sensible forecast of production down to a limiting rate of production as determined by economic conditions. Therefore, methods for forecasting production from gas wells have been emphasized.

PRESSURE AND TEMPERATURE BASES FOR GAS MEASUREMENT

The natural gas industry has been plagued by an inability to agree upon a standard pressure base for gas measurement. It is not uncommon to find that production is reported to the regulatory body on one pressure base, reserves are kept on a second pressure base, and heating value is reported on a third pressure base. Further, the pressure base for reporting production varies from state to state. Arkansas, Kansas, Oklahoma, and Texas have adopted 14.65 psia as the standard pressure base for reporting purposes. Louisiana has adopted 15.025 psia. The Bureau of the Budget adopted 14.73 psia as the standard for Federal reporting purposes while the Federal Energy Regulatory Commission (FERC) and its predecessor, the Federal Power Commission (FPC), adopted 14.735 psia as the standard pressure base. The latter is almost equivalent to 30 inches of mercury at 32°F. The American Gas Association reports its meter factors at a pressure base of 14.73 psia. The Province of Alberta, which has adopted the SI metric system, uses standard pressure and temperature bases of 101.325 *kPa* and

15°C. These are equivalent to 14.696 psia and 59.0°F. The Society of Petroleum Engineers recommends for its SI metric system 100 *kPa* and 15°C which are equivalent to 14.504 psia and 59.0°F. Fortunately, the industry in the United States has not been plagued with a multitude of temperature bases for measurement purposes.

The author has arbitrarily selected a pressure base of 14.65 psia and a temperature base of 60°F; all gas volumes used herein exist at these standard conditions unless stated otherwise. This has been done in the belief that most readers are required to report test results and production as standard cubic feet at 14.65 psia and 60°F. Recently, as a result of the marked increase in the sales price of natural gas, gas sales contracts are being written with the British thermal unit as the unit of measurement for pricing purposes.

PHYSICAL CONSTANTS

The physical constants used in the second edition are those listed in Table 1–1 along with the most recently accepted values. The changes in the physical constants since publication of the first edition have been considered minor or negligible when used in engineering calculations. The single exception has been the heating values for the components of natural gas. The heating values and methods for calculating the heating values of natural gas mixtures are those set out in the Gas Processors Association GPA Standard 2172–84 (Revised 1984). These methods are given in Chapter 2.

Table 1–1 ***Values of physical constants used compared to values recently proposed in industry standards.***

Constants	*Used*	*Most Recent*
Standard atmosphere, psia	14.696	14.696
Gas constant, R, cu ft, psia, °R^{-1}	10.732	10.7316
Molar mass of air (molecular weight)	28.964*	28.9625
Vapor pressure of water at 60°F	0.25636	0.25636
0°F in absolute units, °R	459.67**	459.67

* Except for Table 2–4.

** A value of 460 was used in all examples.

CALCULATIONAL PROCEDURES

Calculational procedures for any task requiring lengthy arithmetic are undergoing or have undergone a complete revolution more fundamental than generally recognized. The electronic computer, from the largest and fastest machine to the personal computer and the hand-held programmable calculator, has made tedious and lengthy computations a triviality with respect to time and cost. As an example, the method for calculating bottom-hole pressures for a flowing gas well, as illustrated in Table 5–3 of Chapter 5, would require from 2.5 to 4 hours if done entirely by hand with an old-fashioned desk calculator. The use of a programmable calculator reduces the time to about 30 minutes or less. A personal computer with a program designed for efficiency reduces the time for the calculation to 10 seconds or less. The question then becomes pertinent—why learn or set out the computational procedures? In order to make full use of the principles and possibilities of natural gas engineering, one must know the computational procedures.

NOTES FOR PROBLEMS

Problems have been included at the end of most of the chapters for the purpose of giving the student an insight into the practice of natural gas engineering. Many times the engineer working with field problems finds he must improvise to solve his daily assignments. Therefore, some of the problems at the end of the chapters are not too explicit in giving all the information needed for a nice, neat solution. For example, problem No. 2 of Chapter 2 requires the gross heating values for butanes and the pentanes and heavier. These values are not to be found in any reference work; therefore, reasonable estimates must be made in order to solve the problem. When such estimates are made, the estimated values should be clearly indicated as estimates so that corrections can be made if additional information becomes available.

NOMENCLATURE*

Symbol	*Description*	*Oil-field Units*
A	constant, equations 5–7 and 5–8	—
A	cross-sectional area	sq ft

* Standard symbols proposed by SPE-AIME. Exceptions are made where strict conformance with the standard may be confusing.

Symbol	*Description*	*Oil-field Units*
A_1–A_8	constants, equation 2–11	—
A_x	availability function, equation 9–23	—
API	gravity, degrees API	—
b	gas constant for 1 lb of gas	cu ft, psia, °R^{-1}, lb^{-1}
$\sqrt{b}$	summation factor	$psia^{-2}$
B_g	formation volume factor for gas	—
B_1	constant, equation 5–7	—
B_2	constant, equation 5–8	—
B	virial coefficient	$psia^{-1}$
cf	volume	cu ft
c′	coefficient, equations 4–4 and 4–5	—
c_g	compressibility of gas	$psia^{-1}$
c_t	total compressibility of a system	$psia^{-1}$
C	orifice-plate coefficient, problem 4–1	—
C	coefficient, equations 7–1, 7–3, 7–7, etc.	—
°C	temperature, degrees Celsius	—
C(t)	coefficient, equation 7–2	—
d	diameter	in.
d	internal diameter of pipe	in.
d_1	outside diameter of inner pipe, annular space	in.
d_2	inside diameter of outer pipe, annular space	in.
D	internal diameter of pipe	ft
E	internal energy	ft-lb
f	coefficient of friction	—
$\sqrt{1/f}$	transmission factor	—
F	frictional resistance	ft-lb
°F	temperature, degrees Fahrenheit	°F
F_a	orifice thermal expansion factor	—
F_b	basic orifice factor	—

Symbol	*Description*	*Oil-field Units*
F_g	specific gravity factor	—
F_l	gauge location factor	—
F_m	manometer factor	—
F_{nD}	non-Darcy flow factor	—
F_p	basic orifice prover factor	—
F_{pb}	pressure base factor	—
F_{pv}	supercompressibility factor	—
F_r	Reynolds number factor	—
F_r	factor defined by equation 5–11	—
F_t	flowing temperature factor	—
F_{tb}	temperature base factor	—
F_{tf}	flowing temperature factor	—
g_c	gravitational conversion factor (32.17)	ft/sec^2
G	volume of gas at standard conditions (14.65 psia and 60°F)	cu ft
G_i	gas initially in place	cu ft
G_p	gas produced	cu ft
h	thickness	ft
h_w	differential pressure	in. of water
H	elevation or difference in elevation	ft
I_n	factor used in Chapter 5	—
ID	internal diameter	in.
k	permeability (Chapter 6 only)	darcy
k	reservoir permeability	md
k	absolute roughness characteristic	in.
K	discharge coefficient	—
K	term in equations 2–21 and 4–3	—
ln	logarithm to base e	—
log	logarithm to base 10	—
L	length	ft

Symbol	*Description*	*Oil-field Units*
m	slope of straight-line, semilog coordinates	psia^2/cycle or psia^2/cp/cycle
m(p)	real gas potential	psia^2/cp
M	thousands	—
M	molar mass or molecular weight	lb/lb-mole
M_a	average molecular weight air = 28.964	lb/lb-mole
M_i	molar mass or molecular weight of component i	lb/lb-mole
M_g	molar mass or molecular weight of gas	lb/lb-mole
Mcf	thousands of cubic feet	10^3 cf
MMcf	millions of cubic feet	10^6 cf
Mcfd	flow rate	10^3 cf/day
MMcfd	flow rate	10^6 cf/day
n	quantity of gas	lb-mole
n	coefficient in equations 7–1, 7–2, 7–3, etc.	—
N	mass rate of flow	lb/sec
N_{Re}	Reynolds number	—
OD	outside diameter	in.
p	pressure	psia
(p)	indicates preceding variable is a function of pressure	—
Δp^2	difference in pressures squared	psia^2
p_b	base pressure for gas measurement	psia
p_{cf}	pressure on casing at surface—no packer with flow-through tubing	psia
$\bar{p}_{cs}$	stabilized shut-in pressure at surface on casing—no packer	psia
p_m	low base pressure for real gas potential	psia
p_m	measurement pressure	psia
p_{pc}	pseudocritical pressure	psia

Symbol	*Description*	*Oil-field Units*
p_{pr}	pseudoreduced pressure	—
$\bar{p}_R$	stabilized shut-in reservoir pressure	psia
$\bar{p}_{Ri}$	shut-in reservoir pressure before flow test	psia
p_{tf}	surface pressure on tubing, flow through tubing	psia
p_{tfd}	deliverability pressure	psia
$\bar{p}_{ts}$	stabilized shut-in pressure at surface on tubing, equation 7–7	psia
p_{wf}	bottom-hole flowing pressure	psia
p_{ws}	bottom-hole shut-in pressure	psia
q	energy lost to surroundings	ft-lb
q	volumetric rate of flow	cfd
q_d	deliverability	Mcfd
q_D	dimensionless rate of flow	—
q_h	rate of flow	cf/hr
q_k	rate of flow	Mcfd
q_m	rate of flow	MMcfd
q_o	rate of flow for oil	bbl/day
q_{of}	open flow potential, equation 7–8	Mcfd
r	radius	in.
r	internal radius of pipe	in.
r_a/r_g	ratio of aquifer radius to radius of gas reservoir	ratio
r_w	wellbore radius	ft
r'_w	effective wellbore radius	ft
R	gas constant	cu ft, psia, °R^{-1}, lb mole^{-1}
°R	°F + 460, degrees Rankine	°F + 460
R_g	gas to hydrocarbon liquid ratio	cu ft/bbl
s	factor in equations 5–19 and 5–20	—
s	van Everdingen skin effect, equation 8–23	—
S_w	water saturation	fraction

Symbol	Description	Oil-field Units
t	time (check usage)	days, years
t	reciprocal reduced temperature (see equations 2–14, 2–15)	—
(t)	preceding variable is function of time	—
t_D	dimensionless time	—
T	absolute temperature, °F + 460 = °R	°R
T_b	base temperature for gas measurement 60°F or (°F + 460) depending on usage	°F, °R
T_{pc}	pseudocritical temperature	°F + 460
T_{pr}	pseudoreduced temperature	—
u	velocity	ft/sec
$\overline{u}$	average velocity	ft/sec
v	specific volume	cu ft/lb
V	volume (Chapter 6, cc)	cu ft
V_g	volume containing gas	cu ft
V_L	vapor volume equivalent of hydrocarbon liquid	cu ft/bbl
W	weight (Chapter 6)	gm
W	energy supplied to system	ft-lb
W_f	energy loss caused by friction or irreversible energy loss	ft-lb
x	ratio of pressures $\overline{p}_R/p_{wf}$	—
x_f	fracture length	ft
x	mole fraction	—
X	see equation 2–21	—
X	length of pipe	miles
y_i	mole fraction of component i in gas mixture	—
y	reduced density see equations 2–13, 2–15	—

Symbol	Description	Oil-field Units
Y	see equation 2–21	—
Y	expansion factor in gas measurement	—
z	compressibility factor	—
$\overline{z}$	compressibility factor at average conditions	—
γ_g	gas specific gravity or molar mass ratio (air = 1.000)	—
γ_{gf}	gas gravity of flowing fluid in a well	—
γ_o	specific gravity of oil referred to water	—
Δ	increment of the following variable	—
Δp^2	difference in pressures squared	$psia^2$
μ	viscosity	m N/m or cp
$\overline{\mu}$	viscosity at average conditions	m N/m or cp
ρ_a	density of dry air	lb/cu ft
ρ_g	density of gas	lb/cu ft
ρ_{pr}	pseudoreduced density, equation 2–10	—
ρ_L	density of liquid	lb/cu ft
ρ_1	density (Chapter 2)	gm/cc
ϕ	porosity	fraction

Subscripts

a	air	—
b	standard or base conditions for gas measurement	—
b	bulk (Chapter 6)	—

Symbol	Description	Oil-field Units
cs	casing, shut-in (with pressure)	—
cf	casing, flowing (with pressure)	—
d	deliverability (with pressure)	—
e	external boundary	—
f	flowing	—
f	friction	—
f	fracture	—
g	gas	—
i	initial	—
i	i^{th} component	—
k	thousands	—
L	liquid	—
m	millions	—
m	measurement (with pressure)	—
ma	matrix	—
n	n^{th} term	—
nD	non-Darcy flow	—
o	oil	—
of	open flow	—
p	pore, see equation 6–1	—
p	prover	—
pc	pseudocritical	—
pr	pseudoreduced	—
r	related to radius, equation 5–11	—
R	reservoir	—

Symbol	Description	Oil-field Units
t	total system	—
tfd	deliverability (with pressure)	—
tfp	pipeline pressure	—
w	water	—
w	wellbore	—
wf	bottom-hole flowing (with pressure)	—
ws	bottom-hole shut in, (with pressure)	—
x	indicates a function of *x*, equation 9–23	—
1,2,3 etc.	indicates time or position	—

2 Properties of Natural Gas

A knowledge of natural gas properties is fundamental to applied natural gas engineering. The engineer concerned with the production, transportation, and sale of natural gas is interested in those properties that influence flow and sales value. These are pressure-volume-temperature relationships, viscosity, and composition of the gas. The heating value, or its quality for use as a fuel, controls the sales value—providing other variables remain constant.

Although the sales price of natural gas may be related to its heating value, usually expressed in British thermal units (Btu) per cubic foot, the engineering principles relating to flow in porous media and pipe, material balance, and availability of the gas are related to volume. Pure carbon dioxide or nitrogen flowing in a pipe or being produced from a well exhibit the same general characteristics as an essentially all hydrocarbon gas—even though both carbon dioxide and nitrogen have a heating value of zero. Therefore, the unit of measurement for present purposes is the cubic foot, which is defined as the quantity of gas contained in one cubic foot of volume at given base pressure and temperature. It is convenient to use units of a thousand cu ft (Mcf) and a million cu ft (MMcf) in engineering calculations. Unfortunately, there is no industry-wide standard base pressure; however, all gas volumes referred to as "standard" herein exist at a base pressure of 14.65 psia and a base temperature of 60°F unless otherwise specified. When the sales value is based on heating value, the water vapor content of the natural gas becomes important, especially at today's higher prices. Thus, a natural gas in equilibrium with water at 14.696 psia and 60°F contains 0.01744 mole fraction of water vapor which can make a difference of 15.1 Btu (see Table 2–6) between the heating value per ideal cubic foot on a dry basis and the

value on a water saturated basis. At $3.00 per million Btu, this difference is equivalent to $0.0453 per Mcf or $45.30 per MMcf. However in most reservoir engineering work the natural gas is assumed tacitly to be dry, or free of water vapor, unless gas hydrate problems are under consideration.

Natural gas is generally measured in producing operations at pressures several times greater than the base pressure and at temperatures substantially above the base temperature. Frequently, engineering studies require calculating the volume of a given quantity of gas at specified pressures and temperatures. These calculations are made by applying the gas laws.

GAS LAWS

A gas may be defined as a homogeneous fluid that fills any container in which it is placed. The physical laws that relate the effects of pressure and temperature on the volume of a given quantity of gas are well defined. The laws were first determined empirically in the seventeenth and eighteenth centuries and have been given the names of the persons credited with their discovery. Boyle's law states that under conditions of constant temperatures, the volume of a gas varies inversely with the pressure. Difficulties with the formulation of a scientific concept of temperature delayed the discovery of the relationship between volume and temperature for nearly a century. Charles' or Gay-Lussac's law states that under conditions of constant pressure, the volume of a gas varies directly as the absolute temperature.

When these laws are combined with Avogadro's hypothesis, which states that equal volumes of different ideal gases at the same pressure and temperature contain the same number of molecules, the ideal or perfect gas laws can be expressed by the equation:

$$pV/T = nR \qquad (2\text{–}1)$$

Equation 2–1 relates pressure, volume, and temperature to the quantity of gas under consideration, n, and the gas constant, R. The development of the kinetic-molecular theory of gases gave the gas laws a theoretical basis.

Experimental work has shown that the ideal gas equation describes the pressure-volume-temperature behavior of natural gas only as a first approximation. For engineering purposes, the deviations from ideal behavior for natural gas have been correlated by means of the ratio pV/nRT, which is called the compressibility factor, z. For nonideal gases, the gas laws may be stated as follows:

$$pV = nzRT \tag{2–2}$$

where n is the quantity of gas in moles and is defined as:

$$n = \frac{\text{mass of gas}}{\text{molar mass of gas}} = \frac{\text{mass gas}}{\text{M}}$$

Gas Constant, R

The value of the constant R is dependent upon the units used in equation 2–2. For the units:

p = absolute pressure, psia
V = volume, cu ft
z = the compressibility factor, ratio
n = number of lb-moles
T = absolute temperature, (°F + 459.67), known as degrees Rankine or °R

the value of the gas constant, R, is:

R = 10.732 (cu ft, psia, °R^{-1}; lb-mole^{-1})

and

R = 1,545.4 (cu ft, lb/sq ft, °R^{-1}, lb-mole^{-1})

For 1 lb of gas:

$$pV = bzT; \quad b = \frac{R}{M} \tag{2–3}$$

Taking the average molecular weight of air as 28.964 and γ_g, the specific gravity of the gas referred to air, the molar mass or molecular weight of the gas is as follows (see reference 5):

$$M = 28.964\,\gamma_g$$

then:

$$b = \frac{10.732}{28.964\,\gamma_g} = \frac{0.37053}{\gamma_g} \tag{2–4}$$

in units of cu ft, psia, °R^{-1}, and lb^{-1}, and:

$$b = \frac{(10.732)(144)}{28.964\,\gamma_g} = \frac{53.356}{\gamma_g} \tag{2–5}$$

in units of cu ft, lb/sq ft, °R^{-1}, and lb^{-1}.

Compressibility Factor, z

In practice, the compressibility factor, z, is a variable, and its value depends upon the pressure, the temperature, and the composition of the gas. Plots of compressibility factors as a function of pressure for a nonideal gas at constant temperature take the form illustrated in Figure 2–1.

Studies of the compressibility factors for natural gases of various compositions have shown that compressibility factors can be generalized with sufficient accuracies for most engineering purposes by introducing the con-

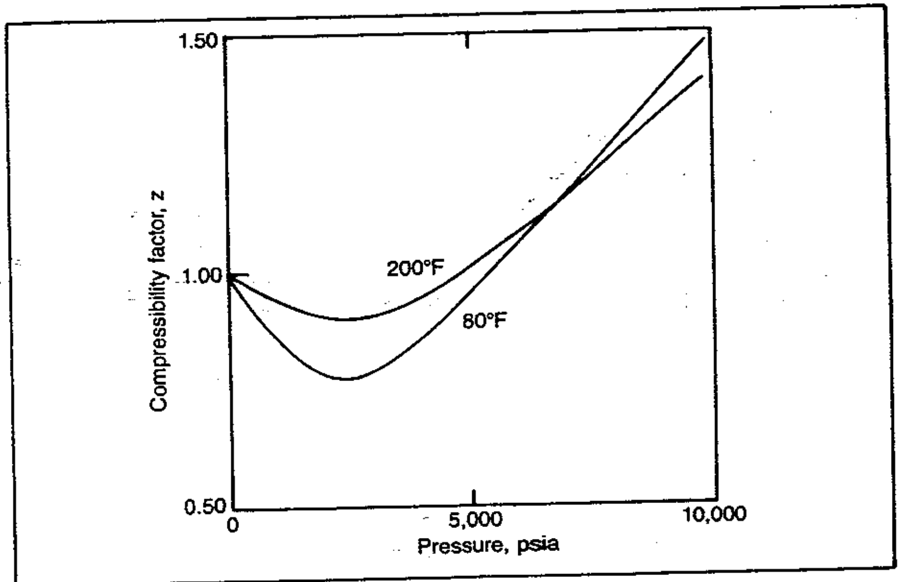

Figure 2–1 *Compressibility factors for a natural gas as a function of pressure at constant temperature.*

cepts of pseudoreduced pressures and temperatures based on pseudocritical pressures and temperatures as follows:

$$p_{pr} = p/p_{pc};\ T_{pr} = T/T_{pc}$$

where all pressures are in absolute units, psia, and temperatures are absolute, such as degrees Rankine (°F + 460).*

In natural gas engineering it has become common practice to use the compressibility chart of Standing and Katz for compressibility factors.[1] This chart was converted to a table by Poettmann and Carpenter and has been duplicated as Table A–3 in Appendix A.[2] Poettmann and Carpenter's table was expanded into a still larger table of compressibility factors by the Interstate Oil Compact Commission in its *Manual of Back-Pressure Testing of Gas Wells*.[3]

Table A–3 in Appendix A gives values of compressibility factors for pseudoreduced pressures from 0.20 to 15.00 in increments of 0.05 and for values of pseudoreduced temperatures from 1.05 to 3.00 in convenient increments. A correlation from reference 3 between the specific gravity (air = 1.00) of the natural gas and pseudocritical pressures and temperatures is given as Table A–1, and corrections to the pseudocritical properties for carbon dioxide, nitrogen, and hydrogen sulfide are given in Table A–2 in Appendix A. If the gas under consideration contains more than 5.0 vol% of carbon dioxide or hydrogen sulfide, the reader should consult the work of Robinson et al.[4] or the work of Wichert and Aziz that is summarized on pages 16–10 through 16–15 of reference 5.

The information in Tables A–1 and A–2 can be approximated in the range of specific gravities from 0.55 to 0.90 by the following equations:

$$p_{pc} = 689.60 - 30.48\,\gamma_g + 4.35\,(\%\,CO_2) - 1.68\,(\%\,N_2) + 4.44\,(\%\,H_2S) \tag{2–6}$$

$$T_{pc} = 152.89 + 339.29\,\gamma_g - 1.68\,(\%\,CO_2) - 2.69\,(\%\,N_2) + 0.00\,(\%\,H_2S) \tag{2–7}$$

Other means for calculating compressibility factors for natural gases have been proposed by Hall and Yarborough, Yarborough and Hall, Gopal, and Garb.[6–9] The equations proposed in references 6, 7, 8, and 9 are readily adaptable to large computers, but Garb and Meehan and Lyons give equations and programs for small, hand-held programmable calculators.[10,11] Garb credited his information to the Energy Resources Conservation Board of Alberta, Canada (Bulletin 75–34). The equations presented by Garb were published originally by Dranchuk, Purvis, and Robinson.[12] They are:

* This approximation will be used for absolute temperature.

$$p_{pc} = 702.5 - 50\,\gamma_g \tag{2–8}$$

$$T_{pc} = 167 + 316.67\,\gamma_g \tag{2–9}$$

$$\rho_{pr} = 0.27\,p_{pr}/zT_{pr} \tag{2–10}$$

$$\begin{aligned} z = 1 &+ (A_1 + A_2/T_{pr} + A_3/T_{pr}^3)\,\rho_{pr} \\ &+ (A_4 + A_5/T_{pr})\,\rho_{pr}^2 \\ &+ (A_5A_6\,\rho_{pr}^5)/T_{pr} \\ &+ (A_7\,\rho_{pr}^2/T_{pr}^3)(1 + A_8\,\rho_{pr}^2)\exp(-A_8\,\rho_{pr}^2) \end{aligned} \tag{2–11}$$

where:

$A_1 = 0.31506237$
$A_2 = -1.0467099$
$A_3 = -0.57832729$
$A_4 = 0.53530771$
$A_5 = -0.61232032$
$A_6 = -0.10488813$
$A_7 = 0.68157001$
$A_8 = 0.68446549$

The compressibility-factor chart published by Standing and Katz has been accepted by wide usage for natural gas and gas condensate systems under well and reservoir conditions.[1] Hall and Yarborough and Yarborough and Hall derived a new equation of state (which they called an augmented hard-sphere equation) that reproduced the compressibility chart within an average of 0.3% except for pseudoreduced temperatures of 1.05, 1.10, and 1.15.[6,7] The Hall and Yarborough equations are:

$$z = p/\rho RT \tag{2–12}$$

$$y = b\rho/4 \tag{2–13}$$

$$z = \frac{1 + y + y^2 - y^3}{(1 - y)^3} - (14.76t - 9.76t^2 + 4.58t^3)\,y + (90.7t - 242.2t^2 + 42.4t^3)\,y^{(1.18+2.82t)} \tag{2–14}$$

$$bp_{pc}/RT_{pc} = 0.245\,exp\,[-1.2(1 - t)^2] \tag{2–15}$$

where:

t = reciprocal reduced temperature, T_{pc}/T

y = "reduced" density defined by equations 2–13 and 2–15

b = term in equations 2–13 and 2–15.

The solution to equations 2–12 to 1–15 for the compressibility factor, z, is accomplished by solving equation 2–15 for b and substituting into equation 2–13 then rearranging and solving for the density. The expression is substituted into equation 2–12 and the result is then substituted into equation 2–14. The resulting equation is multiplied by y and solved by trial and error. The Newton-Raphson method works satisfactorily. Although the foregoing is not convenient for hand calculations, the procedure can be done easily with a personal computer.

Comparison of Methods for Estimating Compressibility Factors

Thus, with no more than a hand-held programmable calculator, compressibility factors may be determined by interpolating data from Table A–3 or from the tables in the Interstate Oil Compact Commission Manual or by calculations using methods given by Garb and Meehan and Lyons. A comparison is given in Table 2–1 between compressibility factors from Tables A–1, A–2, and A–3 and the methods of Garb, Hall and Yarborough, and Yarborough and Hall for a gas having a specific gravity of 0.641 and carbon dioxide and nitrogen contents of 2.0 and 3.0 mole %, respectively.

Although there are large differences between the sets of compressibility factors in Table 2–1, the equations given by Hall and Yarborough fit the factors given in the table very closely. The important characteristic of a system for estimating compressibility factors for engineering purposes is

Table 2–1 *Compressibility factors for a natural gas interpolated from Tables A–1, A–2, and A–3 and calculated from references 6, 7, and 9.*

$\gamma_g = 0.641$, $CO_2 = 2.0\%$. $N_2 = 3.0\%$

	Temperature								
	100°F			*150°F*			*200°F*		
Pressure (psia)	*Tables*	*Ref. 9*	*Ref. 6, 7*	*Tables*	*Ref. 9*	*Ref. 6, 7*	*Tables*	*Ref. 9*	*Ref. 6, 7*
500	0.936	0.929	0.935	0.954	0.947	0.953	0.967	0.960	0.966
1,000	0.878	0.864	0.878	0.913	0.902	0.913	0.938	0.928	0.938
1,500	0.834	0.813	0.833	0.881	0.867	0.882	0.917	0.904	0.917
2,000	0.807	0.785	0.805	0.861	0.848	0.863	0.903	0.892	0.905
2,500	0.798	0.781	0.799	0.852	0.844	0.858	0.898	0.891	0.902
3,000	0.812	0.797	0.811	0.859	0.855	0.867	0.906	0.900	0.910

that the system be continuous and smooth enough for material-balance calculations. For example, using one compressibility factor from the tables with another from the Garb or Hall and Yarborough systems could introduce a large error into a material balance tabulation.

If equations 2–6 and 2–7 are used to calculate pseudocritical properties of a gas, with equation 2–11, very good agreement with the tables is obtained at pressures up to 2,000 psia, but the divergence becomes large at higher pressures.

Supercompressibility Factor, F_{pv}

The term used to express the deviation of a gas from ideal behavior appears in gas measurement equations as the square root of the reciprocal of the compressibility factor, z. This term has been called the supercompressibility factor, F_{pv}:

$$F_{pv} = \sqrt{1/z} \tag{2–16}$$

DENSITY AND SPECIFIC VOLUME, ρ and *v*

Density, ρ, and specific volume, v, may be calculated using equations 2–2 and 2–4 when 1 lb of gas is considered.

$$pv = bzT \tag{2–2}$$

$$b = \frac{0.37053}{\gamma_g} \tag{2–4}$$

$$\frac{1}{\rho} = v = \frac{0.37053\, zT}{p\, \gamma_g} \tag{2–17}$$

$$\rho = \frac{1}{v} = 2.6988\, \frac{p\, \gamma_g}{zT} \tag{2–18}$$

where the units are lb/cu ft for density and cu ft/lb for specific volume.

Example 2–1. Determine the density and specific volume of a gas (γ_g = 0.641, CO_2 = 2.0%, and N_2 = 3.0%) at 2,765 psia and 179°F. From Tables A–1 and A–2:

$$p_{pc} = 670 + 8 - 5 = 673$$

$$p_{pr} = 2{,}765/673 = 4.11$$

$$T_{pc} = 372 - 3 - 9 = 360$$

$$T_{pr} = (460 + 179)/360 = 1.78$$

	T_{pr}	
p_{pr}	*1.70*	*1.80*
4.10	0.857	0.893
4.15	0.857	0.894

z at p_{pr} = 4.11 and at T_{pr} = 1.78 = 0.886

$$\rho = \frac{(2.6988)(2{,}765)(0.641)}{(0.886)(639)} = 8.45 \text{ lb/cu ft}$$

$$v = 1/8.45 = 0.118 \text{ cu ft/lb}$$

COMPRESSIBILITY, c_g, FOR GASES

Many reservoir engineering equations contain terms defined as the compressibilities of various fluids. The isothermal compressibility of a substance is defined as the change in volume per unit volume for a unit change in pressure, or:

$$c_g = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

The general gas equation (equation 2–2) can be differentiated and solved for compressibility with the following result:

$$c_g = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T \tag{2–19}$$

Inspection of equation 2–19 shows that the compressibility for an ideal gas would be the reciprocal of the absolute pressure. The value of $\partial z/\partial p$ can be determined graphically from curves showing the compressibility factor plotted against pressure as illustrated in Figure 2–2.

Note: Compressibility, c_g, must not be confused with the compressibility factor, z.

Example 2–2. Refer to example 2–1 and calculate the compressibility of the same gas at 1,800 psia and 179°F. Since the compressibility calculation requires $(\partial z/\partial p)_T$ at the required pressure and temperature, it is necessary to construct a curve (see Fig. 2–2) showing the relationship between the compressibility factor and pressure.

From Figure 2–2:

$$\left(\frac{\partial z}{\partial p}\right)_T = \left(\frac{\Delta z}{\Delta p}\right)_T = \frac{0.8927 - 0.8987}{1{,}900 - 1{,}600} = -2.0 \times 10^{-5}\ \text{psia}^{-1}$$

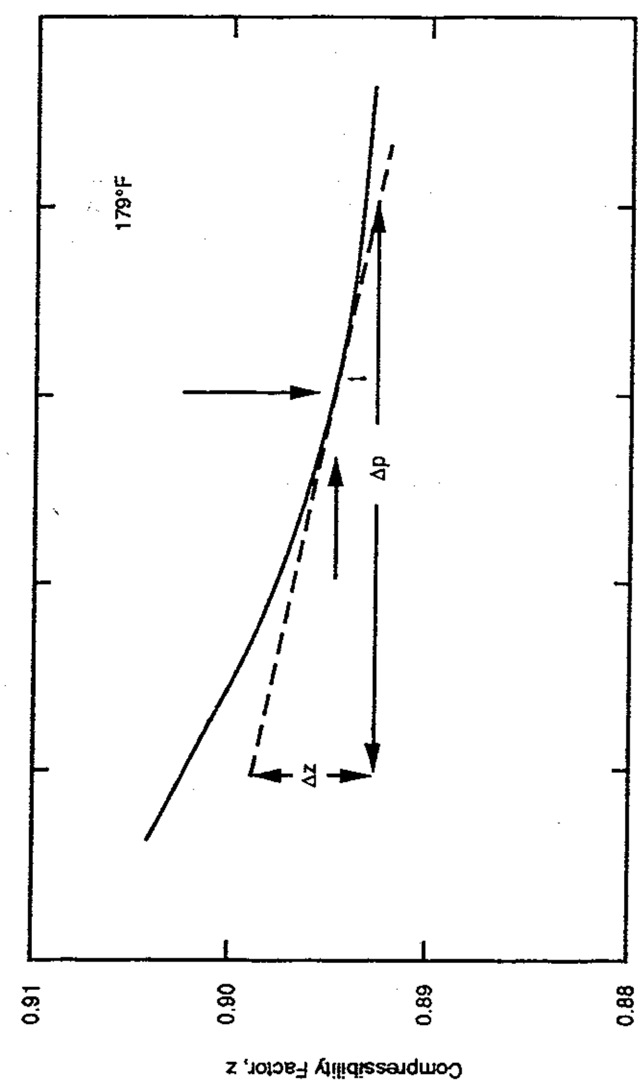

Figure 2–2 *Determination of slope $(-\partial z/-\partial p)_T$ for a gas at 1,800 psia and 179°F.*

From equation 2–19:

$$c_g = \frac{1}{1{,}800} - \left[\frac{1}{0.895}\right]\left(-2.0 \times 10^{-5}\right)$$

$$= 5.56 \times 10^{-4} + 0.22 \times 10^{-4} = 5.78 \times 10^{-4}, \text{ psia}^{-1}$$

FORMATION VOLUME FACTOR, B_g

The reservoir volume, B_g, occupied by a unit volume of gas at standard conditions can be calculated from equation 2–2 with the following result:

$$B_g = \frac{p_b\, z\, T}{p\, z_b\, T_b} \tag{2–20}$$

where the subscript b denotes standard or base conditions of pressure and temperature. Usually, the compressibility factor at standard conditions is assumed to be 1.000.

Example 2–3. Calculate the formation factor for the gas in example 2–2 at 1,800 psia and 179°F for a pressure base of 14.65 psia and 60°F.

$$B_g = \frac{(14.65)(0.895)(460 + 179)}{(1{,}800)(1.000)(520)} = 0.00895$$

Thus, 1 cu ft of gas at 14.65 psia and 60°F would occupy 0.00895 cu ft at 1,800 psia and 179°F.

Note: 460 was used to convert °F to absolute temperature instead of the value of 459.67 given in the discussion of the gas constant, R. This approximation will be used in all examples.

MOLE FRACTION, VOLUME FRACTION, AND MASS FRACTION

For many engineering purposes, the approximation that natural gas and air are ideal gases at atmospheric pressure and ordinary ambient temperatures of about 60°F or at the commonly used standard pressure and temperature conditions can be made. Actually, the compressibility factors for natural gases range from 0.997 to 0.998 at atmospheric pressure and temperature of 60°F, and the compressibility factor for dry air is about 0.9996 at atmospheric pressure and 60°F. Using the approximation that the gases are ideal gases, the number of moles of gas, n, is directly proportional to the volume, and the mole fraction of a constituent in the gas is equivalent to the volume fraction.

To convert mole fraction to mass fraction, one must multiply the mole fraction of each component in the gas by the molar mass or molecular weight of the component, add the masses for the individual components, and divide the mass of each component by the total. Thus:

$$\text{mass fraction} = \frac{(y_i \times M_i)}{\Sigma\,(y_i \times M_i)}$$

where y is the mole fraction of the i^{th} component in the mixture. Mass fractions can be converted to mole fractions by the inverse operation. Molecular weights for the components commonly found in natural gas are given in many reference books but are included in Table 2–4.

APPARENT MOLECULAR WEIGHT OR MOLAR MASS

A gas mixture behaves as if its molecular weight is the same as that for a pure gas. The weight of 1 mole of a gas mixture and, thus, its average molecular weight is:

$$\text{average molecular weight} = \Sigma\,(y_i \times M_i)$$

Example 2–4. A gas consists of 17 lb of methane, 2 lb of ethane, and 1 lb of propane. Calculate the compressibility factor at 1,000 psia and 100°F.

Component	(1) Weight (lb)	(2) Mass Fraction	(3) Molecular Weight	(1) ÷ (3) (Moles)
Methane	17	0.85	16.043	1.0597
Ethane	2	0.10	30.070	0.0665
Propane	1	0.05	44.097	0.0227
Total	20	1.00		1.1489

$$\text{Average molecular weight} = \frac{\text{mass}}{\text{moles}} = \frac{20}{1.1489} = 17.41$$

$$\text{Molar mass ratio} = \frac{\text{average molecular weight}}{\text{average molecular weight air}} = \frac{17.41}{28.964} = 0.601$$

From table A–1, pseudocritical pressure = 671 psia
pseudocritical temperature = 358°R

$$\text{Pseudoreduced pressure} = \frac{1000}{671} = 1.49$$

$$\text{Pseudoreduced temperature} = \frac{(460 + 100)}{358} = 1.56$$

From Table A–3, the compressibility factor z = 0.878

Example 2–5. A gas reservoir has an area of 4 sq mi and a net pay thickness of 10 ft. The porosity is 10%, connate water is 25%, reservoir temperature of 85°F, and reservoir pressure is 700 psia. Calculate the volume of gas in place at 14.65 psia and 60°F and the gas remaining in place when the reservoir pressure has been reduced to 50 psia. The gas composition is given as mass fractions in the following table:

Component	Mass Fraction	Molecular Weight	Moles/lb	Mole Fraction
Methane (C_1)	0.94	16.043	0.5859	0.973
Ethane (C_2)	0.03	30.070	0.00100	0.017
Propane (C_3)	0.02	44.097	0.00045	0.007
n-Butane (n-C_4)	0.01	58.123	0.00017	0.003
Total	1.00		0.06021	1.000

$$\text{Molecular weight} = \frac{\text{mass}}{\text{moles}} = \frac{1.000}{0.06021} = 16.61$$

$$\text{Specific gravity} = \gamma_g = \frac{M_g}{M_a} = \frac{16.61}{28.964} = 0.573$$

$p_{pc} = 672$ psia
$T_{pc} = 346$ °R

At $p = 700$ psia and 85°F:
$p_{pr} = 700/672 = 1.04$
$T_{pr} = (85 + 460)/346 = 1.58$
$z = 0.916$

At $p = 50$ psia and 85°F:
$p_{pr} = 50/672 = 0.07$
$T_{pr} = (85 + 460)/346 = 1.58$
$z = 0.994$

Volume holding gas in reservoir

= (area)(thickness)(porosity)(1 − water saturation)

$$V = 4(5{,}280)^2(10)(0.10)(1 - 0.25) = 83.6 \times 10^6 \text{ cu ft}$$

where V is the actual volume of gas in reservoir at 700 psia and 85°F:

$$\frac{p_b G}{z_b T_b} = \frac{pV}{zT}$$

$$\frac{(14.65)\,G}{(1.00)(60 + 460)} = \frac{(700)(83.6 \times 10^6)}{(0.914)(85 + 460)}$$

$$G = 4{,}170 \times 10^6 \text{ cu ft at 14.65 psia and 60°F}$$

Volume of gas, G, in reservoir at 50 psia and 85°F:

$$\frac{(14.65)G}{(1.00)(60 + 460)} = \frac{(50)(83.6 \times 10^6)}{(0.994)(85 + 460)}$$

$$G = 274 \times 10^6 \text{ cu ft at 14.65 psia and 60°F}$$

Gas removed from the reservoir to reduce pressure from 700 to 50 psia is:

$$4{,}170 \times 10^6 - 274 \times 10^6 = 3{,}896 \times 10^6 \text{ cu ft at 14.65 psia and 60°F}$$

VOLUME OF A POUND-MOLE

Occasionally in natural gas engineering computations, it is convenient to know the volume of a pound-mole of an ideal gas at the various standard pressure bases for gas measurement. Taking the volume of 1 lb-mole of an ideal gas as 379.49 cu ft at 14.696 psia and 60°F, the volumes at other commonly used pressure bases are given in Table 2–2 (see reference 5). If the compressibility factor is known for the actual gas, the volumes given in Table 2–2 should be multiplied by the compressibility factor for the gas at the base pressure and temperature.

Table 2–2 *Volumes of 1 lb-mole of an ideal gas at various pressure bases for gas measurement and at 60°F (based on* GPSA Engineering Data Book).

Pressure Base, psia	*Volume at 60°F, cu ft*
15.025	371.18
14.730	378.61
14.696	379.49
14.650	380.68
14.400	387.29

RELATIVE DENSITY, MOLAR MASS RATIO, AND SPECIFIC GRAVITY OF NATURAL GAS

The specific gravity referred to air for natural gas should be the ratio of the density of the gas to that for dry air. However, in practice the ratio of the molar mass (or molecular weight) of the gas to the molar mass of air has been used in place of the actual specific gravity. The relative density is defined as:

$$\text{Relative density} = (M_g z_a)(p_a' T)(M_a z_g)(p_g' T_g)$$

At pressures near atmospheric and temperatures of 60°F, the compressibility, z_a, is about 0.9996 for air and about 0.998 for natural gas, z_g. Thus, there is a ratio of about 1.0016 between the relative density and the molar mass ratio. For example, in Table 2–6 for the dry gas the molar mass ratio was 0.63260 and the relative density was 0.63355—a difference of 0.00095 or about 0.15%. Although this difference is real, its use in general engineering is probably not justified in view of the accuracy of many of the other variables. It is for this reason the terms specific gravity and molar mass ratio are used interchangeably herein.

In field practice, the specific gravity of a gas is readily determined using recording instruments or a gas-gravity balance. In many cases, the specific gravity of the gas is known when the actual composition is not. If the composition is known, the specific gravity can be calculated from the composition as illustrated in a following section, "Gas Calorimetry."

Occasionally, the only information available may be a bottom-hole pressure survey run with a subsurface pressure gauge. The pressure gradient information can be used with equation 5–20 in Chapter 5 to estimate the specific gravity of the gas.

VISCOSITY OF NATURAL GASES

The viscosity of a Newtonian fluid is defined as the ratio of the shear force per unit area to the local velocity gradient. A convenient chart for determining the viscosities of natural gases is duplicated in Figure 2–3. Other charts for estimating viscosity have been presented by Katz et al.[13] A convenient set of empirical equations for calculating the viscosity, μ, of natural gases, mN/m or centipoises, has been published by Lee et al.[14] Adjusting these equations to give the viscosity results in:

$$\mu = 10^{-4} K \exp (X \rho_1{}^Y) \tag{2–21}$$

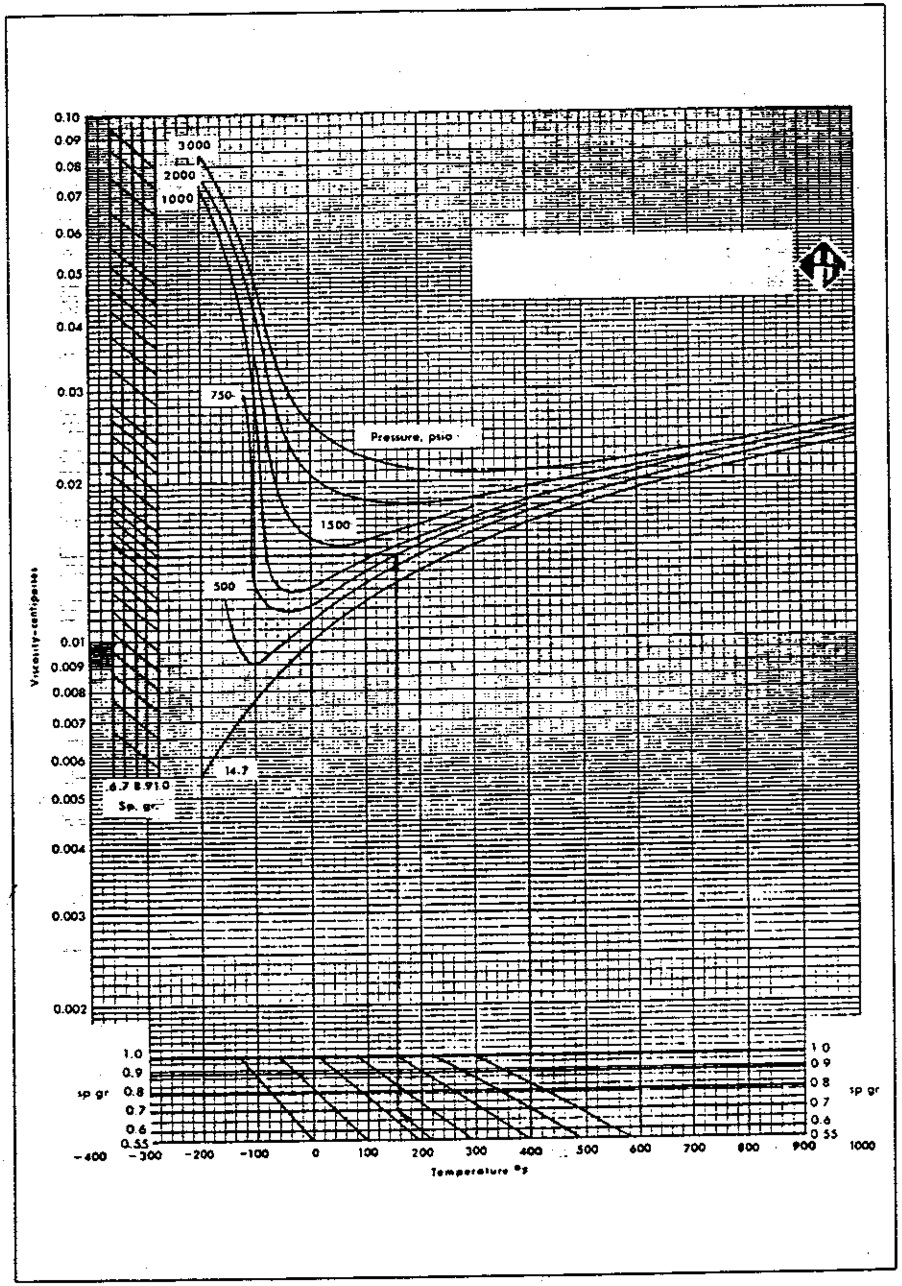

Figure 2–3 *Hydrocarbon gas viscosity* (*source:* GPSA Engineering Data Book).

where:

$$K = \frac{(9.4 + 0.02M)\,T^{1.5}}{209 + 19M + T}$$
$$X = 3.5 + 986/T + 0.01M$$
$$Y = 2.4 - 0.2X$$

and:

ρ_1 = density in gm/cc

$$\rho_1 = (1.4926 \times 10^{-3})\frac{pM}{zT} \qquad (2\text{–}22)$$

Lee et al. measured experimental viscosities for four natural gases from 100 to 340°F and pressures from 100 to 8,000 psia. They found that equation 2–21 reproduced the experimental data with a standard deviation of ± 2.7% and a maximum deviation of 9.0%.

Compressibility factors from Tables A–1, A–2, and A–3 were used to calculate densities using equation 2–22 which, in turn, were used in equation 2–21 to calculate the viscosities at 220°F of Natural Gas No. 2 from reference 14. These viscosities are shown in Table 2–3 along with viscosities taken from Figure 2–3 and the experimentally measured viscosities.

The agreement between experimental and calculated viscosities and those from Figure 2–3 are believed to be very good. Values from either source are probably accurate enough for most engineering calculations. The viscosities of natural gases are used in calculating Reynolds numbers and in flow equations in calculations with the permeability of the reservoir rock.

Table 2–3 ***Viscosities at 220°F for Natural Gas No. 2 in reference 14.***

Pressure (psia)	*Viscosity at 220°F, cp*		
	Experimental	*Calculated*	*Figure 2–3*
700	0.01426	0.01408	—
1,000	0.01486	0.01465	0.0152
1,500	0.01613	0.01588	0.0165
2,000	0.01772	0.01745	0.0186
3,000	0.02157	0.02133	0.0222
4,000	0.02555	0.02522	—
5,000	0.02950	0.02894	—
6,000	0.03315	0.03236	—

CALCULATION OF THE GAS GRAVITY OR MOLAR MASS RATIO OF THE FLOWING FLUID IN A WELL, γ_{gf}

In calculating subsurface pressures from measured wellhead pressures, one must know the specific gravity or molar mass ratio (air = 1.000) of the flowing fluid, γ_{gf}, in order to account for the weight of the column of gas. If the well produces dry gas (no hydrocarbon liquids), the specific gravity can be measured directly or calculated from the analysis. For wells equipped with a separator for removing liquid from the gas stream and a stock tank for storing the liquid, one must calculate the specific gravity of the flowing fluid from that of the separator gas, the gas-to-hydrocarbon liquid ratio, and the specific gravity of the hydrocarbon liquid.

In test example 1, Tables 7–3 and 7–4 in Chapter 7, the specific gravity of the gas from the separators is 0.625 and the gas-to-hydrocarbon liquid ratio is 193,000 cu ft/bbl. Since the hydrocarbon liquid has a gravity of 50.2° API in the stock tank and the gas-to-liquid ratio is relatively high, the hydrocarbon liquid probably condensed out of the flowing fluid as it moved up the wellbore. At any rate, as it moved up the wellbore, the flowing fluid lifted the hydrocarbon liquid, either as a vapor or as a liquid; therefore, it is necessary to calculate the specific gravity, γ_{gf}, of the flowing fluid from the gas-to-hydrocarbon liquid ratio, R_g, in cu ft/bbl from the specific gravity of the separator gas referred to air, γ_g, from the specific gravity of the hydrocarbon liquid, γ_o, referred to water, and from the vapor volume equivalent of the hydrocarbon liquid, V_L, in cu ft/bbl. The gas volume and weight associated with 1 bbl of hydrocarbon liquid is:

$$\text{Volume, cu ft (14.65 psia and 60°F)} = R_g + V_L$$

$$\text{Weight, lb} = R_g\,\gamma_g\,\rho_a + 5.6146\,\rho_L$$

where:

ρ_a = density, lb/cu ft at 14.65 psia and 60°F of dry air

ρ_L = density, lb/cu ft at 60° of the separator liquid

Here, one customarily assumes that the density of the hydrocarbon liquid in the separator is equal to that of the liquid in the stock tank; this is done to save computations.

$$\rho_a = \frac{\text{average molecular weight of air}}{\text{cu ft lb-mole (14.65 and 60°F)}} = \frac{28.964}{380.68} = 0.07608$$

$$\rho_{gf} = \frac{R_g\,\gamma_g\,(0.07608) + 5.6146\,\rho_L}{R_g + V_L} \qquad (2\text{–}23)$$

and:

$$\gamma_{gf} = \frac{\text{density of flowing fluid}}{\text{density of air}}$$

since:

$$\rho_L = (\gamma_o)(\text{density of water})$$
$$\text{density of water} = (7.4805 \text{ gal/cu ft}) \times (8.3372 \text{ lb/gal at 60°F}) = 62.3664 \text{ lb/cu ft}$$

From equation 2–23:

$$\gamma_{gf} \doteq \frac{R_g\,\gamma_g + 4603\,\gamma_o}{R_g + V_L} \qquad (2\text{–}24)$$

The specific gravity and the approximate vapor volume of the hydrocarbon liquid can be calculated from the API gravity by:

$$\gamma_o = \frac{141.5}{131.5 + (\text{API})} \qquad (2\text{–}25)$$

and:

$$V_L = 369 + 5(\text{API}) + 0.04(\text{API})^2 \qquad (2\text{–}26)$$

where (API) = degrees API.

Except for a slight difference in the constant, equation 2–24 is the same as an equation in the IOCC Manual, and equation 2–26 is given in that manual.[15] Equation 2–25 is given in many textbooks. Thus, for test example 1, Tables 7–3 and 7–4 in Chapter 7:

$$V_L = 369 + 5(50.2) + 0.04(50.2)^2 = 721$$

$$\gamma_o \doteq 141.5/(131.5 + 50.2) = 0.7788$$

and:

$$\gamma_{gf} = \frac{193{,}000(0.625) + 4{,}603(0.7788)}{193{,}000 + 721} = 0.641$$

Note: Equation 2–24 is approximate because it does not take into account the hydrocarbon vapor that escapes from the liquid in a separator-stock tank system when the hydrocarbon liquid moves from the pressure and temperature of the separator to that of the stock tank.

GAS CALORIMETRY

The heating value of natural gas determines its value as a fuel and under many recent contracts it determines the sales price. As a result the determination of the heating value has become important economically. The Gas Processors Association established a standard for calculation of gross heating values from compositional analyses.[16] The 1984 standard was revised in 1986.[17] Consequently, the gas production engineer must have an elementary knowledge of the calculational procedures. (Actual measurement of heating values with calorimeters will not be discussed.)

Definitions

A British thermal unit, Btu, (International Table) is equal to 1055.055 852 62 joule (exact). It has been described as the quantity of heat that must be added to one pound of pure water to raise its temperature from 58.5 to 59.5°F under standard pressure. The description is included to give an idea as to the magnitude of a Btu.

The total or gross heating value or total or gross calorific value is the number of Btu's evolved by the complete combustion at constant pressure of one cubic foot with air, with temperature of the gas, air, and products of combustion at 60°F and all water formed by the combustion and originally in the gas as a vapor being condensed to the liquid state. The reader should note that under this definition water vapor that may have been originally in the gas or formed as a result of the combustion contributes to the gross heating value because the condensation of water vapor to the liquid state releases heat. At 14.696 psia and 60°F the heat released from condensation of water vapor is 50.4 Btu per cubic foot of ideal gas.

A standard cubic foot is the unit volume for purposes of measurement and is one cubic foot of gas at a temperature base of ___ (usually 60°F) and a pressure base of ___ (usually 14.65, 14.696, 14.73, or 15.025 psia).

One standard cubic foot of gas in equilibrium with liquid water contains water vapor in the volume ratio that the partial pressure of the water vapor bears to the standard pressure. Thus, at 60°F and 14.696 psia:

$$\text{Vapor pressure of water at } 60°\text{F} = 0.25636 \text{ psia}$$

$$\text{Water vapor mole fraction} = 0.25636/14.696 = 0.01744$$

$$\text{Gas mole fraction} = (14.696 - 0.25636)/14.696 = 0.98256$$

Calculations

An example of the calculation of the gross heating value per ideal cubic foot at 60°F and 14.696 psia from the compositional analysis is given in Table 2–4. The steps in calculating the molar mass ratio of the gas from the molar mass of the pure components and from the molar mass of air are shown also in Table 2–4.

The gross heating value for the gas composition given in Table 2–4 is for an ideal cubic foot of the gas at a pressure of 14.696 psia and 60°F. The gas laws are used to convert the value to another pressure base. Thus, at a pressure base of 15.025 psia the gross heating value of 915.1 Btu per ideal cubic foot at a pressure base of 14.696 psia becomes:

$$(915.1)(15.025)/14.696 = 935.6 \text{ Btu/ideal cf.}$$

Table 2–4 *Calculation of molar mass (molecular weight), gross heating value per ideal cubic foot at 14.696 psia and 60°F, and ideal specific gravity (molar mass ratio) from gas composition.*

Component	(1) Mole Fraction	(2) Molar Mass	(3) = (1) × (2)	(4) Gross Heating Value	(5) = (1) × (4)
Helium (He)	0.0054	4.0026	0.0216	0	0
Carbon Dioxide (CO_2)	0.0003	44.010	0.0132	0	0
Nitrogen (N_2)	0.1297	28.0134	3.6333	0	0
Methane (C1)	0.8283	16.043	13.2884	1010.0	836.6
Ethane (C2)	0.0230	30.070	0.6916	1769.8	40.7
Propane (C3)	0.0088	44.097	0.3881	2516.2	22.1
Isobutane (iC4)	0.0013	58.123	0.0756	3252.1	4.2
n-Butane (nC4)	0.0021	58.123	0.1221	3262.4	6.9
Isopentane (iC5)	0.0004	72.150	0.0289	4000.9	1.6
n-Pentane (nC5)	0.0003	72.150	0.0216	4008.8	1.2
n-Hexane (nC6)	0.0002	86.177	0.0172	4756.2	1.0
n-Heptane (nC7)	0.0002	100.204	0.0200	5502.5	1.1
n-Octane (nC8)	0	114.231	0	6249.1	0
n-Nonane (nC9)	0	128.258	0	6996.4	0
n-Decane (nC10)	0	142.285	0	7743.2	0
	1.0000		18.3216		915.4

Molar mass ratio or ideal specific gravity = (molar mass gas)/(molar mass air) = 18.3216/28.9625 = 0.63260

Gross heating value (dry basis) = 915.4 Btu/ideal cubic foot at 14.696 psia and 60°F.

Gross heating value (water saturated) = 915.4(1 − (0.25636/14.696)) + 50.3(0.25636/14.696) = 900.3 Btu at 14.696 psia and 60°F.

Notes: A value of 28.9625 has been used for the molar mass of air.

Molar masses and gross heating values taken from reference 16.

The gross heating values for the pure components given in column 4 of Table 2–4 can be converted to another pressure base by the same method.

Conversion of the gross heating value per ideal cubic foot to a real cubic foot requires division by the compressibility factor at the given pressure and temperature base. Reference 16 gives a procedure for calculating compressibility factors from virial coefficients for natural gas components and a truncated virial equation of state:

$$z(T,p) = 1 + Bp \tag{2–27}$$

where B is the second virial coefficient for the natural gas mixture. B is calculated as follows:

$$B = x_1^2B_{11} + x_2^2B_{22} + \cdots + x_n^2B_{nn} + 2x_1x_2B_{12} + \cdots + 2x_{n-1}x_nB_{n-1,n} = \sum_{i=1}^{n} \Sigma x_ix_jB_{ij} \tag{2-22}$$

where x_i is the mole fraction of component i in the mixture, B_{ii} is the second virial coefficient for component i, and $B_{ij} = B_{ji}$ is the second cross virial coefficient for components i and j. A tabulation of second virial coefficients from reference 16 for natural gas components is given in Table 2–5. Division of the ideal heating value by the compressibility factor does not produce a real heating value for the gas, but ~~only~~ allows calculation of the ideal energy released upon combustion using the real gas flow rate.

The calculation of the compressibility factor for a natural gas with several components is tedious by hand; therefore, a computer is recommended for solution of equations 2–27 and 2–28. Table 2–6 gives the results of the computations for specific gravity or molar mass ratio, compressibility factor, and gross heating values for the dry and water-saturated natural gas from Table 2–4. Calculations with the truncated equation of state (Table 2–6) gave a compressibility factor of 0.99810 whereas the methods of references 6 and 7 gave a compressibility factor of 0.99798. The close agreement may be coincidental.

For the reader desiring to calculate the compressibility factor by hand, reference 16 gives an alternate approximation which is:

$$B = -[x_1\sqrt{b_1} + x_2\sqrt{b_2} + \cdots + x_n\sqrt{b_n}]^2 \tag{2-29}$$

where $\sqrt{b_i}$ is the summation factor for component i.

Values of $\sqrt{b_i}$ at 60°F are given in Table 2–7. Equation 2–29 is not valid for mixtures containing hydrogen and/or helium. Values of the relative densities, compressibility factors, and gross heating values per real cubic foot in Table 2–6 and marked with an asterisk were calculated by the "hand procedures." The summation factor for the small concentration of helium in the gas was assumed to be zero for illustration purposes.

Note: At this point a warning is believed to be appropriate regarding use of the calculational methods for gas calorimetry. The impetus to calculate

Table 2–5 *Second virial coefficients for natural gas components,* B_{ii}, B_{ij}.

	C_1	C_2	C_3	iC_4	nC_4	iC_5	nC_5	nC_6	nC_7	nC_8	nC_9	nC_{10}	H_2	He	H_2O	CO	N_2	O_2	H_2S	Ar	CO_2
C_1	−0.135	−0.281	−0.425	−0.457	−0.560	−0.632	−0.675	−0.793	−0.962	−1.114	−1.149	−1.307	0.022	0.070	−0.224	−0.021	−0.060	−0.052	−0.241	−0.076	−0.181
C_2		−0.569	−0.833	−1.005	−1.106	−1.293	−1.379	−1.652	−1.953	−2.355	−2.614	−2.901	0.032	0.057	−0.422	−0.106	−0.152	−0.124	−0.445	−0.164	−0.385
C_3			−1.183	−1.522	−1.666	−1.953	−2.068	−2.542	−2.872	−3.274	−4.093	−4.639	0.016	0.098	−0.589	−0.178	−0.237	−0.201	−0.842	−0.244	−0.618
iC_4				−2.097	−1.982	−2.566	−2.614	−3.317	−4.007	−4.782	−5.500	−6.204	0.000	0.075	−0.704	−0.241	−0.213	−0.270	−1.112	−0.264	−0.819
nC_4					−2.289	−2.671	−2.915	−3.404	−4.254	−5.113	−5.888	−6.678	0.026	0.128	−0.761	−0.253	−0.250	−0.293	−1.152	−0.345	−0.862
iC_5						−3.375	−3.504	−4.452	−5.371	−6.376	−7.454	−8.545	0.069	0.075	−0.862	−0.313	−0.326	−0.391	−1.436	−0.388	−1.062
nC_5							−3.978	−4.739	−5.746	−6.850	−7.755	−8.904	−0.012	0.075	−0.899	−0.319	−0.280	−0.474	−1.476	−0.391	[illegible]
nC_6								−6.434	−7.238	−8.674	−9.996	−11.45	−0.010	0.080	−1.023	−0.408	−0.373	−0.508	−1.676	−0.416	−1.379
nC_7									−8.904	−10.77	−12.39	−14.13	−0.121	0.086	−1.120	−0.483	−0.452	−0.572	−2.269	−0.511	−1.623
nC_8										−12.93	−14.69	−17.23	0.032	0.095	−1.235	−0.585	−0.474	−0.678	−2.709	−0.618	−1.910
nC_9											−17.71	−20.39	0.072	0.101	−1.321	−0.675	−0.618	−0.761	−2.131	−0.732	−1.183
nC_{10}												−23.64	−0.083	0.106	−1.407	−0.747	−0.658	−0.870	−3.605	−0.819	−2.485
H_2													−0.041	0.046	0.000	0.038	0.035	0.032	0.002	0.020	−0.066
He														0.034	0.057	0.052	0.060	0.063	0.043	0.051	0.051
H_2O															−3.678	−0.172	−0.132	−0.224	−1.020	−0.121	−0.689
CO																−0.028	−0.003	−0.011	−0.149	−0.043	−0.124
N_2																	−0.019	−0.037	−0.135	−0.037	−0.144
O_2																		−0.053	−0.161	−0.019	−0.118
H_2S																			−0.641	−0.136	−0.416
Ar																				−0.051	−0.134
CO_2																					−0.388

Units are 1000 $psia^{-1}$
Example: For $B_{nC_4N_2}$ go to row nC_4 and across to N_2 column, the value is −0.250/1000
Used by permission of the Gas Processors Association.

Table 2–6 *Molar mass, molar mass ratio, relative density, compressibility factor, and gross heating values for the dry and water saturated natural gas of Table 2–4. Calculations conform to Gas Processors Association Standard GPA 2172–84.*

Natural Gas Properties from Composition
Conditions: 60°F, 14.696 psia

DRY ANALYSIS

Methane	= 0.82830	Hydrogen	= 0.00000
Ethane	= 0.02300	Helium	= 0.00540
Propane	= 0.00880	Water	= 0.00000
i-Butane	= 0.00130	Carbon monoxide	= 0.00000
n-Butane	= 0.00210	Nitrogen	= 0.12970
i-Pentane	= 0.00040	Oxygen	= 0.00000
n-Pentane	= 0.00030	Hydrogen sulfide	= 0.00000
Hexanes	= 0.00020	Argon	= 0.00000
Heptanes	= 0.00020	Carbon dioxide	= 0.00030
Octanes	= 0.00000		
Nonanes	= 0.00000		
Decanes	= 0.00000		

Molar mass	= 18.322	
Molar mass ratio	= 0.63260	
Relative density	= 0.63355	0.63354*
Compressibility factor	= 0.99810	0.99813*
Gross heating value, Btu/lb	= 18959.5	
Gross heating value, Btu/ideal cf	= 915.4	
Gross heating value, Btu/real cf	= 917.1	917.1*

* Indicates computations by hand procedures

WATER SATURATED ANALYSIS

Methane	= 0.81385	Hydrogen	= 0.00000
Ethane	= 0.02260	Helium	= 0.00531
Propane	= 0.00865	Water	= 0.01744
i-Butane	= 0.00128	Carbon monoxide	= 0.00000
n-Butane	= 0.00206	Nitrogen	= 0.12744
i-Pentane	= 0.00039	Oxygen	= 0.00000
n-Pentane	= 0.00029	Hydrogen sulfide	= 0.00000
Hexanes	= 0.00020	Argon	= 0.00000
Heptanes	= 0.00020	Carbon dioxide	= 0.00029
Octanes	= 0.00000		
Nonanes	= 0.00000		
Decanes	= 0.00000		

Molar mass	= 18.316	
Molar mass ratio	= 0.63241	
Relative density	= 0.63341	0.63355*
Compressibility factor	= 0.99804	0.99783*
Gross heating value, Btu/lb	= 18652.5	
Gross heating value, Btu/ideal cf	= 900.3	
Gross heating value, Btu/real cf	= 902.1	902.2*

* Indicates computations by hand procedures

Table 2–7 *Summation factors at 60°F for components of natural gas.*

Mixtures, Units: psia^{-2}

Component	$\sqrt{b_i}$	*Component*	$\sqrt{b_i}$
Methane	0.0116	Hydrogen	
Ethane	0.0239	Helium	
Propane	0.0344	Water vapor	0.0623
Isobutane	0.0458	Carbon monoxide	0.0053
n-Butane	0.0478	Nitrogen	0.0044
Isopentane	0.0581	Oxygen	0.0073
n-Pentane	0.0631	Hydrogen sulfide	0.0253
Hexane	0.0802	Argon	0.0071
Heptane	0.0944	Carbon dioxide	0.0197
Octane	0.1137		
Nonane	0.1331		
Decane	0.1538		

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compressibility factors and heating values to five significant figures places an accuracy burden on the gas analysis, gas measurement, the question as to whether the analysis is representative of the gas stream, and the calculational procedures. In general, such computations for a chain of procedures are no more accurate than the least accurate of the analyses of procedures. Therefore, all steps in the procedures become critical with respect to the accuracy of the final result.

PROBLEMS

1. A tank with a volume of 40 cu ft is filled with a carbon dioxide and air mixture. The pressure within the tank is 30.0 psia at 70°F. It is known that 2.0 lb of carbon dioxide was placed in the tank. Assume that air is 80% nitrogen and 20% oxygen and use the ideal gas laws. Calculate:

   a. Mass of air in the tank
   b. Mass percent of each constituent
   c. Volume percent of each constituent
   d. Mole fraction of each constituent
   e. Partial pressure of each constituent

f. Average molecular weight of the mixture
g. Density of the mixture at 20 psia and 70°F, at 14.7 psia and 60°F, and at 14.7 psia and 32°F
h. Specific gravity of the mixture (air = 1.000)

2. Given the following gas:

Component	Mole (%)
Methane	58.7
Ethane	16.5
Propane	9.9
Butanes	5.0
C_{5+}	3.5
Hydrogen sulfide	6.4
	100.0

Calculate:

a. Average molecular mass
b. Density at 32°F and 14.7 psia
c. Density at 0°C and 760 mm mercury
d. Specific gravity and gross heating value at 14.696 and 60°F

3. Construct a chart of compressibility factors plotted against pressure for pressures at 500 psi intervals to 3,000 psia and at temperatures of 100, 150, and 200°F for natural gas with the composition given in Table 2–6.

4. Repeat problem 3 for a natural gas with the specific gravity $\gamma_g = 0.650$, $CO_2 = 3.5\%$, and $N_2 = 2.0\%$.

5. Using the results of problem 4, calculate the compressibility and the formation volume factor at 2,700 psia and 200°F.

6. Calculate the viscosity of the natural gas in problem 4 at 2,700 psia and 200°F using equation 2–21; compare the answer with the value read from Figure 2–3.

7. Calculate the gross heating in Btu per ideal cu ft for the gas composition given below on a dry basis and for the gas in equilibrium with liquid water at a pressure base of 14.73 psia and temperature base of 60°F. The composition follows:

Component	Mole %	Component	Mole %
Helium	0.00120	i-Butane	0.00280
Nitrogen	0.02200	n-Butane	0.00830
Carbon dioxide	0.00210	i-Pentane	0.00130
Methane	0.88000	n-Pentane	0.00210
Ethane	0.05760	Hexanes	0.00270
Propane	0.01760	Heptanes	0.00230
		Total	1.00000

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3 Application of Gas Laws to Reservoir Engineering

The general gas laws and the concept that a gas reservoir is an idealized container form the basis for estimating the volume of gas contained in reservoirs. The extent to which a gas reservoir departs from the behavior of an idealized container depends upon the saturation and mobility of other fluids; the porosity and especially the permeability of the reservoir rock, and the capacity of the rock and interstitial liquids to permit the flow of the gas to production wells. In considering the ideal gas reservoir, the following assumptions will be made.

1. The volume of pore space in the reservoir containing gas remains constant throughout the productive life of the reservoir (i.e., no significant water or oil migration occurs, and there is no appreciable compressibility of the rock matrix).
2. Shut-in pressures are representative of actual reservoir pressures.
3. Gas production data include all gas produced from the reservoir.
4. No gas is added to the reservoir by the evolution of solution gas from oil or water.

VOLUMETRIC DETERMINATION OF GAS IN PLACE

For a given quantity of gas, the terms n and R may be eliminated from the general gas equation (equation 2–2) to give the following:

$$\frac{pV_g}{zT} = \frac{p_b G}{T_b z_b} \tag{3–1}$$

where:

V_g = volume of space-holding gas, cu ft
G = volume of gas in the reservoir at standard conditions, cu ft
b = standard conditions for gas measurement

If 1 acre-ft of reservoir rock is under consideration, the volume in cubic feet, V_g, capable of holding gas when the porosity, ϕ, and the water saturation, S_w, are known, is:

$$V_g = 43{,}560\,\phi(1 - S_w) \tag{3–2}$$

Combining equations 3–1 and 3–2 and solving for the gas in place, G (expressed in cubic feet at standard conditions), the result is:

$$G = 43{,}560\,\phi(1 - S_w)pT_b/(zp_bT) \tag{3–3}$$

if the approximation has been made that z_b is unity and with pressure and temperature bases of 14.65 psia and 60°F, equation 3–3 becomes:

$$G(\text{per acre-foot}) = 1{,}546{,}000\,\phi(1 - S_w)p/(zT) \tag{3–4}$$

where G is expressed in standard cubic feet. If one follows the natural gas industry convention by expressing volume in thousands of cubic feet (Mcf), equation 3–4 becomes:

$$G_k = Mcf = 1{,}546\,\phi(1 - S_w)p/(zT) \tag{3–5}$$

Equation 3–5 is used in volumetric estimates of the quantity of gas in place per acre-foot of reservoir.

Example 3–1. The gas used in example 2–5 had a specific gravity of 0.573, p_{pc} = 672, and T_{pc} = 346. Calculate the gas in place per acre-foot when the reservoir pressure was 600 psia at the reservoir temperature of 85°F. Porosity = 10% and S_w = 25%.

$$p_{pr} = 600/672 = 0.89$$
$$T_{pr} = (460 + 85)/346 = 1.58$$
$$z = 0.926$$
$$G_k = Mcf = \frac{(1{,}546)(0.10)(0.75)(600)}{(0.926)(85 + 460)} = 137.9$$

or 137.9 Mcf/acre-ft at standard pressure and temperature of 14.65 psia and 60°F, respectively. At a standard pressure of 14.73 psia, the volume would be:

$$\frac{(137.9)(14.65)}{(14.73)} = 137.2 \text{ Mcf/acre-ft.}$$

MATERIAL BALANCE

Pressure-production methods for estimating gas reserves consist of various types of plots of reservoir pressure corrected for the compressibility of the gas against cumulative production and extrapolating the curve to an abandonment pressure to arrive at the recoverable gas reserve. These methods and their variations are based on a material balance concept and the general gas law. The direct application of material balance principles to determine the quantity of gas in place involves the assumptions that the reservoir is closed and that there is no water drive or intrusion of oil. The material balance equation is:

$$G_p = G_i - G \tag{3–6}$$

This states that the quantity of gas produced is equal to the initial quantity in the reservoir minus the gas remaining in the reservoir at the time under consideration. The applicable gas laws for a reservoir that has a volume of space, V, containing gas are:

$$\frac{p_iV}{z_iT} = \frac{p_bG_i}{T_b} \text{ and } \frac{pV}{zT} = \frac{p_bG}{T_b}$$

where z_b is assumed to be 1.000. Substituting the above into equation 3–6 yields the equations that are commonly used in work with pressure-production curves for gas wells. The resulting equation is:

$$\frac{p}{z} = \frac{p_i}{z_i} - \frac{p_b T}{V T_b}(G_p) \qquad (3\text{–}7)$$

Equation 3–7 represents the form $y = mx + b$ and is a straight line when plotted on cartesian coordinate paper. This equation is the basis for plotting p/z against cumulative production on cartesian coordinates. If the reservoir meets with the requirements set out at the start of Chapter 3, the result is a straight line. Further rearrangement of equation 3–7 results in the following:

$$\log(p_i/z_i - p/z) = \log(G_p) + \log(p_b T/V T_b) \qquad (3\text{–}8)$$

This is an equation of the form ($\log y = \log x + \log b$), and a plot of ($p_i/z_i - p/z$) against cumulative production on log-log coordinates results in a straight line with a slope of unity. It forms a 45° angle with either axis. The term on the right in equation 3–8 is constant for any given reservoir if it is a closed reservoir. This approach was discussed in detail by Gruy and Crichton (see reference 5).

Example 3–2. The following pressures and cumulative production are for a gas well with a reservoir temperature of 112°F. Gas composition is 0% carbon dioxide, 3.0% nitrogen, and 0% hydrogen sulfide. Specific gravity is 0.666. Prepare plots of the pressure-production relationships on cartesian and log-log coordinates and estimate the original gas in place.

	Given		*Calculated*			
Time	*Pressure (psia)*	*Cumulative Production (MMcf)*	p_{pr}	z	p/z	$(p_i/z_i - p/z)$
0	1,556	0	2.34	0.825	1,886	0
1	1,543	19	2.32	0.826	1,868	18
2	1,538	45	2.31	0.826	1,862	24
3	1,504	108	2.26	0.829	1,814	72
4	1,443	214	2.17	0.833	1,732	154
5	1,432	275	2.15	0.834	1,717	169
6	1,423	319	2.14	0.835	1,704	182
7	1,360	427	2.05	0.840	1,619	267
8	1,274	522	1.92	0.848	1,502	384

$p_{pc} = 670 + 0 - 5 + 0 = 665$

$T_{pc} = 378 - 0 - 9 - 0 = 369$

$T_{pr} = (112 + 460)/369 = 1.55$

The original gas in place derived from Figure 3–1 is 2,780 MMcf and from Figure 3–2, 2,950 MMcf.

PROBLEMS IN USING MATERIAL BALANCE METHODS

Material balance methods for estimating gas reserves are widely used and, in general, are much preferred by the author over volumetric methods where conditions are favorable and production information is available. Although plotting pressure divided by compressibility, p/z, against cumulative production on cartesian coordinates is generally accepted, the log-log plot is of historical interest, but the method has serious disadvantages—and some compensating advantages—when applied to new wells in new reservoirs. Regardless of the type of plot used to interpret p/z and cumulative production data, pressures representative of the true reservoir pressure are a necessity. The simple material balance method illustrated by Figures 3–1 and 3–2 is virtually useless for reservoirs that have active water drives. Also, applying the method to wells in low-permeability reservoirs is difficult

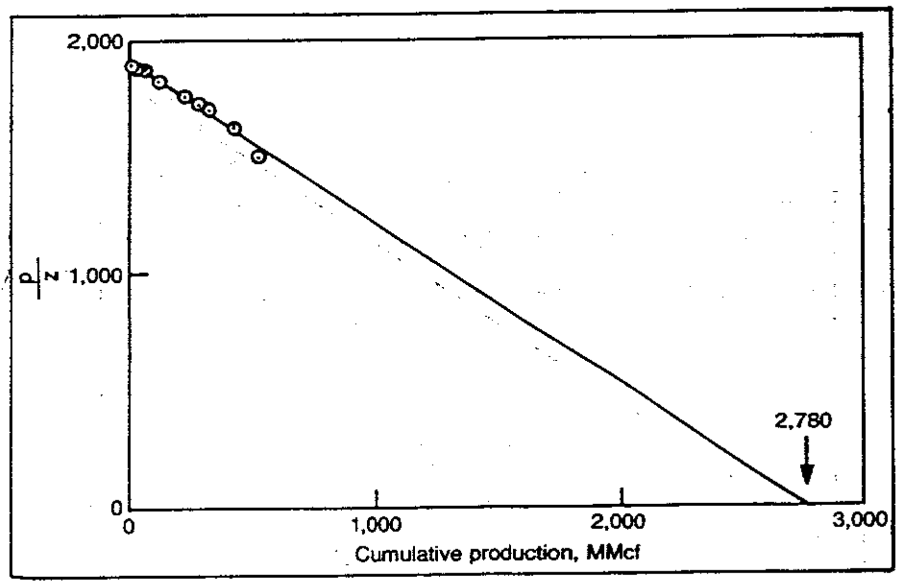

Figure 3–1 *Pressure-production curve for a gas well on cartesian coordinates.*

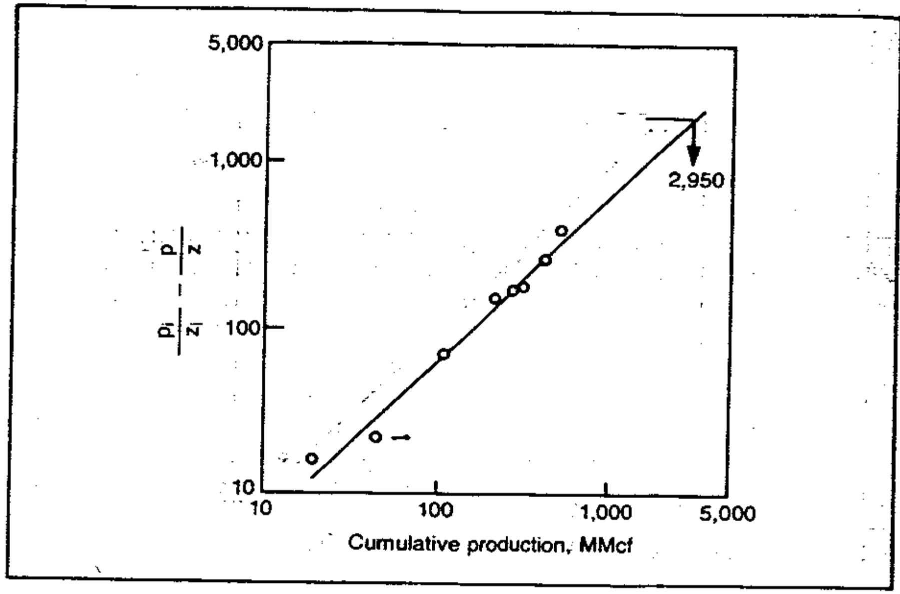

Figure 3–2 *Pressure-production curve for a gas well on log-log coordinates.*

where pressure builds slowly after the well is shut in. Liquid production and accumulation in the wellbore add to the difficulties.

The effect of water influx on *p/z*-cumulative-gas-production curves has been described in detail by Bruns et al. and has been reviewed from the viewpoint of obtaining the most gas from the reservoir by Agarwal et al.[1,2] The conclusions of Bruns et al. are illustrated in Figure 3–3, in which the performance of a gas reservoir containing 469 billion cu ft of gas in place has been calculated for aquifer sizes 1.5 and 10 times the size of the gas reservoir. After the calculated production of 300 billion cu ft, or 64% of the gas in place, the *p/z*-cumulative-production curve for the gas reservoir in contact with the small aquifer indicates gas in place of 475 billion cu ft, or 101.3% of the gas actually in place. For a gas reservoir in contact with an aquifer 10 times the size of the gas reservoir, the curve would indicate 990 billion cu ft or 211% of the actual gas in place (see Fig. 3–3). Thus it was concluded that it is dangerous to extrapolate *p/z*-cumulative-production curves by using a straight line without considering the possibility of water influx.

Unfortunately, similar apparent behavior can be caused early in the production life of a well by a combination of flow rates, unsteady-state behavior, and insufficient time for buildup of the shut-in pressure. This behavior is characteristic of wells in reservoirs with low permeability. At first, the well is put on production at very high rates of flow. The short-term shut-in pressures show rapid decrease from the original shut-in pressure. Then, the operator cuts the production rate and shut-in pressures tend to stabilize. Again, interpreting the *p/z*-cumulative-production curve becomes difficult. For an introduction to such unsteady-state behavior, the reader is referred to the treatises on pressure buildup and flow tests in wells by Matthews and Russell and by Earlougher.[3,4] Regardless of reservoir characteristics, stabilization time, or producing techniques, a minimum of 5% of the gas in place should be produced before confidence can be placed in a material balance forecast.

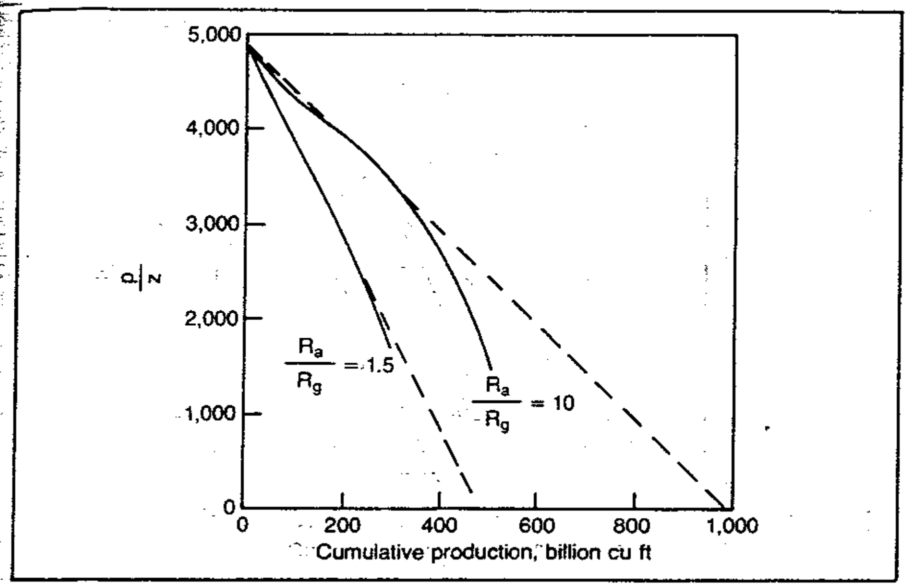

Figure 3–3 *Effect of aquifer size on p/z-cumulative production curves.*[1]

PROBLEMS

1. Given a pressure-production history as follows, make two types of plots and determine the initial gas in place. The specific gravity of the gas = 0.740, CO_2 = 0.00%, N_2 = 6.17%, and the reservoir temperature = 152°F.

Note: First determine z for each pressure at 152°F, then calculate p/z.

Time	Reservoir Pressure (psia)	Cumulative Production (MMcf)
0	2,066.5	0
1	1,997.1	50
2	1,782.2	479
3	1,728.4	597
4	1,647.3	722
5	1,597.6	837
6	1,533.1	945

Repeat the problem but assume ideal gas laws.

2. A gas reservoir at a temperature of 105°F with gas having a specific gravity of 0.610, CO_2 = 0.00%, and N_2 = 0.00% had the following production history. Calculate the original gas in place.

Reservoir Pressure (psia)	Cumulative Production (Billion cu ft)
2,080	0.0
1,885	6.9
1,620	14.0
1,205	23.7
888	31.0
645	36.2

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4 Gas Measurement

In production and transportation practices, natural gas is measured by volume in cubic feet at an operating pressure and temperature and is corrected to some reference or base pressure and temperature. Also, the field engineer is involved in measurement using orifice meters and critical flow provers. The development of the principles of measurement using these devices starts with an energy balance across a general flow system as illustrated in Figure 4–1 in which points 1 and 2 are any two points between which an energy balance may be considered. At point 1, the internal energy is E_1; the energy needed to move the fluid past point 1 is p_1V_1; the potential energy is H_1 above the datum; and the kinetic energy is $u_1^2/2g_c$. Between points 1 and 2, a pump may supply energy, W, friction may cause an energy loss of W_f, and energy, q, may be gained or lost to the surroundings. The energy balance is:

$$E_1 + p_1V_1 + H_1 + \frac{u_1^2}{2g_c} + q + W - W_f = E_2 + p_2V_2 = H_2 + \frac{u_2^2}{2g_c} \qquad (4\text{–}1)$$

If the energy balance is narrowed to 1 lb of fluid and flow through a typical, flat-plate orifice, equation 4–1 reduces to:

$$p_1V_1 - p_2V_2 = \frac{u_2^2}{2g_c} - \frac{u_1^2}{2g_c} \qquad (4\text{–}2)$$

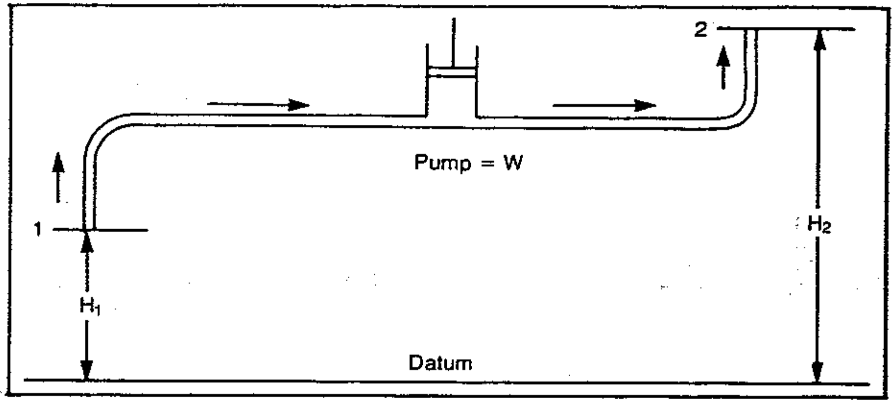


Figure 4–1 *General flow system for energy balance.*

where:

$H_1 = H_2$ and $E_1 = E_2$
W_f, W, and $q = 0$

Equation 4–2 states that, for a velocity meter, the induced change in velocity must be accompanied by a change in the static pressure. This is the principle of the orifice meter.

ORIFICE METER

If one follows the procedure of the American Gas Association Committee Report No. 3, equation 4–2 becomes:[1]

$$q_h = 218.44\, d^2 K \frac{T_b}{p_b} \sqrt{\frac{1}{T_f \gamma_g}} \sqrt{h_w p_f} \qquad (4\text{–}3)$$

where:

q_h = flow rate, cubic ft/hr, at base conditions
d = orifice diameter, in.
K = discharge coefficient, which must be determined experimentally
T_b = absolute temperature at reference or base conditions, °F + 460

p_b = absolute pressure at reference or base conditions, psia
T_f = absolute flowing temperature, °F + 460
γ_g = specific gravity of the flowing gas (air = 1.00)
h_w = differential pressure in inches of water
p_f = absolute static pressure, psia

For convenience, equation 4–3 is reduced to the following equations:

$$q_h = C'' \sqrt{h_w p_f} \qquad (4\text{–}4)$$

where:

$$C'' = F_b \times F_r \times Y \times F_{pb} \times F_{tb} \times F_{tf} \times F_g \times F_{pv} \times F_m \times F_a \times F_l \qquad (4\text{–}5)$$

where:

F_b = basic orifice factor
F_r = Reynolds number factor
Y = expansion factor
F_{pb} = pressure base factor
F_{tb} = temperature base factor
F_{tf} = flowing temperature factor
F_g = specific gravity factor
F_{pv} = supercompressibility factor
F_m = manometer factor
F_a = orifice thermal expansion factor
F_l = gauge location factor

For actual practice in field measurement and gas transportation, this text shall adopt the tables and procedures of the Interstate Oil Compact Commission, so equations 4–4 and 4–5 become:[2]

$$q_k = F_b \times \sqrt{h_w p_f} \times F_t \times F_g \times F_{pv} \qquad (4\text{–}6)$$

where:

q_k = thousands of cubic feet/24 hr (Mcfd) at a pressure base of 14.65 psia and a temperature base of 60°F.
F_b = basic orifice factor

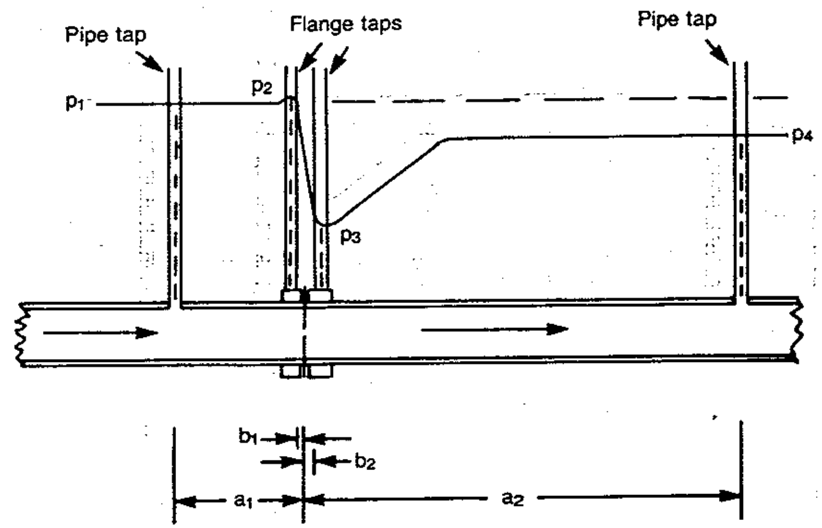
F_t = flowing temperature factor
F_g = specific gravity factor
F_{pv} = supercompressibility factor

Values for the basic orifice factor, F_b, are given in Appendix B, Tables B–1 and B–2, for flange- and pipe-tap types of orifice runs, respectively. Values for the flowing temperature, F_t, and the specific gravity, F_g, factors are given in Table B–4 and B–5, respectively. Values of the supercompressibility factor, F_{pv}, should be calculated using the methods given in Chapter 2.

Example 4–1. Gas with a specific gravity of 0.610, a carbon dioxide and nitrogen content of 2.0 and 3.0%, respectively, is flowing through a 2-in. nominal meter run with an orifice diameter of 0.75 in. The differential pressure is 20.5 in. of water, the flowing pressures is 624 psia, and the temperature is 105°F. Calculate the rate of flow. The meter has flange taps.

$F_b = 2.779$ (Table B–1)
$F_t = 0.9594$ (Table B–4)
$F_g = 1.280$ (Table B–5)
$p_{pc} = 671 + 8 - 5 = 674$
$T_{pc} = 362 - 3 - 9 = 350$
$p_{pr} = 624/674 = 0.926$
$T_{pr} = (105 + 460)/350 = 1.614$
$z = 0.931$
$F_{pv} = \sqrt{1/0.931} = 1.036$
$q_k = (2.779)\sqrt{(20.5)(624)}\,(0.9594)(1.280)(1.036)$
$q_k = 400$ Mcfd at 14.65 psia and 60°F

The meter run illustrated in Figure 4–2 includes upstream and downstream sections of pipe, the flanges between which the orifice is installed, the orifice plate itself, the pressure taps, and the indicating or recording instrument. The term *flange taps* is used to indicate that pressure and differential pressure measurements are made at the flanges. For *pipe taps*, the measurements are made upstream and downstream along the meter run. The use of flange taps for pressure measurement predominates since the construction and setting of the meter are easier because there is less piping and less danger of damage.



a_1 = 2.5 pipe diameters upstream
a_2 = 8.0 pipe diameters downstream
b_1 = 0.1 to 0.5 pipe diameter from orifice plate
b_2 = about 0.4 pipe diameter from orifice plate
p_2 slightly > p_1
p_4 stabilizes about 4 to 6 pipe diameters downstream

Figure 4–2 *Orifice plate showing pressure profile across plate.*

DESCRIPTION OF ORIFICE METERS

Refer to the American National Standard ANSI/API 2530, First Edition for definitive specifications and recommendations for orifice meters.[3] Figure 4–2 shows that the critical components of an orifice meter are the orifice plate, the taps for connecting instruments to measure pressure and the differential pressure across the plate, and placement of the taps with respect to the plate. Unfortunately, the figure cannot show the careful attention to construction details that allows accurate metering of natural gas.

Orifice Plates

The orifice plate should be concentric with and perpendicular to the axis of the meter tube. The upstream face of the plate should be as flat as can be obtained commerically. The surface roughness should not exceed 50 micro-inches. The thickness of the plate should not be greater than ⅛ the diameter of the orifice. The ratio of the orifice diameter to the diameter of the meter tube should be between 0.15 and 0.70 for flange taps and between 0.20 and 0.67 for pipe taps meters. The upstream edge of the measuring orifice should be square and sharp so that it will not show a beam of light when checked with an orifice edge gauge or will not reflect a beam of light when viewed without magnification. The orifice plate should be kept clean at all times. If it is nicked or out-of-shape, it should be replaced. Reference 4 presents tolerances for orifice plate flatness and orifice diameters that should be followed carefully.

Meter Tubes

Meter tubes upstream and downstream from the orifice plate are subject to specifications of diameter, conditions of the inside surface, and length and placement of ells upstream and downstream of the orifice. The purpose of all specifications is to create a region of steady flow conditions, both upstream and downstream from the orifice plate, and to ensure maximum metering accuracy. Needless to say, meter tubes should be free of liquids, dirt, and other foreign material.

Miscellaneous Fittings

The placement of pipe-size reducers, valves, regulators, and straightening vanes with respect to the orifice plate likewise is subject to specifications. Incidental equipment such as orifice flanges, orifice fittings, pressure tap holes, and thermometer wells should meet American National Standard specifications.

PULSATING FLOW

Pulsating flow consisting of sudden changes in velocity and pressure is a serious detriment to accurate gas measurement. It causes the differential pen on the chart to cover large areas of the chart with ink, or to "paint," which causes doubt as to the effective differential pressure reading. The most common sources of pulsation are:

1. Compressors
2. Pressure regulators or worn valves
3. Irregular movements of liquids in the line
4. Dumping of separators and intermitters on wells

Although it may not be possible to eliminate pulsations, it is possible to diminish the effects of pulsations on gas measurement. The meter location can be changed to the suction side of the compressor and the distance from the source can be increased, which increases the volume of gas between the source and the meter. These factors can be considered in the design and installation of a meter. The effect of pulsation on gas measurement can be minimized by using a smaller orifice so that the differential is near the upper range of the meter. If the source is a pressure regulator or valve, one may be able to adjust the regulator or replace or repair the valve so that the pulsations are diminished.

CRITICAL FLOW PROVER

Another type of gas measurement device is the critical flow prover in which measured gas is discharged under pressure through an orifice plate and into the atmosphere. The instrument wastes gas but is justified when testing gas wells that are remote from a pipeline. The testing of such wells is directed to the problem of determining if the availability of gas from the well justifies the expense of connecting the well to a market for the gas.

When gas flows through an orifice, the velocity up to a point is controlled by the differential pressure across the orifice. As the differential pressure increases—either by an increase in the upstream pressure, a decrease in the downstream pressure, or both—the velocity of the gas increases and approaches a value known as the *critical velocity*. After this critical velocity is reached, the differential pressure across the orifice no longer affects the velocity through the orifice. Critical velocity is reached when the ratio of upstream to downstream pressure is approximately 2 or more. Above the critical pressure ratio, no matter how much pressure is applied at the upstream side, the linear velocity at the *vena contracta* remains the same. This does not mean that flow through the orifice will remain the same because the density of the flowing fluid increases as the pressure increases.

For upstream pressure values greater than twice the downstream pressure ($p_1/p_2 > 2$), the rate of flow is directly proportional to the upstream pressure. Thus critical flow is said to exist at this pressure condition. The equation for calculating the rate of flow through a critical flow prover as given in the IOCC well test manual for a Bureau of Mines critical flow prover is:

$$q_k = F_p \times p_m \times F_t \times F_g \times F_{pv} \tag{4-7}$$

where:

q_k = rate of flow, Mcfd
F_p = basic orifice prover factor
p_m = pressure upstream of orifice plate, psia
F_t = flowing temperature factor
F_g = specific gravity factor
F_{pv} = supercompressibility factor

Basic orifice prover factors are given in Appendix B, Table B–3, and F_t and F_g are given in Tables B–4 and B–5, respectively.

Example 4–2. A well is tested using a 2-in. Bureau of Mines critical flow prover. The following data are taken. Calculate the rate of flow using:

Carbon dioxide, %	0
Nitrogen, %	0
Flowing pressure, psig	270
Plate size, in.	3/8
Atmospheric pressure, psia	13.7
Flowing temperature, °F	60
Specific gravity (air = 1.00)	0.73

Then:

F_p = 2.439 (Table B–3)
F_t = 1.000 (Table B–4)
F_g = 1.170 (Table B–5)
$p_{pc} = 668$
$T_{pc} = 401$
$p_{pr} = 283.7/668 = 0.425$
$T_{pr} = (60 + 460)/401 = 1.297$
$z = 0.930$
$F_{pv} = \sqrt{1/0.930} = 1.037$
$q_k = \text{Mcfd} = 2.439 \times 283.7 \times 1.000 \times 1.170 \times 1.037$
$= 840$ Mcfd at $p_b = 14.65$ and $T_b = 60°\text{F}$

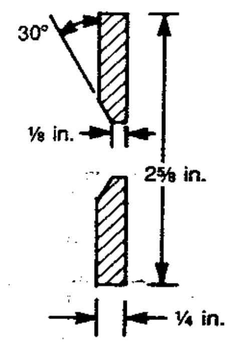


Figure 4–3 *Design of orifice plates for U.S. Bureau of Mines type critical flow prover.*

Bureau of Mines Critical Flow Prover

The specifications for this 2-in. prover are given in Figure 36, page 118, of *Bureau of Mines Monograph 7*, and the specifications for the orifice plates are shown in Figure 4–3.[4] Other types of provers have been made and sold for various purposes.

PROBLEMS

1. The Bureau of Mines gives the following table of orifice prover factors and the formula for calculating rate of flow.[5] Devise a constant multiplier for determining the factors in Table B–3. Note that the pressure bases for gas volumes are different.

Prover coefficients for a 2-in. critical-flow prover from the Bureau of Mines

Orifice Diameter (in.)	*Average Coefficient*	*Maximum Variation (%)*	*Orifice Diameter (in.)*	*Average Coefficient*	*Maximum Variation, (%)*
1/16	1.524	3.61	1/2	101.8	2.29
3/32	3.355	1.14	5/8	154.0	1.56
1/8	6.301	2.25	3/4	224.9	1.03
3/16	14.47	3.88	7/8	309.3	2.31
7/32	19.97	3.82	1	406.7	2.09
1/4	25.86	1.88	1 1/8	520.8	1.26
5/16	39.77	2.13	1 1/4	657.5	3.61
3/8	56.58	2.74	1 3/8	807.8	2.05
7/16	81.09	2.33	1 1/2	1,002.0	6.32

$$q' = \frac{Cp}{\sqrt{\gamma_g T}}$$

q' = Mcfd at 14.4 psia and 60°F
C = plate coefficient, given in table
p = upstream pressure, psia
γ_g = gas gravity (air = 1.00)
T = flowing temperature (°F + 460), °R

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5 Flow of Natural Gas in Circular Pipe and Annular Conductors

The behavior of a flowing gas stream whether in the tubing or the annular space of a well or in a pipeline is similar. The only difference between horizontal pipelines and gas wells is the pressure of the head of gas in the well, which increases the pressure drop in flowing gas wells. Flow in wells and pipelines is affected by friction factors, pipe roughness, pipe diameter, pipe length, temperature, pressure, and the properties of the gas. An accurate description of the behavior of gas flow in the form of a valid flow equation has been an engineering necessity since the early days of the gas industry. During the nearly 100 years in which gas has been produced and transported, over two dozen flow equations have been proposed. In 1935, the Bureau of Mines published its first extensive study as Monograph 6, and a later work, Monograph 9, was published in 1956.[1,2] The equations and methods presented in these publications have been supported by subsequent developments and other laboratory experiments.

For an equation to be useful to the engineer, relatively simple determination of values for the three principal variables must be possible. These three variables—throughput flow rate, pipe diameter, and pressure drop—should be related in such a way that any one is a function of the other two and of the known properties of the gas in the pipe. They should be expressed in practical industry terminology so that they can be used easily.

DEVELOPMENT OF EQUATIONS

The unity of the principles for the flow of gas in horizontal pipelines and gas wells was demonstrated by Cullender and Smith.[3] They showed that horizontal flow in pipelines at practically constant temperature and flow in wells (either production or injection) were special cases of flow in inclined conduits.

The energy balance of equation 4–1 in differential form for flow of gas is:

$$vdp + dH + \frac{udu}{g_c} + dW_f = 0 \tag{5–1}$$

where:

v = specific volume
p = pressure
H = difference in elevation
u = velocity
g_c = conversion factor
W_f = irreversible energy loss

Equation 5–1 is usually applied to the flow of fluids, but it is equally useful in a study of static columns of gas in wells in which there is no flow. In a static gas column, the kinetic energy and irreversible-energy terms are zero. Equation 5–1, which is applicable to a unit mass of fluid, states that the energy available from the flow process is equal to the energy required to lift the unit mass of fluid through the vertical distance, dH, plus the energy involved in the change in velocity, plus the energy transformed into heat by frictional resistance. (Equation 5–1 is applicable to systems in which energy losses such as liquid slippage occur, but this text will designate dW_f as the energy transformed into heat by friction.) If the flow of gas is upward as in producing gas wells, the term dH is positive; for horizontal flow, the term dH is zero.

If equation 5–1 is applied to the upward flow of gas in a well, as illustrated in Figure 5–1, and the frictional resistance term, dW_f, for a circular cross section is defined by:

$$dW_f = \frac{4fu^2dL}{2g_cD} \tag{5–2}$$

where:

f = coefficient of friction, dimensionless
D = internal diameter of pipe, ft

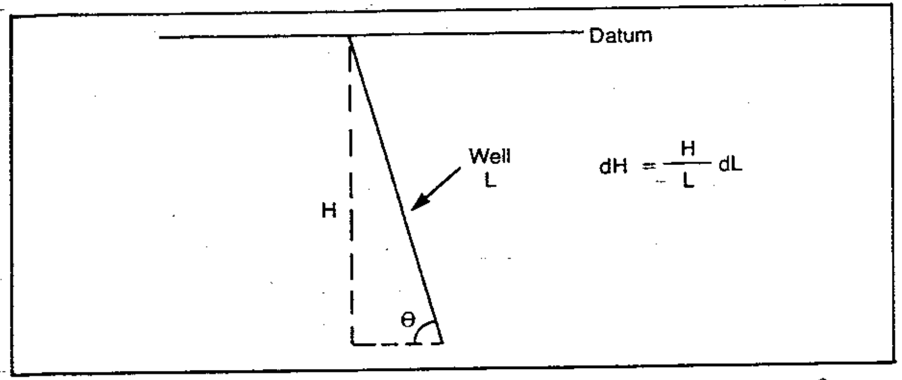

Figure 5–1 *Length of pipe and vertical penetration for a gas well.*[3]

Equation 5–1 becomes:

$$vdp + dH + \frac{udu}{g} + \frac{4fu^2dL}{2g_cD} = 0 \tag{5–3}$$

Making the following substitutions:

$$u = \frac{Nv}{A}$$

$$u^2 = \frac{N^2v^2}{A^2}$$

$$du = \frac{N}{A}dv$$

$$dH = \frac{HdL}{L}$$

$$pv = bTz$$

$$v = \frac{bTz}{p}$$

$$dv = -bTz\frac{dp}{p^2}$$

Setting the compressibility factor, z, at the pressure and temperature bases equal to 1.000, changing signs for integration from position 2 to position 1, making the proper conversion of units, and recalling from equation 2–5 that the gas constant for 1 lb of gas is:

$$b = 53.356/\gamma_g$$

equation 5–3 becomes:

$$\frac{1{,}000\,\gamma_g L}{53.356} = \int_{p_2}^{p_1} \frac{\left[p/Tz - 2.082\dfrac{\gamma_g q_m^{\,2}}{d^4 p}\right] dp}{F^2 + \dfrac{H}{L}\dfrac{(p/Tz)^2}{1{,}000}} \tag{5–4}$$

where:

$$F^2 = \frac{2.6665 f q_m^{\,2}}{d^5} = (F_r q_m)^2$$

q_m = rate of flow, MMcfd, at a standard base pressure of 14.65 psia and standard base temperature of 60°F
γ_g = specific gravity (air = 1.00)
H = difference in elevation, ft
L = length of pipe, ft
p = pressure, psia
T = absolute temperature = °F + 460
z = compressibility factor, dimensionless

d = internal diameter, in., of the circular pipe
f = coefficient of friction, dimensionless

(Equation 5–4 was reported in a rearranged form by Young but with an error in the sign for the kinetic energy term).[4]

Without making assumptions with respect to T and z, equation 5–4 does not lend itself to mathematical integration, but numerical integration over definite limits can be accomplished by the trapezoidal rule.

If:

$$\int_{p_1}^{p_n} \frac{\left[\dfrac{p}{Tz} - 2.082\dfrac{\gamma_g q_m^{\,2}}{d^4 p}\right] dp}{F^2 + \dfrac{H}{L}\dfrac{(p/Tz)^2}{1{,}000}} = \int_{p_1}^{p_n} I dp = \frac{1{,}000\,\gamma_g L}{53.356}$$

$$= \tfrac{1}{2}\left[(p_2 - p_1)(I_2 + I_1) + (p_3 - p_2)(I_3 + I_2) + \ldots (p_n - p_{n-1})(I_n + I_{n-1})\right] \tag{5–5}$$

Then:

$$37.484\,\gamma_g L = \left[(p_2 - p_1)(I_2 + I_1) + (p_3 - p_2)(I_3 + I_2) + \ldots (p_n - p_{n-1})(I_n + I_{n-1})\right] \tag{5–6}$$

Thus far, the only assumptions that have been made are:

1. Flow is completely turbulent.
2. The coefficient of friction, f, is a constant.
3. The compressibility of a flowing gas at the standard base pressure and temperature is 1.000.
4. Only a gas phase is flowing.

Where the variation in temperature with length, L, is known, it is necessary to select appropriate values for the length; determine p_n by trial and error so that equation 5–6 is satisfied.

Before making calculations with equations 5–6 and 5–5, one must examine the coefficient of friction, f, and show its relationship to the Reynolds number, N_{Re}. In connection with equations 5–4, 5–5, and 5–6, it was assumed that the coefficient of friction, f, was a constant. One purpose of the following section is to examine the validity and limits of this assumption.

COEFFICIENT OF FRICTION, f

Of the variables used in calculating flow of natural gas, the one that bothers engineers most is the coefficient of friction. In a practical sense, however, friction factors are known to the same degree of accuracy that many other variables are known. The following discussion is included to acquaint practicing engineers with the use of equations requiring a friction factor.

The coefficient of friction, f, has been defined as:

$$dF = \frac{4fu^2dL}{2g_cD} \qquad (5\text{–}2)$$

where:

f = coefficient of friction, dimensionless
D = internal diameter of pipe, ft

The coefficient of friction for turbulent flow in circular pipe has been the subject of an entire category of technical literature. However, space permits only a brief summary here based on the publications of Smith et al.[2,3,5]

The turbulent flow of fluids in circular conduits falls into two categories, depending upon the relationship between the boundary layer of the fluid and the roughness of the pipe wall. When the boundary layer of the fluid next to the pipe wall covers the roughness of the pipe, the fluid, in effect, is flowing in a conduit, the wall of which is composed of the fluid itself. In this case, the dominant characteristic of the container wall is the viscosity of the fluid, and the coefficient of friction is directly related to the viscosity. Since the nature of the pipe wall is masked in its effect on the flow, flow is said to be smooth even though the flow is classified as turbulent. As the velocity increases, the shear intensity at the stationary boundary layer increases, and the thickness of the boundary layer becomes less. When the roughness of the pipe wall becomes larger than the boundary layer of the fluid next to the pipe wall or the thickness of the boundary layer of fluid

becomes less than the roughness of the pipe wall, the rough elements protrude into the flow stream and interfere with the flow. In this case, flow is said to be rough and the dominant characteristic of the container wall is the roughness of the wall. The coefficient of friction no longer relates to the viscosity of the fluid but is a direct function of the relative roughness of the pipe wall.

Beginning with the theoretical work of von Karman and moving through later publications, the equation for turbulent flow in smooth pipes or pipes in which the boundary layer of the fluid is thicker than the absolute roughness of the pipe is:[6–9]

$$\sqrt{1/f} = A \log N_{Re} \sqrt{f} + B_1 \qquad (5\text{–}7)$$

and the equation for turbulent flow in rough pipes is:

$$\sqrt{1/f} = A \log r/k + B_2 \qquad (5\text{–}8)$$

where:

A = A constant common to both equations, and B_1 and B_2 are constants
N_{Re} = Reynolds number, dimensionless
r = internal radius of pipe
k = absolute characteristic of roughness, length

Equation 5–7 for turbulent flow in smooth pipe with constants indicates that the coefficient of friction is an implicit, logarithmic function of the Reynolds number. It cannot be arranged into an explicit function of the coefficient of friction. However, the constants can be determined from flow in smooth pipes or flow in rough pipes where the wall roughness has no effect on the coefficient of friction.

Equation 5–8 has four constants, one of which is common to equation 5–7. The constant r is the internal radius of the pipe and appears with the absolute characteristic of pipe roughness, k. The principal attempt to define the roughness characteristic of equation 5–8 was made by Nikuradse, who conducted flow tests on pipes in which uniform sand grains were glued to the internal surface.[10] Nikuradse used the ratio of the pipe radius to the diameter of the sand grain as a measure of the quantity, r/k, and determined a value for B_2 in equation 5–8.

Values of the constants for equation 5–7 as determined by various investigations are given in Table 5–1. Nikuradse gave a value of 3.476 for the constant B_2 in equation 5–8. Since that time, investigators have used experimental data and Nikuradse's value for B_2 to calculate an absolute roughness for commercial pipe. Taking the values of the constants from the experimental work of Smith, Miller, and Ferguson, equation 5–7 becomes:

$$\sqrt{1/f} = 4 \log N_{Re} \sqrt{f} - 0.6 \tag{5–9}$$

and equation 5–8, with the value 3.476 from Nikuradse, becomes:

$$\sqrt{1/f} = 4 \log r/k + 3.476$$

or:

$$\sqrt{1/f} = 4 \log \frac{7.4r}{k} \tag{5–10}$$

where:

r = radius of pipe
k = absolute roughness
r/k = relative roughness

In experimental work with gas wells, Smith, Williams, and Dewees recommended an absolute roughness of 0.00065 in. for flow strings in gas

Table 5–1 ***Values of the constants in equation 5–7 as determined by various investigators.***

Investigator	*Values of Constants in Equation 5–7*	
	A	*B_1*
von Karman[6]	4	−0.6
Nikuradse[10]	4	−0.4
Smith et al.[2]	4	−0.6

wells,[5] but later an absolute roughness of 0.0006 in. was used by Cullender and Smith for gas wells.[3] Further work with surface pipelines resulted in an absolute roughness of 0.0007 for commercial pipe.[2] All of these values are in essential agreement, and a value of 0.0006 in. will be used as the absolute roughness for flow in gas wells and horizontal pipe in this work.

The range of flow conditions over which equation 5–9 and equation 5–10, at an absolute roughness of 0.0006 in., are applicable is illustrated in Figures 5–2 and 5–3 in which $\sqrt{1/f}$ and the coefficient of friction, f, are shown as a function of the Reynolds number (ratio of inertial forces to viscous forces) for several sizes of tubing. For example, at Reynolds numbers less than 139,000 in 1-in. tubing (ID = 1.049 in.), the coefficient of friction is determined by equation 5–9, the smooth pipe equation. At these low Reynolds numbers for natural gas, 1-in. tubing acts as if it were a smooth pipe and the natural gas is flowing in a container in which the walls consist of a boundary layer of natural gas. The boundary layer is thicker than the roughness of the pipe. At Reynolds numbers greater than 139,000, the roughness of the pipe wall becomes the dominant factor, and the coefficient of friction is independent of the Reynolds number and is determined by equation 5–10, the rough pipe equation. The lines on Figures 5–2 and 5–3 for various sizes of tubing represent an absolute roughness of 0.0006 in. The minimum Reynolds number at which equation 5–10, the rough pipe equation, is applicable has been tabulated for several sizes of pipe in Table 5–2.

Figures 5–2 and 5–3 indicate an abrupt change between the smooth pipe relationship (equation 5–9) in which the coefficient of friction, f, is a function of the Reynolds number and the rough pipe relationship (equation 5–10) in which the coefficient of friction is a function of the roughness of the pipe wall. If the data of Smith et al. are examined closely, a transition region is found between the two relationships that is mathematically complicated.[2,5] Previously, most investigators adopted the equation proposed by Colebrook for the transition region between smooth and rough turbulent flow,[11] and its use continues. However, none of the data reported by Smith et al. for the commercial pipe used in gas wells and pipelines shows a transition region similar to that reported by Colebrook. Therefore, the decision was made to drop the transition relationship of Colebrook from consideration in this work.

In addition to giving the minimum Reynolds number at which equation 5–10 is applicable (this is also the maximum Reynolds number at which equation 5–9 is applicable), Table 5–2 gives values of $\sqrt{1/f}$ that are useful in computations with equations in which compressibility and temperature are assumed to be constant at average values. Also, the coefficient of friction

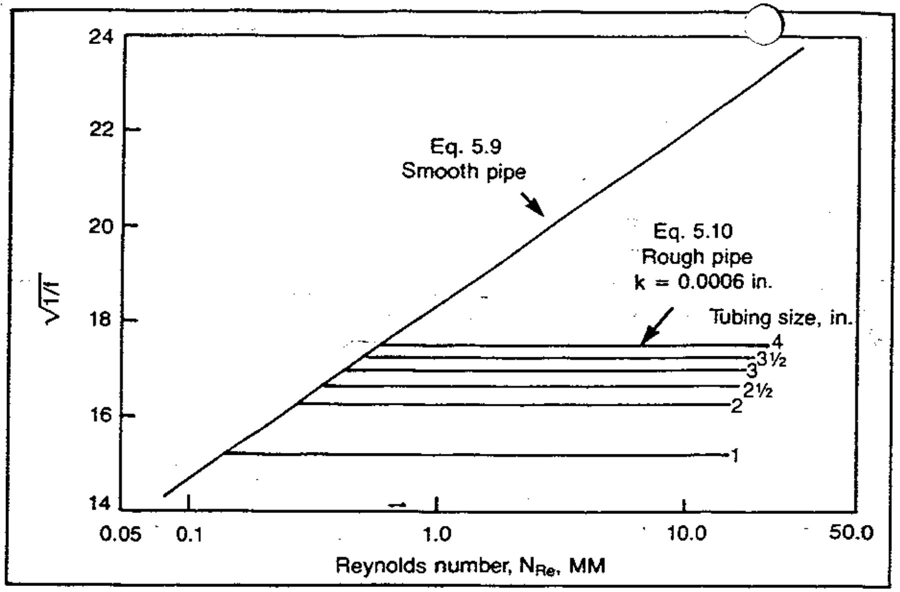

Figure 5–2 *Relationship between $\sqrt{1/f}$ and Reynolds number for turbulent flow.*

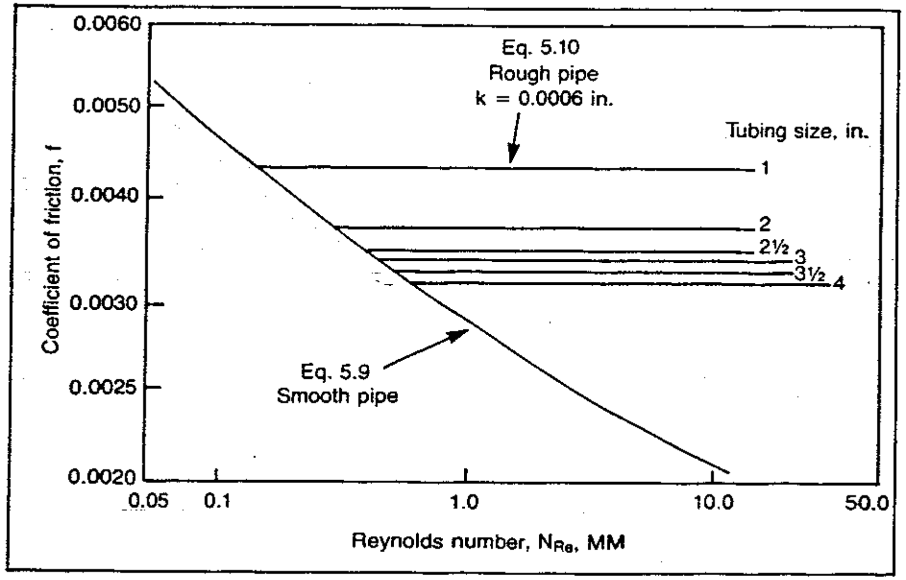

Figure 5–3 *Relationship between the coefficient of friction and Reynolds number for turbulent flow.*

Table 5–2 *Minimum Reynolds numbers at which equation 5–10 is applicable and corresponding $\sqrt{1/f}$ and coefficients of friction for various sizes of pipe (absolute roughness = 0.0006 In.).*

Nominal Pipe Size (in.)	*Weight (lb/ft)*	*OD (in.)*	*ID (in.)*	$\sqrt{1/f}$	*Coefficient of Friction (f)*	F_r	*Minimum Reynolds Number* (N_{re})
Tubing							
1	1.80	1.315	1.049	15.24	0.004304	0.09505	139,000
1¼	2.40	1.660	1.380	15.72	0.004047	0.04643	189,000
1½	2.75	1.990	1.610	15.99	0.003912	0.03106	224,000
2	4.70	2.375	1.995	16.36	0.003736	0.01776	284,000
2½	6.50	2.875	2.441	16.71	0.003581	0.01050	355,000
3	9.30	3.500	2.992	17.06	0.003434	0.006180	445,000
3½	11.00	4.000	3.476	17.32	0.003332	0.004184	525,000
4	12.70	4.500	3.958	17.55	0.003247	0.002985	605,000
4½	16.25	4.750	4.082	17.60	0.003227	0.002755	626,000
	18.00	4.750	4.000	17.57	0.003240	0.002905	612,000
4¾	18.00	5.000	4.276	17.68	0.003198	0.002442	659,000
	21.00	5.000	4.154	17.63	0.003216	0.002633	638,000
Casing							
	13.00	5.000	4.494	17.77	0.003167	0.002146	696,000
	15.00	5.000	4.408	17.74	0.003179	0.002257	681,000
5 3/16	14.00	5.500	5.012	17.96	0.003100	0.001617	784,000
5 3/16	15.00	5.500	4.976	17.95	0.003104	0.001647	777,000
	17.00	5.500	4.892	17.92	0.003115	0.001722	763,000
	20.00	5.500	4.778	17.88	0.003129	0.001830	744,000
	23.00	5.500	4.670	17.84	0.003143	0.001942	725,000
	25.00	5.500	4.580	17.80	0.003155	0.002043	710,000
5⅝	15.00	6.000	5.524	18.13	0.003043	0.001256	872,000
	17.00	6.000	5.450	18.11	0.003050	0.001301	859,000
	20.00	6.000	5.352	18.07	0.003061	0.001363	842,000
	23.00	6.000	5.240	18.04	0.003074	0.001440	823,000
	26.00	6.000	5.140	18.00	0.003085	0.001514	806,000
6¼	20.00	6.625	6.049	18.29	0.002990	0.0009922	964,000
	22.00	6.625	5.989	18.27	0.002996	0.001018	953,000
	24.00	6.625	5.921	18.25	0.003003	0.001049	941,000
	26.00	6.625	5.855	18.23	0.003009	0.001080	930,000
	28.00	6.625	5.791	18.21	0.003015	0.001111	919,000

Table 5–2, continued

Nominal Pipe Size (in.)	Weight (lb/ft)	OD (in.)	ID (in.)	$\sqrt{1/f}$	Coefficient of Friction (f)	F_r	Minimum Reynolds Number (N_{re})
6¼	31.80	6.625	5.675	18.18	0.003027	0.001171	898,000
	34.00	6.625	5.595	18.15	0.003035	0.001215	885,000
6⅝	20.00	7.000	6.456	18.40	0.002954	0.0008380	1,035,000
	22.00	7.000	6.398	18.38	0.002959	0.0008578	1,025,000
	24.00	7.000	6.336	18.37	0.002964	0.0008798	1,014,000
6⅝	26.00	7.000	6.276	18.35	0.002970	0.0009018	1,003,000
	28.00	7.000	6.214	18.33	0.002975	0.0009253	992,000
	30.00	7.000	6.154	18.32	0.002981	0.0009489	982,000
	40.00	7.000	5.836	18.22	0.003011	0.001089	926,000
7¼	26.40	7.625	6.969	18.53	0.002911	0.0006872	1,125,000
7¼	29.70	7.625	6.875	18.51	0.002919	0.0007119	1,108,000
	33.70	7.625	6.765	18.48	0.002928	0.0007423	1,089,000
	38.70	7.625	6.625	18.44	0.002938	0.0007837	1,064,000
	45.00	7.625	6.445	18.40	0.002955	0.0008417	1,033,000
	26.00	8.000	7.386	18.63	0.002880	0.0005911	1,199,000
7⅝	28.00	8.125	7.485	18.66	0.002873	0.0005710	1,216,000
	32.00	8.125	7.385	18.63	0.002880	0.0005913	1,199,000
	35.50	8.125	7.285	18.61	0.002887	0.0006126	1,181,000
	39.50	8.125	7.185	18.59	0.002895	0.0006349	1,163,000
8¼	17.50	8.625	8.249	18.83	0.002822	0.0004438	1,353,000
8¼	20.00	8.625	8.191	18.81	0.002825	0.0004520	1,342,000
	24.00	8.625	8.097	18.79	0.002831	0.0004657	1,326,000
	28.00	8.625	8.003	18.77	0.002837	0.0004801	1,309,000
	32.00	8.625	7.907	18.75	0.002844	0.0004953	1,292,000
	36.00	8.625	7.825	18.73	0.002849	0.0005089	1,277,000
8¼	38.00	8.625	7.775	18.72	0.002853	0.0005174	1,268,000
	43.00	8.625	7.651	18.70	0.002861	0.0005394	1,246,000
8⅝	34.00	9.000	8.290	18.83	0.002819	0.0004382	1,360,000
	38.00	9.000	8.196	18.81	0.002825	0.0004513	1,343,000
	40.00	9.000	8.150	18.80	0.002828	0.0004579	1,335,000
8⅝	45.00	9.000	8.032	18.78	0.002836	0.0004756	1,314,000
9	36.00	9.625	8.921	18.96	0.002781	0.0003623	1,473,000
	40.00	9.625	8.835	18.95	0.002786	0.0003715	1,458,000
	43.50	9.625	8.755	18.93	0.002791	0.0003804	1,444,000
	47.00	9.625	8.681	18.91	0.002795	0.0003888	1,430,000

Table 5–2, continued

Nominal Pipe Size (in.)	Weight (lb/ft)	OD (in.)	ID (in.)	$\sqrt{1/f}$	Coefficient of Friction (f)	F_r	Minimum Reynolds Number (N_{re})
9	53.50	9.625	8.535	18.89	0.002804	0.0004063	1,404,000
	58.00	9.625	8.435	18.86	0.002810	0.0004189	1,386,000
9⅝	33.00	10.000	9.384	19.05	0.002756	0.0003178	1,557,000
	55.50	10.000	8.908	18.96	0.002782	0.0003637	1,471,000
	61.20	10.000	8.790	18.94	0.002789	0.0003764	1,450,000
10	32.75	10.750	10.192	19.19	0.002715	0.0002566	1,704,000
	35.75	10.750	10.136	19.18	0.002717	0.0002602	1,694,000
	40.00	10.750	10.050	19.17	0.002721	0.0002660	1,678,000
	45.50	10.750	9.950	19.15	0.002726	0.0002730	1,660,000
	48.00	10.750	9.902	19.14	0.002729	0.0002765	1,651,000
10	54.00	10.750	9.784	19.12	0.002735	0.0002852	1,630,000

and F_r values for computations by hand using equations of the type of equation 5–5 are given. F_r is defined as (see equation 5–4):

$$F^2 = (F_r q_m)^2 = \frac{2.6665 f q_m^{\,2}}{d^5}$$

or:

$$F_r = \left(\frac{2.6665f}{d^5}\right)^{0.5} \qquad (5\text{–}11)$$

AVERAGE VELOCITY AND REYNOLDS NUMBER FOR FLOW OF NATURAL GAS IN PIPE

If the average velocity in pipe is defined as:

$$\text{average velocity} = \frac{\text{volume flow rate at actual conditions}}{\text{cross-sectional area}}$$

then the average velocity, $\overline{u}$, in feet per second is:

$$\overline{u} = 2{,}122 \frac{p_b q_m z T}{T_b p d^2}$$

or:

$$\overline{u} = 59.79 \frac{q_m z T}{p d^2} \tag{5–12}$$

The Reynolds number, a dimensionless ratio, is commonly used to characterize the conditions of fluid flow in circular pipes. Since the 1840s, a great deal of work has been performed by many investigators to determine the relationship between friction losses, pipe characteristics, fluid properties, and flow variables. Correlations for important variables were proposed long before the basic theory was formulated. In 1883, Reynolds demonstrated the existence of two distinct flow regimes. At low flow rates, he found that flow is essentially laminar. At increased flow rates, he found that random velocity components begin to occur at the center of the flow stream, gradually spreading with increasing velocity outward from the flowing core of the fluid toward the boundary. Under these circumstances, it appeared difficult to develop one single expression for the friction factor as a function of system variables. Thus the correlation of Reynolds numbers, which stated that the type of flow was a function of the magnitude of the Reynolds number, was established for the laminar flow regime, and attention was focused on the behavior in the turbulent flow regime.

In a later study, Reynolds found that the flow rate at which the transition from laminar to turbulent conditions occurred was a function of the cross-sectional area of the flowing stream, the average linear velocity, and the density and the viscosity of the fluid. For flow in pipe, the criteria of similarity was:

$$\frac{D\overline{u}\rho}{\mu}$$

This is now called the Reynolds number. The Reynolds number may be interpreted physically as the ratio of inertial forces to viscous forces. Reynolds' principle of similarity, on which the use of the number is based, states that for flow in or around geometrically similar bodies, the flow patterns of all flows are similar if their Reynolds numbers have the same value.

It would be interesting and helpful if one could examine the laminar flow regime in detail and become familiar with the relationship between flow regime and Reynolds number. However, this text is involved solely with turbulent flow in gas wells and, in most cases, turbulent flow in natural gas pipelines.

If equation 5–12 (average velocity) is combined with equation 2–18 for the density, the Reynolds number may be calculated by the following equation:

$$N_{Re} = \frac{2.001 \times 10^4 \, q_m \gamma_g}{\mu d} \tag{5–13}$$

where the viscosity, μ, is expressed in centipoise. It is interesting to note that the pressure, temperature, and compressibility factor do not appear in equation 5–13. Therefore, the Reynolds number for flow of gas is independent of pressure, temperature, and compressibility except for the influence of pressure and temperature on the viscosity of the gas.

APPLICABILITY OF EQUATIONS FOR COEFFICIENT OF FRICTION

It has been shown from experimental work reported by many investigators that turbulent flow occurs when the Reynolds number is greater than 2,000. For Reynolds numbers greater than 2,000 to 4,000, the smooth pipe equation (equation 5–9) is applicable until the effect of the boundary layer is replaced by the effect of pipe wall roughness. This transition occurs in commercial pipe with an absolute roughness of 0.0006 in. for each size of pipe at the Reynolds numbers given in Table 5–2. At Reynolds numbers larger than those in Table 5–2, the rough pipe equation (equation 5–10) is applicable.

In the previous discussion, it was emphasized that the Reynolds number was independent of pressure and temperature except when viscosity was affected by pressure and temperature. It is believed that the coefficient of friction can be calculated for engineering purposes using equation 5–9, the smooth pipe equation, in the range of Reynolds numbers in which the equation is applicable.

FLOW CALCULATIONS USING EQUATIONS 5–5 AND 5–6

Producing Gas Well—Flow in a Positive Direction

Since equations have been given to describe flow and to compute the coefficient of friction, the means are available for calculating pressures in shut-in and flowing gas wells, injection pressures in gas wells, and pressures along pipelines that have changes in elevation. The degree of assumptions needed to make the calculations feasible depends to some extent upon the sophistication of the means of calculation at hand. If a computer is available, subroutines for determining compressibility factors can be programmed internally, and solutions for equations 5–5 and 5–6 become routine. If one is limited to hand-held programmable computers, it may be necessary to estimate or calculate compressibility factors using a separate step than Tables A–1, A–2, and A–3 or using equations such as equation 2–11. Therefore, the following procedures apply to the case in which compressibility factors are determined separately.

Since equations 5–5 and 5–6 were developed for flow in a producing gas well, it will be helpful to review the algebraic sign convention consistent with those conventions for producing gas wells in which the wellhead pressure is known and one desires to calculate the bottom-hole pressure. In this case, both H and L in equations 5–5 and 5–6 are considered positive in sign as indicated in Figure 5–4a. Other consistent sign designations for various known pressures, injection in wells, and flow in pipelines are shown in Figure 5–4.

Example 5–1 Low-pressure well producing at a high rate. This example is patterned after the example in Table 30–15 of reference 12 in which a gas well (γ_g = 0.615, H = 10,658 ft, L = 10,490 ft of 2½-in. tubing, ID = 2.441 in. and 2-in. tubing, ID = 1.995 in. from 10,490 to 10,658 ft, $H = L$, surface temperature = 117°F, and temperature gradient is 5°F/1,000 ft) is assumed to be producing 11.299 MMcfd at a wellhead pressure of 500 psia and at 117°F. Calculate the flowing bottom-hole pressure.

First, the Reynolds number should be calculated using equation 5–13 at the wellhead in the 2½-in. tubing:

$$N_{Re} = \frac{(2.001)(10^4)(11.299)(0.615)}{(0.0132)(2.441)} = 4.3 \times 10^6$$

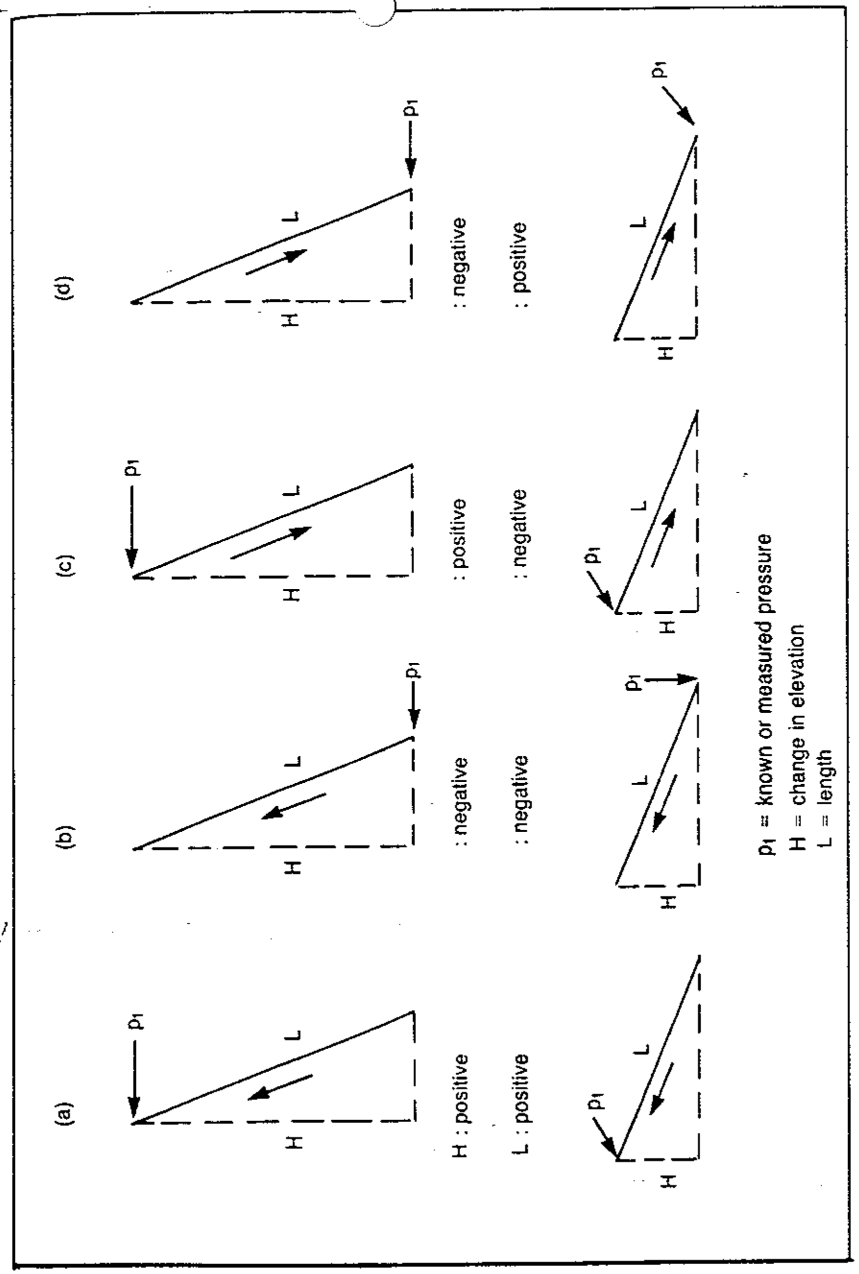

Figure 5–4 *Algebraic sign convention for flow in wells and pipelines.*

The viscosity at 500 psia and 117°F was estimated to be 0.0132 centipoise. Since 4.3×10^6 is much larger than the minimum Reynolds number of 3.55×10^5 given in Table 5–2, one can conclude that the friction coefficient, f, can be calculated using equation 5–10, and its value is 0.003581 according to Table 5–2. The calculations using equations 5–5 and 5–6 can be made conveniently as illustrated in Table 5–3. The step-by-step procedure follows.

1. Enter the well information and calculate line 1 of Table 5–3. Note that p_n on line 1 is the measured wellhead pressure (in the actual example, it was an assumed value).
2. Calculate I_1 (61.358) using the definition of I given in equation 5–5 (see line 1).
3. Estimate the pressure at length L (in this case 1,000 ft) in the flow string. The first estimate of Δp can be made by calculating $37.484\, \gamma_g L$ and dividing it by $2(I_1)$ or 2×61.358. The result is 188.0, which is then used to calculate a value of 81.042 for I_2 at 1,000 ft, line 2.
4. Calculate $(\Delta p)(I_2 + I_1)$, which is 26,771 compared to the value that it should be of 23,053, which is $37.484\, \gamma_g L$ at 1,000 ft, line 2. Use current value of $(I_2 + I_1)$ and divide it into 23,053 for a new Δp (not shown) and continue until $\Delta p\,(I_1 + I_2)$ is in satisfactory agreement with the value of $37.484\, \gamma_g L$, line 3.
5. Go to new L of 2,000 ft and repeat the procedure.
6. In changing the flow string size as was done at L = 10,490 ft, follow the procedure shown in lines 13 and 14 of Table 5–3.

The net result of these calculations is a flowing pressure of 1,683 psia at the depth or length of 10,658 ft. In this example, the pressure at each interval is found by trial and error to be within 0.1 psia. In actual practice, this degree of precision may not be required.

It is readily recognized that the number of calculation intervals directly affects the length of computations. If the calculations for this example were made in four steps instead of the thirteen steps illustrated in Table 5–3, the resulting bottom-hole pressure would be 1,706 psia, causing a difference of 23 psi in a total pressure difference of (1,683 − 500) or 1,183 psi. The difference was caused by the nature of the change of I_n with the length, L. Examination of the values of I_n in Table 5–3 shows that I_n is not a linear function of L. The closer the function is to a linear relationship, the fewer the number of

Table 5–3 *Worksheet for calculating subsurface flowing pressure using equations 5–5 and 5–6.*

Company ______ Lease ______ Well No. ___ B ___ Date of test ______

γ_g 0.615 % CO_2 2.5 % N_2 — p_{pc} 679 T_{pc} 361 Equations used 5–5 & 5–6

q_m 11.299 H 10,658 L 10,490 / 10,658 d 2.441 in. / 1.995 Temperature gradient 5°F/1,000 ft

H	L	d	p_n	t	z	I_n	Δp	$(\Delta p) \times (I_n + I_{n-1})$	$\Sigma(\Delta p) \times (I_n + I_{n-1})$	$37.484 \times \gamma_g L$	Line
0	0	2.441	500.00	117	0.937	61.358	—	—	—	0	1
1,000	1,000	2.441	688.0	122	0.924	81.042	188.0	26,771	26,771	23,053	2
1,000	1,000	2.441	664.7	122	0.927	78.608	164.7	23,052	23,052		3
2,000	2,000	2.441	800.6	127	0.916	91.083	135.9	23,061	46,113	46,105	4
3,000	3,000	2.441	920.9	132	0.908	100.469	120.3	23,044	69,156	69,158	5
4,000	4,000	2.441	1,031.6	137	0.902	107.756	110.7	23,050	92,206	92,211	6
5,000	5,000	2.441	1,135.8	142	0.898	113.463	104.2	23,051	115,256	115,263	7
6,000	6,000	2.441	1,235.4	147	0.894	118.070	99.6	23,061	138,317	138,316	8
7,000	7,000	2.441	1,331.6	152	0.893	121.630	96.2	23,059	161,376	161,369	9
8,000	8,000	2.441	1,425.2	157	0.891	124.554	93.6	23,043	184,419	184,421	10
9,000	9,000	2.441	1,516.9	162	0.891	126.819	91.7	23,051	207,470	207,474	11
10,000	10,000	2.441	1,607.2	167	0.890	128.666	90.3	23,070	230,540	230,527	12
10,490	10,490	2.441	1,650.9	169	0.890	129.426	43.7	11,279	241,819	241,822	13
10,490	10,490	1.995	1,650.9	169	0.890	60.125	—	—	241,819	241,822	14
10,658	10,658	1.995	1,683.0	170	0.889	60.856	32.1	3,883	245,702	245,695	15

steps for comparable precision. Young recommends an integration interval of 1,000 ft, as was used in example 5–1 to ensure accurate trapezoidal integration of equation 5–5.[4]

If the gas well had been drilled at an angle instead of vertically as assumed, L would have been larger than H for the nonvertical sections, and the calculations would have been carried out in the same manner.

Shut-in Gas Well

In equation 5–5, the definition of I_n contained the rate of flow, q_m, in the term accounting for the change in kinetic energy, or the second term in the numerator, and in the term F in the denominator. For a shut-in gas well, the rate of flow, q_m, is zero and equations 5–5 and 5–6 become.

$$\int_{p_1}^{p_n} \frac{\left(\frac{p}{Tz}\right) dp}{\frac{H}{L}\frac{(p/Tz)^2}{1{,}000}} = \int_{p_1}^{p_n} I dp \tag{5–14}$$

$$= \tfrac{1}{2}\left[(p_2 - p_1)(I_2 + I_1) + (p_3 - p_2)(I_3 + I_2) + \ldots (p_n - p_{n-1})(I_n + I_{n-1})\right]$$

and:

$$37.484\,\gamma_g L = \left[(p_2 - p_1)(I_2 + I_1) + (p_3 - p_2)(I_3 + I_2) \ldots (p_n - p_{n-1})(I_n + I_{n-1})\right] \tag{5–15}$$

Numerical solutions for equations 5–14 and 5–15 follow the same procedural steps given in example 5–1. It is obvious that equation 5–14 could be simplified; but with the hand-held calculator, it is more expedient to substitute $q_m = 0$ into equation 5–5, which in effect makes the computations using equation 5–14.

Example 5–2 Shut-in low-pressure gas well. The gas well described in example 5–1 was shut-in, and the resulting wellhead pressure was 1,522 psia. Calculate the bottom-hole pressure at a depth of 10,658 ft. Table 5–4 illustrates the calculations that were made by following steps 1 through 5 in example 5–1.

It is interesting, when comparing the calculations for a shut-in gas well (Table 5–4) with those for the flowing well in the previous example[7] (Table 5–3), to note that in calculations using two steps (not given in text), the bottom-hole pressure was 1,925.6 psia instead of the 1,926.1 psia calculated using twelve steps in Table 5–4. In the case of the shut-in gas well, the change in I_n for each 1,000-ft interval was small and of a second-order relationship. Therefore, larger calculation intervals for this shut-in case do not have as much influence on the calculations as they did in example 5–1.

Injection in a Gas Well—Flow in a Negative Direction

For an injection well in which gas flows in a negative direction (downward), H is considered positive and L is considered negative (see Fig. 5–4). Equations 5–5 and 5–6, after the sign change is made, become:

$$-\frac{1{,}000\,\gamma_g L}{53.356} = \int_{p_1}^{p_n} \frac{\left[\frac{p}{Tz} - 2.082\frac{\gamma_g q_m^{\,2}}{d^4 p}\right]}{+F^2 - \frac{H}{L}\frac{(p/Tz)^2}{1{,}000}} dp = \int_{p_1}^{p_n} I dp \tag{5–16}$$

and:

$$-37.484\,\gamma_g L = (p_2 - p_1)(I_2 + I_1) + (p_3 - p_2)(I_3 + I_2) + \ldots (p_n - p_{n-1})(I_n + I_{n-1}) \tag{5–17}$$

where:

$$F^2 = 2.6665\frac{f q_m^{\,2}}{d^5}$$

The procedures for obtaining a numerical solution for equations 5–16 and 5–17 are the same as those illustrated in examples 5–1 and 5–2.

The denominator of the I term in equation 5–16 is of interest because the term F^2 represents the energy loss caused by friction, and the term on the right represents the energy gained as the gas flows down the wellbore.

Table 5–4 *Worksheet for calculating subsurface shut-in pressure using equations 5–5 and 5–6.*

Company ______ *Lease* ______ *Well No.* B *Date of test* ______

γ_g 0.615 $\%CO_2$ 2.5 $\%N_2$ — p_{pc} 679 T_{pc} 361 *Equations used* 5–5 & 5–6 or 5–13 & 5–14

q_m 0 H 10,658 L 10,658 d — *Temperature gradient* 5°F/1,000 ft

H	L	d	p_n	t	z	I_n	Δp	$(\Delta p)x(I_n + I_{n-1})$	$\Sigma(\Delta p)x(I_n + I_{n-1})$	$37.484x\gamma_g L$	Line
0	0	—	1,522.0	117	0.850	322.240	—	—	—	0	1
1,000	1,000	—	1,557.8	122	0.852	315.576	35.8	22,834	22,834	23,053	2
1,000	1,000	—	1,558.0	122	0.852	318.270	36.0	23,058	23,058		3
2,000	2,000	—	1,594.4	127	0.855	314.780	36.4	23,043	46,101	46,105	4
3,000	3,000	—	1,631.2	132	0.857	311.025	36.8	23,030	69,131	69,158	5
4,000	4,000	—	1,668.5	137	0.861	308.071	37.3	23,092	92,223	92,211	6
5,000	5,000	—	1,706.1	142	0.864	304.864	37.6	23,046	115,269	115,263	7
6,000	6,000	—	1,744.1	147	0.867	301.742	38.0	23,061	138,320	138,316	8
7,000	7,000	—	1,782.5	152	0.870	298.704	38.4	23,057	161,377	161,369	9
8,000	8,000	—	1,821.2	157	0.874	296.100	38.7	23,019	184,396	184,421	10
9,000	9,000	—	1,860.4	162	0.877	293.213	39.2	23,101	207,497	207,474	11
10,000	10,000	—	1,899.9	167	0.880	290.415	39.5	23,053	230,550	230,527	12
10,658	10,658	—	1,926.1	170	0.882	288.490	26.2	15,167	245,717	245,695	13

Since the terms have different signs, they can cancel each other exactly. This means that a discontinuity can develop for certain ranges of data. When this occurs, the pressure drop across the interval should be assumed to be zero, and an inflection point occurs in the pressure traverse.

Example 5–3 Injection of gas in a well. It is proposed to inject gas into a storage well at the rate of 25.0 MMcfd at a wellhead pressure of 1,200 psi. Calculate the bottom-hole pressure at a depth of 3,000 ft. The flow string is 3-in. tubing with an ID of 2.992 in., γ_g is 0.600, CO_2 is 0%, N_2 is 2%, the wellhead temperature is 140°F, and the temperature gradient is -8°F/1,000 ft. Recalculate using the same values except the rate of injection is 10.0 MMcfd.

The Reynolds numbers for the rates of flow are 6.47×10^6 and 2.59×10^6, respectively. The results of the calculations are given in Table 5–5 for both rates of flow.

Note: For an injection rate of 25.0 MMcfd, the value of the term I_n is positive, which means that the friction loss term, F^2, in equation 5–16 is larger than the energy gained from the loss in elevation. For an injection rate of 10.0 MMcfd, I_n is negative, which means that the friction loss is less than the energy gained from the loss in elevation. The balance between the loss in elevation and friction energies occurs at a flow rate of about 11.58 MMcfd in example 5–3.

Flow in a Pipeline

The convention for the algebraic signs for elevation, *H*, and length, *L*, illustrated in Figure 5–4 can be used with equations 5–5 and 5–6 to solve various gas pipeline problems using the same calculation procedures outlined previously. The advantage of doing so is that temperature gradients along the pipeline can be used without simplifying assumptions.

Example 5–4 Flow in a pipeline. Gas (γ_g = 0.600, CO_2 = 0%, and N_2 = 2.0%) is flowing at a rate of 20 MMcfd in a pipeline (ID = 10.25 in.) with an inlet pressure of 800 psia. The elevation increases by 4,000 ft over a horizontal distance of 100 mi. The temperature gradient is indicated in Figure 5–5. Calculate the pressure after the gas has traveled the horizontal distance of 100 mi. The length, *L*, is 528,015 ft. The calculations are outlined for five-step and one-step intervals in Table 5–6. In this case, both *H* and *L* are negative. The Reynolds number is 1.63×10^6 at the inlet.

Table 5–5 *Worksheet for calculating subsurface injection pressure using equations 5–5 and 5–6.*

Company ______ *Lease* ______ *Well no.* ______ *Date of test* ______

γ_g 0.600 %CO_2 0.00 %N_2 2.00 p_{pc} 668 T_{pc} 352 *Equations used* 5–5 & 5–6 or 5–16 & 5–17

q_m 25.00 H 3,000 L −3,000 d 2.992 *Temperature gradient −8°F/1,000 ft*

H	L	d	p_n	t	z	I_n	Δp	$(\Delta p)x$ $(I_n + I_{n-1})$	$\Sigma(\Delta p)x$ $(I_n + I_{n-1})$	$37.484x$ $\gamma_g L$	Line
q_m = 25.00											
0	0	2.992	1,200.0	140	0.901	116.755	—	—	—	0	1
1,000	−1,000	2.992	1,103.7	132	0.903	104.849	− 96.3	−21,340	−21,340	−22,490	2
1,000	−1,000	2.992	1,098.2	132	0.903	104.097	−101.8	−22,483	−22,483		3
2,000	−2,000	2.992	982.6	124	0.906	90.455	−115.6	−22,490	−44,973	−44,981	4
3,000	−3,000	2.992	846.8	116	0.912	75.242	−135.8	−22,502	−67,475	−67,471	5
q_m = 10.00											
0	0	2.992	1,200.0	140	0.901	−2,001.96	—	—	—	0	6
1,000	−1,000	2.992	1,205.6	132	0.895	−1,674.25	+ 5.6	−20,587	−20,587	−22,490	7
1,000	−1,000	2.992	1,206.1	132	0.895	−1,669.66	+ 6.1	−22,397	−22,397		8
2,000	−2,000	2.992	1,213.4	124	0.888	−1,412.59	+ 7.3	−22,500	−44,897	−44,981	9
3,000	−3,000	2.992	1,222.0	116	0.880	−1,209.06	+ 8.6	−22,546	−67,443	−67,471	10

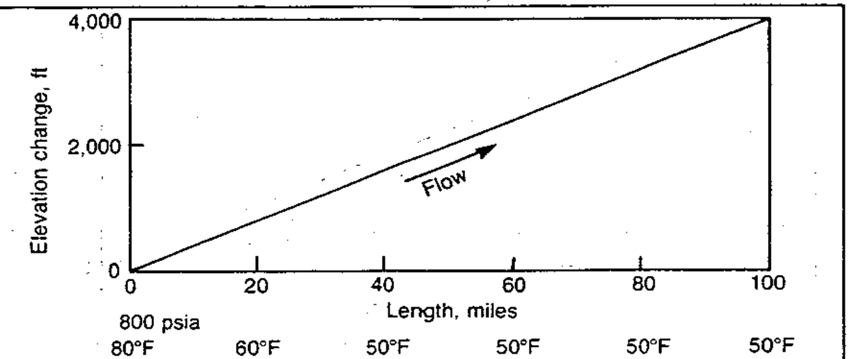

Figure 5–5 *Hypothetical pipeline showing inlet pressure and temperature profile.*

For the five-step calculations (Table 5–6, lines 1–7), the required pressure at 100 mi. is 632.3 psia and is 631.0 psia for the one-step calculation (Table 5–6, lines 8–10) for a difference of 1.3 psi.

SIZE OF INTEGRATION INTERVAL

An examination of the foregoing examples raises a question of the number of integration intervals or calculation steps needed to restrict computational errors. Young recommends an interval of 1,000 ft in gas wells, which is probably conservative.[4] Also, the integration interval depends to some extent on the sophistication of the means of calculation available to the user or, in reality, the cost of the computations. For this reason, no hard-and-fast recommendations will be made here, but the user is encouraged to set his own rules for his particular problems. The integration interval for pipelines can be much larger than 1,000 ft.

KINETIC ENERGY TERM IN FLOW EQUATIONS

From the start of efforts to calculate flow of gas in pipe, it has been customary to ignore the kinetic energy term because kinetic energy is insignificant in most flow problems. While logical, this procedure leaves an arbitrary decision that may or may not be reasonably made. Therefore, some investigation of the problem is desirable, starting with the numerator of the integrand in equation 5–5, which is:

Table 5–6 *Worksheet for calculating pressures along a pipeline using equations 5–5 and 5–6.*

Company ______ Line ______ Date of test ______

γ_g 0.600 % CO_2 0.00 % N_2 2.00 p_{pc} 668 T_{pc} 352

q_m 20.00 H −4000 L −528015 d 10.25 Equations used 5–5 & 5–6

Temperature gradient figure 5

H	L	d	p_n	t	z	I_n	Δp	$(\Delta p)x$ $(I_n + I_{n-1})$	$\Sigma(\Delta p)x$ $(I_n + I_{n-1})$	37.484x $\gamma_g L$	Line
(−800)	(−105,603)	10.25	800.0	80	0.893	35,741.8	—	—	—	0	1
−800	−105,603	10.25	766.7	60	0.882	35,768.4	−33.3	−2,381,289	−2,381,289	−2,375,054	2
−800	−105,603	10.25	766.8	60	0.882	35,768.8	−33.2	−2,374,152	−2,374,162		3
−1,600	−211,206	10.25	733.5	50	0.896	35,603.0	−33.3	−2,376,680	−4,750,832	−4,750,107	4
−2,400	−316,809	10.25	700.1	50	0.885	35,419.4	−33.4	−2,372,148	−7,122,980	−7,125,161	5
−3,200	−422,412	10.25	666.4	50	0.890	35,043.3	−33.7	−2,374,593	−9,497,573	−9,500,215	6
−4,000	−528,015	10.25	632.3	50	0.895	34,541.9	−34.1	−2,372,854	−11,870,427	−11,875,269	7
(−4,000)	(−528,015)	10.25	800.0	80	0.893	35,741.8	—	—	—	0	8
−4,000	−528,015	10.25	632.3	50	0.895	34,541.9	−167.7	−11,786,576	—	−11,875,269	9
−4,000	−528,015	10.25	631.0	50	0.895	34,522.2	−169.0	−11,874,616	—	—	10

$$\frac{p}{Tz} - 2.082\frac{\gamma_g q_m^{\,2}}{d^4 p}$$

The first term represents the energy available from the expansion of the gas, and the second term represents the energy converted for changing the kinetic energy of the gas. If one makes the arbitrary decision that the kinetic energy term in equation 5–5 should be used whenever the kinetic energy term is equal to or greater than $\frac{1}{100}$ of the available energy term, the criterion is:

$$\frac{p}{Tz} = (100)(2.082)\frac{\gamma_g q_m^{\,2}}{d^4 p}$$

or:

$$\frac{p^2 d^4}{Tz\,\gamma_g q_m^{\,2}} = 208.2 \qquad (5\text{–}18)$$

If this criterion is applied to the preceding flow examples, the only place where the ratio of equation 5–18 is approached is from line 1 of Table 5–3 where the ratio is 209.1. When the kinetic energy term is omitted, the pressure drop from a depth ($L = H$ in this example) of 1,000 ft to 0 ft is calculated at 163.6 psi instead of 164.7 psi given on line 3 of Table 5–3. This is a difference of 1.1 psi for a pressure drop of 164.7 psi, or 0.7%, which is well within the tolerance of the methods. The kinetic energy term therefore could have been neglected in all of the examples.

The equations for flow in wells and pipelines given by Cullender and Smith can be derived from equations 5–5 and 5–6 by omitting the kinetic energy term and keeping the term $[(1{,}000\ \gamma_g\ L)/53.356]$ in equation 5–4 positive by dividing both sides of the equation by (−1) where necessary.[3]

FLOW AT CONSTANT TEMPERATURE AND COMPRESSIBILITY

If one assumes that the temperature and compressibility variables can be approximated using an average but constant temperature and compressibility and that the change in kinetic energy can be neglected, equation 5–5 reduces to:

$$p_{wf}^2 = e^s p_{tf}^2 + \frac{L}{H}(F_r\, q_m\, \overline{T}\overline{z})^2 (e^s - 1) \tag{5-19}$$

for flowing gas wells in which pressure squared is expressed in thousands, and:

$$s = 0.037484\,\frac{\gamma_g H}{\overline{T}\overline{z}}$$

For shut-in gas wells, $q_m = 0$, and the equation becomes:

$$p_{ws}^2 = e^s\, p_{ts}^2 \tag{5-20}$$

Since $H = 0$ for horizontal pipelines, equation 5–5 becomes:

$$q_k = 1.3762\,\sqrt{\frac{1}{f}}\,\sqrt{\frac{1}{\gamma_g}}\,\sqrt{\frac{1}{\overline{T}}}\,\sqrt{\frac{1}{\overline{z}}}\,(d^{2.5})\left(\frac{p_1^2 - p_2^2}{X}\right)^{0.5} \tag{5-21}$$

where:

X = length, miles

Solution of equation 5–19 is by trial and error on the average compressibility factor, z.

WEYMOUTH FORMULA

Comparing the historic Weymouth formula with the foregoing relationships is of interest because a discussion of flow in pipe would not be complete without mentioning the relationship published by Weymouth in 1913.[13] The Weymouth formula has been used widely in engineering calculations and is still in use today. Smith, Miller, and Ferguson showed that in the Weymouth formula the coefficient of friction was related to the inside pipe diameter by:

$$f = 0.008/d^{1/3} \tag{5-22}$$

or:

$$\sqrt{1/f} = 11.18\, d^{1/6}$$

If the above are substituted into equation 5–21 and the rate of flow is changed to cubic feet per hour, one has the following:

$$q(\text{cu ft/hr}) = 641.1\, d^{8/3}\,\sqrt{\frac{1}{\gamma_g}}\,\sqrt{\frac{1}{\overline{T}}}\,\sqrt{\frac{1}{\overline{z}}}\left(\frac{p_1^2 - p_2^2}{X}\right)^{0.5} \tag{5-23}$$

where the length, X, is expressed in miles.

FLOW IN ANNULAR SPACES

Frequently, gas zones are completed so that production flows through the annular space between casing and tubing. Since it is impracticable to run subsurface pressure gauges in annular spaces, any knowledge of subsurface pressures, either shut-in or flowing, must be gained from surface pressures. Therefore, it is important to have appropriate friction factors for calculating subsurface pressures in annular spaces.

If one starts with the basic differential equation (equation 5–1) for flow and develops the equations for flow in annular spaces, the only difference between the two equations is in the definition of the friction term, F_r, except for the kinetic energy term. Ignore this difference for flow in annular spaces. The friction term is:

$$F_r = \left[2.6665\, f\, \frac{(d_2 + d_1)}{(d_2^2 - d_1^2)^3}\right]^{0.5} \tag{5-24}$$

Take the effective diameter of a noncircular cross section as:

$$d_{eff} = (d_2 - d_1)$$

where d_2 is the inside diameter of the outer pipe and d_1 is the outside diameter of the inner pipe. Substituting the effective diameter into equation 5–10, the coefficient of friction, f, can be calculated from the following:

$$\sqrt{\frac{1}{f}} = 4\log\frac{7.4\,(d_2 - d_1)}{2k} \tag{5–25}$$

This equation brings the value of absolute roughness, k, into question. Previously, 0.0006 in. was used for the absolute roughness of the inside of commercial pipe. Since the outside of tubing at best has upset joints and at worst collars, and since no known experimental data exist for annular spaces, it is reasonable to suggest an absolute roughness for annular spaces other than that for the inside of normal, commercial pipe. For lack of better information, use an absolute roughness of 0.001 in. for annular spaces. To this end, F_r values have been calculated for an absolute roughness of 0.001 in. for various annuli and are given in Table 5–7.

SUMMARY

The development of equations for flow in gas wells and in pipelines has shown that both are special cases of the same problem. In gas wells, change in elevation is a significant variable, but in pipelines the change in elevation can be a minor variable. In either case, the pressure loss caused by friction is the principal concern of the engineer. As a result of the unity of the problems, the same calculational procedure can be used for either problem. Also, the pressure loss caused by changes in the kinetic energy of the gas can be taken into account very easily. Consequently, there is no need to neglect the kinetic energy term in any of the calculations, even when using a hand-held programmable calculator.

PROBLEMS

1. Using the appropriate equations, calculate at least five lines of data in Table 5–2; start with the inside diameters of the pipe as given values.

2. Calculate the average velocity and the Reynolds number for a gas, $\gamma_g = 0.615$, $CO_2 = 4.0\%$, and $N_2 = 3.0\%$, flowing at a wellhead in 2-in. tubing (I.D. = 1.995 in.) at a pressure of 1,427 psia and temperature of 127°F at a rate of 2.15 MMcfd.

3. Calculate the bottom-hole flowing pressure for the well in problem 2 at a depth of 8,248 ft in 2-in. tubing. The temperature gradient is 2.5°F/1,000 ft. How many calculation steps did you use?

Table 5–7 F_r values for various annuli (k = 0.001 in.).

Casing ID (in.)	*Tubing OD (in.)*							
	1.900	*2.375*	*2.875*	*3.500*	*4.000*	*4.500*	*4.750*	*5.000*
4.154	0.005082	0.006901	0.01093					
4.276	0.004576	0.006087	0.009268					
4.408	0.004107	0.005356	0.007867					
4.494	0.003838	0.004948	0.007119					
4.580	0.003593	0.004583	0.006473	0.01250				
4.670	0.003361	0.004242	0.005886	0.01086				
4.778	0.003109	0.003880	0.005281	0.009289				
4.892	0.002872	0.003544	0.004738	0.007980				
4.950	0.002761	0.003390	0.004492	0.007419				
4.976	0.002713	0.003324	0.004389	0.007187				
5.012	0.002649	0.003235	0.004251	0.006883	0.01245			
5.140	0.002438	0.002946	0.003809	0.005947	0.01012			
5.240	0.002289	0.002746	0.003509	0.005343	0.008738			
5.352	0.002137	0.002545	0.003213	0.004770	0.007506			
5.450	0.002016	0.002385	0.002983	0.004342	0.006634			
5.524	0.001931	0.002274	0.002825	0.004055	0.006074	0.01098		
5.595	0.001854	0.002175	0.002684	0.003806	0.005601	0.009783		
5.675	0.001773	0.002070	0.002538	0.003552	0.005133	0.008658		
5.791	0.001663	0.001930	0.002346	0.003226	0.004552	0.007351	0.01017	
5.836	0.001623	0.001880	0.002277	0.003111	0.004354	0.006924	0.009455	

Table 5–7, continued

Casing ID (in.)	Tubing OD (in.) 1.900	2.375	2.875	3.500	4.000	4.500	4.750	5.000
5.855	0.001607	0.001859	0.002249	0.003065	0.004274	0.006755	0.009176	
5.921	0.001551	0.001790	0.002155	0.002911	0.004012	0.006215	0.008301	
5.989	0.001497	0.001722	0.002064	0.002764	0.003768	0.005726	0.007528	
6.049	0.001452	0.001665	0.001988	0.002643	0.003570	0.005341	0.006935	0.009582
6.154	0.001376	0.001572	0.001865	0.002450	0.003260	0.004757	0.006057	0.008132
6.214	0.001336	0.001522	0.001799	0.002349	0.003100	0.004466	0.005630	0.007451
6.276	0.001296	0.001472	0.001735	0.002251	0.002947	0.004193	0.005235	0.006837
6.336	0.001259	0.001427	0.001676	0.002161	0.002810	0.003952	0.004892	0.006313
6.366	0.001241	0.001405	0.001647	0.002119	0.002745	0.003839	0.004734	0.006074
6.398	0.001222	0.001382	0.001618	0.002074	0.002678	0.003724	0.004573	0.005835
6.445	0.001195	0.001349	0.001576	0.002012	0.002584	0.003565	0.004352	0.005508
6.456	0.001189	0.001342	0.001566	0.001998	0.002563	0.003529	0.004302	0.005436
6.625	0.001099	0.001234	0.001429	0.001796	0.002266	0.003041	0.003639	0.004486
6.765	0.001032	0.001153	0.001327	0.001651	0.002057	0.002710	0.003201	0.003879
6.875	0.0009830	0.001095	0.001255	0.001549	0.001912	0.002486	0.002910	0.003486
6.969	0.0009439	0.001049	0.001198	0.001469	0.001800	0.002316	0.002692	0.003196
7.185	0.0008619	0.0009524	0.001079	0.001306	0.001577	0.001987	0.002276	0.002655
7.285	0.0008273	0.0009120	0.001030	0.001240	0.001487	0.001857	0.002116	0.002450
7.385	0.0007946	0.0008739	0.0009839	0.001178	0.001405	0.001740	0.001972	0.002268
7.386	0.0007943	0.0008736	0.0009834	0.001177	0.001404	0.001739	0.001970	0.002266

4. Calculate the shut-in pressure for the well in problems 2 and 3 at a depth of 8,248 ft if the wellhead shut-in pressure is 2,048 psia and the wellhead temperature and the temperature gradient are the same as in problems 5–2 and 5–3.

5. Assume that a natural gas, $\gamma_g = 0.700$, $CO_2 = 1.5\%$, and $N_2 = 10.0\%$, is being injected into a well in a storage reservoir at a depth of 3,500 ft. The wellhead pressure is 1,368 psia, the temperature is 125°F, the rate of injection is 28.5 MMcfd, and the flow string is a $5\frac{3}{16}$-in. nominal size of 25.00 lb/ft of casing. Calculate the bottom-hole flowing pressure at a depth of 3,500 ft in the flow string. Assume the temperature in the flowing gas stream at 3,500 ft is 120°F.

6. Use the information in the section "Kinetic Energy Term in Flow Equations" to calculate the fraction of the energy available from the expansion of the gas that is being converted to kinetic energy at the wellhead for problems 2 and 5. Could you have neglected the kinetic energy term?

7. Assume that the horizontal distance over which a pipeline with an internal diameter of 10.192 in. moves gas (characteristics given in problem 2) is 21.5 mi, the temperature is constant at 65°F, the inlet elevation is 620 ft above sea level, and the elevation is 2,650 ft after a horizontal distance of 4.5 mi from the inlet end, decreasing to 878 ft at the delivery end. The delivery pressure must be at least 314 psia. Calculate the pressure at the inlet end at several rates of flow between 1,000 and 10,000 Mcfd. Calculate the difference between the inlet pressure squared and the outlet pressure squared and plot the data on log-log paper against the rate of flow. Express the results as:

$$q_k = C(p_i^2 - p_o^2)^n$$

What are the values of C and n?

8. Change the internal diameter of the pipeline to 8.191 in. and repeat problem 7. Use 100,000 Mcfd as the upper limit to the rate of flow. What are the values of n and C in the empirical equation?

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6 Flow of Gas in Porous Media (A Review)

This chapter reviews some of the elementary principles of flow in porous media and, as such, is by no means complete. Refer to Chapter 2 of Amyx, Bass, and Whiting for a more comprehensive treatment of the subject.[1] However, an attempt has been made herein to show that approximate solutions to complicated production problems sometimes can be outlined through recourse to the elementary principles.

POROSITY

From a petroleum and natural gas production viewpoint, the two basic properties of petroleum-bearing rock are its volumetric capacity to hold fluids and its capability to transmit fluids. The quantitative measures of these properties are known as porosity and permeability, respectively. Porosity is defined as 1.0 minus the fraction of the bulk volume of the rock's solid matter. If the rock contains no fluids, porosity is the fraction of the bulk volume that is void space. Sometimes, rock contains fluids in pores that were sealed during sedimentation and, as a result, not all pore space is interconnected. Thus there is a concept of total, or absolute, porosity and effective porosity. It is common usage to use the term *porosity* for effective porosity because effective porosity represents the space that contains mobile fluids. These concepts can be expressed as:

$$\phi = \frac{V_p}{V_b} = \frac{V_b - V_{ma}}{V_b} = V_b - \frac{W}{\rho_{ma}} \tag{6–1}$$

where:

V_p = connected pore volume
V_b = bulk volume
V_{ma} = volume of matrix materials (grain volume)
W = weight
ρ_{ma} = density of matrix materials (quartz = 2.59 to 2.66 gm/cc)

Thus, porosity may be determined by measuring any two of the three quantities—matrix volume, void volume, and bulk volume.

PERMEABILITY

The introduction to API Code RP 27 states that permeability is a property of the porus medium and is a measure of the capacity of the medium to transmit fluids.[2] The concepts of permeability or the fluid conductivity of porous medium arose out of the work of H. Darcy in 1856. Darcy found through experimental work that the flow rate of water through sand filters was directly proportional to the area of the sand bed and to the hydraulic or pressure gradient between the inlet and outlet faces of the bed and inversely proportional to the thickness or length of the bed in the direction of flow. Darcy's relationship has been made to apply to fluids in general by including the viscosity of the fluid. The expanded version has become known as Darcy's law.

In order to make practical use of the empirical relationship known as Darcy's law, a proportionality constant known as permeability is necessary. Permeability as defined by API Code RP 27 has the units of the darcy and the millidarcy—the millidarcy being one thousandth of a darcy. A porous medium has a permeability of one darcy when a fluid of one centipoise flows at an apparent velocity of one centimeter per second under a pressure gradient of one atmosphere per centimeter under conditions of viscous flow. Specifically, the permeability constant has the dimensions of length squared (L^2) of those of an area.

Much work has been done on the permeability concept in which the limitations of Darcy's law have been defined. Numerous attempts have been made to relate permeability to the properties of a porous medium, but none has been entirely successful. Permeability measurements on thousands of core samples from oil and gas reservoir rocks have demonstrated the diversity of such media. Permeability has been found to vary widely, both laterally and vertically. The variation is generally much larger vertical to the bedding plane of stratified rocks than parallel. In spite of these variations, the concepts

of permeability and Darcy's law have proved to be immensely useful in reservoir engineering.

DARCY'S LAW

The basis for a quantitative statement of permeability was formulated by H. Darcy in 1856 and was later generalized by the inclusion of viscosity, the driving forces both of pressure and of hydraulic gradients, and the application of the statement to multidimensional flow systems. Stating Darcy's law in differential form for horizontal flow is most convenient for present purposes, thus:

$$u = -\frac{k}{\mu}\frac{dp}{dl} \qquad (6\text{–}2)$$

where:

k = the permeability constant
dp/dl = the pressure gradient

Note: The velocity, u, is not the actual velocity but the apparent velocity that is equivalent to q/A, or the rate of flow divided by the cross-sectional area.

If the flowing fluid is assumed to be noncompressible, equation 6–2 can be integrated as follows for linear flow:

$$\frac{q}{A}\int_{o}^{L} dl = -\frac{k}{\mu}\int_{p_2}^{p_2} dp$$

$$q = \frac{kA\,(p_1 - p_2)}{\mu L} \qquad (6\text{–}3)$$

Note: If the permeability constant, k, has the units given in the quotation from API Code RP 27, the units of the remainder of the terms are fixed in equation 6–3. Thus:

$$1 \text{ darcy} = \frac{(cc/sec)(cp)(cm)}{(sq\ cm)(atm)}$$

For radial flow, the equation becomes:

$$q = \frac{2\pi\, kh\,(p_e - p_w)}{\mu \ln r_e/r_w} \qquad (6\text{–}4)$$

The specimen of porous medium is assumed to be cylindrical in shape with a hole of radius, r_w, and thickness, h, as shown in Figure 6–1. Again, the fluid is noncompressible and the units are those of the definition of the permeability constant.

The foregoing equations are for horizontal flow and complete saturation of a porous medium with flowing fluid. If a second fluid that is immiscible with the first fluid is introduced into the porous media, the permeability of the medium to the first fluid will change. Also, the definitions require that the flowing fluid not interact with the porous medium. As an example of interaction, a porous medium containing clays may lose all permeability when fresh water is introduced.

FLOW OF GASES IN POROUS MEDIA

When a gas is used to measure the permeability of cores in the laboratory, apparent permeability varies with the mean pressure of the

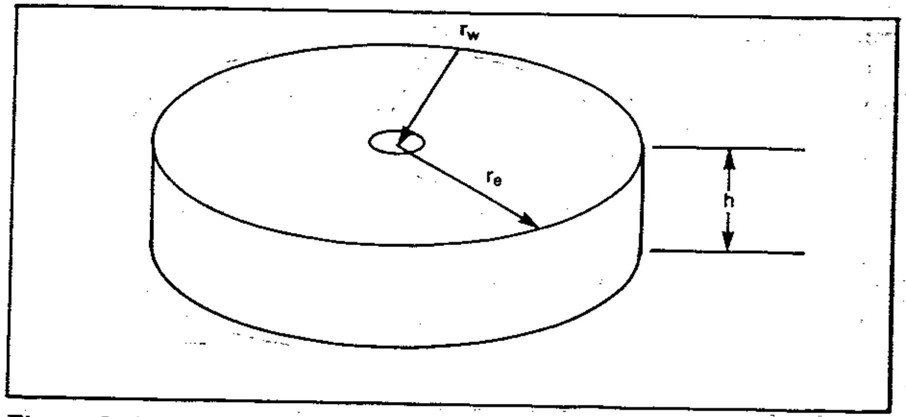

Figure 6–1 *Shape of porous medium specimen to illustrate radial flow to a central well bore.*

measurement and the size of the gas molecules. Klinkenberg has reported variations in permeability using different gases as the flowing fluid.[3] He attributed the variations to slippage—a phenomenon observed during the flow of gases in capillaries. Consequently, slippage in porous media is related to the mean free path of the gas molecules and is controlled by temperature, pressure, and the nature of the gas. Klinkenberg and others found that the observed permeability for a gas was a straight-line function of the reciprocal of the mean pressure of the measurement, thus:

$$k_g = k_L\left(1 + \frac{b}{\overline{p}}\right) \qquad (6\text{–}5)$$

where:

k_g = permeability to gas
k_L = permeability of medium to a nonreactive liquid
b = constant for a given gas in a given medium
$\overline{p}$ = mean pressure of measurement.

Thus, as the pressure of a measurement approaches an infinite value, the observed permeability approaches a value for a nonreactive liquid. All permeabilities are for a completely saturated medium containing a single phase.

Note: All symbols thus far have the dimensions given in the definition of permeability following equation 6–3.

EQUATIONS FOR RADIAL FLOW OF GASES

Although the relatively simple equations for steady-state flow and horizontal flow of gases in porous media have limited practical application to gas production problems, they illustrate several important problems. For this reason, the following equations are introduced.

If one starts with the definition of Darcy's law as given in equation 6–2 and derives the flow equation for radial flow of a gas, the result is:

$$q = \frac{707.8\, kh\,(p_e^2 - p_w^2)}{\overline{\mu}\,\overline{z}\, T_f \ell n\, r_e/r_w} \qquad (6\text{–}6)$$

or:

$$q = 307.4\,\frac{kh\,(p_e^2 - p_w^2)}{\overline{\mu}\,\overline{z}\, T_f \log r_e/r_w} \qquad (6\text{–}7)$$

where:

q = cu ft/day at 14.65 psia and 60°F
k = permeability, darcies
h = thickness, ft
p_e = pressure at external radius, r_e, psia
p_w = pressure at inner radius, r_w, psia
$\bar{\mu}$ = average viscosity, cp
$\bar{z}$ = average compressibility factor, ratio
T_f = flowing temperature, °F + 460
r = radius, ft

Equation 6–6 and 6–7 are limited to steady-state flow conditions; therefore, they do not show how pressure may vary with time. However, the equations do have some practical value in that they can be used to illustrate some of the limitations inherent in the radial flow of gas.

If one calculates the pressure in a radial direction for the flow of gas by means of equations 6–6 or 6–7 for the conditions listed below, the result is the pressure distribution given in Figure 6–2, where:

k = 0.01 darcy
h = 20 ft
$\bar{\mu}$ = 0.02 cp
$\bar{z}$ = 0.90
T_f = 580°F abs
r_e = 2,640 ft
r_w = 0.25 ft
p_e = 3,000 psia
p_w = 1,000 psia
(q_k is calculated to be 11,708 Mcfd)

Pressures are shown as a function of radial distance. The most immediate reaction is that about 50% of the total pressure drop of 2,000 psi occurs within 10 ft of the wellbore. A question then presents itself—what would the rate of flow be if the wellbore were increased from the initial radius of 0.25 to 10 ft with pressures and all other conditions held constant? Calculations show that the new rate of flow would be 19,454 Mcfd. This represents a 1.66-fold increase in rate of flow for an increase in wellbore radius of 40-fold.

If one assumes that all of the conditions remain constant except the

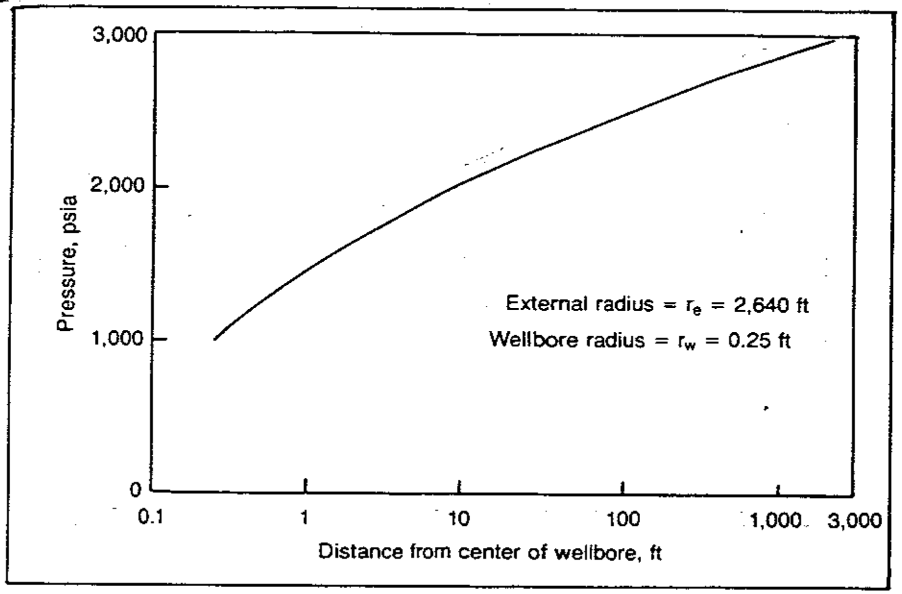

Figure 6–2 *Pressure drop along a radius for radial flow of a gas.*

radius of the wellbore, the relationship between the rate of flow before the wellbore is enlarged (subscript 1) to that after enlargement is:

$$\frac{q_2}{q_1} = \frac{\ell n\, r_e/r_{w1}}{\ell n\, r_e/r_{w2}} = \frac{\log r_e/r_{w1}}{\log r_e/r_{w2}} \qquad (6\text{–}8)$$

If all other factors remain the same, the increase in productive capacity is a function of the logarithm of the ratio of the external radius to the radius of the wellbore before and after the enlargement. This relationship of equation 6–8 is illustrated in Figure 6–3. If the wellbore radius before enlargement is 0.25 ft, it is necessary to enlarge the wellbore to 26 ft to double the rate of flow, provided the other conditions remain the same.

Thus, from the nature of radial flow the stimulation of wells by the enlargement of the wellbore requires relatively large increases in the size of the wellbore to bring about modest increases in the productive capacities of the well, even though most of the pressure drop occurs near the wellbore.

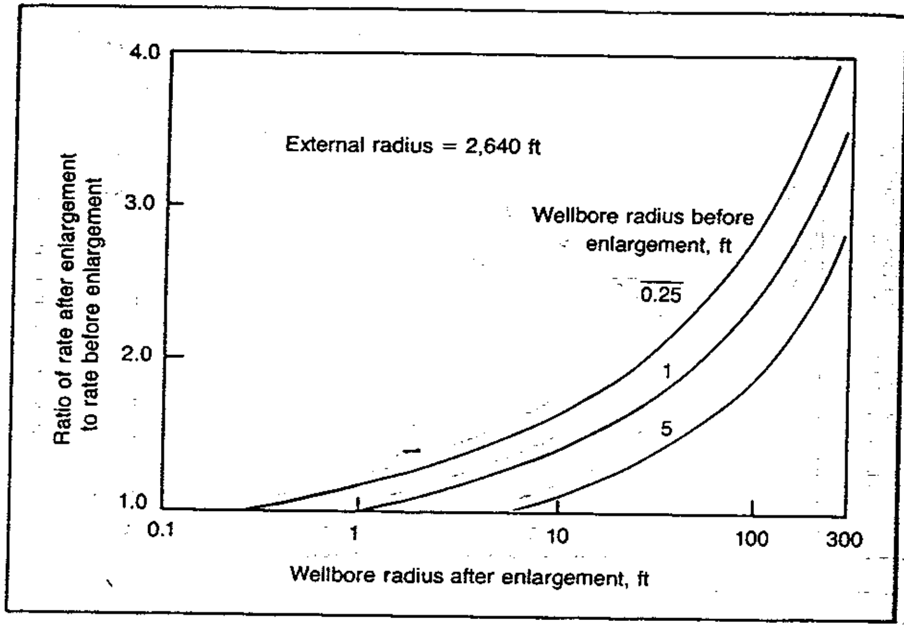

Figure 6–3 *Effect of wellbore enlargement on producing rate.*

RADIAL FLOW OF GAS IN PARALLEL BEDS

If one considers radial flow in parallel beds as illustrated in Figure 6–4, the flow through each of the separate beds is as follows:

$$q_1 = \frac{707.8\, k_1\, h_1\, (p_e^2 - p_w^2)}{\mu z\, T_f\, \ell n\, r_e/r_w}$$

$$q_2 = \frac{707.8\, k_2 h_2\, (p_e^2 - p_w^2)}{\mu z\, T_f\, \ell n\, r_e/r_w}$$

$$q_3 = \frac{707.8\, k_3 h_3\, (p_e^2 - p_w^2)}{\mu z\, T_f\, \ell n\, r_e/r_w}$$

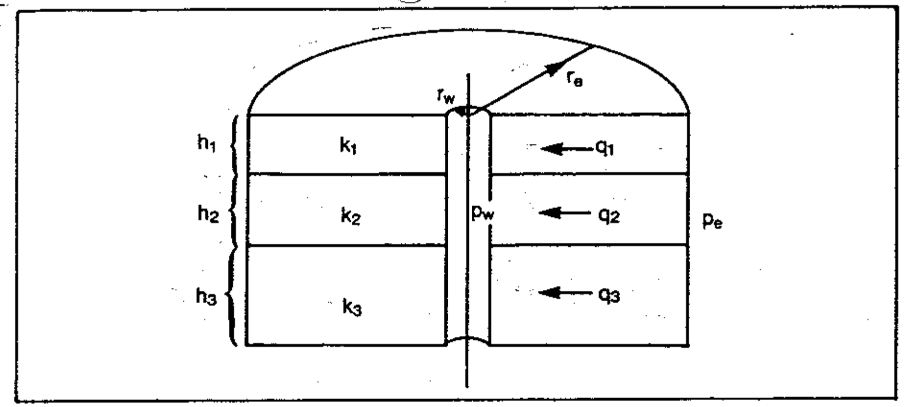

Figure 6–4 *Radial flow in parallel beds.*

Then:

$$q_1 + q_2 + q_3 = \frac{707.8\, (p_e^2 - p_w^2)}{\mu z\, T_f\, \ell n\, r_e/r_w}\left[k_1 h_1 + k_2 h_2 + k_3 h_3\right] \qquad (6\text{–}9)$$

or:

$$\bar{k} = \frac{(k_1 h_1 + k_2 h_2 + k_3 h_3)}{(h_1 + h_2 + h_3)} \qquad (6\text{–}10)$$

where the bar over the symbol indicates an average value. In a more-general form, the result is:

$$\bar{k} = \frac{\sum_1^n k_i\, h_i}{\sum_1^n h_i} \qquad (6\text{–}11)$$

RADIAL FLOW OF GAS IN SERIES BEDS

Referring to Figure 6–5 and by applying equation 6–6, the result is:

$$q = \frac{707.8\, k_2 h\,(p_e^2 - p_1^2)}{\overline{\mu}_2 \overline{z}_2\, T_f\, \ell n\, r_e/r_1} + \frac{707.8\, k_1 h\,(p_1^2 - p_w^2)}{\overline{\mu}_1 \overline{z}_1\, T_f\, \ell n\, r_1/r_w}$$

$$= \frac{707.8\, \overline{k}\, h\,(p_e^2 - p_w^2)}{\overline{\mu}\, \overline{z}\, T_f\, \ell n\, r_e/r_w}$$

But:

$$(p_e^2 - p_1^2) + (p_1^2 - p_w^2) = p_e^2 - p_w^2$$

and:

$$\overline{k} = \frac{\overline{\mu}\, \overline{z}\, \ell n\, r_e/r_w}{\dfrac{\overline{\mu}_2 \overline{z}_2}{k_2}\, \ell n\, r_e/r_1 + \dfrac{\overline{\mu}_1 \overline{z}_1}{k_1}\, \ell n\, r_1/r_w} \quad (6\text{–}12)$$

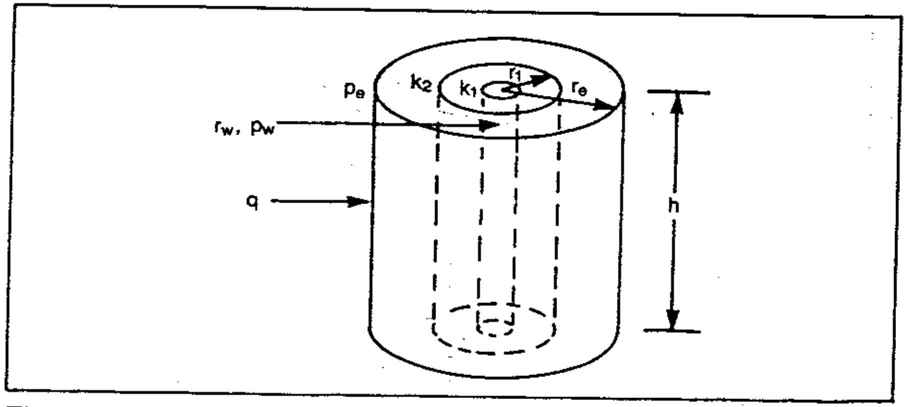

Figure 6–5 *Radial flow in series beds.*

In more general form the result is:

$$\overline{k} = \frac{\overline{\mu}\, \overline{z}\, \ell n\, r_e/r_w}{\displaystyle\sum_{1}^{n} \frac{\mu_i z_i}{k_i}\, \ell n\, r_i/r_{i-1}} \quad (6\text{–}13)$$

SUMMARY

The steady-state equations for flow in porous media have practical application to many gas producing problems. They are especially applicable to problems involving wellbore effects such as the effect of changes in permeability on well productivity. These applications are best illustrated by the problems that follow.

PROBLEMS

1. Calculate permeability for the following core, length = 2 cm and diameter = 1 cm. A flow of 60 cc of water/min results from an upstream pressure of 2.3 atm and a downstream pressure of 1 atm.

2. A 6-in. wellbore produces 100 Mcfd against a pressure of 500 psia. If, at an outer boundary that is 2,640-ft distant, the pressure is estimated to be 1,000 psia, and if the effective wellbore radius were increased to 125 ft, what would be the resulting rate of flow? How might the effective wellbore radius be increased to 125 ft?

3. A 6-in wellbore is drilled into a 500-md pay on a spacing of 640 acres (r_e = 2,640 ft). The drilling mud penetrated the formation to a distance of 1 ft and reduced the permeability to 10% of its original value. Assume that the gas is ideal and that the viscosity is constant. Calculate the average permeability for the well system.

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7 Gas Well Testing

The testing of gas wells began as an effort to provide a comparative measure of the productive capacity of a well. In the early days, wells were opened to the atmosphere and allowed to "blow," and the open flow was estimated by taking impact pressure readings with a pitot tube. Although the open flows taken in this manner may have given some basis for comparing the productive capacities of wells, engineers quickly realized that the measured open flow was a poor indicator of the rate at which a gas well would produce into a pipeline under normal operating conditions. It also was a poor means for studying gas production problems. Furthermore, such open-flow testing was likely to damage the wells, was wasteful of gas, and was generally an unsafe practice.

In the 1920s, much effort was given to developing a method for measuring productive capacity by conducting tests at reasonable, controlled rates of flow. The first generally accepted test was published by Pierce and Rawlins in 1929.[1] Later, the test was set out thoroughly in Bureau of Mines Monograph 7 by Rawlins and Schellhardt in 1935.[2] The methods of Rawlins and Schellhardt quickly became standard practice and became known as the conventional back-pressure or multipoint test. Sometimes the test is referred to as the flow-after-flow back-pressure test. Rawlins and Schellhardt found that, when the flow rates are plotted on logarithmic coordinates against the corresponding values of the difference of the squares of a formation shut-in pressure, $\bar{p}_{Rl}$, and the flowing sand-face pressure, p_{wf}, the relationship may be represented empirically by a straight line. The equation is:

$$q = C\,(\bar{p}_{Ri}^{\,2} - p_{wf}^{\,2})^n \qquad (7\text{–}1)^*$$

where C was considered a constant and sometimes is called the performance coefficient, and the exponent n is a constant. The industry, by common usage, refers to n as the slope of the back-pressure curve but by the same common usage makes plots on log-log coordinates that show $(p_{Ri}^{\,2} - p_{wf}^{\,2})$ as the dependent variable. In these cases, n is the reciprocal of the mathematical slope of the line on the plot. Hereafter, n will be referred to as the exponent of the back-pressure or performance curve.

Almost from the start, so-called stabilization problems beset the early well testing procedures. At this late date, it is difficult to determine whether the stabilization problems were caused by liquid removal, by liquid accumulation in the wellbore, or by the unsteady-state behavior of the wells, but probably they were a combination of the three phenomena. Testing became a procedure for cleaning the wellbore of liquids and achieving stabilization by clever testing procedures or by defining stabilization in such a way that it could be achieved easily. Typically, a flowing well was considered stabilized when the flowing wellhead pressure changed no more than a certain number of psi in a period of 15 min. This was the state of affairs until 1953–1955 when Cullender presented his paper on the isochronal method of determining the flow or performance characteristics of gas wells.[3] Essentially, Cullender pointed out that, for many wells in low-permeability reservoirs, the movement of the pressure disturbance or radius of drainage away from the wellbore upon opening the well to flow was slow. The pressure and rate effects of the movements could be observed for hours and even days after the well started flowing. Changing the rate of flow without shutting in the well only added another transient effect to the one already imposed upon the reservoir in the vicinity of the well.

Cullender proposed that the coefficient C in equation 7–1 was a variable with respect to time and that it was not a function of pressure or the rate of flow. In order to start the well on a different rate of flow, it was necessary to shut in the well and essentially erase the effect of the pressure transient on pressures in the reservoir. Thus, the isochronal back-pressure equation became:

$$q = C(t)\,(\bar{p}_{Ri}^{\,2} - p_{wf}^{\,2})^n \qquad (7\text{–}2)$$

* In considering unsteady state flow, outlined in Chapter 8, it is necessary to distinguish between the shut-in pressure measured before a test, $\bar{p}_{Ri}$, and the reservoir pressure, $\bar{p}_R$, at any time—whether measured or not.

where $C(t)$ indicates that C is a function of time when each rate of flow is started with the well shut in and no prior appreciable pressure transient effects exist within the radius of drainage affected by the tests. The isochronal performance curves consist of a family of parallel curves on logarithmic coordinates in which the difference in pressures squared is plotted against the rate of flow, and straight lines join the points for equal duration-of-flow time.

In effect, Cullender showed that transient pressure effects were a natural occurrence in low-permeability reservoirs and could not be negated by testing techniques. This in effect meant that the so-called absolute open flow potential of a gas well was a function of time and was affected by the pressure transients that were in existence around the wellbore at the time the well was tested. Cullender mentioned that certain gas wells stabilize so rapidly that obtaining isochronal test data was not necessary. This merely meant that the permeability in the reservoir around these wells was so high and that the pressure disturbance moved away from the well so fast that the transient effects were not observed.

At this stage of thought regarding gas well test technology, the Interstate Oil Compact Commission test manual and related test manuals were written and published.[4,5,6] Yet as these manuals were being written, there was a growing understanding of unsteady-state flow behavior as it applied both to oil and to gas wells. The first of a series of four manuals was published by the Oil and Gas Conservation Board of Alberta, Canada, in February 1964.[7] The fourth edition of *Gas Well Testing—Theory and Practice,* published in 1979, presents a comprehensive review of unsteady-state flow theory as applied to the flow of gas into gas wells.[8]

Unsteady-state flow or transient flow theory began with investigations of flow of water in aquifers and flow of hydrocarbon liquids in reservoirs. The work of Aronofsky and Jenkins, Jenkins and Aronofsky, and Bruce et al. showed that numerical solutions for transient gas flow in porous media for various rates of flow in a radial system almost coincide with the solution for transient liquid flow.[9,10,11] These findings opened the way for applying the theory of transient liquid flow to gas well behavior. A comprehensive bibliography is included in reference 8. A significant point in subsequent developments was made by Smith, who confirmed by test results that inertial turbulent flow effects could be treated as an additional rate-dependent pressure loss near the wellbore of gas wells.[12] This finding was additional to the van Everdingen skin effect.[13] Another development that cannot go unmarked was that of Al-Hussainy, Ramey, and Crawford, who introduced the concept of real gas potential $m(p)$, which includes the dimensions of pressure^2/viscosity and, according to reference 8, could be defined as a modified pressure squared.[14] In this connection, it must be pointed out that the flow of gas in porous media involves three variables—pressure, viscosity, and compressibility—when the temperature is constant. Thus, Al-Hussainy, Ramey, and Crawford have attempted to account for changes in pressure, viscosity, and compressibility simultaneously.

TYPES OF GAS WELL TESTING

The testing of gas wells falls into two general classifications depending upon the status (shut-in or flowing) of the well prior to each rate of flow. If rates of flow are imposed in succession without allowing a shut-in period between the flow periods, the test is a multipoint type of test. If the well is shut in between the various flow periods, the test is isochronal. A multipoint test is sometimes referred to as a three-, four-, or five-point test, depending upon the number of rates of flow used in the test. A one-point test usually means that the well was started from shut-in at a predetermined rate of flow and was allowed to produce for an extended period of time. Deliverability tests are usually one-point tests run under specified conditions or tests in which the results are corrected to correspond to a specific set of conditions. More formal definitions, examples, and test procedures follow.

Multipoint Test (Flow After Flow)

Starting with a shut-in well, a series of flow rates—usually in increasing sequence—are imposed on the well at fixed time intervals. The objective of the test is to determine the open-flow potential of a well or to determine the exponent (slope) to be used with a one-point test. The exponent from the original multipoint test may be used with subsequent one-point tests to determine the open-flow potential for a well as the shut-in pressure decreases with production and time.

One-point Test

Starting from shut in, the well is opened to flow with producing pressures, temperatures, and rates of flow measured at specified time intervals. The duration of flow is one to three days or longer.

Isochronal Test

The isochronal test consists of a series of one-point tests, each starting with the well shut in and with the shut-in pressure constant or nearly constant

with time.* Producing pressures, temperatures, and rates of flow are measured and recorded at specified time intervals after the well is opened to flow. Usually the time intervals are 0.5, 1, 2, 3, 6, 24, 48, 72 hr, etc.

Information Test

The information test is a special one-point test especially designed for wells that reach stabilization slowly. The well is started from shut in to flow against a constant pressure, which can be accomplished for a small-capacity, slow stabilization well by flowing the well at capacity into a pipeline of constant or nearly constant pressure. If a pipeline is not available, the well is opened to flow through a choke or orifice of constant size. It is important not to disturb the well by changing the choke setting or orifice size after starting the flow. Producing pressures, temperatures, and rates of flow are measured and recorded at specified time intervals after the well is opened to flow.

DEFINITIONS OF TERMS

Open-flow potential is the rate of flow that would be obtained if the bottom-hole pressure opposite the sand face were reduced to zero pressure. The open-flow potential is independent of well equipment. (The time dependency aspect should be kept in mind.)

Deliverability is the rate of flow from a well against a specified pressure (usually a working pressure at the wellhead) after a specified period of time following a specified shut-in period.

Official tests are tests required by regulatory agencies for allowable purposes. The tester should obtain the test and calculation procedures from the appropriate regulatory agency before testing a well for allowable purposes. No attempt will be made to outline official testing.

PREPARATION OF A WELL FOR TESTING

The wellbore should be cleaned of liquids by flowing at a high rate to a pipeline for a period of 24 hr. (The determination of the rate of flow necessary

* Many investigators have called a nearly constant pressure, either shut-in or flowing, a stabilized pressure, but the author prefers to use stabilization in a more-narrow sense. That is, a well is stabilized when the pressure disturbance known as the radius of investigation or influence reaches the outer boundary of the reservoir or the anticipated drainage area. This concept will be discussed later.

to clean the wellbore of liquids is discussed in detail in Chapter 11.) If the well does not have a pipeline connection, it may be necessary to produce the well to the atmosphere for a short period of time if such action is considered safe. Extra precautions should be taken on new wells to remove drilling mud, solids, and stimulation fluids from the wellbore. If the well has a low capacity to produce gas, extra care should be taken in swabbing the fluids from the wellbore during completion. The time and expense involved in cleaning the well can be kept to a minimum by installing a carefully sized tubing string (see Chapter 11) in the well during completion. The well should be shut in for an appropriate period of time to equalize the reservoir pressure around the wellbore. Even with the largest capacity wells, the shut-in period should not be less than 24 hr, if possible. For small wells, the period between completion and cleanup and connection to the pipeline should be used as the shut-in period. Here, a balance must be maintained between shut-in time and the economic necessity for placing a well on production.

During the shut-in period, gas measurement equipment should be prepared for use. If the gas is to be measured by an orifice meter, the meter should be calibrated, the diameters and condition of the run and plate should be verified and recorded, and the differential pen should be zeroed in accordance with good meter practice. If a critical flow prover (see Chapter 4) is used, it should be placed in a vertical position at the wellhead or downstream from the separator so gas will flow vertically away from the test area. If a separator is used, the rate of flow should be controlled by a production choke, and pressure should be maintained on the separator by the critical flow prover or a back-pressure regulator when an orifice meter is used. If a separator is not used, the rate of flow can be controlled at the wellhead by the critical flow prover. Always install thermometer wells at the wellhead and at gas measuring equipment so that temperatures may be measured by a thermometer or calibrated recording meter. Thermometer wells should be filled with water or lubricating oil to obtain accurate temperature measurement.

One cannot place too much emphasis on proper procedures for preparing the well for testing. The purpose and goal of the preparation is to obtain accurate test results and an absolute-open-flow potential that is indicative of the performance of the well at its best operating condition. Such tests are most valuable in studies of possible remedial work and of the installation of compressors. The natural gas engineer has the responsibility to see that highest quality tests are taken and reported.

The chief difference between testing methods for high-pressure gas wells, gas condensate wells, and low-pressure wells is the care used in taking the data and methods used in computing the results. The preparation of a

high-pressure well for testing usually requires more time and patience on the part of the tester. The effect of liquids on such wells is more pronounced in high-pressure wells; consequently, special care should be used to measure gas-liquid ratios. Often, one must measure the gas-liquid ratio at each rate of flow during the multipoint or isochronal test. If the ratio is not constant, the well probably is accumulating liquid in the wellbore or unloading liquid during testing. In either case, the test probably is not acceptable, and the well should be cleaned by flowing at a high rate and then retested at rates of flow high enough to keep the well free of liquid (see Chapter 11).

A representative multipoint test can usually be obtained by testing the well in accordance with the following procedure:

1. Verify that the wellbore is free of liquids before shutting in the well for testing.
2. Do not start the test until the shut-in pressure of the well is representative of the stablized pressure of the producing formation surrounding the wellbore.
3. The duration of each flow should be the same length of time; usually 3 hr* is sufficient.
4. The flows should be conducted in an increasing-rate sequence.

All shut-in or flowing pressures should be measured using deadweight or piston gauges or other gauges of similar accuracy because bourdon gauges usually are not accurate enough for use in gas well testing. Subsurface pressures in gas wells may be measured directly by pressure gauges or calculated from surface pressures (see Chapter 5). If conventional subsurface gauges are lowered into the well on a wire line, flow rates will be limited to velocities insufficient to cause the gauges to move up the producing string.

TESTING PRACTICES

The following summary lists the principal testing practices that are necessary to obtain maximum engineering and economic benefits from testing.

* Recently, the more-sophisticated methods for analyzing well tests have shown that wellbore storage effects in wells without packers or linear flow to stimulated wells could last easily 1 or 2 hr after the well is opened to flow or after a change is made in the rate of flow. Therefore, the author recommends a 3-hr duration for each rate of flow.

1. The tester should understand the purpose of testing and know what information will be recorded. The tester should have the proper tools for testing such as deadweight gauges, critical flow provers, and thermometers.
2. All surface pressures at the wellhead should be measured by a deadweight gauge.
3. A thermometer should be installed at the wellhead, and wellhead temperatures should be recorded at the same time that the wellhead pressure is measured.
4. The well should be conditioned for testing by removing liquids from the wellbore prior to measuring the shut-in pressure (see Chapter 11).
5. If an official test will be run, the procedures of the regulatory body should be followed closely, and every ethical and economic means should be used to obtain the test with the well in its best operating condition. When the results of the well influence the allowable assigned to the well, there should be no hesitation to retest if it is justified.
6. Multipoint tests should be run in increasing rate of flow sequence, and each flow period should last the same period of time. During each rate of flow, wellhead flowing pressures and flow rate data should be recorded after each 15-min period to permit a determination of the degree of stabilization. In case of high-liquid-ratio wells or unusual temperature conditions, a decreasing-flow-rate sequence may be used if an increasing-flow-rate sequence results in poor alignment of points.
7. The rate of liquid production should be observed at frequent intervals during testing to ensure good liquid-gas ratio information.
8. Multipoint tests on deep, large-capacity wells in reservoirs with high temperature should be preceded by a preflow test at a high rate to ensure temperature stabilization.

SHUT-IN PRESSURE

Determining the shut-in pressure at the reservoir level is one of the most important measurements that can be made on a gas well. The wellhead pressure can be measured and used to calculate the pressure at the level of the reservoir, or the pressure can be measured directly by a subsurface pressure gauge. (If the gauge cannot be lowered to the reservoir level, the reservoir pressure can be determined by extrapolating a series of pressure measurements to the desired depth in the reservoir). If the shut-in pressure at the reservoir level will be calculated from wellhead pressure, several precautions should be observed.

Pressure measurements at the wellhead should be taken using a deadweight tester or piston gauge at convenient time intervals until the pressure reaches a maximum or is constant (or almost constant) with time. If the well has a large production capacity and is producing from a high-pressure, high-temperature reservoir, the shut-in pressure will be influenced by the temperature of the gas in the flow string. For example, a deep, high-pressure well is observed to have a wellhead pressure of 4,173 psia at a wellhead temperature of 177°F. After an extended shut-in time, the wellhead pressure is 4,140 psia. The change in pressure is caused by the cooling of the column of gas in the well because no gas is being produced from the reservoir. Although the effects of temperature changes are minor and for the most part negligible for wells completed in low-pressure, low-temperature reservoirs, they become critical in initial tests upon new wells completed in deep, high-pressure, high-temperature reservoirs. The temperature behavior of such a well is given in Table 7–1. The multipoint test obtained with the data in the table was unsatisfactory because of the temperature changes. Subsequently, the well was produced at a high rate for 6 hr to warm it up before the data given in Table 7–2 were taken. The temperature data given in Table 7–2 gave a satisfactory multipoint test.

A second source of error in calculating subsurface pressures from wellhead pressures is the presence of liquid—either hydrocarbon or water—standing in the wellbore. This problem can be avoided by taking care to

Table 7–1 *Temperature behavior of a deep well without preflow to stabilize its temperature.*

Time	*Flow Rate* (q_m)	*Wellhead Temperature (°F)*
10:20 a.m.	opened well	—
11:20	11.0	108
12:20 p.m.	13.1	110
1:20	17.7	118
2:20	23.8	128
3:20	28.4	149

Table 7–2 *Temperature behavior of the same well (Table 7–1) after six hours of high-rate flow to stabilize its temperature.*

Time	*Flow Rate* (q_m)	*Wellhead Temperature (°F)*
2:00 p.m.	opened well	—
3:00	10.5	130
4:00	13.3	128
5:00	17.7	133
6:00	23.9	142

flow the liquid out of the wellbore before the well is shut in or by using subsurface gauges to measure the subsurface pressures directly. Direct measurement is sometimes necessary because one cannot always free the wellbore of liquid by flowing the well before shut-in.

A third problem encountered in determining reservoir pressure for a gas well is caused by unsteady-state effects in the reservoir which may persist for days, weeks, or even months after the well is shut in. The pressure in the wellbore and at the wellhead of a shut-in gas well may become steady in a matter of minutes or in an hour or two for large-capacity wells in high-permeability reservoirs or, if in a very low permability reservoir, it may climb steadily for months after the well is shut in. For the latter type of well, determining a stabilized reservoir pressure may not be an economic possibility. However, methods are described in the literature for estimating stabilized reservoir pressure from build-up pressure analyses. These methods will not be described herein, but the reader is referred to Matthews and Russell and Earlougher for details of these methods.[15,16]

In testing gas wells, reservoir pressure is taken as the pressure at the midpoint of the productive section in the wellbore that is open to flow into the well. This point may be a large distance above or below a common datum plane in a reservoir to which reservoir pressures may be referred for other purposes.

MULTIPOINT TEST AND EXAMPLE

A multipoint test is taken so that a plot of equation 7–1:

$$q = C(\bar{p}_{Ri}^{\,2} - p_{wf}^{\,2})^n \tag{7–1}$$

can be extrapolated for estimating the rate of flow that would be obtained if the working pressure, p_{wf}, at the sand face in the wellbore were reduced to zero. In special cases, the equation used is:

$$q = C(\bar{p}_{cs}^{\,2} - p_{cf}^{\,2})^n \tag{7–3}$$

where flow is produced through the tubing, no packer is in the well, and pressures are measured at the wellhead on the casing. A four-point test of a constant 3-hr time duration is normally satisfactory for determining the open-flow potential of a well. Rates of flow should be taken in an increasing-rate sequence.

The four rates of flow should be evenly distributed over the test range. For average- to low-capacity gas wells, the first rate of flow should lower the pressure at the wellhead about 5%, and the pressure reduction at the fourth rate of flow should be 25%. After opening the well to flow at the first rate, the test rate should be continued for 3 hr. Each succeeding rate of flow should last for the same period of time. During each flow rate, the working pressure and temperature at the wellhead, all flow rate measurements, and temperatures should be taken and recorded at the end of each 15 min. If a separator and tanks are used during testing, the rate of liquid production for both hydrocarbon and water should be reported. The specific gravity of the gas flowing from the prover or separator should be measured and reported. For new wells, a sample of gas from the separator or prover should be taken for analyzing hydrocarbon and nonhydrocarbon components and for calculating heating value.

Table 7–3 is a copy of the data taken during a four-point test on a medium- to low-capacity gas well. All data recorded on Table 7–3 are needed for calculating the results of the test or for understanding the results of the test. The principal information that is missing is the length of time that the well was shut in before the wellhead pressure of 1,864 psig was measured. Also, no other shut-in pressures are given to indicate whether the shut-in pressure was building up. From another source, we learn that the shut-in time was 72 hr, but we are left to speculate the rate of pressure buildup. However, these data should have been recorded on the field data sheet.

The steps in calculating the results of a multipoint test are:

1. Calculate the rate of flow for each rate imposed on the well.
2. If the well produced hydrocarbon liquid and the specific gravity of the separator gas was measured, calculate the

Table 7–3 *Field data sheet and convenient form for multipoint test*[6] *(Reproduced by permission of the Corporation Commission of The State of Oklahoma).*

TYPE TEST: 4 Point	☐ INITIAL	☐ ANNUAL	☐ SPECIAL	TEST DATE 11-3-56		
COMPANY Test Example 1			CONNECTION			
FIELD	RESERVOIR		LOCATION		UNIT	
COMPLETION DATE	TOTAL DEPTH	PLUG BACK TD	ELEVATION		FARM OR LEASE NAME	
CSG. SIZE 7" OD	WT. 23.00#	d 6.366	SET AT 8293	PERFORATIONS: FROM 8112 TO 8148	WELL NO.	
TBG. SIZE 2 3/8" OD	WT. 4.70#	d 1.995	SET AT 8134	PERFORATIONS: FROM 8127 TO 8132	SEC.	TWP. RGE.
TYPE COMPLETION (DESCRIBE)				PACKER SET AT 8100	COUNTY	
PRODUCING THRU Tubing	RESERVOIR TEMP. F 155 @ 8130	MEAN GROUND TEMP. F 60		BARO. PRESS.—P_a 14.4	STATE	
L 8130	H 8130	G_g 0.625	%CO_2 2.0	%N_2 3.0	%H_2S	PROVER METER RUN 4" TAPS Pipe

DATE Time of Reading	ELAP. TIME Hrs.	WELLHEAD WORKING PRESSURE Tbg. Psig	Csg. Psig	Temp. F	METER OR PROVER Pressure Psig	Diff.	Temp. F	Orifice	REMARKS (Include liquid production data: Type—API Gravity—Amount)
11-3-56									
									St Tk 3' 7 1/8" 60F
7:45A		1864.0	Packer						
8:00A									72 hour Shut-in Pressure
8:30A		1809.0		74	718.0	7.0	60	1.75	Start 1st Flow
9:00A		1793.0		74	720.0	8.0	59		
9:30A		1779.0		74	721.0	8.4	64		
10:00A		1772.0		74	721.0	8.3	66		St Tk 3' - 7 1/2" 60F
10:00A									
10:30A		1765.0		74	719.0	15.0	54	1.75	Increased Rate for 2nd Flow
11:00A		1742.0		74	719.0	15.5	54		
11:30A		1728.0		74	722.0	15.7	55		
12:00N		1698.0		74	729.0	17.5	56		St Tk 3' - 8 1/4" 60F
12:00N									
12:30P		1705.0		74	739.0	24.0	43	1.75	Increased Rate for 3rd Flow
1:00P		1672.0		74	740.0	25.5	46		
1:30P		1633.0		74	742.0	27.5	50		
2:00P		1608.0		74	744.0	29.3	54		St Tk 3' - 9 1/8" 60F
2:00P									
2:30P		1630.0		74	745.0	38.0	44	1.75	Increased Rate for 4th Flow
3:00P		1587.0		74	746.0	39.3	46		
3:30P		1539.0		74	749.0	41.5	52		API - 50.2° at 60°F
4:00P		1512.0		74	759.0	41.5	55		End Tk 3' - 10 1/4" 60F
									Tank Size - 12' Diameter
									1.68 Barrels/inch

PAGE 1 OF 1 DATA BY

specific gravity of the flowing fluid using the method given in Chapter 2.

3. Calculate the shut-in pressure, $\overline{p}_{Ri}$, the flowing bottom-hole pressure, p_{wf}, for each rate of flow, and the difference of squares between the shut-in pressure and the flowing pressure for each rate of flow, $\overline{p}_{Ri}^2 - p_{wf}^2$.
4. Plot the rate of flow, q, against the corresponding pressure difference squared on log-log coordinate paper using the rate of flow as the abscissa.
5. Calculate the exponent, n, for the back-pressure curve.

The calculations are illustrated in Table 7–4, in which the field test data are summarized in the upper five rows of numbers. The calculations for each of the four rates of flow are given in the next four rows. The calculations for arriving at the supercompressibility factor, F_{pv}, are given in the left half of the third group of four numbers. Since the well produced hydrocarbon liquid at the ratio of 193,000 cu ft/bbl and it was assumed that all of the liquid was in the vapor phase during flow in the well, it was necessary to correct the separator gas gravity of 0.625 for the additional liquid. The gas gravity of the flowing fluid was calculated at 0.641, as indicated in the right half of the third group of four rows of numbers. (A procedure for making this calculation is given in Chapter 2.) The calculations of the difference of squares for wellhead and bottom-hole pressures are given in the fourth group of numbers.

Figures 7–1 and 7–2 show the wellhead and bottom-hole performance curves, respectively, on logarithmic coordinates for the multipoint test example given in Tables 7–3 and 7–4.

Note: When comparing the curves in the two figures, the wellhead performance is steeper than the bottom-hole curve.

For a given well, the values of exponent n and coefficient C in equation 7–1 must be determined by measuring pressures and flow rates at different rates of flow and solving for average values of n and C. If equation 7–1 is written as a logarithmic function, the result is:

$$\log q = \log C + n \log (\overline{p}_{Ri}^2 - p_{wf}^2) \qquad (7\text{–}4)$$

This forms a straight line when plotted on logarithmic coordinates. The open-flow potential is found by extrapolating the curve to $p_{wf} = 0$ or the rate of flow at $\overline{p}_{Ri}^2$ as shown in Figure 7–2. Here, the average reservoir

Table 7–4 *Calculation of results from test example 1 in Table 7–3[6] (Reproduced by permission of the Corporation Commission of the State of Oklahoma).*

TYPE TEST: 4 Point	[X] INITIAL	[ ] ANNUAL	[ ] SPECIAL	TEST DATE: 11-3-56
COMPANY: Test Example 1			CONNECTION	
FIELD	RESERVOIR		LOCATION	UNIT
COMPLETION DATE	TOTAL DEPTH	PLUG BACK TD	ELEVATION	FARM OR LEASE NAME
CSG. SIZE: 7" OD	WT.: 23.00#	d: 6.366; SET AT: 8293	PERFORATIONS: FROM 8112 TO 8148	WELL NO.
TBG. SIZE: 2 3/8" OD	WT.: 4.70#	d: 1.995; SET AT: 8134	PERFORATIONS: FROM 8127 TO 8132	SEC. TWP. RGE.
TYPE COMPLETION (DESCRIBE)			PACKER SET AT: 8100	COUNTY
PRODUCING THRU: Tubing	RESERVOIR TEMP. F: 155 @ 8130	MEAN GROUND TEMP. F: 60	BARO. PRESS.—P_a: 14.4	STATE
L: 8130	H: 8130	γ_g: 0.625; %CO2: 2.0	%N2: 3.0; %H2S	METER RUN: 4"; TAPS: Pipe

FLOW DATA

NO.	(LINE) SIZE X ORIFICE SIZE	PRESS. PSIG	DIFF. (INCHES)	TEMP. F	TUBING DATA PRESS. PSIG	TUBING DATA TEMP. F	CASING DATA PRESS. PSIG	CASING DATA TEMP. F	DURATION OF FLOW HR.
SI.					1864.0		Packer		72
1.	4 x 1.750	721.0	8.3	66	1772.0	74			2
2.	4 x 1.750	729.0	17.5	56	1698.0	74			2
3.	4 x 1.750	744.0	29.3	54	1608.0	74			2
4.	4 x 1.750	759.0	41.5	55	1512.0	74			2
5.									

RATE OF FLOW CALCULATIONS

NO.	COEFFICIENT (24—HOUR)	$\sqrt{h_w p_f}$	PRESSURE p_f	FLOW TEMP. FACTOR, F_t	GRAVITY FACTOR F_g	SUPER COMPRESS. FACTOR, F_{pv}	RATE OF FLOW Q
1.	17.23	78.13	735.4	0.9943	1.265	1.059	1793
2.	17.23	114.1	743.4	1.004	1.265	1.065	2659
3.	17.23	149.1	758.4	1.006	1.265	1.067	3488
4.	17.23	179.2	773.4	1.005	1.265	1.068	4192
5.							

NO.	P_{pr}	TEMP. R	T_{pr}	z
1.	1.09	526	1.49	0.891
2.	1.10	516	1.46	0.882
3.	1.13	514	1.46	0.879
4.	1.15	515	1.46	0.877
5.				

GAS LIQUID HYDROCARBON RATIO 193 Cf/BBL
API GRAVITY OF LIQUID HYDROCARBONS 50.2 DEG
SPECIFIC GRAVITY SEPARATOR GAS 0.625 | X X X X X X X X X X
SPECIFIC GRAVITY FLOWING FLUID X X X X X | 0.641
CRITICAL PRESSURE 674 PSIA | 673 PSIA
CRITICAL TEMPERATURE 353 R | 360 R

P_c 1878.4 P_c^2 3528.4

NO.	P_w	P_w^2	$P_c^2 - P_w^2$
1.		3268.5	259.9
2.		3102.6	425.8
3.		2927.4	601.0
4.		2758.1	770.3
5.			

(1) $\dfrac{P_c^2}{P_c^2 - P_w^2} =$ 4.581 (2) $\left[\dfrac{P_c^2}{P_c^2 - P_w^2}\right]^n =$ 3.136

$\text{WAOF} = Q\left[\dfrac{P_c^2}{P_c^2 - P_w^2}\right]^n =$ 13 150

	$\bar{p}_{ts}$, psia	$\bar{p}_{ts}^2$ thou.	p_{tf}, psia	$(\bar{p}_{ts}^2 - p_{tf}^2)$, M	$\bar{p}_{Ri}$, psia	$\bar{p}_{Ri}^2$, M	p_{wf}, psia	$(\bar{p}_{Ri}^2 - p_{wf}^2)$, M
1	1,878.4	3,528.4	1,786.4	337.2	2,313.6	5,352.7	2,224.3	405.2
2			1,712.4	596.1			2,164.8	666.4
3			1,622.4	896.2			2,099.4	945.3
4			1,526.4	1,198.5			2,034.8	1,212.3

Wellhead curve, n = 0.647

Bottom-hole curve, n = 0.765
Open-flow potential, q_k = 12,600

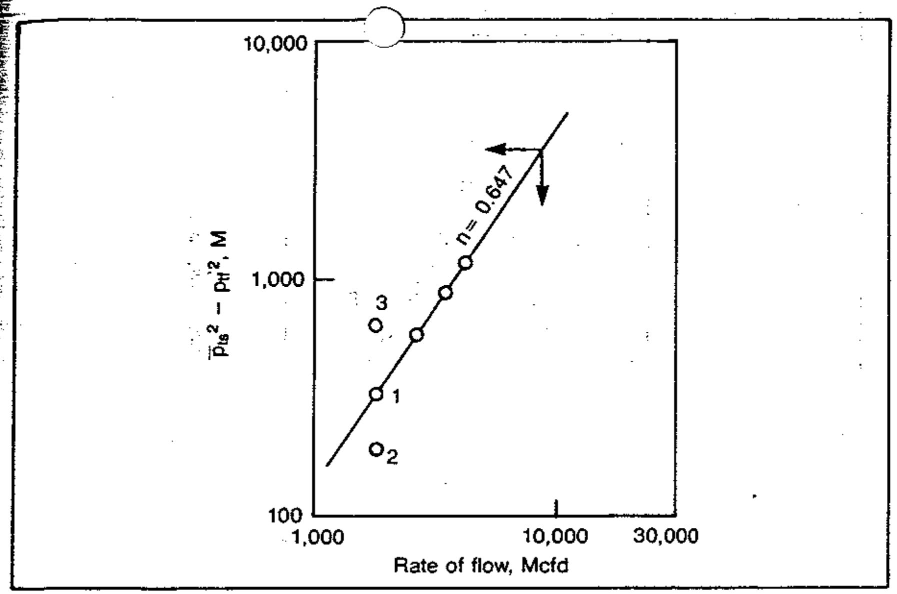

Figure 7–1 *Wellhead performance curve for multipoint test example in Tables 7–3 and 7–4.*

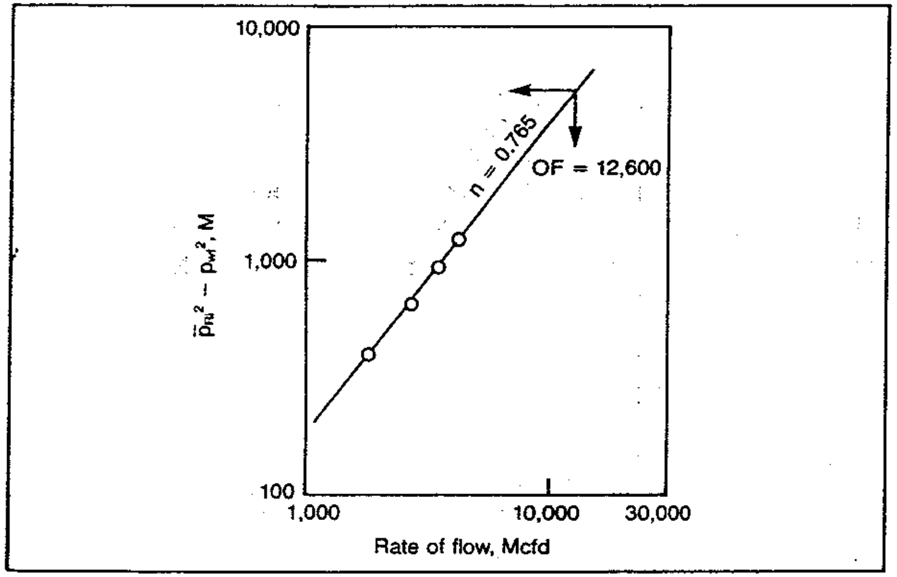

Figure 7–2 *Bottom-hole performance and open-flow potential for multipoint test example in Tables 7–3 and 7–4.*

pressure, $\overline{p}_{Ri}$, was 2,313.6 psia, $\overline{p}_{Ri}^2$ was 5,352.7 (thousands), and the open-flow potential was 12,600 Mcfd.

Determination of Exponent n

If $(\overline{p}_{Ri}^2 - p_{wf}^2)$ is the independent variable, the mathematical slope of equation 7–4 is n. However, by common practice for plotting the results of tests on gas wells, q is considered the independent variable and $(\overline{p}_{Ri}^2 - p_{wf}^2)$ is the dependent variable. Under these circumstances the slope is $1/n$. The value of n is given by:

$$n = \frac{\log q_2 - \log q_1}{\log (\overline{p}_{Ri}^2 - p_{wf}^2)_2 - \log (\overline{p}_{Ri}^2 - p_{wf}^2)_1}$$

$$= \frac{\log q_2/q_1}{\log (\overline{p}_{Ri}^2 - p_{wf}^2)_2/(\overline{p}_{Ri}^2 - p_{wf}^2)_1} \tag{7–5}$$

If the determination is made graphically and the value of the difference of squares is taken exactly one cycle apart, n becomes:

$$n = \frac{\log q_2/q_1}{\log 10} = \log q_2/q_1 \tag{7–6}$$

At $(\overline{p}_{Ri}^2 - p_{wf}^2)$ values of 2,000 and 200, the corresponding values of q are 6,000 and 1,030 Mcfd, respectively, as shown in Figure 7–2, and the logarithm of 6,000/1,030 = $\log$ 5.825 = 0.765, as shown in Figure 7–2.

Determination of Coefficient C

Graphically, the value of coefficient C may be determined by extrapolating the performance curve until the value of $(\overline{p}_{Ri}^2 - p_{wf}^2)$ is 1. At that point, the value of C is equal to the rate of flow. Since this would require extrapolating across two more cycles in Figure 7–2, one may use the knowledge of the value of n at 0.765 and read the intersection of the performance curve at $(\overline{p}_{Ri}^2 - p_{wf}^2)$ = 1,000 (thousands). Here, the value of q_k is 3,520 Mcfd. C is calculated from equation 7–1 as follows:

$$3{,}520 = c(1{,}000)^{0.765} \text{ and } C = 17.85$$

Or from equation 7–4:

$$\log C = \log q_k - n \log (\overline{p}_{Ri}^2 - p_{wf}^2)$$
$$\log C = \log 3{,}520 - 0.765 \log 1{,}000$$
$$C_k = \text{antilog}\,(3.547 - 2.295) = 17.85 \text{ (for } q \text{ expressed in Mcfd and } \Delta p^2 \text{ expressed in thousands)}$$

Thus, the equation for the performance curve in Figure 7–2 is:

$$q_k = 17.85\,(\overline{p}_{Ri}^2 - p_{wf}^2)^{0.765}$$

The difference in pressure squared is expressed in thousands, and the open-flow potential is 12,700 Mcfd although it was previously estimated to be 12,600 Mcfd, reading the value from the curve. The values of n and C could be determined without the use of plots or graphs by using the method of least squares on the data given in Table 7–4; however, the increase in accuracy probably would not be worth the effort.

Wellhead Performance Curve

The wellhead performance curve for the multipoint test example for Tables 7–3 and 7–4 is shown in Figure 7–1 where $(\overline{p}_{ts}^2 - p_{tf}^2)$ has been plotted against the corresponding rates of flow. The difference between the wellhead and bottom-hole performance curves reflects the effects of the weights of the gas columns and the friction losses on the wellhead pressure. The utility of the wellhead curve lies in the ease in calculating the data and in comparing previous tests. For example, the well in the multipoint test example should perform as well as it did in Figure 7–1 over a period of time if, for each test, flow is started after a shut-in period of 72 hr and the rate of flow is approximately that of the first flow rate of 1,793 Mcfd after a flow period of 2 hr. Points 2 and 3 represent hypothetical attempts to duplicate point 1. If the new performance point lies to the right of the curve as point 2 does, the performance probably has improved as a result of liquid removal from the wellbore. If the new point lies to the left of the curve as point 3 does, something has happened to the well to damage its productivity. Wellhead curves are especially useful in determining the flow rate of a well against different line pressures. The general equation for the wellhead performance curve is:

$$q = C(\overline{p}_{ts}^2 - p_{tf}^2)^n \tag{7–7}$$

The specific equation for the curve illustrated in Figure 7–1 is:

$$q_k = 42.96\,(\bar{p}_{ts}^{\,2} - p_{tf}^{\,2})^{0.647}$$

The difference in pressure squared is expressed in thousands.

Critique of Multipoint Test Example

Although both the wellhead and the bottom-hole performance data (see Fig. 7–1 and 7–2) fit nicely to straight lines, there is no doubt that the position of the points after the first rate of flow was influenced by unsteady-state effects. Referring to Table 7–3, note that for each of the four rates of flow, the wellhead pressure decreased 7, 30, 25, and 27 psi for the respective rates of flow during the last 30 min of the flow period. The well could not have approached stabilization during each of the flow periods. Further examination of the wellhead pressures during each of the flow periods shows that the pressure decrease during the 2-hr flow periods was somewhat erratic. This could have been caused by the cyclic unloading of liquid from the well. As for the wellhead temperature, a constant temperature of 74°F was recorded throughout the testing. Experience indicates that the temperature should have increased during the testing. Thus, the multipoint test example was run under unsteady-state conditions, which means that if the tester had varied the time period from the 2-hr period used, the resultant open-flow potential would have been different than the 12,600 Mcfd reported. In other words, under the unsteady conditions that prevailed, the open-flow potential for the well in the multipoint test example was time dependent. In addition, smaller errors probably were introduced by the presence of liquids and perhaps faulty temperature measurement also.

In defense of the tester, it must be pointed out that wellhead temperatures are subject to large errors under the best conditions. Thermometer wells are seldom placed on wellheads, and even so, the thermometers or other measuring devices often are influenced by ambient temperatures.

Pressure, temperature, and rate of flow data should be taken in 15-min intervals, but data were only taken every 30 min in the multipoint test example. Although this is not catastrophic in its effect, data taken every 15 min would have shed more light on the degree of stabilization and whether the well was unloading liquid from the wellbore in cycles.

SIGNIFICANCE OF OPEN-FLOW POTENTIALS

The use of open-flow potentials in connection with gas wells constitutes, for the most part, an attempt to characterize the complex behavior of wells with a single number. During the time the multipoint test was under development (1925–1935), most gas wells were relatively shallow with correspondingly low reservoir pressures and temperatures. In addition, reservoir permeabilities were relatively high and unsteady-state effects were small. Whenever such effects were recognized, the well was said not to have reached stabilization. The development of the Hugoton fields brought about an emphasis on the effects of stabilization on open-flow potentials with the result that much effort was wasted trying to devise tests to eliminate the unsteady-state effects. The principal factors and circumstances that affect the interpretation of tests and the estimation of open-flow potentials are:

1. Unsteady-state conditions
2. Liquids (oil, water, and condensate) in the wellbore and flow string during testing, which affect the wellhead pressures
3. Temperature changes of the flowing or shut-in column of gas
4. Errors in calculating bottom-hole pressures

Unfortunately, some of the factors listed above combine naturally or can be used by the tester to give inflated open-flow potentials. In many cases, the factors may cause an unwary tester to report data that result in deflated open-flow potentials. The evaluation of an open-flow potential value or test requires a judgment as to the extent that the results have been influenced by the factors listed above. While the natural gas engineer may be concerned with inflated potentials in the purchase of wells or in the evaluation of acreage from a study of offset wells, his principal responsibility is to prevent the reporting of deflated open-flow potential tests or one-point tests to state regulatory bodies. The loss in allowables and income can be substantial where allowables are restricted by poor testing. The most common causes of poor or deflated test results are inadequate cleaning of liquids from the wellbore prior to the test and failure to compare well performance with that of prior tests.

The most critical tests with respect to allowables are:

1. Multipoint tests in which production is limited to 25% of open flow

2. Multipoint tests and one-point tests in which potential is part of the proration formula
3. Deliverability tests in fields in which deliverability is part of the proration formula
4. The one-point test in which the allowable is limited to the test rate reported on the official test
5. Shut-in pressure tests in which pressure is part of the proration formula or is used in determining a contractural quantity of gas to be purchased

Open-flow potentials or, more accurately, the concept of a maximum producing capacity for a gas well can be valuable when considering the effect of delivery pressure on the capacity of a well to deliver gas. Starting with equation 7–7, a relationship can be developed between wellhead flowing pressure as a fraction of shut-in wellhead pressure and rate of flow as a fraction of the wellhead open-flow potential.

$$q = C(\bar{p}_{ts}^{\,2} - p_{tf}^{\,2})^n \tag{7–7}$$

The open-flow potential or maximum producing capacity at the wellhead is:

$$q_{of} = C(\bar{p}_{ts}^{\,2})^n \tag{7–8}$$

where q_{of} is the open-flow potential at the wellhead or maximum producing capacity when the wellhead pressure is zero. Then:

$$\frac{q}{q_{of}} = \frac{C(\bar{p}_{ts}^{\,2} - p_{tf}^{\,2})^n}{C(\bar{p}_{ts}^{\,2})^n}$$

or:

$$\frac{q}{q_{of}} = \left[1 - \left(\frac{p_{tf}}{\bar{p}_{ts}}\right)^2\right]^n \tag{7–9}$$

A plot of equation 7–9 is shown in Figure 7–3 in which the flowing pressure is a fraction of the shut-in pressure, and the rate of flow is a fraction of the maximum producing capacity or open-flow potential.

Figure 7–3 is of very practical value when working with expected rates of flow against various line pressures for a given well. For example, assume a well has been producing, without restriction, about 150 Mcfd against a pipeline pressure of 1,000 psia for a long period of time. The well has a current shut-in pressure at the wellhead of 3,100 psia and exponent n is 0.75. The current pressure ratio is 1,000/3,100 = 0.323, and the corresponding ratio of 150 Mcfd to the maximum producing capacity is 0.92 from Figure 7–3 or equation 7–9. One can then infer that the maximum producing capacity is 163 Mcfd. If the wellhead pressure is reduced to 500 psia as a result of reducing the pipeline pressure, the pressure ratio is 500/3,100 = 0.161, and the ratio of the rate of flow to the maximum producing capacity is 0.981. The new rate of flow would be 163 × 0.981 = 160 Mcfd. This corresponds to a gain in producing rate of 6.7% for a reduction in pipeline pressure of 50%.

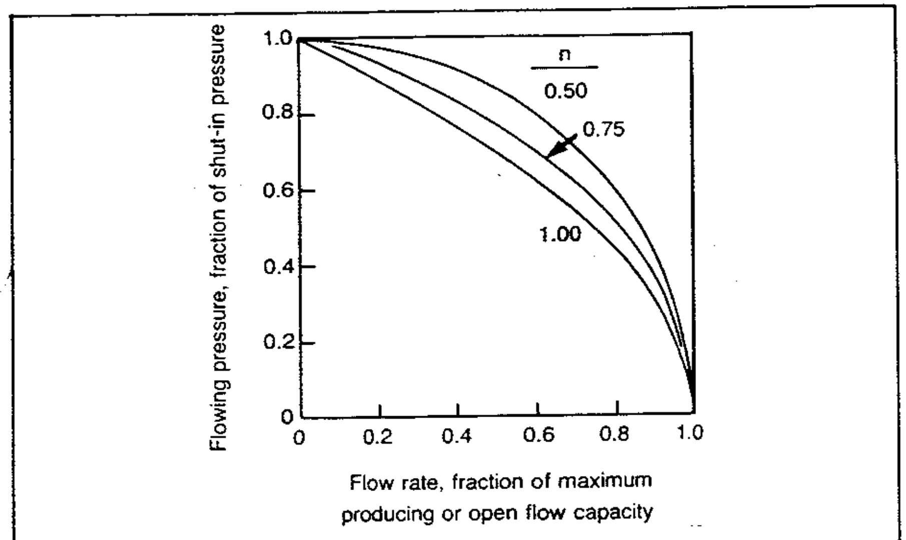

Figure 7–3 *Relationship between rate of flow as a ratio to maximum producing capacity and flowing pressure as a ratio to shut-in pressure for a gas well.*

Note: The results would have been much different if the shut-in wellhead pressure had been 1,100 psia instead of 3,100 psia. In fact, there would have been a 3.13-fold increase in rate of flow to 469 Mcfd, or 313% for a 50% reduction in pipeline pressure.

PRESSURE ADJUSTMENT OF OPEN-FLOW POTENTIALS.

At times, it may be desirable to adjust the open-flow potential of a well to a new shut-in pressure. Such an adjustment is permissible at times when open-flow potentials are required periodically by regulatory bodies. The adjustment can be made readily by squaring the new pressure and reading the adjusted open-flow potential from the performance curve as illustrated in Figure 7–2, but calculation using an equation is preferable. If q_{of_1} is associated with $\bar{p}_{R_1}$, the new open-flow potential, q_{of_2}, can be calculated using the following equations:

$$q_{of_2} = C(\bar{p}_{R_2}{}^2)^n$$

and:

$$q_{of_1} = C(\bar{p}_{R_1}{}^2)^n$$

by dividing the two equations and solving them. The result is:

$$q_{of_2} = q_{of_1}\left(\frac{\bar{p}_{R_2}}{\bar{p}_{R_1}}\right)^{2n} \tag{7–10}$$

In the multipoint test example, the shut-in pressure was 2,313.6 psia, exponent n was 0.765, and the open-flow potential was 12,600 Mcfd. At a new shut-in pressure of 2,050 psia, the corresponding open-flow potential is:

$$q_{of_2} = 12{,}600\left(\frac{2{,}050}{2{,}313.6}\right)^{1.53} = 12{,}600\,(0.831) = 10{,}471\ Mcfd$$

ONE-POINT TEST

The one-point test as used herein is intended to include deliverability testing and tests to determine the maximum allowable or the contractual maximum quantity for a given well. The test, usually with production into a gathering system, extends over a period of 72 hr, with the production during the last 24-hr period being taken as the test rate. If the rules of the regulatory body permit, the 72-hr test should start with a shut-in well. The same precautions for the shut-in time, measurement of shut-in pressure, and preparation of the well for testing as were outlined previously should be observed for the one-point test. Regardless of the information required by the regulatory body or the gas sales contract, the engineer should use the one-point test as an opportunity to gain useful performance information on the well. For this purpose, record the shut-in pressure to establish the pressure relationship with cumulative production. After flow is started, the wellhead flowing pressure and temperature and the instantaneous rates of flow should be taken at 0.5, 1, 2, 3, 6, 23, 24, 47, 48, 71, and 72 hr after starting the well flowing. Also, liquid production should be noted and measured if equipment is available. The rate of flow will be set according to the purpose of the test.

Deliverability Test

The deliverability of a gas well is defined as the capacity of the well to produce over a specified period of time against a given wellhead pressure called the *deliverability pressure*. Thus, the rate of flow or the deliverability of the well is determined by the capacity of the well to produce against the deliverability pressure. If the deliverability pressure should be more than the shut-in pressure for the well, the well is said to have a deliverability of zero. For the rate of flow, it is seldom possible to set the flowing wellhead pressure exactly equal to the deliverability pressure. The deliverability, q_d, against the designated deliverability pressure, p_{td}, is calculated by the following:

$$q_d = q\left(\frac{\bar{p}_{ts}{}^2 - p_{td}{}^2}{\bar{p}_{ts}{}^2 - p_{tf}{}^2}\right)^n \tag{7–11}$$

where q is the observed producing rate. The value of the exponent n may be set at an arbitrary value by the regulatory body and, likewise, the deliverability pressure can be set at 80% of the average shut-in pressure of a large number of wells in the same field or at 80% of the shut-in pressure for the well. Also, it is possible to calculate a subsequent deliverability, q_{d1}, from a previous test or deliverability with the subsequent shut-in pressure, p_{ts1}, and deliverability pressure, p_{td1}.

$$q_{d1} = q_d \left(\frac{\overline{p}_{ts1}^{\,2} - p_{td1}^{\,2}}{\overline{p}_{ts}^{\,2} - p_{td}^{\,2}} \right)^n \tag{7–12}$$

Tests for Maximum Take or Allowable

Frequently, it is necessary to file information with a regulatory body that shows a well is capable of producing an assigned allowable. Similar information may be required by a purchaser to show that a well is capable of producing at some factor (for example, 1.25) times the average contract quantity. Here, the principal precaution is to maintain the well in good condition to produce, if possible, more than the highest allowable to be assigned during the proration year or at a rate more than the factor times the average contract quantity. Unfortunately, not every well is capable of meeting these requirements. But it is the responsibility of the engineer to maintain the wells in the best condition and to see that tests of the highest quality are taken and reported.

ISOCHRONAL TEST AND EXAMPLE

The isochronal test is a series of one-point tests, each of which starts with the well shut in and the shut-in pressure stablized or built up to 95% of the stabilized pressure. The precautions set out previously for preparing the well for testing should be observed. After flow is started in the well, flowing pressures, temperatures, and instantaneous rates of flow are measured at specified time intervals. Convenient time intervals are 0.5, 1, 2, 3, 6, 24 hr, etc. The isochronal test usually consists of 2 to 4 one-point tests with a minimum duration of 3 hr for each rate of flow. Occasionally, one of the series may be extended to 24 hr or more. Each shut-in pressure should be at least 95% of the highest shut-in pressure observed on the well during the series of one-point tests. The resultant data are presented on log-log coordinates in which the difference between the squares of the shut-in pressure and the flowing pressure is the ordinate, and the rate of flow is the abscissa. The performance curves are drawn to connect the points of equal times—hence the isochronal designation.

Isochronal tests can involve a great deal of shut-in time for a well and are least inconvenient between the time the well is completed and the time it is connected to a pipeline. In this way, an isochronal test is run on a new well and subsequent testing would consist of one-point tests. Since the isochronal test determines the true value of the exponent n for the well, there should be no difficulty in correlating the subsequent one-point tests with the original isochronal test.

Data for an isochronal test of a gas well in the Hugoton field were given by Cullender and are set out in Table 7–5 and illustrated in Figure 7–4. The field data for an isochronal test can be recorded conveniently on the form illustrated by Table 7–3 and the results calculated on the form shown in Table 7–4. However, these forms are omitted for this test, and the summary of the isochronal data is shown in Table 7–5.

An examination of the isochronal test results in Figure 7–4 shows that the data behave in a systematic way with time. The four curves are parallel for all practical purposes, and if each of the curves were extrapolated to

Table 7–5 *Isochronal performance for a gas well in the Hugoton field, taken from Cullender (Table 4, copyrighted 1955, Society of Petroleum Engineers of AIME, used by permission[3]).*

Date	*Shutin Pressure (psia)*	*Duration of Flow (hrs)*	*Rate of Flow (Mcfd)*	$\overline{p}_{Ri}^2 - p_{wf}^2$ *(thousands)*
12/03/51	352.4	0.5	983	5.37
		1.0	977	6.69
		2.0	970	8.93
		3.0	965	10.19
12/04/51	352.3	0.5	2,631	15.54
		1.0	2,588	19.82
		2.0	2,533	24.63
		3.0	2,500	27.72
12/05/51	351.0	0.5	3,654	21.63
		1.0	3,565	27.40
		2.0	3,453	34.03
		3.0	3,390	37.97
12/06/51	349.5	0.5	4,782	28.84
		1.0	4,625	35.96
		2.0	4,438	43.98
		3.0	4,318	48.96

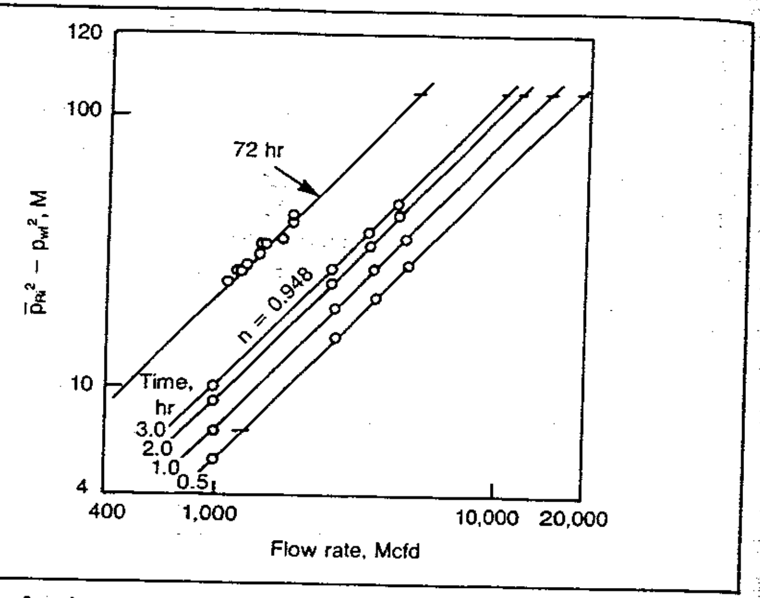

Figure 7–4 *Isochronal test of Table 7–5 for a gas well in the Hugoton Field (From Cullender, Table 4, copyrighted 1955, Society of Petroleum Engineers of AIME, used with permission).*[3]

obtain the open-flow potential, it is apparent that the potential of the well would diminish with time as the performance curve moved to the left. Also, if the well were started from shut-in to flow into a pipeline, the rate of flow would fall off rapidly. Fortunately, Cullender gives the results of 10 annual one-point tests (annual production tests) starting from shut-in with performance data taken at the end of a flow period of 71.75 to 72.5 hr. These annual test data, taken over a period of eight years, have been added to Figure 7–4. Applying the method of least squares to the isochronal performance curves in Figure 7–4 gave the values of the coefficient $C(t)$ and the exponent n shown in Table 7–6. Also, the rates of flow expressed in Mcfd for the various times at $(\bar{p}_{Ri}^2 - p_{wf}^2) = 20.00$ (thousands) are given in the fourth column of Table 7–6.

Table 7–6 shows that the coefficient $C(t)$ in equation 7–2:

$$q = C(t)(\bar{p}_{Ri}^2 - p_{wf}^2)^n \tag{7–2}$$

Table 7–6 *Values of the coefficient* C(t) *and exponent* n *for each of the isochronal performance curves shown in Figure 7–4 and for Rate of Flow,* q_k, *at* $(\bar{p}_{Ri}^2 - p_{wf}^2) = 20.00$ *(thousands).*

Time (hr)	*Coefficient [C(t)]*	*Exponent (n)*	*Flow rate $\Delta p^2 = 20.00$ (Mcfd)*
0.5	201.2	0.9414	3,376
1.0	155.3	0.9458	2,641
2.0	120.4	0.9522	2,087
3.0	105.1	0.9548	1,836
	Average	0.948	
72.0	52.02	0.948	890

is definitely a function of time as was discussed previously in this chapter. Although the values for the exponent n show a slight increase from the 0.5 to the 3.0-hr curve, and the average value of 0.948 was used for the 72-hr curve, the general belief is that the n value is constant for the isochronal curves. The author knows of no data or proof to confirm the belief that n is constant. Since the coefficient $C(t)$ is the value of the rate of flow expressed in Mcfd at $(\bar{p}_{Ri}^2 - p_{wf}^2) = 1.00$ (thousands), q_k at $(\bar{p}_{Ri}^2 - p_{wf}^2) = 20.00$ follows the same rate of change with time. If the well in the isochronal test in Table 7–5 and Figure 7–4 were set to produce at a constant back pressure starting from shut-in so that the difference of pressure squared is 20.00 (thousands), the producing rate would vary with time as shown in the fourth column of Table 7–6 and illustrated by Figure 7–5. The open circles are data points derived from Table 7–6, and the crosses are from data on the same well that was presented by Cullender.

The data in Figure 7–5 show that the gas well in the Hugoton field (Cullender's well No. 3) did not reach stabilization after 214 hr of flow because the coefficient $C(t)$ was still a variable with time. At this point, the reader is justified to inquire as to the time required for stabilization for this particular gas well. This time becomes a subject for Chapter 8 as mentioned previously.

Comments on Isochronal Testing

The advantage of isochronal testing gas wells is that it provides a method for eliminating the complicated pressure gradients in the reservoir that so often confuse the results of multipoint tests. It thereby permits the determination of the true value of the exponent n for the performance curves.

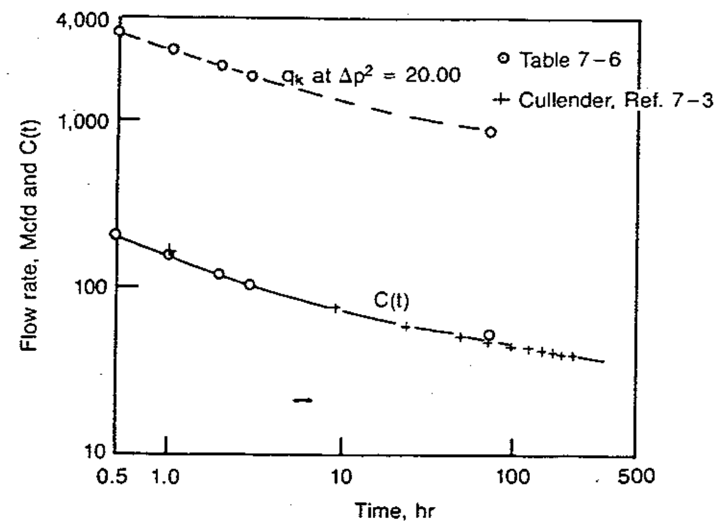

Figure 7–5 *Variation of rate of flow, Mcfd, and coefficient* C(t) *for test well in Table 7–6.*

The results of an isochronal test are good measures of well performance over a period of years. For example the results of the 72-hr production test (one-point test) shown in Figure 7–4 indicate that the performance of the well was constant for a period of about 10 yr. This shows that the well had been maintained in its original operating condition. The principal difficulty with the isochronal test is the amount of time required for the shut-in pressure to build between the flow rates imposed upon the well.

INFORMATION TEST

The results of an information test on an unconnected gas well are given in Table 7–7 and illustrated as an isochronal test in Figure 7–6. The well was opened to flow to the atmosphere through a ¼-in orifice in a 2-in. critical flow prover and was allowed to flow for 120 hr. Then it was shut in, and build-up pressures were taken for 72 hr. The well was not equipped with tubing. The well produced through 4,490 ft of 5½-in. casing with an ID of 5.012 in. The bottom-hole temperature was 95°F, and the surface temperature

Table 7–7 *Example of an information test including pressure buildup on a gas well not connected to a pipeline.*

Elapsed Time (hr)	*Wellhead Pressure,* $\bar{p}_{ts}$ *or* p_{tf} *(psia)*	*Bottom-Hole Pressure,* $\bar{p}_{Ri}$ *or* p_{wf} *(psia)*	$\bar{p}_{Ri}^2 - p_{wf}^2$ *(thousands)*	*Rate of Flow (Mcfd)*
0.0	1,894	2,114.0	0.0	0
0.25	1,603	1,785.9	1,279.3	2,892
0.50	1,453	1,616.7	1,855.0	2,509
0.75	1,342	1,491.7	2,243.8	2,290
1.0	1,260	1,399.4	2,510.6	2,136
2.0	1,079	1,196.1	3,038.2	1,791
3.0	1,009	1,117.6	3,219.9	1,670
4.0	969	1,072.7	3,318.2	1,589
5.0	937	1,036.9	3,393.8	1,534
6.0	910	1,006.6	3,455.5	1,487
24.0	655	722.2	3,947.3	1,038
48.0	584	643.3	4,055.0	921
72.0	554	610.1	4,096.7	867
96.0	530	583.5	4,128.4	828
120.0	516	567.9	4,146.3	805
Well shut in				
0.25	661	728.8	—	0
0.50	780	861.3	—	0
0.75	884	977.4	—	0
1.0	975	1,079.3	—	0
2.0	1,242	1,378.9	—	0
3.0	1,405	1,562.4	—	0
4.0	1,497	1,666.1	—	0
5.0	1,554	1,730.4	—	0
6.0	1,592	1,773.2	—	0
24.0	1,757	1,959.4	—	0
48.0	1,803	2,011.3	—	0
72.0	1,826	2,037.3	—	0

was about 60°F. An inspection of the results of the test shown in Figure 7–6 shows that the well stabilized very slowly and that stabilization was not reached after 120 hr of flow. If an arbitrary exponent n of 0.700 is assigned to the data in Figure 7–6, the open-flow potential decreases from 6,600 Mcfd after 0.25 hr of flow to 860 Mcfd after 120 hr of flow. Although the arbitrary assignment of the 0.700 exponent may have caused considerable error in the extrapolated open-flow potential of 6,600 Mcfd after 0.25 hr of flow, it

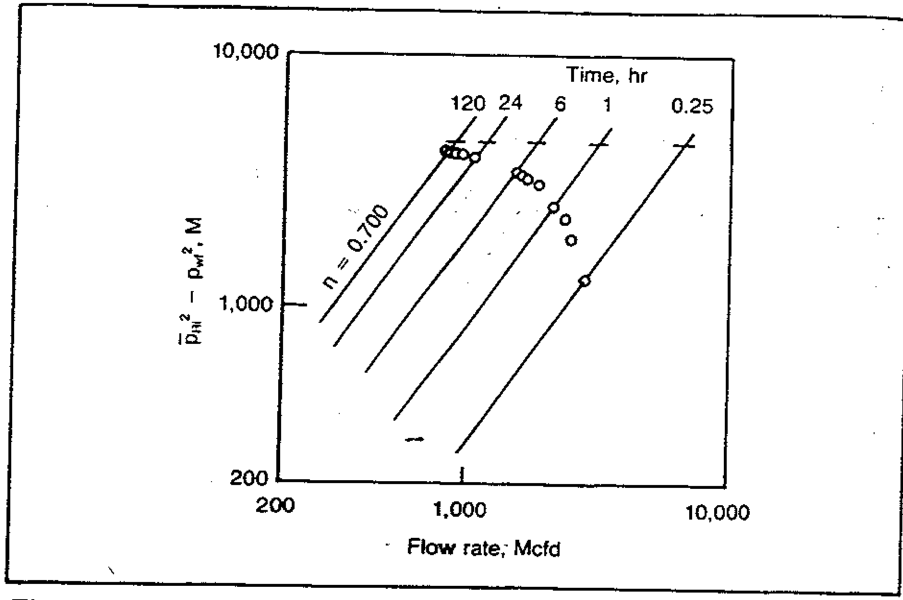

Figure 7–6 *Well test data from Table 7–7 plotted as isochronal test with arbitrary exponent of 0.700 assigned.*

caused little error in the potential of 860 Mcfd after 120 hr of flow. The well produced at a very large fraction of its open-flow potential after it had been flowing for 24 hr.

In the course of the test, the well was estimated to have produced a total of about 5,010 Mcf, but the average pressure in the casing was reduced from 2,004 to 542 psia. Thus, gas stored in the casing and in the open hole below the casing was reduced by an estimated 80 Mcf, and the formation and fractures must have given up about 4,930 Mcf during testing. The decrease in gas storage in the wellbore, although significant, was not a dominant factor but was only a contributing factor affecting the test results on the well.

The well can be called a small-capacity well, and one can conclude that the capacity of the well to produce into a pipeline was somewhat less than 800 Mcfd if the pressure in the pipeline was about 600 psia. Further, one must conclude that the well was still in unsteady-state flow, even after 120 hr.

The test given in Table 7–7 was introduced under well testing as an example of information testing.

Note: The size of the orifice plate in the critical flow prover was not changed during the test, and the working wellhead pressure and rate of flow did not change nearly so rapidly after the first 24 hr of the test although both decreased throughout the test. Now, the crucial question is at what point does the decrease in rate of flow and flowing pressure cease to be a result of unsteady-state flow and become a result only of the decrease in reserves for the well? In other words, when does the radius of pressure disturbance or the radius of investigation reach the outer boundary of the reservoir? These matters will be discussed in the next chapter.

COMPARISON WITH THEORY

Thus far in describing the flow behavior of gas wells, practically no attempt has been made to explain the behavior on a theoretical basis. It has been known for many years that the flow of gas was related to the difference of the squares of the pressures at the inlet and the outlet of the pipe or porous medium through which the gas was flowing. Rawlins and Schellhardt expressed this relationship for gas wells as equation 7–1:[2]

$$q = C(\bar{p}_{Ri}^2 - p_{wf}^2)^n \tag{7–1}$$

in which C and n were considered constants for a given gas well. The exponent n could take on values between 0.5 and 1.0. Later, Cullender came to the conclusion that the coefficient C was a function of time until the well reached a pseudosteady state of flow.[3] Cullender's work did not change the concept that the exponent n could vary between 0.5 and 1.0. However, Cullender made the statement that he had observed a number of wells that at low flow rates perform with an exponent of 1.00 and exhibit a breaking point above the flow rate at the breaking point, the well performs with a characteristic exponent less than 1.00.

Returning to Chapter 6 and the statements of Darcy's law as given in equations 6–6 and 6–7, it can be concluded that gas wells should perform with an exponent of 1.00 in equation 7–1. This apparent paradox has hindered the progress of natural gas engineering for several decades. It also has been the source of much controversy.

For a long time many investigators held to the idea that gas wells do perform with an exponent of 1.00 in equation 7–1, but experimental results from testing actual gas wells were clouded by many factors other than strictly flow phenomena. Among these factors have been unsteady state effects, the change in compressibility and viscosity of the gas with the decrease in pressure as it flows to the wellbore, condensation of hydrocarbon liquids in the reservoir around the wellbore, and wellbore storage effects. However, there have been attempts as early as 1948 to explain gas well behavior by the introduction of a turbulance factor into gas flow equations.[17] In 1961, Smith concluded that gas well behavior was influenced by a rate-dependent non-Darcy flow effect[12] that occurs near the wellbore and that it could be considered as an additional "skin effect." This was in addition to the van Everdingen skin effect.[13] Other investigators have enlarged upon the non-Darcy flow effect.[18–20] Odeh et al. in 1975 reviewed the methods for determining the non-Darcy flow effects and presented a method for determining the rate-dependent non-Darcy flow constant from a one flow drawdown and buildup test sequence for a gas well.[21] In 1978, Odeh presented a pseudosteady-state equation for flow into gas wells which follows:[22]

$$q_k = \frac{703 \times 10^{-6}\, kh(\overline{p}_{Ri}^{\,2} - p_{wf}^2)}{\mu T_f z(\ln r_e/r_w - 0.75 + s + F_{nD}\, q_k)} \tag{7–13}$$

where:

s = van Everdingen skin factor
F_{nD} = non-Darcy flow factor.

If the following replacements are made:

$$C_1 = \frac{703 \times 10^{-6}\, kh}{\mu\, T_f z}, \text{ and}$$

$$C_2 = \ln r_e/r_w - 0.75 + s$$

equation 7–13 becomes:

$$q_k = \frac{C_1(\overline{p}_{Ri}^{\,2} - p_{wf}^2)}{C_2 + F_{nD}\, q_k}, \text{ or}$$

$$F_{nD}\, q_k^2 + C_2\, q_k - C_1(\overline{p}_{Ri}^{\,2} - p_{wf}^2) = 0 \tag{7–14}$$

Equation 7–14 may be solved for q_k:

$$q_k = \frac{-C_2 + \sqrt{C_2^2 - 4\,F_{nD}\, C_1(\overline{p}_{Ri}^{\,2} - p_{wf}^2)}}{2\,F_{nD}} \tag{7–15}$$

and equation 7–14 may be written as:

$$(\overline{p}_{Ri}^{\,2} - p_{wf}^2) = C_3\, q_k + C_4\, q_k^2 \tag{7–16}$$

Comparison with Well Test Data

An examination of the terms in equation 7–13 indicates that every factor known to influence flow under present technology has been included. Therefore, it is desirable to determine how well actual well test data can be modeled by equation 7–16. Test data for this purpose should be for a well that reached the pseudosteady-state flow in a very short time; it should be a large flow capacity well with no liquid production and a low shut-in pressure; and above all, the test data should cover a wide range of flow rates and should have been taken carefully and accurately. In other words the data should be representative of the flow performance of the well. A search of the published literature showed that the test data published by Cullender for his well No. 5 met these requirements.[3] The data were used with the method of least squares to determine the constants in equations 7–1 and 7–16. The results were:

$$q_k = 49.488\,(\overline{p}_{Ri}^{\,2} - p_{wf}^2)^{0.54946} \tag{7–17}$$

for equation 7–1, and

$$0.2665\, q_k + 127.8 \times 10^{-6}\, q_k^2 = \overline{p}_{Ri}^{\,2} - p_{wf}^2 \tag{7–18}$$

or

$$q_k = \frac{-0.2665 + [0.07102 + 511.2 \times 10^{-6}\,(\overline{p}_{Ri}^{\,2} - p_{wf}^2)]^{0.5}}{255.6 \times 10^{-6}}$$

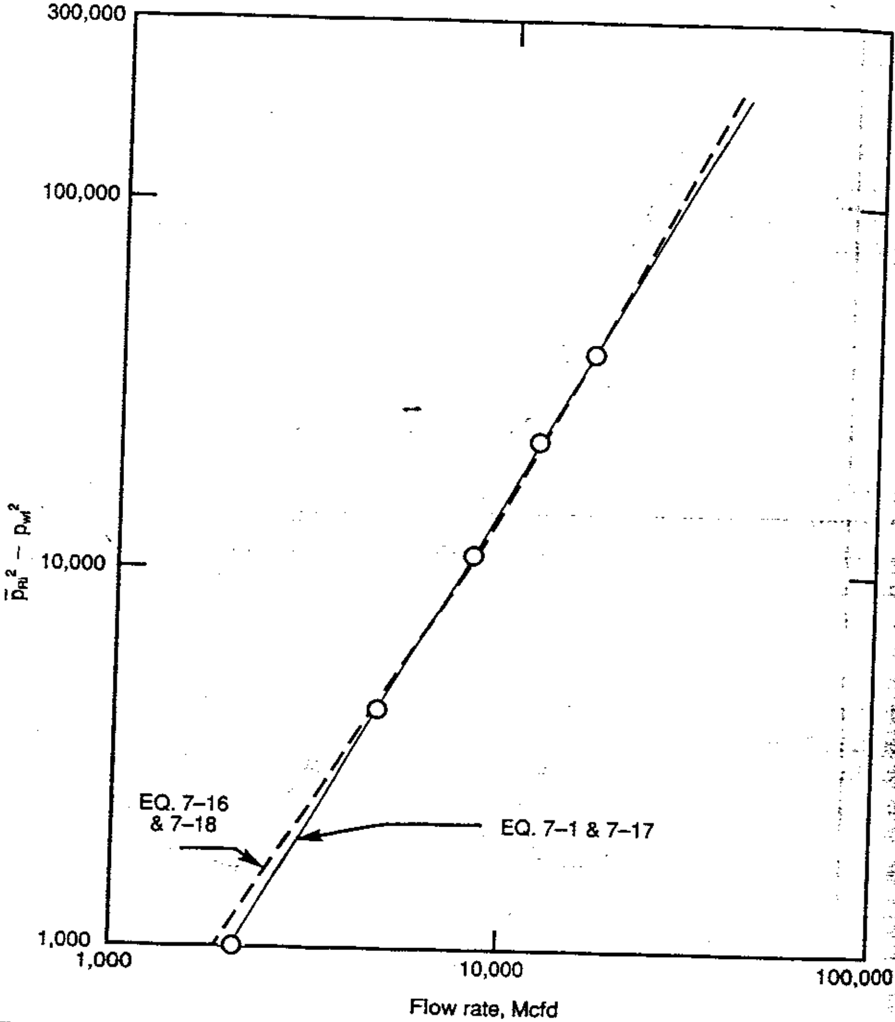

Figure 7–7 *Comparison of equations 7–1 and 7–16 with actual test data from a low-pressure large capacity gas well.*

for equation 7–16. Both equations 7–17 and 7–18 fit the experimental data remarkably well over the range of flow rates from 2,231 to 16,817 Mcfd as shown in Figure 7–7. The differences between the experimental values and the values calculated by equations 7–17 and 7–18 were probably less than the experimental errors.

Equations 7–13 to 7–16 have the real advantage over equation 7–1 in that the constants in equations 7–13 and 7–16 are related to the factors known to influence the flow of gas into a wellbore. Therefore, these equations should find wider acceptance in the interpretation of test results. In addition, engineering acceptance of the non-Darcy flow effects now seems to be widespread.

SUMMARY

Flow testing gas wells falls into three and, at most, four types of testing. The first and oldest type of testing is the multipoint test in which three or four rates of flow are imposed upon a gas well without shut-in periods between the rates of flow. Usually, the well is shut in until the pressure at the wellhead reaches a maximum before the first rate of flow is imposed upon the well. Although the multipoint test has been applied indiscriminately to all types of wells and reservoirs, the most meaningful information concerning the capacity of a well to produce gas is obtained when the radius of investigation reaches the reservoir boundary within the period of time selected for each rate of flow. The literature contains several methods for unscrambling the unsteady-state effects from a multipoint test, but the effort seldom seems worth the results.

Because wells were tested in which the radius of investigation did not reach the reservoir boundary within a relatively short period of time, one-point and isochronal tests were devised and used. The one-point test was used principally to replace the open-flow potential test in regard to proration formulas with a deliverability rate of flow. For this use, the one-point or deliverability test became a test run in which flow time period, flow rate, and flowing pressures were held at specified values or ranges of values, and the results were corrected to a standard working wellhead pressure at a specified time or narrow range of time. For this test, the well is usually shut in and the pressure is allowed to build to near its maximum value prior to imposing the rate of flow on the well.

The isochronal test is a series of one-point tests in which the well is shut in between each rate of flow. The shut-in period as originally recommended by Cullender should be long enough to permit the pressure to return to a pressure comparable to that existing before the well was first opened. Good practice in the Hugoton fields has been to allow the pressure to return to at least 95% of the shut-in pressure before the well was first opened to flow. The isochronal test method permits a determination of the true exponent of the performance curve for a particular gas well. Also, it permits the determination of the variation of the rate of flow from a well with time for a given pressure differential or the variation of the performance coefficient, $C(t)$, with time. The principal disadvantage is the lengthy shut-in periods between the various rates of flow that may be required for some wells.

The information test is designed specifically to determine the unsteady-state behavior of a well and to determine at which point in time the unsteady-state behavior ends and pressure depletion of the reservoir begins. Ideally, the well should begin producing unrestrictedly from a shut-in condition against a constant back pressure until the transition from unsteady-state flow to pressure depletion occurs. Ideal conditions are most closely followed in practice when the well is first connected to a pipeline that has a nearly constant pressure. In the case of an unconnected well (as was the case in Table 7–7), the well should be opened to flow through a properly sized orifice and no further adjustments should be made in the rate of flow during the test. In either case, flowing pressures, temperatures, and rates of flow are measured at time intervals to give an even spacing of points when the data are plotted as in Figure 7–5. Interpretation will be discussed in the following chapter.

Actual test data from gas wells can be fitted closely to the theoretically derived equation 7–13. The terms in equation 7–13 are defined precisely and are acceptable in reservoir engineering. Thus, test data for a well producing under pseudosteady-state flow can be used to determine the properties of the reservoir and the completion characteristics of the well.

PROBLEMS

1. Given a well with a wellbore radius of 0.25 ft in the center of a cylindrical reservoir with a radius of 2,640 ft in which the pressure is maintained constant at the outer edge of the cylinder, other values are as follows:
   k = 500 millidarcies
   h = 20 ft
   $\overline{\mu}$ = 0.02 cp
   $T_f = 460 + 120 = 580$
   $\overline{z} = 0.90$

   Calculate a multipoint test using equation 6–7 from Chapter 6, plot the data on log-log paper, and determine the exponent n and the coefficient C.

2. Assume that drilling mud infiltration has damaged the reservoir to a radius of 2 ft so that the permeability has become 50 md in the damaged portion. Recalculate the performance curve of problem 1 above. To what extent was the performance of the hypothetical well damaged?
   Note: Review equations 6–12 and 6–13 in Chapter 6.

3. At a flow rate of 5,000 Mcfd, what percentage of the total pressure drop occurred out to the radius of 2 ft in the undamaged case and in the damaged case?
   Note: Assume that p_e^2 = 1068.36 thousands or p_e = 1033.6 psia.

4. Given the following test data on a gas well, construct a performance curve and write the equation of the curve.

Rate of Flow (Mcfd)	$\overline{p}_{Ri}^2 - \overline{p}_{wf}^2$ *(thousands)*
14.0	6.5
60.0	44.0
210.0	230.0

5. With $\overline{p}_{Ri}$ = 679 psia and the following test data, construct and write the equation for the performance curve and determine the open-flow potential and rate of flow against a bottom-hole pressure of 50 psia.

q_k	p_{wf}
6	671
14	656
40	583
80	400

   Answer: $q_k = 0.96(\overline{p}_{Ri}^2 - p_{wf}^2)^{0.777}$

   Open-flow potential = 112.7 Mcfd and q_k into line = 112.2 Mcfd.

6. A well produces separator gas with a specific gravity, γ_g, of 0.650 and hydrocarbon liquid with a gravity of 61.0° API at a ratio of 159,600 cu ft/bbl. Calculate the specific gravity of the flowing fluid.

Note: See Chapter 2.

Answer: $\gamma_{gf} = 0.668$

7. Cullender gives the following isochronal test data on Gas Well No. 1 (Table 1, copyrighted 1955, Society of Petroleum Engineers of AIME, used by permission).[3]

Date	$\bar{p}_{Ri}$	Duration of Flow (hr)	q_k	$\bar{p}_{Ri}^2 - p_{wf}^2$, M
10/03/44	435.2	24.0	9,900	97.70
10/24/44	436.8	1.0	4,656	22.52
—	—	3.0	4,587	27.26
—	—	23.0	4,440	38.67
12/11/45	394.7	0.1	2,016	4.61
		0.2	2,009	5.68
		0.5	2,001	7.10
		1.0	1,994	8.26
		3.0	1,980	10.38
		24.0	1,947	14.56
1/11/46	390.7	0.1	2,991	7.64
		0.2	2,977	9.21
		0.5	2,956	11.56
		1.0	2,941	13.28
1/15/46	394.0	0.2	7,327	26.09
		0.5	7,199	31.25
		1.0	7,092	35.52
		3.0	6,887	42.99
1/17/46	391.1	0.5	2,952	11.23
		1.0	2,937	12.87
		3.0	2,905	15.72
12/05/46	381.0	0.1	4,158	10.61
		0.2	4,130	12.72
		0.5	4,086	15.91
		1.0	4,052	18.40
		3.0	3,989	22.17

Construct isochronal performance curves and calculate the rate of flow from Gas Well No. 1 into a pipeline that would reduce the bottom-hole pressure, p_{wf}, to 366.7 psia. Delivery rates should be calculated after 0.1, 0.2, 0.5, 1.0, 3.0, and 24.0 hr of flow to the pipeline. Assume the well was opened to flow on December 5, 1946.
Note: Save your results to work a problem in chapter 8.

8. Use a selected rate of flow from the table in problem 7. Calculate p_{wf}^2 and plot the answer on semilog paper with time on the log scale. Extrapolate to 72 and 240 hr and estimate production rates for $(\bar{p}_{Ri}^2 - p_{wf}^2) = 10.69$ M.

9. Figure 2 of reference 3 lists multipoint tests of 24-hr duration on Gas Well No. 1 for each rate of flow from which the following coefficients, *C*, and exponents, *n*, can be calculated. (Figure 2, copyrighted 1955, Society of Petroleum Engineers of AIME, used by permission.)[3]

Curve	n	C	Description
A	1.097	66	Reverse sequence
B	0.701	336	Normal sequence
C	0.776	234	Normal sequence
D	0.867	190	Isochronal test

Plot the performance curves and decide whether a production rate could have been calculated from the multipoint data.
Note: The difference in the multipoint tests is due to the difference in rates of flow and the order in which the rates of flow were imposed on the well.

10. Calculate the absolute open-flow potentials from the isochronal data for Well No. 1, problem 7, for the various periods of time. Which potential is more nearly representative of the capacity of the well to produce? If you, as an engineer, had an opportunity to purchase Well No. 1, which type of test data (multipoint or isochronal) would be most helpful?

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8 Unsteady-State Flow Behavior of Gas Wells

The treatment of gas flow in Chapter 6 applies to steady-state conditions in which the driving pressure is maintained constant at the inlet to the system. In a radial system the pressure at the external radius, r_e, remains constant as if there were an infinite supply of gas keeping the pressure constant at the external edge of the porous medium. If the system is closed at the external boundary and there is no gas added to the system, continuation of the flow after the pressure disturbance reaches the outer boundary continues to reduce the amount of gas contained in the system, and the pressure at the outer boundary is reduced. Thus, the system has reached a series of steady-state conditions which has been called pseudosteady-state flow. Pressures within the system are decreasing, but the steady-state flow equations hold at any instant. Before the pressure disturbance caused by starting a well to flow reaches the outer boundaries of the reservoir or the drainage area of another well, the well is said to be under unsteady-state flow conditions. The radius of drainage is increasing, resulting in a decrease in the flow rate if the wellbore pressure is held constant or a decrease in the wellbore pressure if the flow rate is held constant. If the valve or choke controlling the rate of flow from the well is not changed, both the flow rate and the wellbore pressure decrease. Thus, it is possible to discuss unsteady-state flow against a constant wellbore pressure or at a constant rate of flow or any possible combination of the two. Mathematically, it is easier to deal with flow against a constant inner boundary pressure or flow at a constant rate with a variable inner boundary pressure than to deal with a combination of the two main types of unsteady-state flow. Gas wells under production practices tend to be produced at a nearly constant rate with the well having excess productive capacity or against a constant back pressure. Under test conditions the wells are generally produced through a choke or valve with the result that both the flow rate and the wellbore pressure tend to decrease with time.

In the preceding chapter, conventional methods based on steady-state behavior were inadequate for testing and analyzing test results when a well exhibited any marked degree of unsteady-state flow behavior. The unsteady-state flow behavior of water wells and oil wells has received an immense amount of study since the first papers on unsteady flow in oil reservoirs were published in the 1930s. The work of van Everdingen and Hurst and that of Miller, Dyes, and Hutchinson on flow in reservoirs[1,2] and of Horner on pressure buildup in wells[3] started an avalanche of papers on unsteady-state flow that continues until this day. The work of Aronofsky and Jenkins; Jenkins and Aronofsky; Bruce, Peacemen, Rachford, and Rice; and Smith showed that the numerical solutions for transient flow of gases in porous media could be approximated by the solutions for transient liquid flow.[4,5,6,7]

Later, Al-Hussainy, Ramey, and Crawford introduced the concept of the real gas potential, $m(p)$, which has the dimensions of (pressure)^2/viscosity.[8] At times, the real gas potential has been called real gas pseudo-pressure and pseudopressure. It is defined as:

$$m(p) = 2 \int_{p_m}^{p} \frac{p}{\mu(p)z(p)} dp \tag{8–1}$$

where a symbol followed by (p) indicates that the symbol is a function of pressure, p_m is a low base pressure, and the flow is considered to be isothermal. Under isothermal conditions, $\mu(p)$ and $z(p)$ are functions of pressure alone for a given gas. The real gas potential $m(p)$ can be substituted for pressure in liquid solutions for unsteady-state flow in porous media.

It is beyond the scope of this book to deal with the fundamentals of unsteady-state flow of gases. Therefore, refer to a comprehensive review of the subject and to pertinent literature presented by the Energy Resources Conservation Board of Alberta, Canada.[9]

Before further discussing the important aspects of unsteady-state gas flow, it will be helpful to ask what the practicing engineer requires of gas well testing and what questions management expects the gas well testing to answer. Returning to the example in Chapter 7 of the information test on a gas well given in Table 7–7 and Figure 7–6, a gas well produces at a rate of 2,829 Mcfd after 0.25 hr of flow although the rate had decreased to

805 Mcfd after 120 hr of flow. Keeping in mind that the well was not connected to a pipeline, the primary questions are:

1. If connected to a pipeline is it possible to make a reasonable forecast of production?
2. Will connecting the well to a pipeline be profitable?
3. Assuming the geology is favorable, do the reserves justify drilling a second well to the same reservoir?

In this case, every question hinges on the production forecast. The same questions can be asked in a different way—was the decrease in producing rate over the period of 120 hr principally caused by depletion of the reserve or was it caused by unsteady-state effects?

To some extent, the question about reserves can be answered by comparing the pressure before the test to the pressure observed after the 72-hr buildup. The difference of 76.7 psi between the before and after pressures caused by the production of 5,010 Mcf of gas indicates that the gas in place after the test was at least:

$$\frac{2{,}037.3}{76.7} \times 5{,}010 = 133{,}100\ Mcf$$

This calculation assumes that the pressures given were representative of stabilized pressures in the reservoir. Note also that the assumption was made that the gas in the reservoir acted as an ideal gas. Unfortunately, one is left with a vague idea of the reserves and without a production forecast. A good production forecast is a necessity for an economic analysis.

In posing the primary questions about reserves and profitability, information gained thus far from testing concerning the characteristics of the reservoir and the efficiency of well completion has been ignored. These are equally important to the reservoir engineer and will be treated in later discussions.

Referring to Figure 7–6, note that, as time went on, the well produced closer to its current open-flow potential. If the well had been producing into a pipeline with a constant pressure of 600 psia, the well would have been "floating" on the line after little more than 24 hr. In other words, the wellhead pressure would have become constant at 600 psia, and the rate of flow would have been the only variable. This type of operation, production against a constant back pressure, is a common mode of operation for small-capacity gas wells. Therefore, consider the constant inner boundary pressure case as the principal interest in considering unsteady-state effects.

BEHAVIOR AGAINST A CONSTANT WELLBORE PRESSURE

Solutions to the unsteady-state theory for predicting decreasing producing rates against constant wellbore pressures were published for liquid (slightly compressible), single-phase systems by Moore, Schilthuis, and Hurst and by Hurst.[10,11] The results were presented in graphical form for radial systems in terms of a dimensionless flow rate, q_D, and a dimensionless time, t_D. Tabular values of q_D and t_D are found in reference 12 for the infinite reservoir and in references 13 and 14 for finite reservoirs. The dimensionless flow rate, q_D, and dimensionless time, t_D, are defined as follows:

$$q_D = \frac{141.3\, q_o(t)\mu B}{kh(p_i - p_{wf})} \quad (oil) \qquad (8\text{–}2)$$

$$t_D = \frac{0.00634\, kt}{\phi\mu\, c_t r'^{\,2}_w} \qquad (8\text{–}3)$$

where $q_o(t)$ is the rate of oil production (b/d), t is time (days), and c_t is the total compressibility (psi^{-1}). Fetkovich and Fetkovich & Thrasher have collected the tabular values and prepared convenient charts of q_D as a function of t_D for plane radial systems with infinite and finite outer boundaries and constant pressure at the inner boundary.[15,16] The charts of Fetkovich and Fetkovich & Thrasher are reproduced from tabular values in Figures 8–1A and 8–1B where q_D varies in the range of 0.001–1.00 and t_D varies from 0.0001–10^9. In addition to the tabular values for plane radial systems, Fetkovich and Thrasher have added the constant pressure solution for a vertical fracture with infinite conductivity given by Agarwal, Carter, and Pollock which is shown as a dashed line on Figure 8–1A.[17]

It is the intention here to use the charts illustrated on Figures 8–1A and 8–1B for type curve matching, but because they have been reduced in size, they are much too small.* However, selected numerical values obtained

* The full-size type curves published by Fetkovich with a grid suitable for actual use are available on written request from SPE Book Order Department, P.O. Box 833836, Richardson, Tx 75083-3836. Specify SPE 9086 and include $5.00 prepayment for each order of "Type Curves for Decline Curve Analysis Using Type Curves."

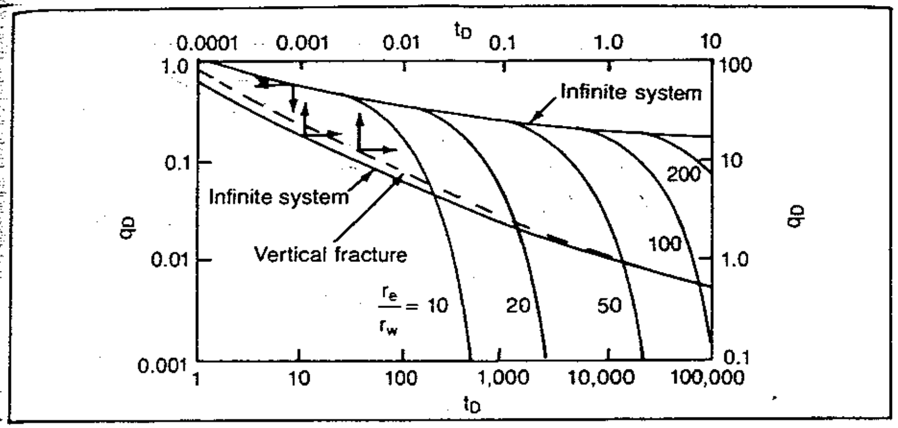

Figure 8–1A *Dimensionless flow rate* (q_D) *and time function* (t_D) *for plane radial systems with infinite and finite outer boundaries and constant pressure at the inner boundary, after Fetkovich and Fetkovich & Thrasher.*[15,16]

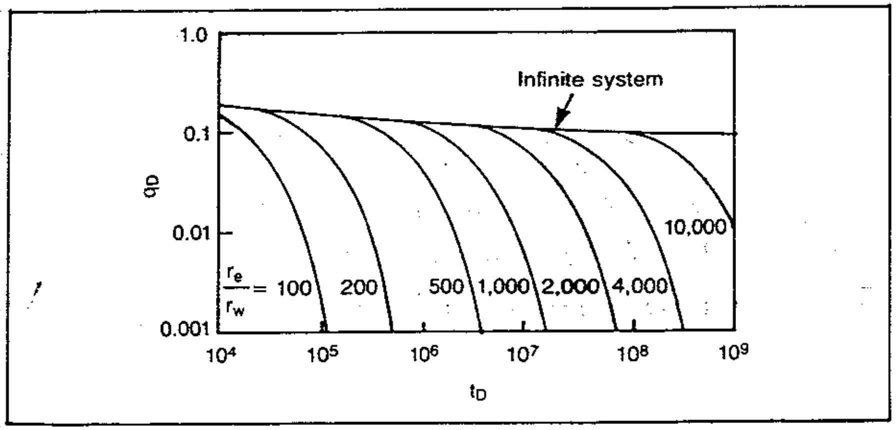

Figure 8–1B *Dimensionless flow rate* (q_D) *and time function* (t_D) *for plane radial systems with infinite and finite outer boundaries and constant pressure at the inner boundary, after Fetkovich.*[15]

for q_D as a function of t_D by Ferris et al.[12] for a plane radial system in an infinite isotropic porous medium with constant pressure at the inner boundary are given in Table 8–1. The constant pressure solution for a plane radial system with an infinite conductivity vertical fracture taken by Fetkovich and Thrasher from Agarwal, Carter, and Pollock was shown as a dashed line on

Table 8–1 *Values of q_D as a function of t_D for a plane radial system, infinite outer boundary with constant pressure at the inner boundary.*

	t_D						
Multiplier	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10	10^{2}
1	56.9	18.34	6.13	2.249	0.985	0.534	0.346
2	40.4	13.11	4.47	1.716	0.803	0.461	0.311
4	28.7	9.41	3.30	1.333	0.667	0.405	0.283
6	23.5	7.77	2.78	1.160	0.602	0.377	0.268
8	20.4	6.79	2.46	1.057	0.562	0.359	0.258
10	18.3	6.13	2.25	0.985	0.534	0.346	0.251

	t_D						
Multiplier	10^{3}	10^{4}	10^{5}	10^{6}	10^{7}	10^{8}	10^{9}
1	0.251	0.1964	0.1608	0.1360	0.1177	0.1037	0.0927
2	0.232	0.1841	0.1524	0.1299	0.1131	0.1002	0.0899
4	0.215	0.1733	0.1449	0.1244	0.1089	0.0968	0.0872
6	0.206	0.1675	0.1408	0.1213	0.1066	0.0950	0.0857
8	0.200	0.1636	0.1380	0.1192	0.1049	0.0937	0.0846
10	0.196	0.1608	0.1360	0.1177	0.1037	0.0927	0.0838

Source: Ferris et al., *Theory of Aquifer Tests.*[12]

Figure 8–1A. Values of q_D and corresponding t_D values were read from the dashed curve on a plot much larger than that in Figure 8–1A and are given in Table 8–2. In a similar fashion, the values of q_D and corresponding t_D values were obtained for Table 8–3. It must be emphasized that the values given in Tables 8–2 and 8–3 are not numerical solutions to the appropriate flow functions but were read from large plots of the numerical solutions. The values in Tables 8–2 and 8–3 are included here for convenience to enable the reader to construct duplicates of Figures 8–1A and 8–1B.

Perhaps a more thorough understanding of unsteady-state flow can be gained by discussing the significance of Figures 8–1A and 8–1B. If flow is started in a well against a constant back pressure in a plane radial flow regime from a reservoir that is in a steady state of pressure equilibrium, the variation of the flow rate with time on a dimensionless basis will follow the lower solid line in Figure 8–1A at the lesser values of dimensionless time. If the well should have a vertical fracture of infinite conductivity, the variation of the flow rate with time will follow the dashed curve in Figure 8–1A. In

Table 8–2 *Values of q_D as a function of t_D for a plane radial system with an infinite conductivity vertical fracture, infinite outer boundary and constant pressure at the inner boundary.*

	t_D				
Multiplier	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1
1	76.0	23.8	7.9	2.71	1.08
2	53.0	17.1	5.7	1.98	0.86
4	37.5	12.2	4.15	1.52	0.69
6	30.8	10.0	3.42	1.30	0.61
8	26.5	8.8	3.00	1.18	0.56
10	23.8	7.9	2.71	1.08	0.53

Note: the above values were read from charts and are not numerical solutions of the flow functions.

Source: Fetkovich and Thrasher, "Constant Well Pressure Testing and Analysis in Low Permeability Reservoirs."

either case, the reservoir with respect to flow will act as if it were an infinite system. In other words, the pressure disturbance caused by starting the well flowing will not have reached the boundary of the reservoir. Flow in the reservoir with the fracture will become identical to that in the reservoir without a fracture at dimensionless time, t_D, between 1 and 10. If the reservoir boundary is at $r_e/r_w = 1{,}000$, the variation of q_D with t_D will continue as if the reservoir were of infinite extent until t_D is almost 10^6 (see Fig. 8–1B) and then will follow the stem labelled 1,000. At t_D somewhat over 10^7, the value of q_D will be 0.001. Since the curves in Figure 8–1A and 8–1B are dimensionless, the solutions to the flow functions are general for a plane radial system.

Conversion to Gas Terminology

Equation 8–2, as given previously, is expressed in terms of barrels per day, and after multiplication by the formation volume factor, B, the flow rate becomes reservoir barrels per day. For flow of liquids, the volume of a unit weight of the liquid and the viscosity are relatively constant over the range of pressures found in oil wells, and it is inconsequential to assume them to be constant. Usually for gas flow, it is necessary to assume that the volume flowing was constant at an average pressure. Also, it is expedient to express the volume in standard cubic feet. Thus:

Table 8–3 *Values of t_D as a function of q_D for a plane radial system with r_e/r_w and constant pressure at the inner boundary.*[16]

q_D	r_e/r_w 10	20	50	100	200	500	1,000	2,000	4,000	10,000
0.4	2.90×10^1	—	—	—	—	—	—	—	—	—
0.3	5.15×10^1	1.51×10^2	—	—	—	—	—	—	—	—
0.2	8.70×10^1	3.40×10^2	1.65×10^3	4.50×10^3	—	—	—	—	—	—
0.15	1.09×10^2	4.75×10^2	2.85×10^3	1.04×10^4	3.30×10^4	1.28×10^5	—	—	—	—
0.10	1.43×10^2	6.60×10^2	4.45×10^3	1.80×10^4	7.00×10^4	4.03×10^5	1.43×10^6	4.90×10^6	1.62×10^7	6.40×10^7
0.08	1.60×10^2	7.62×10^2	5.31×10^3	2.24×10^4	9.00×10^4	5.50×10^5	2.12×10^6	8.00×10^6	3.00×10^7	1.60×10^8
0.06	1.83×10^2	9.00×10^2	6.50×10^3	2.78×10^4	1.15×10^5	7.50×10^5	3.00×10^6	1.20×10^7	4.75×10^7	2.79×10^8
0.04	2.17×10^2	1.08×10^3	8.20×10^3	3.60×10^4	1.54×10^5	1.03×10^6	4.30×10^6	1.76×10^7	7.10×10^7	4.50×10^8
0.03	2.43×10^2	1.22×10^3	9.25×10^3	4.07×10^4	1.80×10^5	1.23×10^6	5.15×10^6	2.15×10^7	8.85×10^7	5.80×10^8
0.02	2.75×10^2	1.40×10^3	1.08×10^4	4.92×10^4	2.16×10^5	1.50×10^6	6.42×10^6	2.71×10^7	1.14×10^8	7.35×10^8
0.015	3.00×10^2	1.54×10^3	1.20×10^4	5.45×10^4	2.43×10^5	1.70×10^6	7.30×10^6	3.10×10^7	1.31×10^8	8.50×10^8
0.010	3.32×10^2	1.72×10^3	1.36×10^4	6.28×10^4	2.80×10^5	1.96×10^6	8.50×10^6	3.65×10^7	1.55×10^8	1.07×10^9
0.008	3.50×10^2	1.83×10^3	1.44×10^4	6.70×10^4	3.00×10^5	2.12×10^6	9.20×10^6	3.95×10^7	1.68×10^8	—
0.006	3.75×10^2	1.95×10^3	1.57×10^4	7.25×10^4	3.26×10^5	2.31×10^6	1.00×10^7	4.35×10^7	1.83×10^8	—
0.004	4.04×10^2	2.14×10^3	1.72×10^4	8.05×10^4	3.63×10^5	2.60×10^6	1.13×10^7	4.88×10^7	2.10×10^8	—
0.003	4.28×10^2	2.27×10^3	1.83×10^4	8.60×10^4	3.90×10^5	2.80×10^6	1.22×10^7	5.30×10^7	2.28×10^8	—
0.002	4.61×10^2	2.46×10^3	2.00×10^4	9.40×10^4	4.28×10^5	3.08×10^6	1.34×10^7	5.85×10^7	2.52×10^8	—
0.0015	4.85×10^2	2.60×10^3	2.12×10^4	1.00×10^5	4.55×10^5	3.26×10^6	1.43×10^7	6.25×10^7	2.71×10^8	—
0.0010	5.20×10^2	2.80×10^3	2.30×10^4	1.06×10^5	4.92×10^5	3.60×10^6	1.56×10^7	6.80×10^7	2.98×10^8	—

Note: the above values were read from charts and are not numerical solutions of the flow functions.
Source: Fetkovich, "Decline Curve Analysis Using Type Curves," *JPT*, June 1980.

$$q_o(t)B = \frac{178.1071(2)\, q_k p_b\, \bar{z}\, T}{(p_i + p_{wf})\, T_b} \tag{8–4}$$

and by substitution, equation 8–2 becomes

$$q_D = \frac{q_k p_b\, \bar{z}\,\bar{\mu}\, T}{19.87 \times 10^{-6} kh\,(p_i^2 - p_{wf}^2)\, T_b} \tag{8–5}$$

for gases. The equation defining the dimensionless time, t_D, for gases remains the same as in equation 8–3.

In the case of a vertical fracture, the dimensionless time based on the fracture half-length, X_f, according to Fetkovich and Thrasher is:

$$t_{DX_f} = \frac{0.00634\, k\, t}{\phi\, \mu\, C_t X_f^2} \tag{8–6}$$

The equivalent wellbore radius, r_w', is

$$r_w' = \frac{X_f}{2} \tag{8–7}$$

Therefore:

$$t_{Dr_w'} = 4t_{DX_f} \tag{8–8}$$

Since:

$$\frac{p_i^2 - p_{wf}^2}{\bar{\mu}\bar{z}} = m(p_i) - m(p_{wf}) \tag{8–9}$$

equation 8–5 becomes:

$$q_D = \frac{q_k\, p_b\, T}{19.87 \times 10^{-6}\, kh\,[m(p_i) - m(p_{wf})]T_b} \tag{8–10}$$

on the basis of the real gas potential, $m(p)$.

In a later section, type curves will be used to determine dimensionless time, t_D, as defined in equation 8–3. After the values for the variables other than the wellbore radius, r_w, in equation 8–3 have been determined, it is possible to solve for the wellbore radius. The result is not the wellbore radius as the well was drilled, but it is the effective wellbore radius which includes any damage to the permeability of the reservoir rock or any effects of stimulation treatment. It also includes the non-Darcy effects around the wellbore.

After the effective wellbore radius, r_w' is known, the sum of the non-Darcy effect, $F_{nD}q_D$, and the van Everdingen skin factor, s, can be calculated by the following:[18]

$$F_{nD}q_D + s = -\ell n\,(r_w'/r_w) \tag{8–11}$$

or:

$$F_{nD}q_D + s = -2.3031 \log (r'_w/r_w)$$

where r_w is the radius of the wellbore as drilled.

EXAMPLE AND ANALYSIS OF UNSTEADY-STATE BEHAVIOR

The test results given in Table 7–7 and illustrated in Figure 7–6 are typical of the unsteady-state behavior of a gas well. Also, they are representative of the type of testing and results encountered frequently by the natural gas engineer. The test, however, was not run at a constant back pressure, which makes it desirable to correct the test results to a constant back pressure, but, unfortunately, there has been no generally accepted method for the correction. Winestock and Colpitts normalized test results by dividing $(p_R^2 - p_{wf}^2)$ with and without a correction for turbulence by the rate of flow, q_k.[19] In this section, several approaches will be examined.

Referring to Figure 7–6, note that as time went on the well produced closer and closer to its open-flow potential. If the well were producing into a pipeline with a constant pressure of about 600 psia, the well would have been floating on the line after little more than 24 hr. In other words, the wellhead pressure would have become constant at about 600 psia and the rate of flow would have become the only variable. If one assumes that the bottom-hole pressure, p_{wf}, was constant at 662 psia, it is possible to calculate

Table 8–4 *Normalization of test results from Table 7–7 to give the rate of flow, q_2, at a constant back pressure, p_{wf}, of 662 psia.*

Elapsed Time (hr)	$(\bar{p}_{Ri}^2 - p_{wf}^2)$ *(thousands)*	q_1	q_2 at $(\bar{p}_{Ri}^2 - p_{wf}^2)$ = *4,031 (thousands)*		
			n = 0.70	*n = 0.85*	*n = 1.00*
0.25	1,279.3	2,829	6,317	7,504	8,914
0.50	1,855.0	2,509	4,320	4,853	5,452
0.75	2,243.8	2,290	3,451	3,768	4,114
1.0	2,510.6	2,136	2,975	3,194	3,430
2.0	3,038.2	1,791	2,183	2,278	2,376
3.0	3,219.9	1,670	1,954	2,021	2,091
4.0	3,318.2	1,589	1,821	1,875	1,930
5.0	3,393.8	1,534	1,730	1,776	1,822
6.0	3,455.5	1,487	1,656	1,695	1,735
24.0	3,947.3	1,038	1,053	1,057	1,060
48.0	4,055.0	921	917	916	916
72.0	4,096.7	867	857	855	853
96.0	4,128.4	828	814	811	808
120.0	4,146.3	805	789	786	783

the rate of flow for the well producing against the bottom-hole pressure, provided one also assumes values for the exponent n of 1.00, 0.85, and 0.70 for the isochronal type curve for the well. This is accomplished by the following equation:

$$q_2 = q_1 \left(\frac{\Delta p_2^2}{\Delta p_1^2}\right)^n \tag{8–12}$$

where $\Delta p^2 = \bar{p}_{Ri}^2 - p_{wf}^2$. The results of the calculations are shown in Table 8–4 and in Figure 8–2.

TYPE CURVE MATCHING

In the preceding section, the purpose was to adjust the test results so that the results more nearly conformed to data that would have been obtained if the well had been producing against a constant back pressure at the sand face in the well. It is evident from Figure 7–6, Table 8–4, and Figure 8–2 that the value of the exponent n selected for the normalization process makes

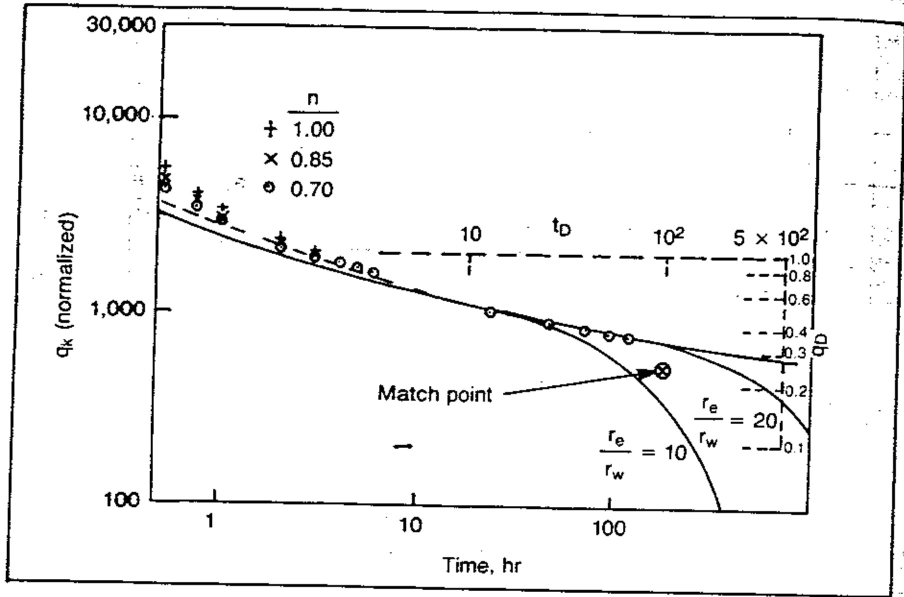

Figure 8–2 *Normalized test data from Table 8–4 imposed on flow functions from Figures 8–1A and 8–1B; pressures squared used.*

very little difference in the final results. After about 3 hr of test time, the normalized points coverage very closely. For purposes of further analysis, the data will be considered identical.

The normalized data points are plotted on log-log tracing paper with 3-in. cycles as illustrated in Figure 8–2. Tracing paper is then superimposed on copies of Figures 8–1A and 8–1B on 3-in. cycle log-log paper, and the tracing paper is moved back and forth, keeping the coordinates parallel, until the points on the tracing paper match the curves in Figure 8–1A. It is possible to match only the points from 24 to 120 hr of the test as shown in Figure 8–2. The underlying grid scale of Figure 8–1A has been shown in part by the dashed lines in Figure 8–2. An arbitrary match point is selected and is indicated in Figure 8–2 by an X inside a circle. The match points at q_k = 535 Mcfd and time = 181 hr or 7.54 days on the tracing paper are equivalent to a dimensionless flow rate, q_D = 0.25, and a dimensionless time, t_D = 100. Rearranging equation 8–5 and setting the basis of the gas measurement to be 14.65 psia and 60°F, the result is:

$$kh = \frac{1{,}418\, q_k\, \bar{z}\bar{\mu}\, T}{q_D(\bar{p}_D^{\,2} - p_{wf}^{\,2})} \tag{8–13}$$

where $\bar{p}_R$ has been substituted for p_i. The value of kh can then be calculated:

$$kh = 1{,}418\,\frac{(535)(0.863)(0.0140)(555)}{(0.25)(4{,}031{,}000)}$$

where:

$$kh = 5.05 \text{ md-ft}$$

or:

$$k = 5.05/31 = 0.16\ md$$

Here, the average compressibility factor, $\bar{z}$, has been taken at the reservoir temperature of 95°F and the average of $\bar{p}_R$ = 2,114 and p_{wf} = 662. The average viscosity, $\bar{\mu}$, is obtained in a similar manner. The thickness, h, is estimated from electric logs to be 31 ft, which gives the permeability as 0.16 md, and the porosity capable of holding gas, $\phi(1 - S_w)$, is estimated to be 9% from electric logs.

If one rearranges equation 8–3 and uses the calculated permeability, 0.16, one can calculate the effective wellbore radius, r_w':

$$r_w' = \left(\frac{0.00634\, k\, t}{\phi\, \mu\, C_t\, t_D}\right)^{0.5} \tag{8–14}$$

$$r_w' = \left[\frac{(0.00634)(0.16)(7.54)}{(0.09)(0.01607)(0.000455)(100)}\right]^{0.5}$$

$$r_w' = 11 \text{ ft}$$

Table 8–5 *Values of the real gas potential,* m(p), *for pressures of interest in test results on a gas well from Table 7–7. Temperature is 95°F.*

Elapsed Time (hr)	q_k *(Mcfd)*	$\bar{p}_{Ri}$ *or* p_{wf} *(psia)*	*Compressibility factor (z)*	*Viscosity,* μ *(cp)*	m(p) $\times 10^6$ *(psia²/cp)*	$m(\bar{p}_{Ri}) - m(p_{wf}) \times 10^6$ *(psia²/cp)*
0.0	0	2,114.0	0.8305	0.01607	346.12	0.0
0.25	2,829	1,785.9	0.8402	0.01508	265.69	98.43
0.50	2,509	1,616.7	0.8482	0.01460	219.73	144.39
0.75	2,290	1,491.7	0.8555	0.01427	188.12	176.00
1.0	2,136	1,399.4	0.8615	0.01404	166.16	197.96
2.0	1,791	1,196.1	0.8766	0.01356	122.15	241.97
3.0	1,670	1,117.6	0.8830	0.01339	106.84	257.28
4.0	1,589	1,072.7	0.8869	0.01330	98.54	265.58
5.0	1,534	1,036.9	0.8900	0.01322	92.10	272.02
6.0	1,487	1,006.6	0.8927	0.01316	86.83	277.29
24.0	1,038	722.2	0.9200	0.01265	44.76	319.36
—	—	662.0	0.9262	0.01250	37.61	326.51
48.0	921	643.3	0.9281	0.01253	35.50	328.62
72.0	867	610.1	0.9316	0.01248	31.92	332.20
96.0	828	583.5	0.9344	0.01244	29.19	334.93
120.0	805	567.9	0.9360	0.01242	27.64	336.48
Well shut in						
0.25	0	728.8	0.9193	0.01266	45.58	—
0.50	0	861.3	0.9062	0.01289	63.65	—
0.75	0	977.4	0.8954	0.01311	81.90	—
1.0	0	1,079.3	0.8863	0.01331	99.71	—
2.0	0	1,378.9	0.8629	0.01399	161.45	—
3.0	0	1,562.4	0.8512	0.01445	205.74	—
4.0	0	1,666.1	0.8457	0.01474	232.78	—
5.0	0	1,730.4	0.8426	0.01492	250.22	—
6.0	0	1,773.2	0.8407	0.01504	262.12	—
24.0	0	1,959.4	0.8341	0.01559	316.31	—
48.0	0	2,011.3	0.8327	0.01575	332.09	—
72.0	0	2,037.3	0.8320	0.01583	340.10	—

Equation 8–14 can be used unchanged to calculate the equivalent wellbore radius as follows:

$$r_w' = \left[\frac{(0.00634)(0.17)(7.29)}{(0.09)(0.01607)(0.000455)(150)}\right]^{0.5} = 8.92 \text{ or } 9 \text{ ft}$$

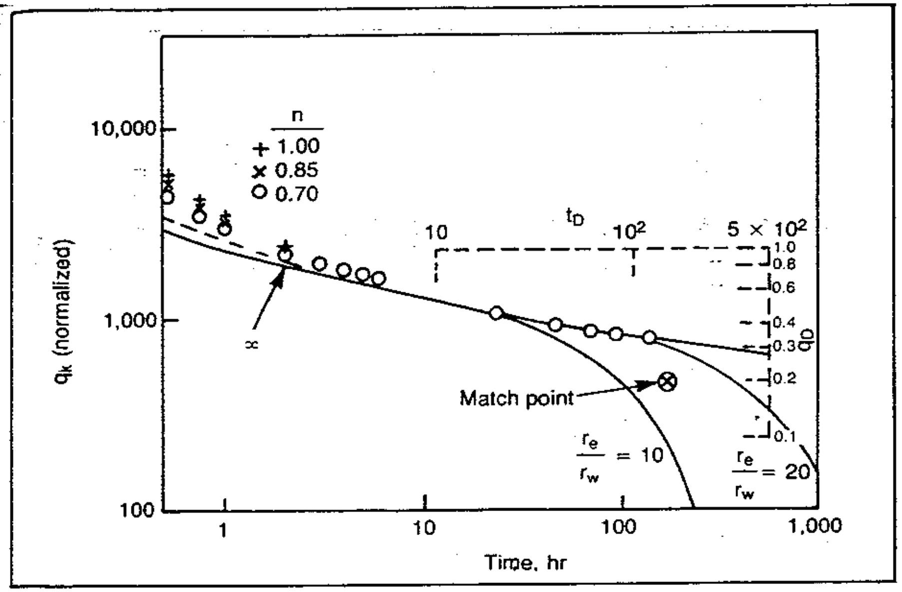

Figure 8–3 *Normalized test data (see Table 8–5) imposed on flow functions from Figures 8–1A and 8–1B; real gas potential used.*

Note: The use of the real gas potential does not require estimating an average compressibility factor or viscosity. The calculation of the effective wellbore radius requires the use of the viscosity and compressibility of the undisturbed portion of the reservoir. The total skin effect, $F_{nD}\, q_D + s$, according to equation 8–11 is:

$$F_{nD}\, q_D + s = -\ln\left(\frac{r_w'}{r_w}\right) = -\ln\frac{9}{0.25} = -3.58$$

assuming the wellbore radius as drilled was 0.25 ft.

Working with pressures squared, one previously calculated the *kh* value to be 5.05 md-ft, the permeability to be 0.16 md, and the effective wellbore radius to be 11 ft. With the real gas potential, the values were 5.42, 0.17, and 9 ft, respectively. The agreement is considered good.

CALCULATING THE REAL GAS POTENTIAL, $m(p)$

The real gas potential is calculated starting from a base pressure of 15 psia and at 50, 100, 200, and 300 psia and in 100-psi increments thereafter. The steps are illustrated in Table 8–6. The first step is to calculate the compressibility factor, z, and viscosity, μ, for each pressure at the reservoir temperature (in this case 95°F). Compressibility factors are calculated using equations 2–8 to 2–11 and viscosities using equations 2–21 and 2–22. The second step is to calculate $p/\mu z$ for each of the pressures.

By definition in Table 8–6, p_m is set at 15 psia and $m(p)$ at 15 psia is set at zero. The $m(p)$ value at 50 psia is:

$$m(50) = 2\left(\frac{4{,}233 + 1{,}267}{2}\right)(50 - 15) + 0 = 0.19250 \times 10^6$$

The $m(p)$ value at 100 psia is:

Table 8–6 *Example calculation of the real gas potential,* m(p), *for the test example from Table 7–7.*

$\gamma_g = 0.560$, temperature = 95°F, and M = 16.22.

Pressure, p (psia)	Compressibility Factor (z)	Viscosity μ (cp)	$p/\mu z$ (psia/cp)	$\Delta m(p)$ (psia²/cp)	$m(p)$ (psia²/cp)
15	0.9982	0.01186	1,267	0.0	0.0
50	0.9942	0.01188	4,233	0.193×10^6	0.193×10^6
100	0.9883	0.01191	8,496	0.636×10^6	0.829×10^6
200	0.9768	0.01199	17,077	2.557×10^6	3.386×10^6
300	0.9654	0.01209	25,073	4.278×10^6	7.664×10^6
400	0.9543	0.01220	34,357	6.006×10^6	13.670×10^6
500	0.9433	0.01233	42,989	7.735×10^6	21.405×10^6
600	0.9326	0.01247	51,593	9.458×10^6	30.86×10^6
700	0.9222	0.01262	60,142	11.174×10^6	42.04×10^6
800	0.9122	0.01278	68,623	12.877×10^6	54.91×10^6
900	0.9025	0.01296	77,066	14.569×10^6	69.47×10^6
1,000	0.8933	0.01315	85,129	16.220×10^6	85.68×10^6
etc.					

$$m(100) = 2\left(\frac{8{,}496 + 4{,}233}{2}\right)(100 - 50) + m(50)$$

$$= 0.63645 \times 10^6 + 0.192500 \times 10^6 = 0.82895 \times 10^6$$

The real gas potential at intermediate pressures may be found by interpolation on a plot of $m(p)$ against pressure or calculating the $m(p)$ value at the desired pressure. The inverse of converting value of the real gas potentials to pressures is done most readily by recourse to the plot of $m(p)$ against pressure.

$$m(p) = 2\int_{p_m}^{p} \frac{p\,dp}{\mu(p)z(p)}$$

PRESSURE BUILDUP BEHAVIOR

Since pressure buildup data were taken (Tables 7–7 and 8–5) on the test well after 120 hr of flow, it is desirable to use the buildup data to calculate the permeability and skin effect as a check on the analysis of the drawdown data. The theory of pressure buildup for wells has been given in detail by Matthews and Russell and by Earlougher,[23,24] but the theory given herein will be only that necessary for analysis of the test example given in Tables 7–7 and 8–5. The equation for the pressure behavior of a well producing oil above the bubble point after shut-in has been given by Matthews and Russell as:

$$p_{ws} = p_i - 162.2\frac{q_o\mu B}{kh}\log\frac{(t + \Delta t)}{\Delta t} \qquad (8\text{–}16)$$

for the case in which the well was producing at a constant rate or a series of discrete constant rates before the well was shut in. Recently, Uraiet and Raghavan have studied the pressure buildup behavior for a well produced against a constant wellbore pressure.[25] They concluded that the same equations can be used for analyzing the buildup from the constant pressure case as has been used for the buildup from the constant rate case, but the production time variable, t, must be defined as the actual production time, and the production rate must be defined as the instantaneous production rate at the time the well was shut in. Equation 8–16 shows that if the pressure, p_{ws}, observed in the well during shut-in is plotted against the logarithm, $\Delta t/(t + \Delta t)$, the result should be a straight line with a slope m equal to the coefficient of the logarithmic term from which it is possible to calculate kh:

$$kh = 162.6 \frac{q_o \mu B}{m} \tag{8–17}$$

For natural gas engineering, it is convenient to substitute the following for the liquid terms in equations 8–16 and 8–17:

$$q_o \times 5.6146 = 1{,}000\, q_k$$

$$B = \frac{p_b \bar{z} T}{p T_b}$$

$$\bar{p} = \frac{p_i + p_{ws}}{2}$$

The result is:

$$p_{ws}^2 = p_i^2 - 57{,}920.4 \frac{q_k \bar{\mu}\bar{z}\, T}{kh} \frac{p_b}{T_b} \log \frac{(t + \Delta t)}{\Delta t} \tag{8–18}$$

Substituting p_b = 14.65 and T_b = 520, the result is:

$$p_{ws}^2 = p_i^2 - 1{,}632 \frac{q_k \bar{\mu}\bar{z}\, T}{kh} \log \frac{(t + \Delta t)}{\Delta t} \tag{8–19}$$

and:

$$kh = 1{,}632 \frac{q_k \bar{\mu}\bar{z}\, T}{m} \tag{8–20}$$

where the slope, m, is expressed in pressure squared per cycle. If the real gas potential, $m(p)$, is used, equations 8–19 and 8–20 become:

$$m(p)_{ws} = m(p)_i - 1{,}632 \frac{q_k T}{kh} \log \frac{(t + \Delta t)}{\Delta t} \tag{8–21}$$

and:

$$kh = 1{,}632 \frac{q_k T}{m} \tag{8–22}$$

where the slope, m, is expressed in the real gas potential, $m(p)$.

The total skin factor, $F_{nD}\, q_D + s$, may be calculated by the following equation given by Earlougher modified to represent the flow of gas:

$$F_{nD}\, q_D + s = 1.1513 \left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k}{\phi \mu\, c_t\, r_w^2}\right) + 3.2275 \right] \tag{8–23}$$

Where p_{1hr} is the pressure in the well after 1 hr of shut-in time, c_t is the total system compressibility, and r_w is the wellbore radius expressed in feet. In working with natural gas terms, pressure squared and the real gas potential can be substituted for the pressure terms in equation 8–23.

Test Example

Returning to the test example given in Table 7–7 and calculating pressures squared and the real gas potential, the results are given in Table 8–7 and in Figures 8–4 and 8–5. The scales in Figures 8–4 and 8–5 were selected to give the appearance of a plot of pressure against time in the buildup, but the changes in no way prevent the use of the buildup equations. Plots such as these are sometimes called Horner plots in deference to the pioneering work of Horner.

In attempting to analyze the results presented in Figures 8–4 and 8–5, one is confronted with the dilemma of choosing a straight line to use in the analysis. This choice often happens when working with pressure buildup curves for gas wells. The technical literature is replete as to causes of such buildup behavior, and it is no surprise that almost any deviation

Table 8–7 *Pressure build-up data for test example given in Table 7–7.*

Shutin Time (hr)	$\frac{\Delta t}{(t + \Delta t)}$	$\bar{p}_R$ or p_{wf}	$\bar{p}_R^2$ or p_{wf}^2 *(thousands)*	m(p)
0.25	0.002079	728.8	531.1	45.58×10^6
0.50	0.004149	861.3	741.8	63.65×10^6
0.75	0.006211	977.4	955.3	81.90×10^6
1.0	0.008264	1,079.3	1,164.9	99.71×10^6
2.0	0.01639	1,378.9	1,901.4	161.45×10^6
3.0	0.02439	1,562.4	2,441.1	205.74×10^6
4.0	0.03226	1,666.1	2,775.9	232.78×10^6
5.0	0.04000	1,730.4	2,994.3	250.22×10^6
6.0	0.04762	1,773.2	3,144.2	262.12×10^6
24.0	0.1667	1,959.4	3,839.2	316.31×10^6
48.0	0.2857	2,011.3	4,045.3	332.09×10^6
72.0	0.3750	2,037.3	4,150.6	340.10×10^6

Gas produced during test = 5,010 Mcf
Instantaneous rate of flow at end of test = 805 Mcfd
Duration of flow = 120 hr

from the ideal conditions for which the mathematics were derived could be the culprit. In attempting to choose between the straight lines evident in Figures 8–4 and 8–5, turn to the work of Earlougher and that of Gringarten, Ramey, and Raghavan.[24,26] The type curve match of the pressure buildup for the test example in Table 8–7 on a real gas potential basis from the theoretical work of Gringarten, Ramey, and Raghavan is shown in Figure 8–6. The buildup data for 4, 5, and 6 hr can be matched well on the curve for the infinite conductivity vertical fracture, and the data fall to the right of the approximate start of the semi-log straight line portion of the curve. Therefore, one can conclude that the buildup data for 4, 5, 6 hr after shut-in define the start of the proper portion of the buildup curve to be used in equations 8–20, 8–22, and 8–23 to calculate *kh* and the total skin factor, $F_{nD}\, q_D + s$. These have been calculated and are given in Table 8–8.

Table 8–8 shows, for comparative purposes, the permeability and the total skin factor obtained from the drawdown portion of the well test as well as the results of the pressure squared analysis and the real gas potential analysis. Since the test results given in Table 7–7 were taken at random from a number of well tests, the agreement between the values for permeability and total skin factor given in Table 8–8 are believed to be good.

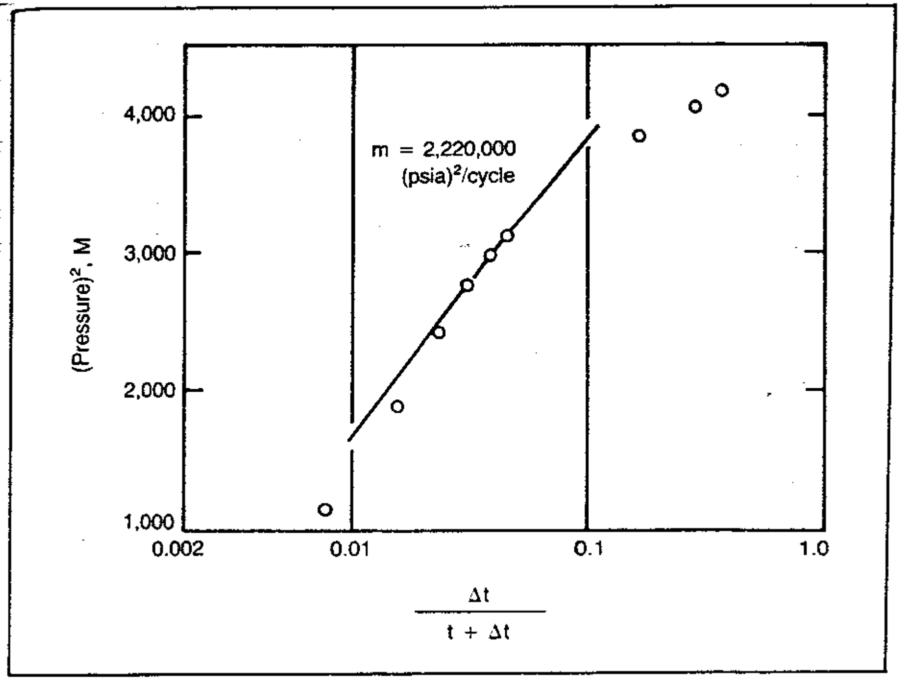

Figure 8–4 *Pressure buildup using pressure squared, well test data from Table 7–7.*

The proper straight-line portion of the pressure buildup curve started at about 4 hr of shut-in time, which resulted in a line with only three points. More data taken at shut-in times of 7, 8, 10, and 12 hr would have been more appropriate for the purpose of the buildup analysis.

However, note that the total skin effect as calculated in the foregoing sections includes, in addition to the true skin effects, the effects of non-Darcy flow near the wellbore. To arrive at the true skin effect, it is necessary to determine the apparent skin at several rates of flow and extrapolate to the skin effect indicated at a zero rate of flow. Details of such calculations have been given by Smith and others.

PRODUCTION FORECASTING

Corollaries to the questions asked early in this chapter about the reserves available for production from the well are: Did the pressure disturbance

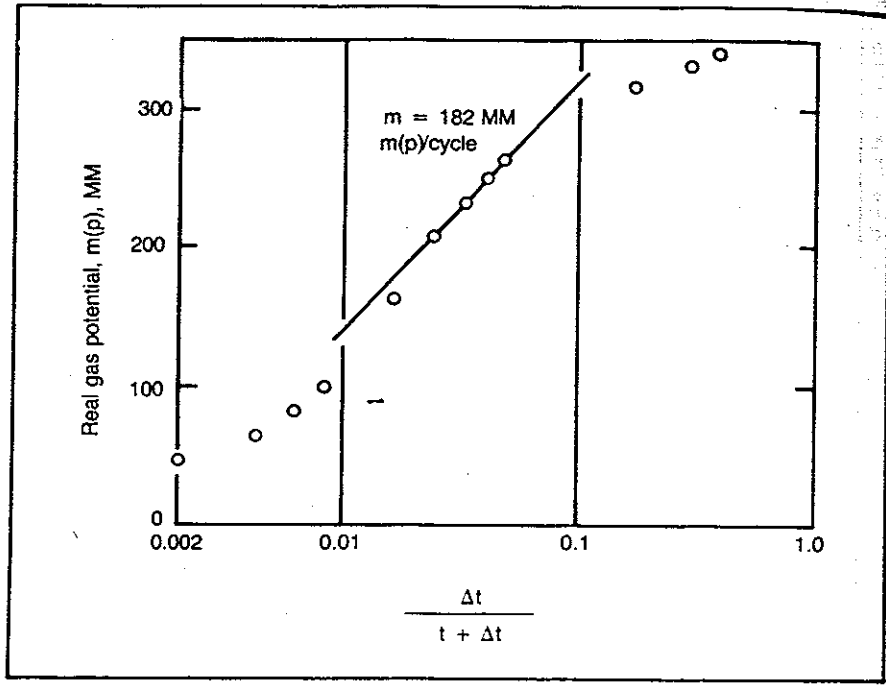

Figure 8–5 *Pressure buildup using real gas potential, well test data from Tables 7–7 and 8–5.*

reach the edge or boundaries of the reservoir, and was the pressure drop observed at the well caused by reserve depletion or by the unsteady-state effects? To answer these questions, return to Figure 8–3 in which the drawdown data have been imposed on the numerical solutions to the constant back pressure case. The drawdown data follow the curve where the reservoir acts as if it were infinite with respect to the pressure disturbance, and the data points have passed the places where the r_e/r_w lines for values of 10 and 20 diverge from the infinite curve. This means that the pressure disturbance did not reach the reservoir boundary at the total test time of 120 hr and that the external radius of the pressure disturbance was slightly in excess of 20 times the effective wellbore radius. Since the effective wellbore radius was calculated to be 9 ft, using the real gas potential method, the pressure disturbance had receded from the wellbore to a distance of about 9 × 20 = 180 ft or it had covered an area of about 2.34 acres. Thus, the pressure disturbance had not reached a reservoir boundary and had covered

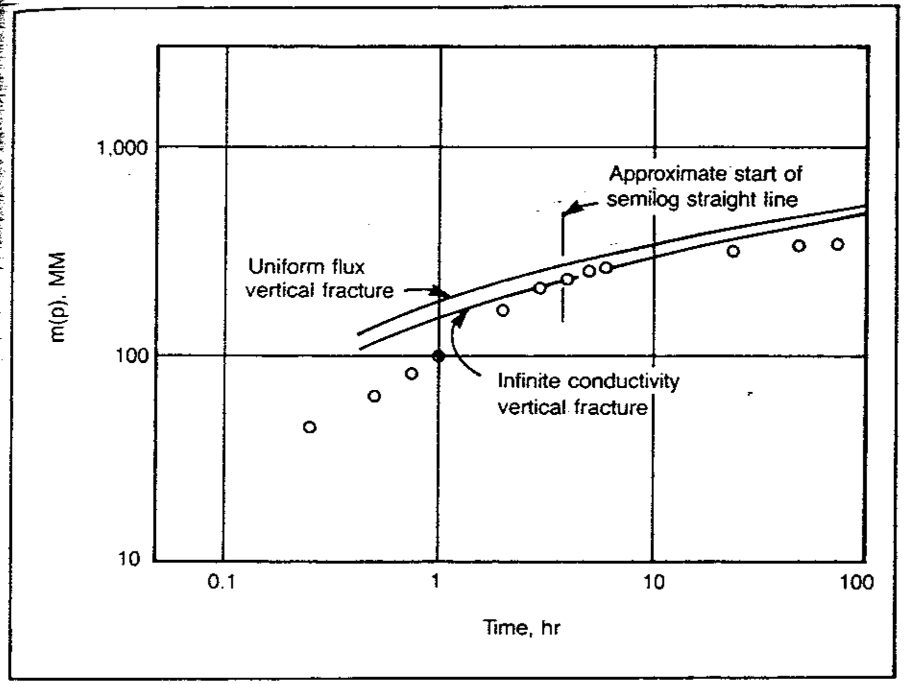

Figure 8–6 *Type curve match for infinite conductivity vertical fracture and pressure buildup test data from Table 8–7.*[24,27]

Table 8–8 *Summary of analyses for permeability and total skin factor for the test example drawdown and buildup results given in Tables 7–7 and 8–7.*

	kh, md-ft	*Permeability, md*	*Total Skin Effect,* $F_{nD}\,q_D + s$
Pressure drawdown			
p^2 analysis	5.05	0.16	−3.78
$m(p)$ analysis	5.42	0.17	−3.58
Pressure buildup			
p^2 analysis	4.14	0.13	−2.90
$m(p)$ analysis	4.01	0.13	−2.87
Thickness, h = 31 ft			

a radius of slightly more than 180 ft. Thus far in the test, all pressure changes have been caused by unsteady-state effects and not by an appreciable depletion of the reserves available to the well.

TIME FOR THE PRESSURE DISTURBANCE TO REACH THE RESERVOIR BOUNDARY

Type Curve Method

Since the reservoir, insofar as the pressure disturbance is concerned, acts as if it were infinite until the pressure disturbance reaches a boundary, one must consult the known geology of the area and the plans for well-spacing in order to calculate the time required for the pressure disturbance to reach the reservoir boundary. If one sees no boundary in his knowledge of the geology and decides that the only boundary foreseen is that imposed by the drainage boundaries of other wells, one can assume a distance of 5,280 ft between wells. The ratio of the external radius to the wellbore radius will be about 2,640/9 = 293, or 300. The dimensionless time, t_D, at which r_e/r_w of 300 diverges from the curve for the infinite system is about 2×10^4 on Figure 8–1B, and the dimensionless rate of flow, q_D, will be about 0.183.

Returning to Figure 8–3 and recalling that the match point is:

$$q_k = 450 \text{ Mcfd}$$

$$t = 7.29 \text{ days}$$

$$q_D = 0.20$$

$$t_D = 150$$

One can calculate that:

$$t = \frac{7.29}{150} t_D = 0.0486\, t_D$$

Or the time for the pressure disturbance to reach the drainage boundary will be:

$$t = \frac{7.29}{150}(2 \times 10^4) = 972 \text{ days, } 23{,}328 \text{ hr, or } 2.66 \text{ years}$$

At that time, the rate of flow would be:

$$q_k = \frac{450(0.183)}{0.20} = 412 \text{ Mcfd}$$

Returning to Table 8–4, the point at which the pressure disturbance reached the drainage boundary has been calculated. After 972 days or 23,328 hr of flow, the rate of flow would be 412 Mcfd and the $(\bar{p}_R^2 - p_{wf}^2)$ value would be 4,031,000. Thus the so-called point of stabilization can be added to Figure 7–6, which has been done on Figure 8–7 and marked with a "+."

Calculational Method

Previously, methods for computing the position for the stabilized curve for the isochronal test have been exceptionally complicated and often of a circular nature. Recently, Poettmann extended and simplified a procedure originally proposed in 1978 by Brar and Aziz.[28] Poettmann's method is direct and much more straightforward than any previously proposed.[29] It has been examined carefully and shown to be applicable to the traditional methods for analyzing isochronal test results.[30,31]

For present purposes the adaption of the method to the isochronal back-pressure will be discussed:[31]

$$q = C_t(\bar{p}_{Ri}^2 - p_{wf}^2)^n$$

Rearranged and for a flow rate of $q = 1$, it becomes

$$\bar{p}_{Ri}^2 - p_{wf}^2 = \frac{1}{C_t^{1/n}} \qquad (8\text{–}24)$$

Since the difference in pressures squared term has been shown to be a straight line function of logarithm of time, t, under unsteady state flow, the term $1/C_t^{1/n}$ is also a straight line function of the logarithm of time when q

is unity. Returning to Table 7–7 and the information test results on a gas well, assuming as done previously that $n = 0.70$ for the isochronal test, calculating the values for $1/C_t^{1/n}$ and plotting on semi-logarithmic coordinates the results are those given on Figure 8–7.

Since the exponent for the isochronal curves was not known, the values for $1/C_t^{1/n}$ were calculated for exponent values of 0.85 and 1.00 in addition to those for a value of 0.70. Straight lines were fitted to each of the three sets of values by the method of least squares. All values for times less than six hours were not considered. The values for an exponent of 0.70 gave the best fit as measured by the coefficient of determination, r^2. However, the values for the other assumed exponents were not far from a good fit to a straight line as shown on Figure 8–8. The equation for an exponent of 0.70 was:

$$1/C_t^{1.429} = -216.9 + 2833 \log t \tag{8–25}$$

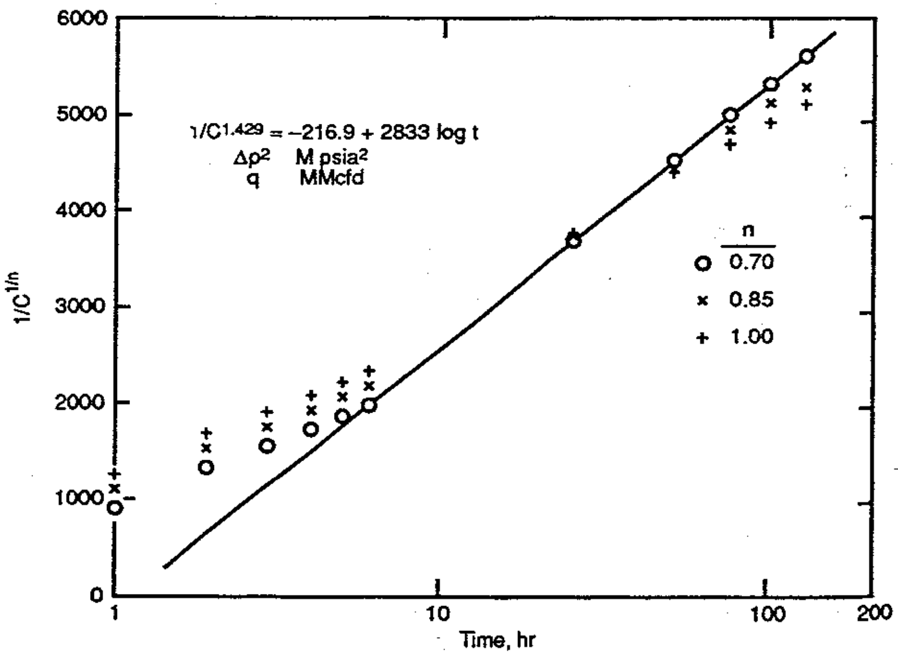

Figure 8–7 *Test data taken from Table 7–7.*

where p^2 was in M psia, q was in MMcfd and log is to base 10. The slope, m, of the line through the points for an exponent of 0.70 was 2833. The slope of the line can be used to calculate the permeability to gas by:

$$k_g = 1{,}632 \frac{\overline{\mu}\,\overline{z}\,T}{m'\,h} \tag{8–26}$$

as follows:

$$k_g = \frac{1{,}632(0.0140)(0.863)(555)}{(2833)(31)} = 0.12 \text{ md}$$

The value of 0.12 md for the permeability to gas compares very favorably with the values of 0.16 and 0.17 for the pressure drawdown analyses and 0.13 for the pressure buildup analyses given in Table 8–8 for the same testing sequence on the well.

Now that we have a value for the permeability it is possible to calculate the time to stabilization for an external radius of drainage of 2,640 feet.

$$t_s = \frac{948\,\phi\,\overline{\mu}\,\overline{c}_t\,r_e^2}{k_g}$$

$$t_s = \frac{948(0.09)(0.0140)(0.000455)(2640)^2}{0.12} \tag{8–27}$$

$$t_s = 31{,}566 \text{ hr or } 1{,}315 \text{ days or } 3.60 \text{ years.}$$

The coefficient C for the stabilized isochronal curve for the test data on the well given in Table 7–7 and an assumed isochronal curve exponent of 0.70 may then be calculated by equation 8–25. Thus:

$$1/C_t^{1.429} = -216.9 + 2833(4.499)$$

$$C_t = 0.00135 \text{ for } q \text{ in MMcfd}$$

The stabilized isochronal curve was then placed on Figure 8–8 where the stabilized curve determined previously by the type-curve method is shown also.

The question arises as to the causes of the difference in the positions of the two estimates of the stabilized curves even though different methods were used in their estimates. The type curves used in the previous analysis are illustrated on Figure 8–3 where the determination of the proper position of the overlay required a judgemental decision. The larger permeability obtained from the calculational method requires that the resultant stabilized curve lie to the right of that for the type curve method, which it does. The

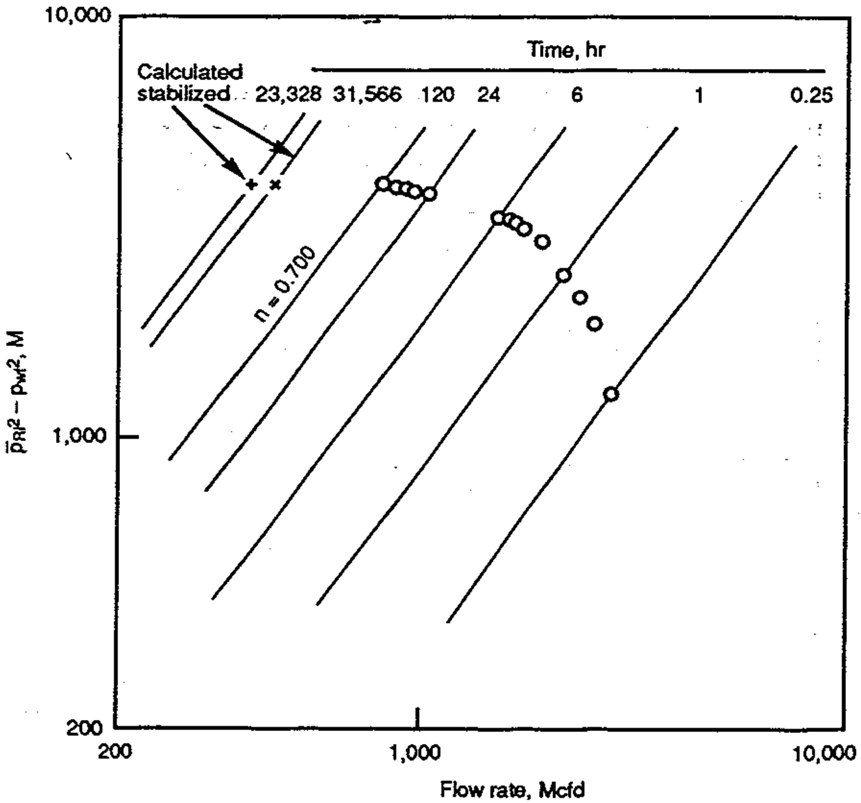

Figure 8–8 *Well test data from Table 7–7 plotted as isochronal test with calculated stabilized curves from two methods added; slope is arbitrary.*

difference might have been reduced by reworking the type curve method, but that was not done.

The principal use of determining the stabilized isochronal curve for a gas well is in production forecasting. The difference between the two curves probably would not have made much difference in the forecasted economics for the well using the two estimates of the stabilized curves.

Production Forecast

The production forecast is made by extending the relationships in Figure 8–3 as has been done in Figure 8–9, but Figure 8–9 returns to the dimensionless rate of flow and time scales. The dimensionless rates of flow and corresponding dimensionless times can be read from the curves in Figure 8–8 and then converted from the dimensionless quantities to the real times and rates of flow by using the match point indicated in Figure 8–3. This has been done in Table 8–9 in which the production rates are instantaneous and are given for the year end.

The production rates of Table 8–9 can be averaged over the time periods to give a cumulative production of about 2,800,000 Mcf. Recalling that the

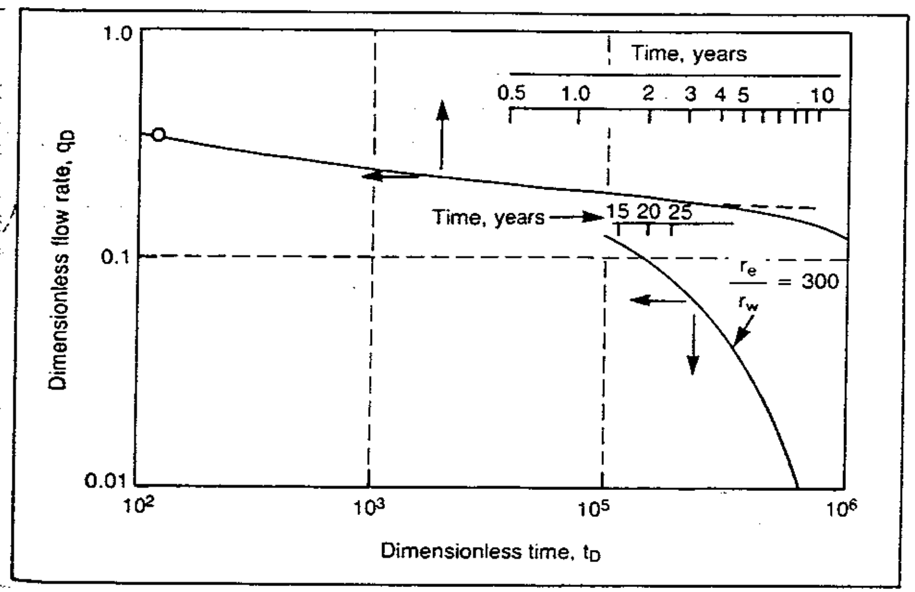

Figure 8–9 *Production forecast for test data given in Table 7–7.*

Table 8–9 *Production forecast for the well from the test example in Table 7–7. Production is against a constant wellbore pressure, p_{wf}, of 662 psia.*

Time (years)	t_D	q_D	*Production Rate, q_x (Mcfd)*	
0.5	3.75×10^3	0.218	491	
1.0	7.51×10^3	0.201	452	
2.0	1.50×10^4	0.188	423	
2.66	2.00×10^4	0.183	412	(Stabilization)
3.0	2.25×10^4	0.181	407	
4.0	3.00×10^4	0.174	392	
5.0	3.75×10^4	0.167	376	
6.0	4.51×10^4	0.161	362	
7.0	5.26×10^4	0.157	353	
8.0	6.01×10^4	0.150	338	
9.0	6.76×10^4	0.146	329	
10.0	7.51×10^4	0.140	315	
15.0	1.13×10^5	0.118	266	
20.0	1.50×10^5	0.099	223	
25.0	1.88×10^5	0.084	189	

Note: Production rates are given for the year end.

original bottom-hole pressure is 2,114.0 psia; the reservoir temperature is 95°F; compressibility factor is 0.8305; the porosity capable of holding gas, $\phi(1 - S_w)$, is 9%; thickness is 31 ft; and the outer boundary is at 2,640 ft; and the original gas in place from equation 3–5 is:

$$G_k = \frac{1{,}546(0.09)(2{,}114)(31)}{(0.8305)(555)} = 19{,}783 \text{ Mcf/acre}$$

or:

$$G_k = 19{,}783 \times \frac{\pi\,(2{,}640)^2}{43{,}560} = 9{,}940{,}000 \text{ Mcf}$$

The percentage of the gas in place that would be produced in 25 years by the test example well is about 28%. Note that the distance to the reservoir boundary is assumed to be 2,640 ft, or a half mile. Since one of the premises is that the well would be producing against a constant wellbore pressure of 662 psia, the amount of gas that would not be produced under the premise would be:

$$G_k = \frac{1{,}546(0.09)(622)(31)}{(0.9262)(555)} \times \frac{\pi\,(2{,}640)^2}{(43{,}560)} = 2{,}790{,}000 \text{ Mcf}$$

The gas available for production under the constant pressure premise is about 7,150,000 Mcf, and the well will produce about 39% of that quantity in 25 years.

SUMMARY

The well of the test example has low productivity, the permeability is about 0.17 md, and the pressure disturbance caused by the testing will never reach an outer reservoir boundary during the test period of 120 hr. In fact, it was calculated that in the 120 hr of testing, the pressure disturbance covered an area of about 2.34 acres. Thus, the question remains unanswered as to whether the reservoir was proved to be commercial by the well test. However, with the ability to make production forecasts, the engineer can estimate the minimum size of the reservoir that is required to meet the standards of economic acceptance. The engineer must then depend upon the known geology of the reservoir for assurance that the reservoir around the well extends to the minimum distance for the well to meet the economic standards.

PROBLEMS

1. Use the isochronal data given below and the remainder of the isochronal data given on the well in Table 7–5, a reservoir thickness of 45 ft, an initial reservoir shut-in pressure, $\bar{p}_R$, of 441.6 psia, a reservoir temperature of 90°F, a porosity capable of holding gas, $\phi(1 - S_w)$, of 0.12, an outer reservoir boundary of 2,640 ft, $\gamma_g = .72$, and a constant wellbore pressure of 400 psia to calculate:
   a. The time required for the pressure disturbance to reach the outer boundary.
   b. The time and the rate of production schedule for the rate of pro-

duction to decline to 400 Mcfd and the percent of the original gas in place recovered at the time.

Additional isochronal test data for problem 1:*

Date	Shut-in Pressure, p_R (psia)	Duration of Flow (hr)	Rate of Flow, q_x (Mcfd)	$\bar{p}_R^2 - p_{wf}^2$ (thousands)
10/11/44	441.6	1.0	1,229	8.62
		9.0	1,202	18.01
		23.5	1,187	23.07
		49.0	1,176	26.71
		70.5	1,171	28.81
		96.5	1,166	30.52
		120.0	1,163	31.56
		144.0	1,161	32.13
		169.0	1,159	32.89
		190.0	1,157	33.54
		214.0	1,156	33.91

2. Recalculate the test example given in the text of Chapter 8 from Table 7–7 with an outer reservoir boundary of 1,320 ft—in effect reducing the reservoir volume to ¼ that given in the text.

3. In the test example in Chapter 8, the effective wellbore radius using the real gas potential was found to be 8.92 ft. If the well were given a fracture treatment to increase the effective wellbore radius to 35 ft, recalculate the time for the pressure disturbance to reach the outer boundary of 2,640 ft and reestimate the production forecast for 25 years.

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* All test data were taken from Cullender. (Table 4, copyrighted 1955, Society of Petroleum Engineers of AIME, used by permission.)

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9 Production Forecasting for Gas Wells

In the previous chapter, the unsteady-state theory of gas flow was used to predict the future production from a gas well when the well was produced at capacity against a constant back pressure or pipeline pressure. To be able to make the prediction for that particular well, the following were needed:

1. An information test of at least 120-hr duration (for this particular well)
2. Independent estimates of the porosity, thickness, and water saturation in the reservoir
3. An estimate of the outer radius of drainage since the test on the well did not last long enough for the pressure disturbance to reach a boundary

Note that a small change in the 662-psia back pressure would make little change in the results (see Fig. 7–3). Nevertheless, the important feature of the example in Chapter 8 was that the well was operating at capacity against a constant back pressure. This is a common mode of operation for many marginal wells or wells with low production capacities. It also applies to many wells located near a large pipeline that serves many wells. It is true that the constant back pressure can be changed with the use of compressors, but there may be a time when further change in the back pressure is no longer feasible from an economic standpoint.

OPERATING CONDITIONS FOR PRODUCTION FORECASTING

In considering the operation of gas wells in general, three principal modes of operation can be identified:

1. The well has excess production capacity and production is restricted to a constant daily (or monthly) quantity of gas, or in other words, production is restricted to the market demand.
2. Production is allocated to various wells, usually located in a proration unit, to a fraction of the open-flow potential or according to some definition of deliverability, or the wells are allowed to produce with a wellhead pressure that is at a constant fraction of the current shut-in wellhead pressure.
3. The well produces against a constant wellhead pressure.

Other modes of operation can be devised, but the author believes that most gas wells are operated in such a manner as to approximate one of the modes mentioned in the foregoing list. In order to make a production forecast for a well, it is necessary to idealize the operations to some extent. In the production forecast example of the previous chapter, there is no way in reality that a well could be turned on and allowed to produce continuously for 25 years against a constant back pressure as indicated in the production forecast.

The daily average production or take from gas wells is usually set by two procedures or a combination of the two. These are the contract quantity and the allowable assigned to the well by a regulatory agency. The contract quantity is the amount of gas usually specified in a legal document that the purchaser has agreed to take from the well or a group of wells. It usually can be stated as a daily average quantity, but in reality the amount of gas that can be taken by the purchaser may vary from day to day, depending upon the purchaser's market demand. If there are competing producers or competing purchasers in a common reservoir, there usually exists a set of rules that specifies the amount of gas to be produced by the well over a period of time, usually a month.

If the well is capable of delivering more gas into the pipeline than the purchaser requires, production from the well must be restrained, and the well is said to have excess productive capacity.

Regarding the second mode of operation, recall that the allowables for many gas wells are limited to 25% of the current open-flow potential. Also, many wells are operated in such a manner that the flowing pressure at the wellhead is an almost-constant fraction of the shut-in pressure. In order to understand why this should occur, the reader is referred to Figure 7–3 in which the flowing pressure for a gas well is shown as a fraction of the shut-in pressure, and the resulting rate of flow is shown as a fraction of maximum

producing capacity. If one assumes that the well is being operated so the flowing pressure is 50% of the shut-in pressure, the well is producing at 75–86% of its productive capacity. Further reduction in the flowing pressure results in little increase in the rate of production. If a group of wells is being produced into a common compressor, the collective rate of production soon reaches a state of balance with the compressor operation so that a further increase in compressor capacity is not justified by the increase in production.

The third mode is operating against a constant back pressure. This type of operation usually occurs where a gas well or a small group of gas wells is near a large pipeline carrying a large quantity of gas. In this instance, the question will be how much gas can be produced into the pipeline before compression is justified.

In forecasting production from a well, the engineer must decide how the well will be operated. The first task is to decide how the average daily take from the well will be determined. If the well is in a prorated field, the decision may have already been made. If so, the average daily allowable should be determined and compared to the productive capacity of the gas well against the expected pipeline pressure at the wellhead. If the well has excess productive capacity, the problem becomes one of determining how much gas can be produced until the productive capacity of the well is equal to the allowable or average daily take from the well. If the operator of the well and the purchaser of the gas have reached agreement as to the daily average contract quantity and there are no other restrictions imposed by regulatory agencies, the problem remains the same.

In developing procedures to make production forecasts for the three listed modes of operation, one will need the following information:

1. A stabilized back-pressure or stabilized isochronal performance for the well
2. An independent estimate of the quantity of gas in place available for production from the well
3. A determination of the market demand for gas from the well

WELLS WITH EXCESS PRODUCTIVE CAPACITY

A well with excess productive capacity can deliver more gas than the purchaser is willing to take. In other words, the contract quantity is less than the capacity of the well to deliver gas into the sales pipeline against the pressure in the pipeline. If the back-pressure equation is known for the wellhead performance of the well when the pressure disturbance reaches the reservoir boundary (the well is said to be stabilized when the disturbance reaches the boundary), one can calculate the minimum shut-in pressure at the wellhead at which the well can produce a given quantity of gas into the pipeline operating at a given pressure. If the performance curve at stabilization is:

$$q_k = C(\bar{p}_{ts}^2 - p_{tf}^2)^n \tag{9–1}$$

The pipeline pressure is p_{tfp} and the minimum shut-in pressure against which the well can produce the market demand, q_{kmd}, is:

$$\bar{p}_{ts}\,(minimum) = (\Delta\bar{p}^2(1{,}000) + p_{tfp}^2)^{1/2} \tag{9–2}$$

where Δp^2 is the difference in pressure squared in thousands for the market demand rate of flow.

In order to illustrate the calculations, consider a hypothetical well that has a stabilized wellhead performance curve with pressure squared in thousands as follows:

$$q_k = 12.0\,(\bar{p}_{ts}^2 - p_{tf}^2)^{0.70} \tag{9–3}$$

The contract quantity is 1,250 Mcfd, the wellhead shut-in pressure is 1,500 psia, and the delivery pressure or pipeline pressure is 1,000 psia. First, calculate the shut-in pressure for the well when the well can no longer produce its contract quantity of gas into the pipeline. If one substitutes the present shut-in pressure and the back pressure or pipeline pressure into equation 9–3, the well is capable of producing 1,766 Mcfd into the pipeline:

$$q_k = 12.0\left(\frac{1{,}500^2 - 1{,}000^2}{1{,}000}\right)^{0.70} = 1{,}766 \text{ Mcfd}$$

Note: The custom of expressing pressure squared in thousands has been followed. Thus, the well is capable of producing more than its contract quantity of 1,250 Mcfd by a margin of 41%. The difference in pressure squared in thousands required for a production rate of 1,250 Mcfd is:

$$1{,}250 = 12.0(\Delta p^2)^{0.7}$$

$$\Delta p^2 = (1{,}250/12.0)^{1/0.7} = 762.9 \text{ M}$$

Substituting this into equation 9–2:

$$p_{ts}\,(minimum) = (762{,}900 + 1{,}000^2)^{1/2} = 1{,}327.7 \text{ psia}$$

If one makes the rough approximation that the material balance can be based on shut-in wellhead pressures, about (1–1,327.7/1,500) × 100, or 11%, of the gas in place can be produced at a constant rate of 1,250 Mcfd. A more realistic estimate of the gas to be produced would be to calculate the shut-in pressures in the reservoir corresponding to the wellhead pressures of 1,500 and 1,327.7 psia, estimate the corresponding compressibility factors, and calculate the gas to be produced by:

$$G_p = \frac{p_1/z_1 - p_2/z_2}{p_1/z_1} G_1 \tag{9–4}$$

where G_p is the gas produced to reduce the pressure from p_1 to p_2 in the reservoir and G_1 is the gas in place when the reservoir pressure was p_1.

The time interval that the well would produce at the constant rate of 1,250 Mcfd would be found by dividing the constant producing rate into the quantity of gas produced according to equation 9–5:

$$t = G_p/q \tag{9–5}$$

For example, if the gas in place available for production from the well is 2.5 billion cu ft, and if the estimate that 11% of the gas in place is accurate enough for the purpose at hand, the time of production at 1,250 Mcfd would be:

$$t = \frac{(2.5 \times 10^9)(0.11)}{1{,}250 \times 10^3} = 220 \text{ days}$$

The well would produce (1,250)(220) = 275,000 Mcf of gas during the interval.

WELLS WITH PRODUCTION RATE LIMITED TO A FRACTION OF THE OPEN-FLOW POTENTIAL

It can be shown that production at a constant fraction of the open-flow capacity or potential is identical mathematically to production against a constant fraction of the current shut-in pressure. From equation 7–1:

$$q = C(\bar{p}_R^{\,2} - p_{wf}^{\,2})^n \qquad (7\text{–}1)^*$$

The open-flow capacity is:

$$q_{of} = C(\bar{p}_R^{\,2})^n \qquad (9\text{–}6)$$

Dividing:

$$\frac{q}{q_{of}} = \left[1 - \left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right]^n = constant \qquad (9\text{–}7)$$

If (q/q_{of}) is a constant, $p_{wf}/\bar{p}_R$ is also a constant. This same relationship was developed in equation 7–9 and illustrated in Figure 7–3 for the wellhead curves. It was repeated here for emphasis.

If $p_{wf}/\bar{p}_R = W$ is set as a constant, equation 7–1 can be rewritten as:

$$q = C[\bar{p}_R^{\,2}(1 - W^2)]^n$$

Since:

$$q = -\frac{dG_i}{dt}$$

* In Chapter 7 the symbol $\bar{p}_R$ was introduced to indicate the stabilized shut-in pressure immediately before a well was placed on test. In Chapter 9 p_R indicates the reservoir pressure as calculated by material balance methods.

Then:

$$-\frac{dG_i}{dt} = C[\bar{p}_R^{\,2}(1 - W^2)]^n \qquad (9\text{–}6)$$

At this point, one must choose the relationship to use between G_i, the gas in place, and $\bar{p}_R$, the average reservoir pressure in the gas reserve available for production from the well. Making the assumption that the compressibility factor, z, is 1.00:

$$C = \frac{q_i}{[\bar{p}_{Ri}^{\,2}(1 - W^2)]^n}$$

This allows the subscript i to denote initial conditions and integrates between initial conditions when $t = 0$ and $\bar{p}_R$ is at t. The result is:

$$\frac{q}{q_i} = \frac{1}{\left[\frac{(2n-1)\,q_i t}{G_i} + 1\right]^{\frac{2n}{2n-1}}} \text{ and } n \neq 0.5 \qquad (9\text{–}9)$$

This is the same as equation 30 published by Fetkovich except for an error in the sign in the denominator of the exponent in the published equation.[1]

If n, the exponent in the back-pressure equation, is 0.5, the equation relating q to q_i becomes:

$$\frac{q}{q_i} = \frac{1}{exp\left(\frac{q_i t}{G_i}\right)} \text{ and } n = 0.5 \qquad (9\text{–}10)$$

Equations 9–9 and 9–10 can be integrated against time by:

$$G_p = \int_0^t q\,dt \qquad (9\text{–}11)$$

Equation 9–9 becomes:

$$\frac{G_p}{G_i} = 1 - \left[\frac{(2n-1)\,q_i t}{G_i} + 1\right]^{\frac{1}{1-2n}} \text{ and } n \neq 0.5 \tag{9–12}$$

Equation 9–10 becomes:

$$\frac{G_p}{G_i} = 1 - exp\left(-\frac{q_i t}{G_i}\right) \text{ and } n = 0.5 \tag{9–13}$$

Table 9–1 *Dimensionless rate of flow, q/q_i, as a function of dimensionless time, $(q_i t)/G_i$, for wells flowing at a constant fraction of open-flow capacity or against a constant fraction of current shut-in pressure.*

Values of $q_i t/G_i$

	n					
q/q_i	*1.000*	*0.900*	*0.800*	*0.700*	*0.600*	*0.500*
1.00	0.000	0.000	0.000	0.000	0.000	0.000
0.95	0.026	0.029	0.032	0.037	0.043	0.051
0.90	0.054	0.060	0.067	0.076	0.089	0.105
0.80	0.118	0.130	0.145	0.165	0.189	0.223
0.70	0.195	0.215	0.239	0.268	0.306	0.357
0.60	0.291	0.319	0.352	0.393	0.444	0.511
0.50	0.414	0.451	0.495	0.548	0.612	0.693
0.40	0.581	0.628	0.683	0.748	0.825	0.916
0.35	0.690	0.743	0.804	0.874	0.956	1.050
0.30	0.826	0.885	0.951	1.026	1.111	1.204
0.25	1.000	1.065	1.136	1.215	1.300	1.386
0.20	1.236	1.306	1.381	1.460	1.538	1.609
0.15	1.582	1.655	1.728	1.799	1.859	1.897
0.10	2.162	2.228	2.286	2.327	2.339	2.303
0.07	2.780	2.826	2.851	2.845	2.788	2.659
0.05	3.472	3.483	3.459	3.384	3.238	2.996
0.03	4.774	4.689	4.541	4.308	3.970	3.507
0.015	7.165	6.832	6.384	5.800	5.068	4.200
0.010	9.000	8.428	7.706	6.819	5.772	4.605

Equations 9–10, 9–12, and 9–13 have been published by Fetkovich.

Note: The equations relating producing rate, q, to time do so in terms of dimensionless rate, q/q_i, and dimensionless time, $(q_i t)/G_i$. Therefore, solutions for the equations can be given conveniently as in Table 9–1.

The solutions have also been shown in Figures 9–1, 9–2, and 9–3 for n values of 0.5, 0.7 and 1.0. Two features are worthy of note. First, the curves cross at the dimensionless time, $q_i t/G_i$, values of about 2.0 and continue to spread beyond that value. Second and most important, the dimensionless rate of flow, q/q_i, is a straight-line function of the time function, $q_i t/G_i$, for an n value of 0.5 on semilogarithmic coordinates as shown on Figure 9–2.

It has been common practice to extrapolate declining production rates from a gas well on semilogarithmic coordinates to estimate future production rates. Since the curves for $n > 0.5$ on Figure 9–2 do not depart much from the straight line for $n = 0.5$, there is support for extrapolating the trend of production rates for gas wells as straight lines on semilogarithmic coordinates.

Numerical Example

In the previous example, it was calculated that a hypothetical gas well would produce 1,250 Mcfd for 220 days against a constant pressure of 1,000 psia before it reached the breaking point. There, if the back pressure were not lowered, the rate of flow would decrease; if the rate of flow were held constant, it would be necessary to lower the back pressure continuously. At the breaking point, the gas remaining in place was 2,500,000,000 cu ft—275,000,000 cu ft = 2,225,000,000 cu ft or 2.225×10^9 cu ft.

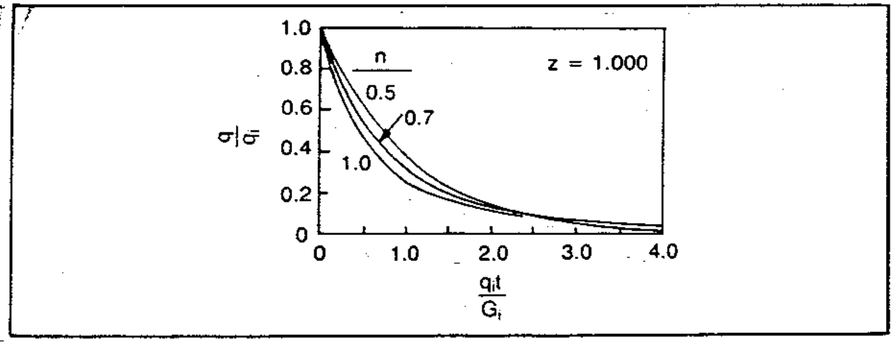

Figure 9–1 *Producing rates for wells flowing at constant fraction of open flow capacity or constant fraction of current shut-in pressure.*

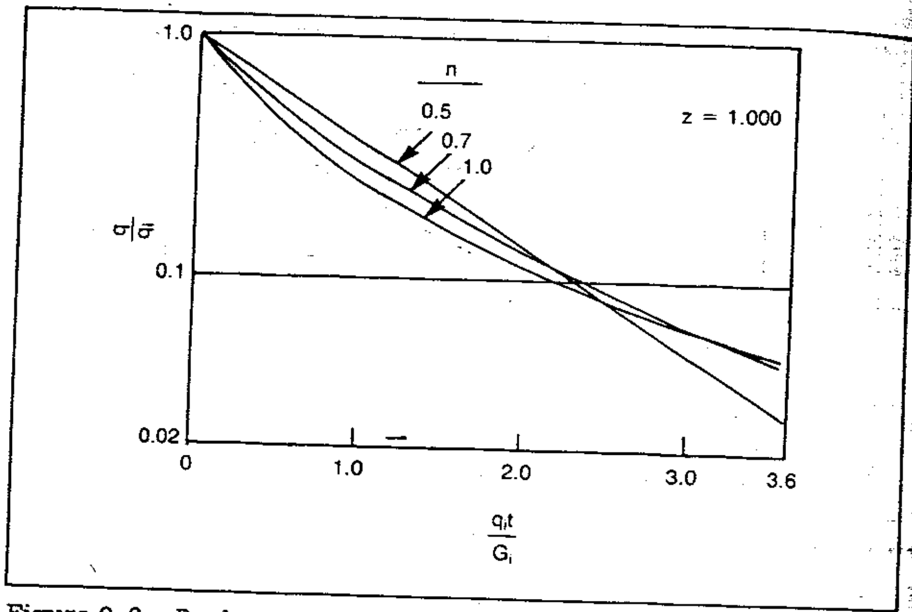

Figure 9–2 *Producing rates for wells flowing at constant fraction of open flow capacity or constant fraction of current shut-in pressure.*

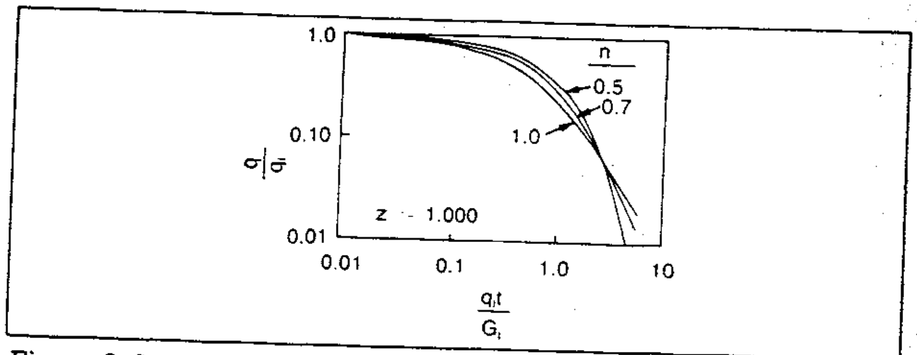

Figure 9–3 *Producing rates for wells flowing at constant fraction of open flow capacity or constant fraction of current shut-in pressure.*

Recalling that the exponent n was 0.70, equation 9–9 can be used to calculate the rates of flow at various times, provided the ratio of the flowing pressure to the current shut-in pressure remains constant throughout the time period for which the calculations are made. At a flow rate of 1,250

Mcfd, the difference in pressure squared is 762.9 thousands, and the shut-in pressure is calculated to be 1,327.7 psia. Thus, at the breaking point the ratio of the flowing pressure to the current shut-in pressure is:

$$1{,}000/1{,}327.7 = 0.7532$$

The value of q_i for use in equation 9–9 is 1,250 Mcfd and G_i is 2.225×10^9 cu ft (z = 1.00 was assumed). Equation 9–9 reduces to:

$$\frac{q(365)}{(1.25 \times 10^6)(365)} = \frac{1}{\left[\frac{(0.4)(1.25 \times 10^6)(365)t}{2.225 \times 10^9} + 1\right]^{3.5}} \qquad (9\text{–}14)$$

At this point, one must be careful with the time units. If time units in years are used, multiply the daily rate in equation 9–9 by 365.

Note: The 365 cancels out of the left side of the equation. Equation 9–14 then reduces to:

$$q = \frac{1.25 \times 10^6}{(0.08202t + 1)^{3.5}} \qquad (9\text{–}15)$$

where the time, t, is expressed in years. The results are given in the second column of Table 9–2 for a period of 8 yr. The total gas produced to a given time can be calculated using equation 9–12:

$$\frac{G_p}{2.225 \times 10^9} = 1 - \left[\frac{(0.4)(1.25 \times 10^6)(365)t}{2.225 \times 10^9} + 1\right]^{-2.5} \qquad (9\text{–}16)$$

or:

$$G_p = 2.225 \times 10^9(1 - [0.08202t + 1]^{-2.5}) \qquad (9\text{–}17)$$

Again, t is expressed in years, and the results are given in the last column of Table 9–2.

Table 9–2 *Production forecast for a well producing against a back pressure that is a constant fraction of its current shut-in pressure.*

$q_i = 1{,}250$ Mcfd Initial Shut-in Pressure = 1,327.7 psia
$G_i = 2.225 \times 10^9$ cu ft Initial Flowing Pressure = 1,000.0 psia
$n = 0.70$ $z = 1.00$

t, Time (years)	q_t At Year End (Mcfd)	G_p, Gas Production (Mcf)	$\bar{q}_t$, Average Production Rate (Mcfd)	G_p, Cumulative Gas Production (Mcf)
0	1,250	0	0	0
1	949	397,986	1,090	397,986
2	735	305,029	836	703,015
3	579	238,228	653	941,244
4	463	189,121	518	1,130,365
5	375	152,300	417	1,282,665
6	308	124,208	340	1,406,873
7	255	102,442	281	1,509,315
8	214	85,345	234	1,594,661

Since the foregoing numerical example is a continuation of the previous example in which the well was produced against a constant back pressure of 1,000 psia, the time scale has been combined on semilogarithmic coordinates for Figure 9–4. The portion of Figure 9–4 labeled *a* shows production when the well had excess capacity, and *b* shows production after the breaking point when the well was produced against a pressure that was a constant fraction of the current shut-in pressure.

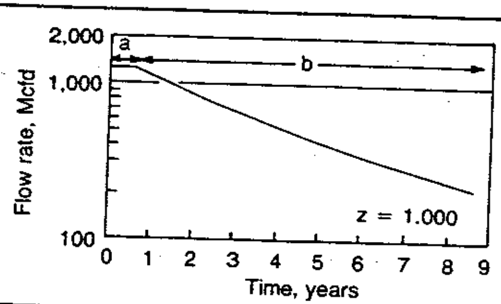

Figure 9–4 *Producing rate for a gas well (a) producing a constant rate against a constant back pressure until the breaking point thereafter (b) against a back pressure that is a constant fraction of the current shut-in pressure.*

PRODUCTION RATES FOR WELLS FLOWING AGAINST A CONSTANT BACK PRESSURE

Starting with the back pressure equation 7–1:

$$q = C(\bar{p}_R^2 - p_{wf}^2)^n \tag{7-1}$$

and setting $x = \bar{p}_R/p_{wf}$. Since p_{wf} is a constant, one can derive the following equation by setting the compressibility factor equal to 1.00:

$$\frac{dx}{(x^2 - 1)^n} = -\frac{q_i x_i \, dt}{G_i(x_i^2 - 1)^n} \tag{9-18}$$

at $t = 0$, $x = x_i$, and at t, $x = x$.

If n is 1.0, equation 9–18 can be integrated between limits to give:

$$\frac{(x_i - 1)(x + 1)}{(x_i + 1)(x - 1)} = exp\left[\frac{2x_i}{(x_i^2 - 1)}\right]\left[\frac{q_i t}{G_i}\right] \tag{9-19}$$

If n is 0.5, equation 9–18, on integration, becomes:

$$\frac{x_i + (x_i^2 - 1)^{0.5}}{x + (x^2 - 1)^{0.5}} = exp\left[\frac{x_i}{(x_i^2 - 1)^{0.5}}\right]\left[\frac{q_i t}{G_i}\right] \tag{9-20}$$

However, if n is between 0.5 and 1.0, one cannot find an analytical solution and must resort to numerical integration. Equation 9–18 then becomes:

$$\int_{x_i}^{x} \frac{dx}{(x^2 - 1)^n} = \left[\frac{x_i}{(x_i^2 - 1)^n}\right]\left[\frac{q_i t}{G_i}\right] \tag{9-21}$$

Or it can be written as:

$$A_x - A_{xi} = \left[\frac{x_i}{(x_i^2 - 1)^n}\right]\left[\frac{q_i t}{G_i}\right] \tag{9–22}$$

Arbitrarily, A_x can be defined as:

$$A_x = \int_{10}^{x} \frac{dx}{(x^2 - 1)^n} \tag{9–23}$$

Values of A_x have been calculated at convenient intervals of x and are given in Table 9–3. Rates of flow corresponding to x can be calculated by the following equation:

$$q = q_i\left[\frac{x^2 - 1}{x_i^2 - 1}\right]^n \tag{9–24}$$

At this point, note the relationship between equation 9–21 for wells producing against a constant back pressure and equation 9–9 for wells producing against a back pressure that is a constant fraction of the current shut-in pressure. If one considers x^2 large with respect to 1 in equation 9–21, equation 9–9 can be derived from equation 9–21. In other words, the limit of equation 9–21 is equation 9–9 as x^2 approaches a large number. Another way of saying the same thing is that equation 9–21 and equation 9–9 become equivalent as the constant back pressure approaches zero.

Numerical Example

Previously, an example of a hypothetical gas well that produced 1,250 Mcfd against a constant pressure of 1,000 psia for a total of 220 days was used. At the breaking point for this well, the current shut-in pressure was calculated to be 1,327.7 psia, and the gas in place (in this case G_i) was 2.225×10^9 cu ft. Now calculate a production forecast for the well if it continues to produce against the constant 1,000 psia back pressure. At time zero, $x_i = 1{,}327.7 \div 1{,}000 = 1.3277$, and A_{xi}, interpolated from Table 9–3, is 1.4466 for an n value of 0.70. Equation 9–22 then becomes:

Table 9–3 *Values of* $A_x = \int_{10}^{x} \frac{dx}{(x^2 - 1)^n}$ ***for wells producing against a constant back pressure.***

	n					
x	*0.50*	*0.60*	*0.70*	*0.80*	*0.90*	*1.00*
1.004	2.9038	2.6152	2.4864	2.5032	2.6695	3.0080
1.005	2.8933	2.5983	2.4593	2.4598	2.6001	2.8966
1.006	2.8837	2.5834	2.4358	2.4230	2.5422	2.8057
1.007	2.8750	2.5698	2.4149	2.3907	2.4924	2.7289
1.008	2.8668	2.5574	2.3960	2.3620	2.4487	2.6624
1.009	2.8592	2.5459	2.3788	2.3360	2.4097	2.6037
1.01	2.8519	2.5352	2.3628	2.3123	2.3744	2.5513
1.02	2.7936	2.4520	2.2442	2.1433	2.1333	2.2072
1.03	2.7489	2.3917	2.1629	2.0334	1.9850	2.0070
1.04	2.7113	2.3428	2.0991	1.9502	1.8765	1.8656
1.05	2.6783	2.3008	2.0458	1.8826	1.7906	1.7565
1.06	2.6485	2.2638	1.9997	1.8253	1.7193	1.6677
1.07	2.6212	2.2304	1.9589	1.7753	1.6583	1.5931
1.08	2.5958	2.1999	1.9221	1.7310	1.6048	1.5287
1.09	2.5721	2.1716	1.8885	1.6910	1.5573	1.4722
1.10	2.5497	2.1452	1.8575	1.6546	1.5145	1.4219
1.11	2.5284	2.1205	1.8287	1.6212	1.4756	1.3766
1.12	2.5081	2.0971	1.8018	1.5902	1.4399	1.3355
1.13	2.4887	2.0750	1.7765	1.5613	1.4069	1.2978
1.14	2.4701	2.0539	1.7526	1.5342	1.3763	1.2631
1.15	2.4521	2.0337	1.7300	1.5088	1.3476	1.2310
1.16	2.4380	2.0144	1.7084	1.4847	1.3208	1.2010
1.17	2.4181	1.9959	1.6879	1.4619	1.2955	1.1730
1.18	2.4019	1.9780	1.6682	1.4403	1.2717	1.1467
1.19	2.3861	1.9608	1.6493	1.4196	1.2491	1.1220
1.20	2.3709	1.9441	1.6312	1.3999	1.2276	1.0986
1.21	2.3560	1.9280	1.6138	1.3810	1.2072	1.0765
1.22	2.3415	1.9124	1.5970	1.3629	1.1877	1.0555
1.23	2.3274	1.8973	1.5807	1.3455	1.1690	1.0355
1.24	2.3136	1.8826	1.5650	1.3288	1.1512	1.0165
1.25	2.3001	1.8682	1.5498	1.3126	1.1341	0.9983
1.26	2.2869	1.8543	1.5351	1.2971	1.1176	0.9809
1.27	2.2740	1.8407	1.5208	1.2820	1.1018	0.9642
1.28	2.2613	1.8275	1.5069	1.2675	1.0865	0.9482
1.29	2.2489	1.8145	1.4934	1.2534	1.0718	0.9329

Table 9–3, continued

x	n					
	0.50	0.60	0.70	0.80	0.90	1.00
1.30	2.2368	1.8019	1.4803	1.2397	1.0576	0.9181
1.35	2.1792	1.7427	1.4194	1.1770	0.9931	0.8518
1.40	2.1262	1.6890	1.3651	1.1221	0.9376	0.7955
1.45	2.0769	1.6399	1.3161	1.0733	0.8888	0.7470
1.50	2.0308	1.5945	1.2714	1.0293	0.8456	0.7044
1.55	1.9874	1.5523	1.2304	0.9894	0.8067	0.6666
1.60	1.9463	1.5127	1.1924	0.9528	0.7716	0.6328
1.65	1.9072	1.4756	1.1570	0.9192	0.7396	0.6023
1.70	1.8700	1.4405	1.1239	0.8880	0.7102	0.5746
1.75	1.8344	1.4073	1.0929	0.8590	0.6831	0.5493
1.80	1.8003	1.3757	1.0636	0.8319	0.6580	0.5260
1.85	1.7676	1.3456	1.0359	0.8064	0.6346	0.5046
1.90	1.7360	1.3168	1.0097	0.7825	0.6128	0.4847
1.95	1.7056	1.2893	0.9848	0.7600	0.5924	0.4662
2.00	1.6763	1.2629	0.9611	0.7386	0.5732	0.4490
2.10	1.6204	1.2131	0.9169	0.6992	0.5381	0.4177
2.20	1.5678	1.1669	0.8761	0.6635	0.5067	0.3901
2.30	1.5182	1.1238	0.8386	0.6309	0.4783	0.3654
2.40	1.4711	1.0833	0.8038	0.6010	0.4526	0.3433
2.50	1.4264	1.0453	0.7714	0.5734	0.4291	0.3233
2.60	1.3838	1.0093	0.7411	0.5478	0.4075	0.3051
2.70	1.3430	0.9752	0.7126	0.5240	0.3877	0.2885
2.80	1.3040	0.9429	0.6858	0.5018	0.3693	0.2733
2.90	1.2665	0.9121	0.6605	0.4810	0.3522	0.2592
3.00	1.2305	0.8827	0.6366	0.4615	0.3362	0.2462
3.25	1.1460	0.8147	0.5818	0.4174	0.3007	0.2177
3.50	1.0684	0.7533	0.5332	0.3789	0.2703	0.1936
3.75	0.9966	0.6973	0.4896	0.3449	0.2438	0.1729
4.00	0.9298	0.6460	0.4502	0.3147	0.2206	0.1551
4.25	0.8673	0.5987	0.4143	0.2875	0.2000	0.1395
4.50	0.8086	0.5547	0.3814	0.2629	0.1815	0.1257
4.75	0.7532	0.5138	0.3511	0.2404	0.1650	0.1134
5.00	0.7008	0.4754	0.3231	0.2199	0.1499	0.1024
5.25	0.6510	0.4394	0.2970	0.2010	0.1363	0.0925
5.50	0.6037	0.4055	0.2727	0.1836	0.1237	0.0835
5.75	0.5585	0.3734	0.2499	0.1674	0.1123	0.0754
6.00	0.5153	0.3430	0.2285	0.1523	0.1017	0.0679
6.25	0.4740	0.3141	0.2083	0.1383	0.0918	0.0611
6.50	0.4342	0.2866	0.1893	0.1251	0.0827	0.0547
6.75	0.3961	0.2604	0.1713	0.1127	0.0742	0.0489
7.00	0.3593	0.2353	0.1542	0.1011	0.0663	0.0435

Table 9–3, continued

x	n					
	0.50	0.60	0.70	0.80	0.90	1.00
7.25	0.3239	0.2114	0.1380	0.0901	0.0589	0.0385
7.50	0.2896	0.1884	0.1226	0.0797	0.0519	0.0338
7.75	0.2566	0.1663	0.1078	0.0699	0.0453	0.0294
8.00	0.2246	0.1451	0.0938	0.0606	0.0392	0.0253
8.25	0.1936	0.1247	0.0803	0.0517	0.0333	0.0215
8.50	0.1635	0.1050	0.0675	0.0433	0.0278	0.0179
8.75	0.1343	0.0860	0.0551	0.0352	0.0226	0.0145
9.00	0.1060	0.0676	0.0432	0.0276	0.0176	0.0112
9.25	0.0784	0.0499	0.0318	0.0202	0.0129	0.0082
9.50	0.0516	0.0327	0.0208	0.0132	0.0084	0.0053
9.75	0.0254	0.0161	0.0102	0.0065	0.0041	0.0026
10.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

$$A_x - 1.4466 = \left[\frac{1.3277}{(1.3277^2 - 1)^{0.7}}\right]\left[\frac{(1{,}250{,}000)(365)t}{(2.225 \times 10^9)}\right]$$

or:

$$A_x = 1.4466 + 0.3291t \qquad (9\text{–}25)$$

Interpolating from Table 9–3 and assuming that the compressibility factor, z, is 1.00, the production forecast is set out in Table 9–4 for intervals of six months. The results also are shown in Figure 9–5 on semilogarithmic coordinates as a continuation of the first portion of the problem when the well had excess capacity and production was limited to 1,250 Mcfd. The instantaneous rate of flow at the end of the time periods (fifth column of Table 9–4) was calculated by means of equation 9–24:

$$q = q_i\left(\frac{x^2 - 1}{x_i^2 - 1}\right)^n \qquad (9\text{–}24)$$

Equation 9–24 is inferred readily from the back-pressure relationship for gas wells.

Table 9–4 *Production forecast for a well producing against a constant back pressure of 1,000 psia.*

$q_i = 1{,}250$ Mcfd
$G_i = 2.225 \times 10^9$ cu ft
$n = 0.70$

Initital shut-in pressure = 1,327.7 psia
Constant back pressure = 1,000.0 psia
$z = 1.00$

t Time (yr)	A_x	x	Shutin Pressure (psia)	q at End of Time Period (Mcfd)	ΔGp Gas Produced in Interval (Mcf)	Gp Cumulative Gas Production (Mcf)	q Average Rate (Mcfd)
0.0	1.4466	1.3277	1,327.7	1,250	0	0	0
0.5	1.6112	1.2115	1,211.5	888	194,731	194,731	1,067
1.0	1.7757	1.1303	1,130.3	616	136,077	330,808	746
1.5	1.9403	1.0751	1,075.1	411	92,506	423,314	507
2.0	2.1048	1.0391	1,039.1	257	60,330	483,644	331

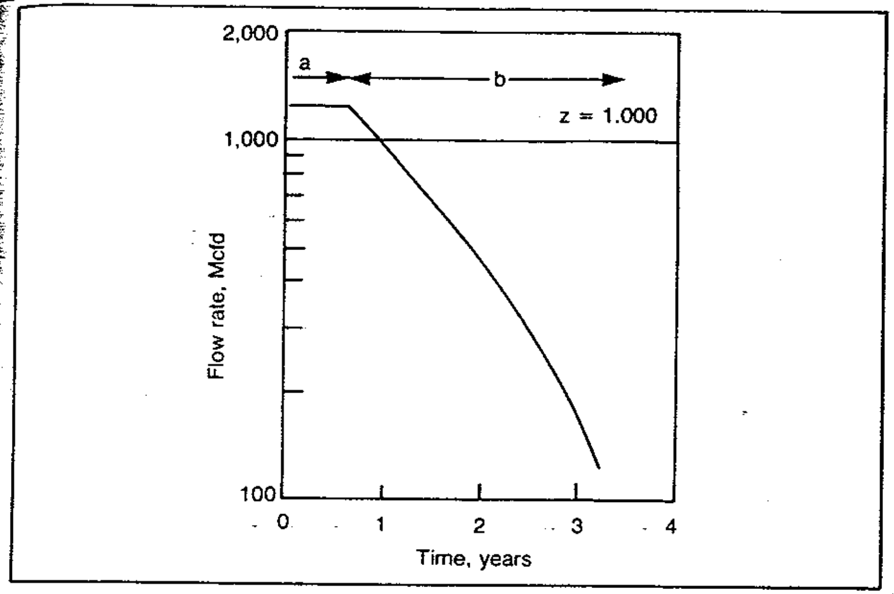

Figure 9–5 *Producing rate for a gas well producing a constant back pressure (a) before breaking point and (b) after breaking point.*

Figure 9–5 shows that the producing rate for the well in this particular example decreases rapidly when the well produces against a constant pressure of 1,000 psia. In other words, production is limited by the high line pressure. In actual practice, a compressor probably would be installed as soon as the drop in producing rate became significant. Also, a comparison between Figures 9–4 and 9–5 shows the effect of the premises as to back pressure conditions on the production forecast.

To the best of the author's knowledge, this is the first published detailed treatment of production forecasting for gas wells, even though forecasting has been done for years. As a matter of fact, the subject has received little attention in the literature except for the information in references 1, 2, and 3 given at the end of this chapter. The methods for forecasting given herein, except for the theoretical method in Chapter 8, are of an empirical nature and evidence of their utility will be given in Chapter 10. The methods were developed to give the practicing engineer a ready and straightforward method for forecasting gas production from wells. As a result of the lack of publications on forecasting methods, one can only speculate as to the actual methods in use in the industry. Probably, most large companies and consulting firms use stepwise material balance methods that require large, detailed computational models. However, the purpose here has been to present methods compatible with small computers.

SUMMARY

The methods presented in this chapter for forecasting production from gas wells are essentially empirical because they are based on the back-pressure equation. These methods do not account for gas produced when the well is producing under unsteady-state conditions. Also, the methods do not fully account for the change in the compressibility factor for the gas as the pressure decreases in the reservoir except as indicated for the method for estimating the time that a well will produce against a constant pressure when it has excess productive capacity. For the cases of a constant fraction of current shut-in pressure and a constant back pressure, the changes in the compressibility factor can be treated adequately only by small steps with large computer facilities. Each of the methods requires an independent means for determining the gas in place available for producing the well. Unfortunately, the engineer may need to make the production forecasts at a time when very little is known about the extent of the reservoir and the volume of gas in place available for producing the well. Thus, ignoring the changes in the compressibility of the gas may be justified in many forecasts. The method for making a production forecast as presented in Chapter 8 overcomes the difficulty caused by the changing compressibility of gas. If it is applied to a well in which the radius of drainage has not been established, it is then necessary to predict when the pressure disturbance will reach equilibrium with the pressure disturbance from other wells.

The methods have been presented without any proof of applicability to actual gas wells. Chapter 10 shall deal with production decline curves for gas wells and will present data on producing wells to show applicability.

PROBLEMS

1. For a gas well, use a stabilized back-pressure equation in which the difference in pressure squared is expressed in thousands (see Fig. 7–1 and Table 7–3 for additional information) at the wellhead:

$$q_{sc} = 42.96\,(\overline{p}_{ts}^{\,2} - p_{tf}^{\,2})^{0.647}$$

The current shut-in wellhead pressure is 1,510 psia, and the gas in place available for production from the well at the current shut-in pressure is 2.1 billion cu ft.

a. Calculate the length of time that the well will deliver a steady production rate of 1,000 Mcfd against a back pressure of 915 psia.

b. Estimate the yearly production rates after (a) if the well continues to produce against the constant pressure of 915 psia.

c. Estimate the yearly production rates after (a) if the back pressure is reduced to the constant fraction of the current shut-in pressure that existed at the time the well failed to make 1,000 Mcfd against the constant back pressure of 915 psia.

2. A well with a very thin section of relatively high-permeability reservoir produces 1,500 Mcfd against a back pressure of 850 psia at a current shut-in wellhead pressure of 3,265 psia. Assume that n is 0.70. The pipeline will take any amount of gas up to about 10,000 Mcfd. The well is in a large common reservoir with several large-capacity wells, and the allowable for the subject well is subtracted from the total allowable to be divided among the capable wells. (In other words, the subject well has very little influence on the pressure production performance of the reservoir.) On the average, the wellhead shut-in pressures of all the wells have been decreasing about 150 psi/year. It is proposed to install a compressor to reduce the pressure on the well to a constant 425 psia. Calculate the increase in gas production per year over the next five years as a result of lowering the back pressure to 425 psia.

3. A gas well in the Texas Hugoton field has a current deliverability of 825 Mcfd with a shut-in pressure of 88 psia. Its average allowable for the past year has averaged 72% of the deliverability, or 594 Mcfd, and is expected to remain constant in the future. Historically, the wellhead pressure has decreased about 10 psi/year. Calculate the time required for the well to produce at 100% of the deliverability.
Note: Deliverability is defined in the Texas Hugoton field essentially as the rate of flow of a well against a pressure that is 80% of the wellhead shut-in pressure. Assume that n is 0.85.

REFERENCES

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2. Stewart, P.R. "Low-Permeability Gas Well Performance at Constant Pressure." *J. Pet. Tech.* (September 1970), p. 1149.
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10 Production Decline Curves for Gas Wells

In Figures 8–1A and 8–1B and the discussion in Chapter 8, the results of the theoretical development of a general production decline curve for gas wells producing against a constant back pressure were given. In Chapter 9, the problem was approached from the traditional empirical concept of gas well behavior. In the present chapter, the empirical approach shall continue to be covered and several actual well histories will be presented to show that the empirical approach is useful in practical natural gas engineering.

In Chapter 9, it was convenient to present production forecasts for a hypothetical gas well on semilogarithmic coordinates with the production rate shown on the logarithmic coordinate and time shown on the normal coordinate. In Figure 9–4 after the well reached the breaking point, the plot of the logarithm of rate against time was very nearly a straight line for a well producing against a pressure that was a constant fraction of the current shut-in pressure. Actually, the calculated rate was slightly more optimistic than the straight line in that the calculated rates were larger than rates found by projecting the straight line. In Figure 9–5 in which the well was producing against a constant back pressure, the calculated rates were less optimistic than a straight line.

This chapter shows how the ideas developed in previous chapters can be consolidated into generalized production decline curves for gas wells and compares the theoretical decline curves with actual gas well behavior.

Equations 9–9, 9–10, 9–19, 9–20, and 9–22 can be rearranged so the dimensionless rate of flow, q/q_i, is a function of the dimensionless time, $q_i t/G_i$, for various values of the exponent n and the ratio x_i, of the original shut-in pressure to the constant back pressure imposed upon the well. As the original shut-in pressure becomes large with respect to the constant back pressure, x_i^2 becomes large with respect to unity, and the problem becomes equivalent mathematically to the solution in which the well produces at a constant fraction of the current open-flow potential or against a back pressure that is a constant fraction of the current shut-in pressure. This is illustrated in Figures 10–1, 10–2, 10–3, 10–4, 10–5, and 10–6 for values of the exponent n from 0.5 to 1.0. The dimensionless rate of flow, q/q_i, has been plotted against the dimensionless time, $q_i t/G_i$, on logarithmic coordinates on these six figures. Numerical values for the relationships are given in Table 10–1 for the reader who wishes to make plots correspond to Figures 10–1 to 10–6, inclusive.* The solutions given in the tables and figures were calculated from analytical solutions when available and otherwise from numerical solutions.

COMPARISON TO THE THEORETICAL DECLINE CURVE

In Chapter 8, a production forecast was made from the theoretical approach to the unsteady-state flow theory for gas wells, but in Chapter 9 an empirical approach was developed. Using the same well example that was used in Chapter 8, one can start at the stabilization point shown in Table 8–9 and make an empirical production forecast for the same gas well using the same premises regarding the reservoir. Here, assume the well is producing against a constant back pressure of 662 psia in the wellbore at the level of the reservoir. Use the availability integral, A_∞, from Table 9–3 for an exponent n value of 0.70, but assume that the compressibility of the gas is 1.000 and use it as such in the material balance. The results are shown in Figure 10–7 and are identified as the empirical method.

The difference in the forecast rates is not too bad for the 10 years after stabilization, and at 10 years the instantaneous predicted production rate is 344 for the empirical method versus 315 Mcfd for the theoretical method. Considering the assumptions involved in the empirical method, the chart reading of the theoretical method, and the widely different approaches to the problem, one could consider the agreement to be good. The theoretical method has a serious flaw in that it ignores the additional pressure loss caused by the rate-dependent non-Darcy flow near the wellbore. However,

* The author recommends coordinates with at least 3-in. cycles.

Table 10–1 Variation of producing rate with time for wells producing against various constant back pressures and various n values, z = 1.00.

	Values of $(q_i t/G_i)$									
	$n = 0.5$					$n = 0.6$				
q/q_i	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.95	0.051	0.051	0.038	0.028	0.018	0.043	0.042	0.032	0.024	0.015
0.90	0.105	0.104	0.078	0.057	0.037	0.089	0.088	0.066	0.048	0.031
0.80	0.223	0.221	0.162	0.118	0.075	0.189	0.187	0.139	0.101	0.064
0.70	0.357	0.352	0.254	0.181	0.114	0.306	0.303	0.220	0.158	0.100
0.60	0.511	0.504	0.353	0.249	0.154	0.444	0.439	0.312	0.221	0.138
0.50	0.693	0.682	0.462	0.320	0.196	0.612	0.603	0.417	0.292	0.180
0.40	0.916	0.899	0.580	0.394	0.238	0.825	0.811	0.539	0.370	0.225
0.35	1.050	1.027	0.643	0.433	0.260	0.956	0.938	0.608	0.413	0.250
0.30	1.204	1.174	0.709	0.472	0.282	1.111	1.088	0.683	0.460	0.276
0.25	1.386	1.344	0.776	0.512	0.304	1.300	1.267	0.766	0.510	0.304
0.20	1.609	1.546	0.846	0.552	0.326	1.538	1.492	0.858	0.564	0.334
0.15	1.897	1.794	0.918	0.593	0.349	1.859	1.785	0.962	0.624	0.367
0.10	2.303	2.105	0.991	0.634	0.371	2.339	2.197	1.082	0.692	0.405
0.07	2.659	2.332	1.036	0.659	0.384	2.788	2.546	1.165	0.739	0.430
0.05	2.996	2.502	1.066	0.676	0.393	3.238	2.853	1.228	0.775	0.449
0.03	3.507	2.685	1.096	0.692	0.402	3.970	3.254	1.301	0.815	0.471
0.015	4.200	2.830	1.118	0.705	0.409	5.068	3.662	1.368	0.853	0.492
0.010	4.605	2.879	1.126	0.709	0.411	5.772	3.835	1.395	0.868	0.500

Table 10–1, continued

q/q_i	n = 0.7					n = 0.8				
	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.95	0.037	0.037	0.028	0.020	0.013	0.032	0.032	0.024	0.018	0.012
0.90	0.076	0.076	0.057	0.042	0.027	0.067	0.066	0.050	0.037	0.024
0.80	0.165	0.163	0.121	0.088	0.056	0.145	0.144	0.107	0.078	0.050
0.70	0.268	0.265	0.194	0.140	0.089	0.239	0.236	0.173	0.125	0.080
0.60	0.393	0.388	0.279	0.199	0.125	0.352	0.348	0.252	0.180	0.113
0.50	0.548	0.540	0.379	0.267	0.165	0.495	0.488	0.346	0.245	0.153
0.40	0.748	0.737	0.500	0.346	0.212	0.683	0.673	0.464	0.324	0.199
0.35	0.874	0.860	0.571	0.392	0.238	0.804	0.791	0.536	0.370	0.227
0.30	1.026	1.007	0.652	0.443	0.267	0.951	0.935	0.619	0.423	0.257
0.25	1.215	1.189	0.744	0.499	0.300	1.136	1.115	0.716	0.485	0.292
0.20	1.460	1.423	0.852	0.565	0.336	1.381	1.350	0.835	0.558	0.334
0.15	1.799	1.741	0.983	0.642	0.379	1.728	1.681	0.986	0.649	0.384
0.10	2.327	2.219	1.147	0.737	0.431	2.286	2.200	1.190	0.768	0.451
0.07	2.845	2.662	1.274	0.809	0.471	2.851	2.708	1.358	0.865	0.504
0.05	3.384	3.089	1.379	0.868	0.503	3.459	3.229	1.507	0.950	0.550
0.03	4.308	3.729	1.513	0.943	0.543	4.541	4.080	1.715	1.067	0.614
0.015	5.800	4.520	1.655	1.022	0.586	6.384	5.295	1.960	1.204	0.688
0.010	6.819	4.920	1.721	1.059	0.606	7.706	5.999	2.086	1.274	0.726

Table 10–1, continued

q/q_i	n = 0.9					n = 1.0				
	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25	$x_i \to \infty$	$x_i = 10$	2	1.5	1.25
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.95	0.029	0.029	0.022	0.016	0.010	0.026	0.026	0.019	0.014	0.009
0.90	0.060	0.059	0.045	0.033	0.021	0.054	0.054	0.040	0.030	0.019
0.80	0.130	0.129	0.096	0.070	0.045	0.118	0.117	0.087	0.064	0.041
0.70	0.215	0.212	0.157	0.114	0.072	0.195	0.193	0.143	0.104	0.066
0.60	0.319	0.315	0.229	0.165	0.104	0.291	0.288	0.210	0.152	0.096
0.50	0.451	0.445	0.318	0.226	0.141	0.414	0.409	0.294	0.210	0.132
0.40	0.628	0.620	0.432	0.303	0.187	0.581	0.573	0.404	0.285	0.177
0.35	0.743	0.732	0.503	0.350	0.215	0.690	0.681	0.472	0.331	0.204
0.30	0.885	0.870	0.586	0.404	0.246	0.826	0.813	0.555	0.385	0.236
0.25	1.065	1.046	0.686	0.468	0.284	1.000	0.983	0.656	0.450	0.274
0.20	1.306	1.280	0.812	0.547	0.328	1.236	1.213	0.785	0.532	0.321
0.15	1.655	1.615	0.977	0.648	0.385	1.582	1.548	0.960	0.641	0.383
0.10	2.228	2.158	1.212	0.788	0.464	2.162	2.102	1.220	0.798	0.471
0.07	2.826	2.710	1.419	0.909	0.530	2.780	2.682	1.459	0.940	0.550
0.05	3.483	3.298	1.612	1.019	0.591	3.472	3.319	1.692	1.075	0.624
0.03	4.689	4.319	1.897	1.181	0.679	4.774	4.470	2.054	1.283	0.738
0.015	6.832	5.930	2.266	1.389	0.792	7.165	6.419	2.558	1.568	0.893
0.010	8.428	6.965	2.471	1.503	0.854	9.000	7.768	2.857	1.735	0.984

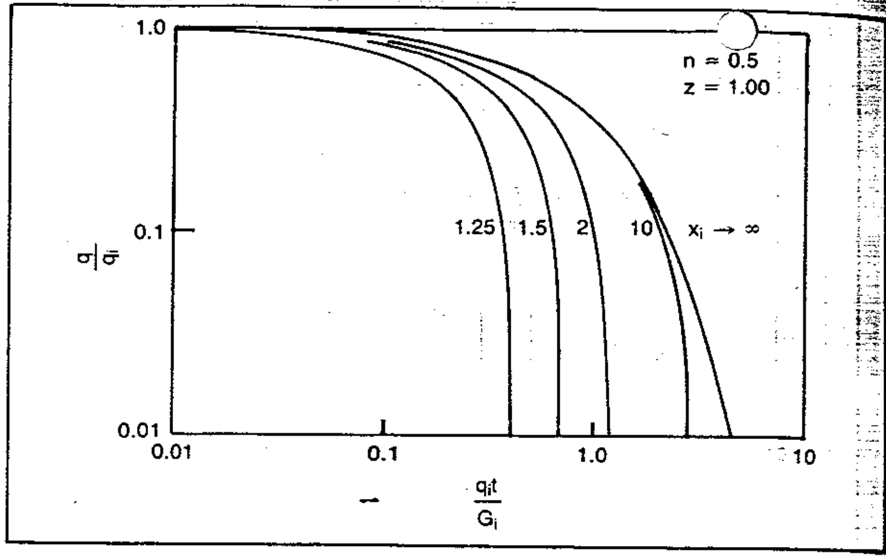

Figure 10–1 *Variation of production rate with time for wells producing against various constant back pressures,* n = *0.5.*

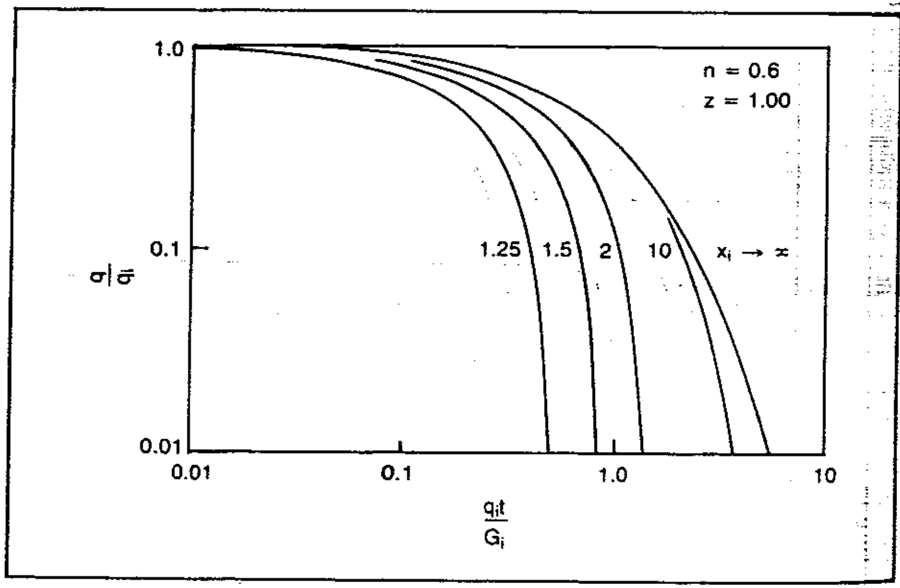

Figure 10–2 *Variation of production rate with time for wells producing against various constant back pressures,* n = *0.6.*

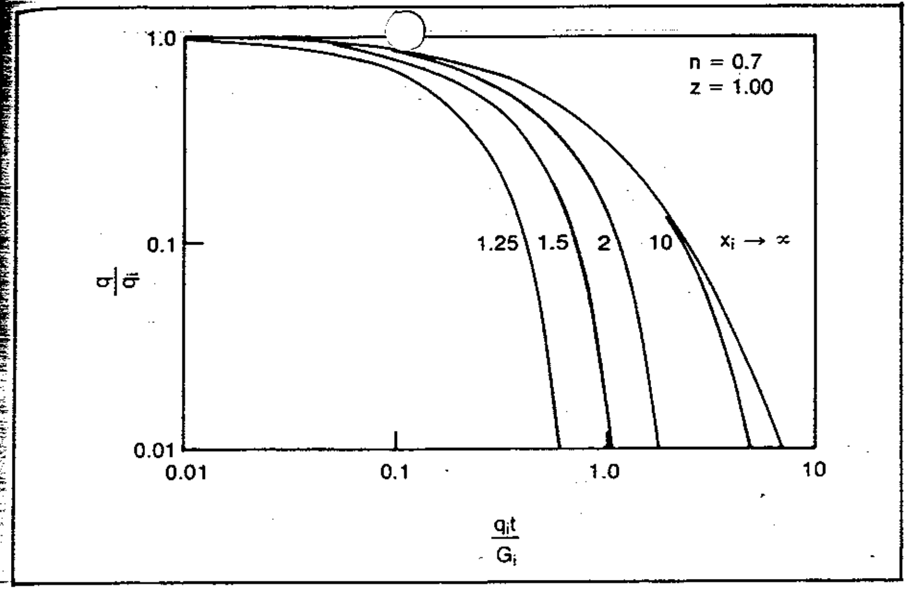

Figure 10–3 *Variation of production rate with time for wells producing against various constant back pressures,* n = *0.7.*

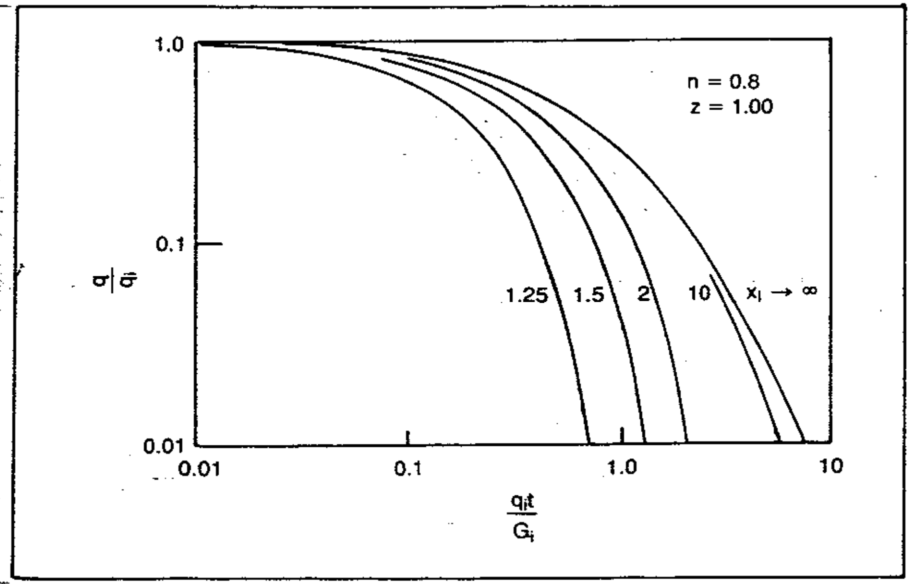

Figure 10–4 *Variation of production rate with time for wells producing against various constant back pressures,* n = *0.8.*

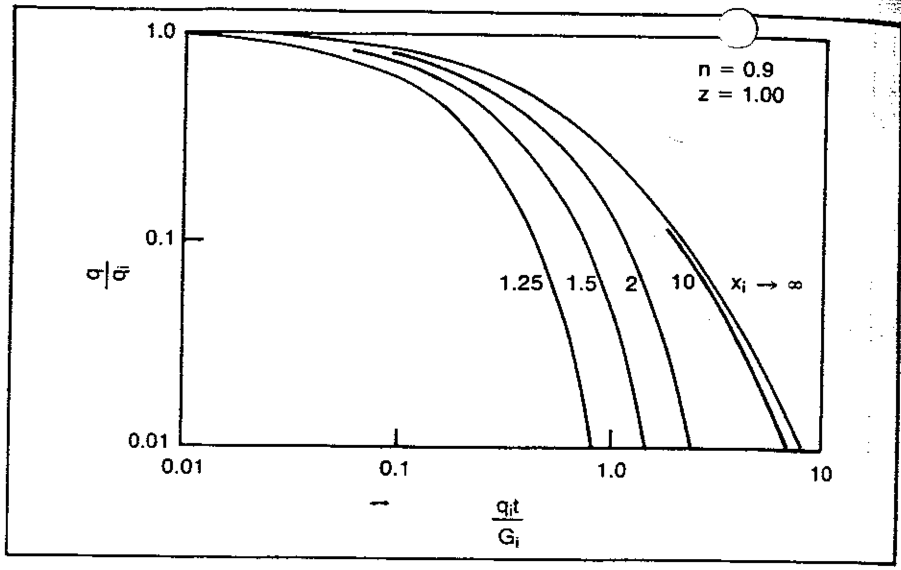

Figure 10–5 *Variation of production rate with time for wells producing against various constant back pressures,* n = *0.9.*

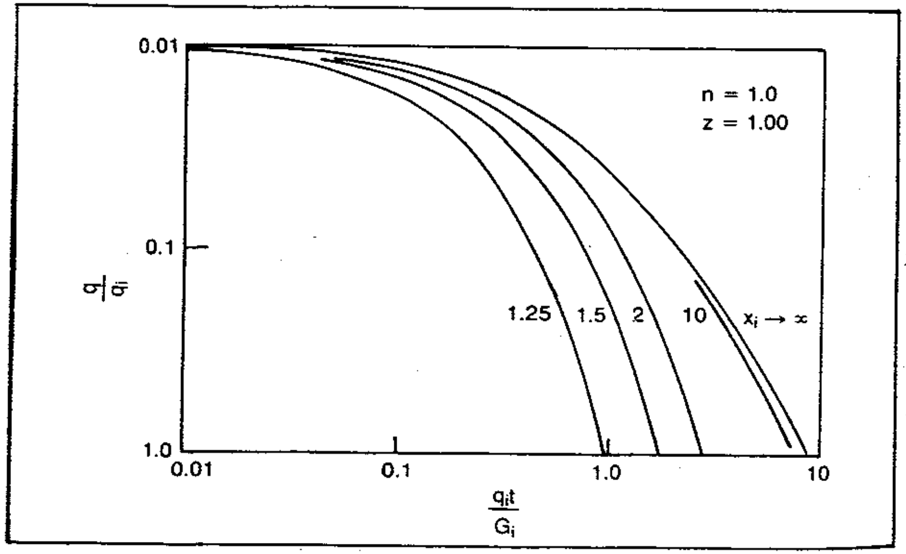

Figure 10–6 *Variation of production rate with time for wells producing against various constant back pressures,* n = *1.0.*

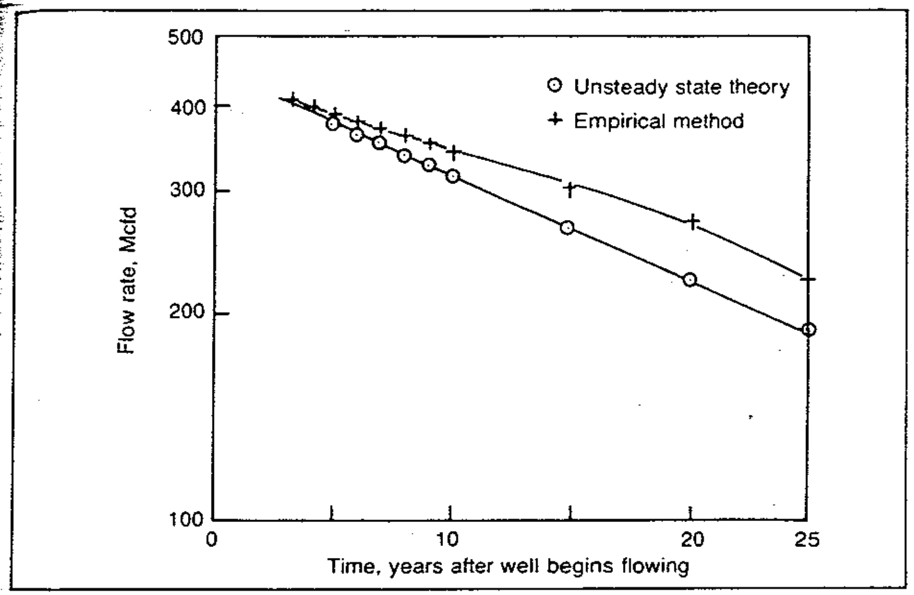

Figure 10–7 *Comparison of production forecasts by the unsteady-state theory with the empirical method.*

the empirical method ignores the compressibility of the gas by assuming it to be 1.000. In the example selected, the exponent *n* is taken as 0.70, which indicates a large pressure drop caused by non-Darcy flow. This, in turn, results in a significant difference in the two methods.

PRODUCTION DECLINE BEHAVIOR OF ACTUAL GAS WELLS

The production for the complete life of a deep well in West Texas is shown in Figure 10–8 in which production rate expressed in Mcf/month is shown plotted against time expressed in months on logarithmic coordinates. The well was remote from other producing wells, but fortunately it was close to a pipeline that operated at about 800 psig. The well produced from a reservoir with good permeability at a depth below 16,000 ft. It was opened into the line and checked periodically. After a producing life of 38 months, it was plugged and abandoned because operating it at a rate of less than 1,000 Mcf/month was uneconomical at the price then paid for gas in the area. Cumulative production to abandonment was 388,259 Mcf. The well

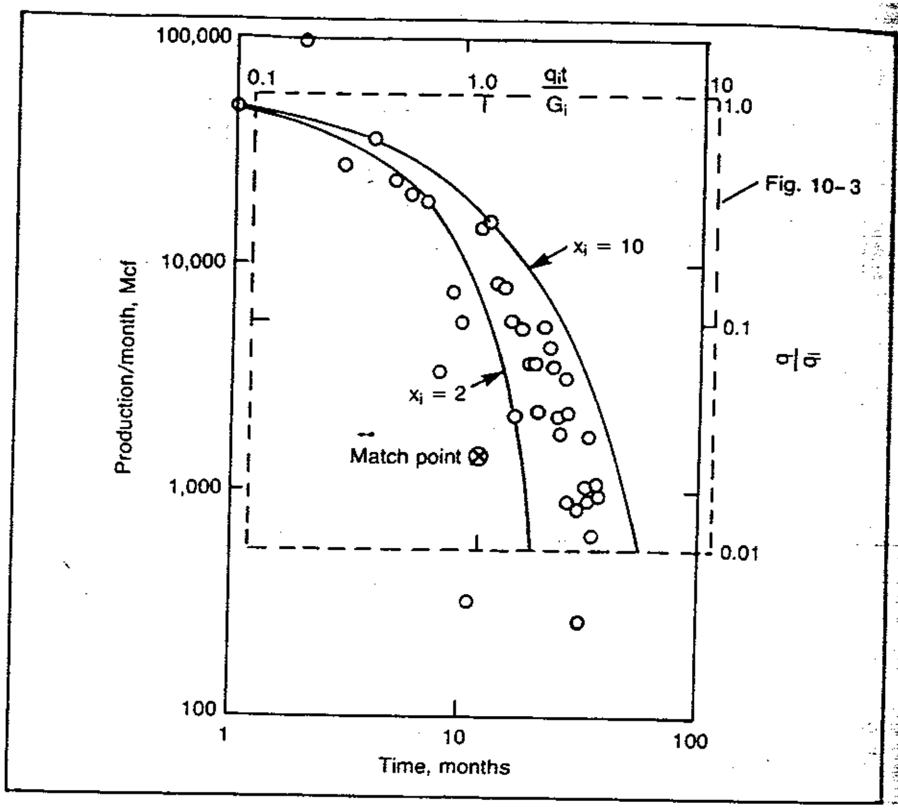

Figure 10–8 *Production decline behavior for a deep, high-pressure well completed in a limited reservoir, West Texas.*

produced small amounts of hydrocarbon liquid and water, and occasionally liquid accumulation in the wellbore interfered with the rate of production, which is apparent from examining the data in Figure 10–8.

Figure 10–3 is used as a type curve and is shown in Figure 10–8 as the dashed line coordinates and the two solid-line curves for x_i = 2 and 10. The pressure in the pipeline probably caused the well to perform as if it were producing against a constant back pressure of x_i = 7 to 8. This indicates that the original shut-in pressure was about 5,500 to 6,400 psia. The fit of the curves in Figure 10–8 has been selected to give the most weight to the points on the right-hand side of the bulk of the data because these points were believed to be least influenced by liquid in the wellbore. The match point, as indicated in Figure 10–8, is:

1,400 Mcf/month is equivalent to q/q_i = 0.0255
12 months is equivalent to $q_i t/G_i$ = 1.00

for which:
q_i = 55,000 Mcf/month
G_i = 659,000 Mcf

At abandonment, the well had produced about 59% of the initial gas in place. The well was completed in a limited reservoir and an offset well did not find the reservoir.

The production history of a well completed in a Mississippian reservoir in Oklahoma is shown in Figure 10–9. The original shut-in pressure at the bottom of the hole was about 2,070 psia. The pipeline pressure at the wellhead varied from 440 to 470 psig, averaging about 450 psig over the 75-month producing life of the well. However, Figure 10–9 shows only the first 38 months of production because production thereafter dropped to 500 Mcf or less per month. These latter production data points have not been shown because the performance of the well was believed to have been damaged by the presence of liquids in the wellbore. Total production to abandonment

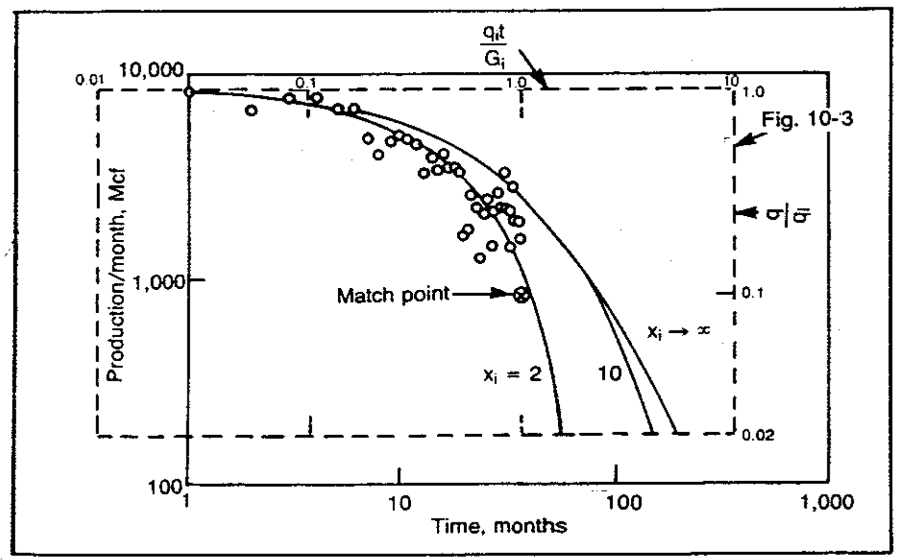

Figure 10–9 *Production decline behavior for a Mississippian well in a limited reservoir, Oklahoma.*

was 155,459 Mcf and 1,934 bbl of condensate. Water production was estimated to be about equal to that of the condensate.

In fitting the type curve on Figure 10–9, most of the weight for fixing the position of the type curve was given to the points indicating the best performance for the well—that is, the largest producing rates for the longest time on the time coordinate. For the match point indicated in Figure 10–9:

830 Mcf/month is equivalent to $q/q_i = 0.10$
38 months is equivalent to $q_i t/G_i = 0.98$

for which:

$q_i = 8{,}300$ Mcf/month
$G_i = 322{,}000$ Mcf

A number of shut-in pressures (seven-day duration) were taken during the life of the well, and it has been possible to plot the relationship between $\bar{p}_R/z$ and cumulative production as shown in Figure 10–10. If the most favorable point (see Fig. 10–10) is taken as an indication of the proper relationship between $\bar{p}_R/z$ and cumulative production, the resultant original gas

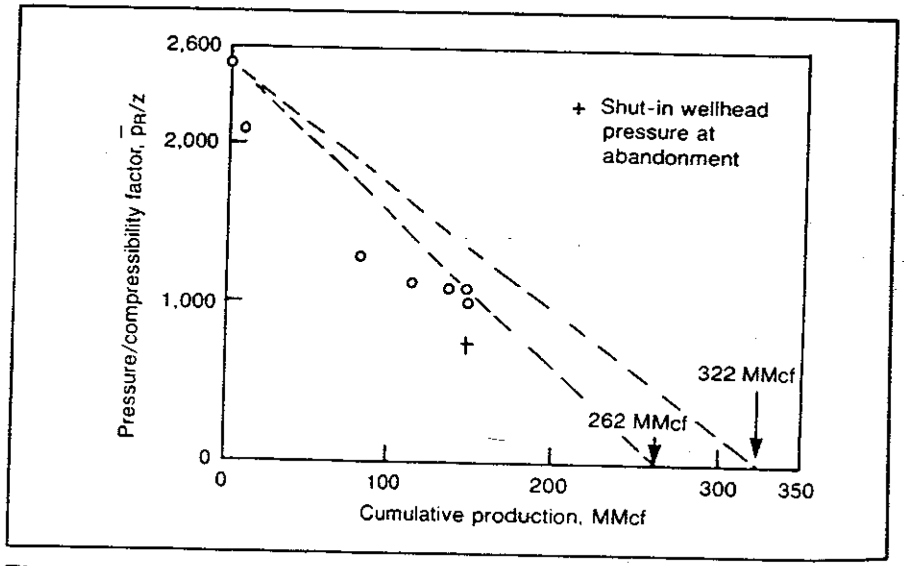

Figure 10–10 $\bar{p}_R/z$ *against cumulative production for a Mississippian well in a limited reservoir, Oklahoma.*

in place is 262,000 Mcf. The relationship between $\bar{p}_R/z$ and cumulative production for the well is by no means a straight line. If one uses the original gas in place as indicated by the production decline curve in Figure 10–9, one can draw a straight line on Figure 10–10 from the initial $\bar{p}_R/z$ of 2,501 to 322,000 Mcf. The result bears a believable relationship to the $\bar{p}_R/z$ data when one realizes that the performance of the well was probably influenced by the presence of liquids in the wellbore and unsteady-state effects. One cannot consider the seven-day pressure measurements shown in Figure 10–10 as completely built-up pressure; therefore, the true $\bar{p}_R/z$ versus cumulative production curve should lie above and to the right of the data points in Figure 10–10.

For the third example, the production history of a gas well in central Pennsylvania is shown in Figure 10–11. The well discovered a long, narrow, and relatively dry gas reservoir in which several additional wells were eventually drilled. Compressors were installed and the back pressure on the well was lowered in an effort to maintain producing rates. The data shown in Figure 10–11 covers the virtual life of the reservoir and the wells over a period of about 100 months. However, not all of the monthly rates are given

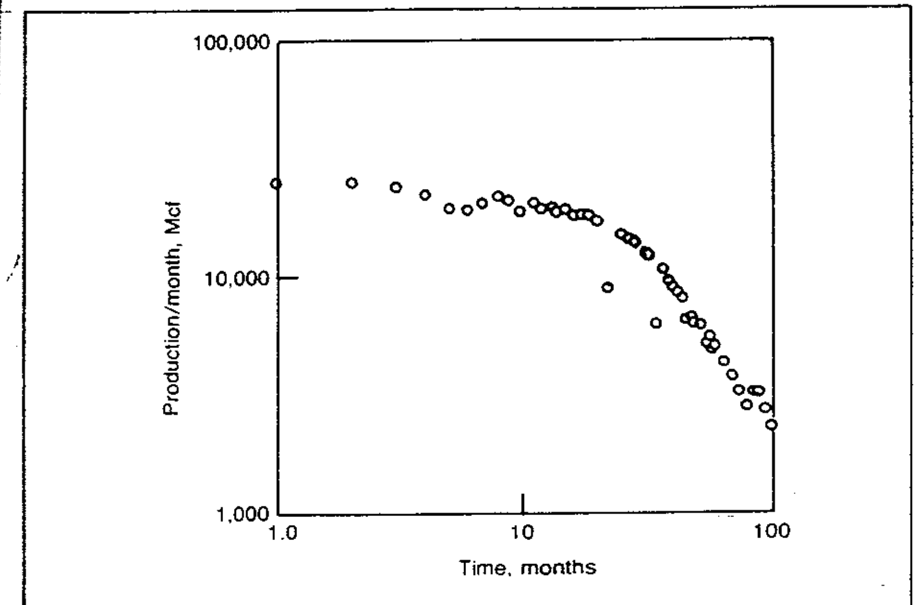

Figure 10–11 *Production history for a gas well in Pennsylvania.*

in the figure in order to reduce the congestion of data points. Monthly production rates are shown up to 20 months, production rates for every other month are given from 20 to 60 months, and rates are for every fifth month from 60 months on for the life of the well.

A comparison of the data in Figure 10–11 with the empirical production decline curves in Figures 10–1 to 10–6 indicates that the data in Figure 10–11 cannot be fitted in entirety to a single curve. However, the producing life up to about 25 months can be fitted very well to the limiting curve as x_i approaches infinity in Figure 10–3 as shown in Figure 10–12. In other words, the well acted as if it were producing against a changing back pressure kept at a constant fraction of its current shut-in pressure for its early producing life. The match point, as indicated in Figure 10–11, is:

16,300 Mcf/month is equivalent to $q/q_i = 0.58$
38 months is equivalent to $q_i t/G_i = 0.69$

for which:

$q_i = 28{,}100$ Mcf/month
$G_i = 1{,}550{,}000$ Mcf

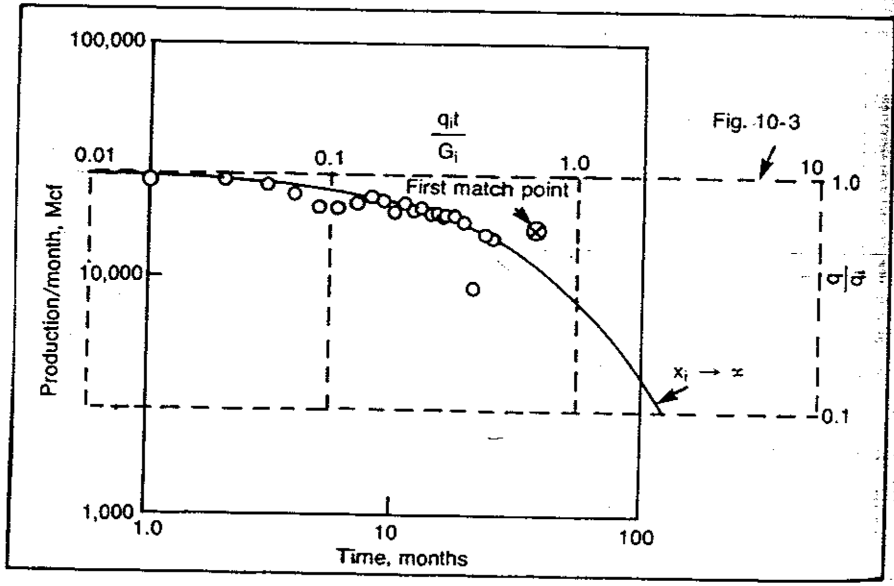

Figure 10–12 *Production decline behavior in early producing life for a gas well in Pennsylvania.*

If one accepts that the production decline relationship could change with time during the life of the well, one makes a second comparison using Figure 10–3, which is shown in Figure 10–13. Here, the fit to the $x_i \to \infty$ curve is considered good, but the most weight has been given to the production points that lie above and to the right of the bulk of the points. This is based on the philosophy that little happens to a gas well to make it perform better than it should. The match point for the performance during the later producing life is:

6,600 Mcf/month is equivalent to $q/q_i = 0.172$
84 months is equivalent to $q_i t/G_i = 2.45$

for which:

$q_i = 38{,}400$ Mcf/month
$G_i = 1{,}320{,}000$ Mcf

The second value of the initial gas in place is 230,000 Mcf less than the indicated early life value of 1,550,000 Mcf, or a 15% decrease.

Recalling that the well in this example was the discovery well in the

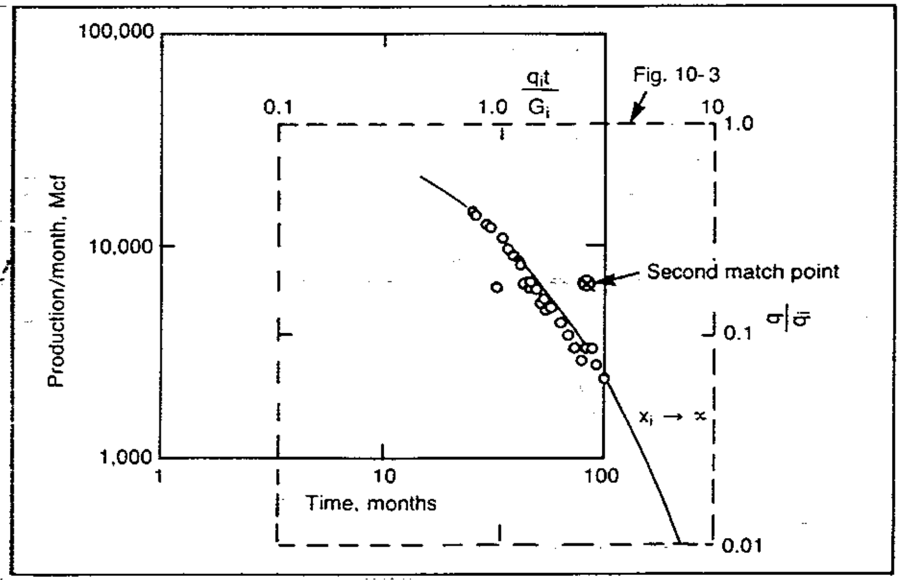

Figure 10–13 *Production decline behavior in late producing life for a gas well in Pennsylvania.*

reservoir and that other wells were completed in the same reservoir and placed on production, it is logical to assume that the completion of other wells in the same reservoir would reduce the amount of gas in place that was available for production from the discovery well in the reservoir. The timing of the occurrence of the reduction of the amount of gas in place would depend on reservoir characteristics such as permeability, porosity, and shape. The size of the reduction would depend, for the most part, on the producing rates of the wells involved and the pressures in the reservoir.

It is well worth noting that the performance of the well in the third example was that of a well producing at a constant fraction of its current open flow potential. This is the same problem mathematically as for a well producing against a back pressure that is a constant fraction of the current shut-in pressure. This performance contrasts with the first and second well examples in which the wells produced against a nearly constant back pressure.

For the fourth example of a production decline history, consider a gas well in the Texas Hugoton field. The well was placed on production in the late 1940s and has produced gas since that time. Production for the past eight years follows:

Year	*Production (Mcf)*
1972	271,588
1973	327,535
1974	362,905
1975	331,972
1976	317,555
1977	250,887
1978	233,503
1979	211,953

Although the production history is a limited amount of information, use type production decline curves to determine the amount of gas in place. Later, additional information on the performance of the well will be introduced to check the conclusions derived from the production decline study.

The first conclusion to be reached from the production history is that the well is a large-capacity producer, considering the present low pressure prevalent in the Texas Hugoton field. Also, note that the producing rate increased to a maximum in 1974 and declined thereafter. Other things being equal, one must conclude that the well reached some sort of breaking point. Since the Texas Hugoton field is prorated, the market demand for gas after being reduced by the amount of gas to be produced by all wells not capable

of producing a full allowable is divided among the capable wells. This means that as pressures in the field decrease, the productivity of the wells decrease and the allowable assigned to the capable wells increases. This is believed to be the reason for the increase in production to 1975 for the well under consideration.

If one uses production for 1974 through 1979 and makes a log-log plot as shown in Figure 10–14 by the data points, an excellent fit can be made to the $x_i \rightarrow \infty$ curve in Figure 10–4 as depicted in Figure 10–14. Note that the yearly production is plotted at the midyear point. The match point is:

145,000 Mcf/year is equivalent to $q/q_i = 0.35$

5.7 years is equivalent to $q_i t/G_i = 0.50$

for which:

q_i = 414,000 Mcf/year

G_i = 4,720,000 Mcf

Since the analysis started with the production for 1974, the production for the period 1974–1979 inclusive must be subtracted from 4,720,000 Mcf to

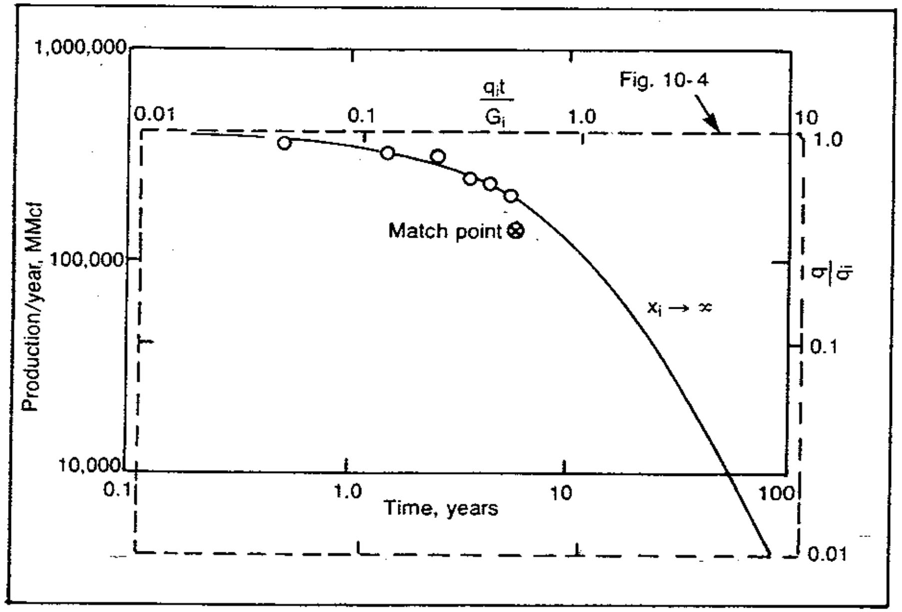

Figure 10–14 *Production decline behavior for a well in the Texas Hugoton Field.*

find the gas in place available for production from the well at the end of 1979. The result is 3,010,000 Mcf.

In order to check this result, information on test pressures and yearly production is obtained, $\bar{p}_R/z$ is calculated, and accumulative production against $\bar{p}_R/z$ is plotted in Figure 10–15. The solid line in the figure represents the least squares fitted to the data points shown. The line indicates the gas in place at the end of 1979 to be 2,550,000 Mcf but, as so often happens, there is a dilemma. The last five data points in Figure 10–15 show an awkward trend. As noted previously from the production data, the example well underwent a change in production trend for the past six years. Also, pressure measurements (taken at the wellhead) may have been influenced by liquid standing in the wellbore. Therefore, the gas in place determination as of the end of 1979 of 3,010,000 Mcf by the production decline method is probably as likely as that of 2,550,000 Mcf from the more conventional $\bar{p}_R/z$ cumulative production method.

SUMMARY

This chapter has attempted to establish the utility of using the empirical production decline curves developed in Chapter 9 to make production forecasts and to estimate the gas in place available for production from a well. Type curves are presented in Figures 10–1 through 10–6 for making comparisons with the production performance of actual wells. Also, the values used to make the plots for the type curves are presented in Table 10–1 so the reader can construct personal type curves. However, log-log coordinates with cycles no less than 3 in. on a side are recommended for the plots.

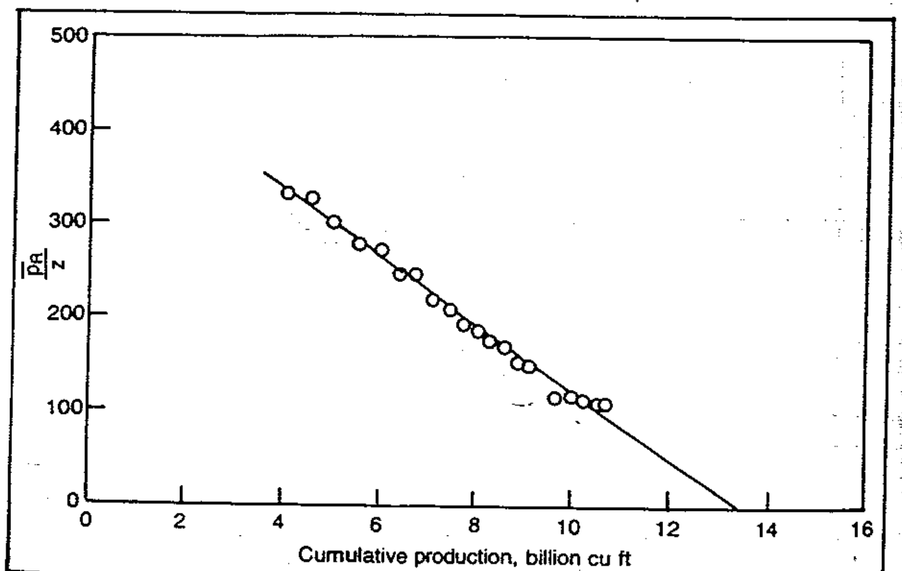

Figure 10–15 *$\bar{p}_R/z$ and cumulative production for the gas well in the Texas Hugoton Field.*

The comparison of the empirical decline method with the theoretical method of Chapter 8 for production forecasting is believed to be good, considering the widely different approaches to the problem and the flaws in each method. However, the empirical method seems to provide the most optimistic answer for the one production forecast presented herein.

Four comparisons of the empirical production decline curves are made using the production histories of actual wells. Three of the four production histories are for the entire productive life of the wells, which provides a comparison over the complete length of the type curve. The fourth comparison is made over a small time interval in the productive life of a well with a large amount of gas in place that is available for production from the well. Where the data are available, the gas in place is estimated by plots of $\bar{p}_R/z$ against cumulative production. Here the comparisons have been excellent.

The examples, except for the Oklahoma well, are from wells that stabilized quickly and were initially capable of producing gas at relatively large rates of flow. Do not attempt to apply the empirical type curves to wells with producing rates affected by unsteady-state conditions.

Unfortunately, the technical literature on production decline curves and production forecasting is practically nonexistent except for the three references given at the end of Chapter 9. It is hoped that the material presented in Chapters 9 and 10 will stimulate interest in the subject and encourage future publications.

A word of caution is necessary regarding the unit of time used in reporting gas production as practiced in some areas: Gas measurements in many places are recorded on eight-day charts, so seven-day recordings can be made on an eight-day chart with some flexibility in installing and removing the charts. Since most gas production reporting is done on a monthly basis and seven-day recordings are not compatible with monthly reporting, the least costly method for making them compatible is to arbitrarily use 28 days in the first and second "months" and 35 days in the third "month." Thus the calendar matches the arbitrary schedule every 91 days. Under this system, a gas well will show an exceptionally good quantity of production every third month. This can be a puzzling circumstance to anyone not aware of the system for making weeks fit nicely into months.

PROBLEMS

1. The production history for a large-capacity gas well follows. Estimate the initial gas in place and calculate the fraction of gas in place produced at the end of the production history.

Month	Production (Mcf)	Month	Production (Mcf)
1	137,792	25	38,195
2	156,079	26	37,856
3	132,681	27	30,186
4	136,731	28	31,671
5	120,615	29	28,547
6	115,589	30	24,549
7	103,547	31	24,018
8	101,154	32	20,904
9	90,616	33	19,660
10	78,505	34	15,847
11	74,353	35	21,624
12	68,654	36	20,655
13	68,500	37	18,012
14	62,803	38	17,638
15	57,499	39	16,076
16	56,763	40	18,732
17	57,599	41	18,244
18	56,193	42	15,752
19	50,461	43	14,640
20	49,463	44	16,198
21	48,055	45	18,392
22	43,747	46	14,758
23	43,482	47	13,082
24	38,089	48	13,692

2. Make similar calculations as in problem 1 for the following production history for an actual gas well.

Month	Production (Mcf)	Month	Production (Mcf)
1	1,302	19	6,209
2	8,400	20	5,855
3	10,838	21	4,933
4	11,559	22	4,569
5	10,659	23	4,227
6	12,159	24	3,639

Month	Production (Mcf)	Month	Production (Mcf)
7	11,949	25	3,510
8	12,563	26	3,135
9	12,283	27	3,125
10	12,746	28	2,664
11	12,398	29	2,631
12	12,125	30	2,440
13	12,439	31	2,385
14	12,146	32	2,148
15	11,711	33	3,017
16	10,254	34	1,719
17	8,152	35	1,673
18	7,866	36	1,562

3. Examine the four examples given in Chapter 10 and your results for problems 1 and 2 to determine the least amount of production history that would have been necessary to derive about the same amount of gas in place as was determined using the given production history.

11 Sizing Flow Strings For Gas Wells

A flow string, or the tubing installed inside the casing of a gas well, can serve at least four important functions. First, if the tubing is installed with a packer near the bottom of the hole, the tubing protects the casing from the pressures of the well fluids in the tubing. Second, it protects the casing from the corrosive effects of the flowing fluids. Third, if sized and set properly, the use of tubing can keep the wellbore free of hydrocarbon liquid and water that otherwise might stand in the wellbore. Fourth, the conductor string must be large enough to permit the well to meet the largest demand for gas within the restraint imposed by the back pressure at the wellhead. The fourth listed function is, within various limits, the most important function. The first two functions are principally subject to mechanical and metallurgical design and will not be discussed here.

TUBING SIZES REQUIRED FOR VARIOUS RATES OF FLOW

The production of gas from a reservoir into a pipeline or gathering system involves the pressure loss from the reservoir into the wellbore, the pressure loss from lifting the gas from the bottom of the well to the surface, overcoming friction in the flow string, and the pressure loss caused by cooling the gas from the reservoir to surface temperatures. The two pressure losses caused by lifting the gas and overcoming friction will be combined to show the effect of the size of the flow string upon the capacity of a well to deliver gas at the wellhead. Assuming that the mechanical features of the tubing design present no difficulties, the sizing is reduced to a balance between well productivity, market demand for gas from the well, lifting liquids from

the wellbore, and costs. The first step in sizing tubing is to establish the back-pressure curve for bottom-hole conditions. The bottom-hole shut-in pressure, $\bar{p}_R$, and selected rates of flow with corresponding flowing pressures, p_{wf}, should be converted to wellhead pressures using the methods outlined in Chapter 5 for several tubing sizes. The next step is to make production forecasts as outlined in Chapters 8 and 9. The production forecasts should be evaluated economically to determine which tubing size gives the best return on investment. However, experience will show that, for wells of moderate productive capacity in known areas, it is not necessary to make the production forecasts and economic analyses, but where the size of tubing influences the sizes of the drilled hole and casing, the complete analysis should be made.

The results of this procedure for tubing design are illustrated for a hypothetical gas well. The shut-in pressures and back-pressure characteristics for the well are given in Table 11–1 and in Figure 11–1. The shut-in pressure and the flowing pressures at the bottom of the hole (depth = 10,250 ft) are converted to wellhead pressures by the methods given in Chapter 5 for the five sizes of tubing. Wellhead back-pressure data are calculated as shown in Table 11–2, and back-pressure performance curves at the wellhead are given in Figure 11–2. If a well with a shut-in pressure of 3,000 psia at the bottom of the hole and a corresponding wellhead pressure of 2,437 psia is opened to flow against a wellhead pressure of 1,000 psia, the rates of flow would vary from 11,500 Mcfd for 2⅜-in. OD tubing to 56,700 Mcfd for 4½-in. tubing (see Table 11–3). Also, flow rates for other pressure conditions are given in Table 11–3.

Table 11–1 *Shut-in pressures and back pressure characteristics for a hypothetical gas well.*

$\bar{p}_{ts}$ (psia)	$\bar{p}_{R}$ or p_{wf} (psia)	$\bar{p}_R^2 - p_{wf}^2$ (M)	q_g (Mcfd)
2,437	3,000	—	0
—	2,990	59.9	3,000
—	2,972	167.2	6,000
—	2,943	338.8	10,000
—	2,844	911.7	20,000
—	2,719	1,607.0	30,000

γ_g = 0.625, H = 10,250 ft, bottom-hole temperature = 251°F, and wellhead temperature = 130°F

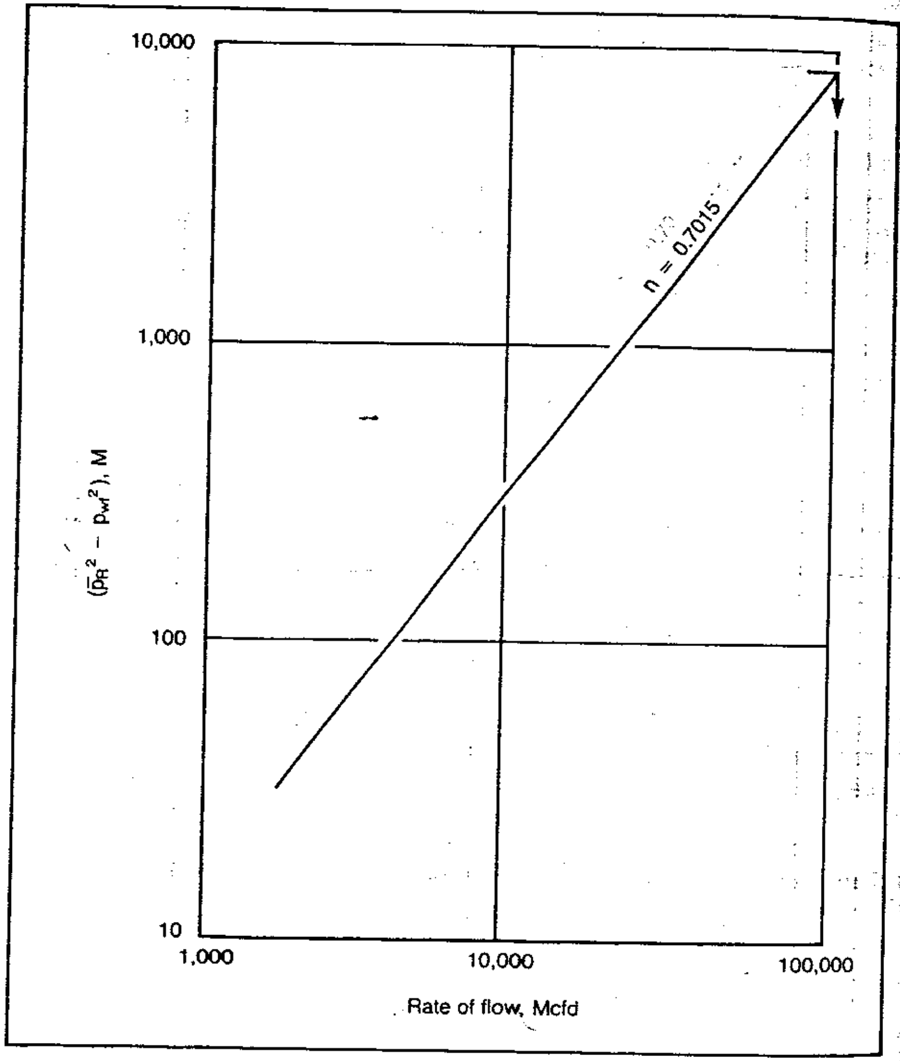

Figure 11–1 *Bottom-hole back-pressure performance curve for a hypothetical well.*

Note that a large-capacity well has been selected for the hypothetical well to demonstrate flow capacities for a wide range of tubing sizes. If the hypothetical well had a contract obligation or an allowable of 3,000 Mcfd, an examination of Table 11–3 shows that 2⅜-in. OD tubing would be ad-

Table 11–2 *Differences of pressure squared (thousands) for wellhead conditions for various sizes of tubing.*

	$(\bar{p}_{ts}^2 - p_{tf}^2)$ (*thousands*)				
q_k (*Mcfd*)	*OD (in.) = 2⅜ ID = 1.995*	*2⅞ 2.441*	*3½ 2.992*	*4 3.476*	*4½ 4.082*
3,000	360	152	78	58	—
6,000	1,386	557	265	182	142
10,000	3,781	1,459	652	420	309
20,000	—	—	2,308	1,382	940
30,000	—	—	5,064	2,815	1,818

equate to produce 3,000 Mcfd against a wellhead pressure of 1,000 psia until the shut-in wellhead pressure was 1,166 psia. In this case, if the demand for gas were 3,000 Mcfd, 2⅜-in. tubing would be adequate. A demand for 30,000 Mcfd gas could be met by 3½-in. tubing for only a short time or until the wellhead shut-in pressure was reduced to 2,428 psia from 2,437 psia. However, the demand for 30,000 Mcfd could be met with 4½-in. tubing against a 1,000 psia pressure at the wellhead until the shut-in wellhead pressure was reduced to 1,466 psia. The minimum wellhead shut-in pressures given above were calculated by equation 11–1 which follows:

$$\bar{p}_{ts} = [p_{tf}^2 + (q/C)^{1/n}]^{0.5} \qquad (11\text{–}1)$$

where the symbols are for wellhead conditions.

Equation 11–1 was used to calculate the minimum shut-in wellhead pressures required to produce the designated flow rate against a back pressure of 1,000 psia for the hypothetical well. The results are illustrated in Figure 11–3. For example, if the demand for gas were 10,000 Mcfd, the well could produce that quantity of gas through 2⅜-in. OD tubing until the shut-in wellhead pressure was 2,185 psia, through 2⅞-in. until the pressure was 1,568 psia, and through 3½-in. until the pressure was 1,293 psia. The hypothetical well could not produce 15,000 Mcfd through 2⅜-in. OD tubing if the initial shut-in wellhead pressure were 2,437 psia or 3,000 psia at the bottom of the hole. The incentive for installing tubing with larger than 3½-in. OD tubing is small if the demand for gas does not exceed 10,000 Mcfd. However, demands for gas of 15,000 Mcfd or more require economic analysis to make the decision to install tubing larger than 3½-in. OD.

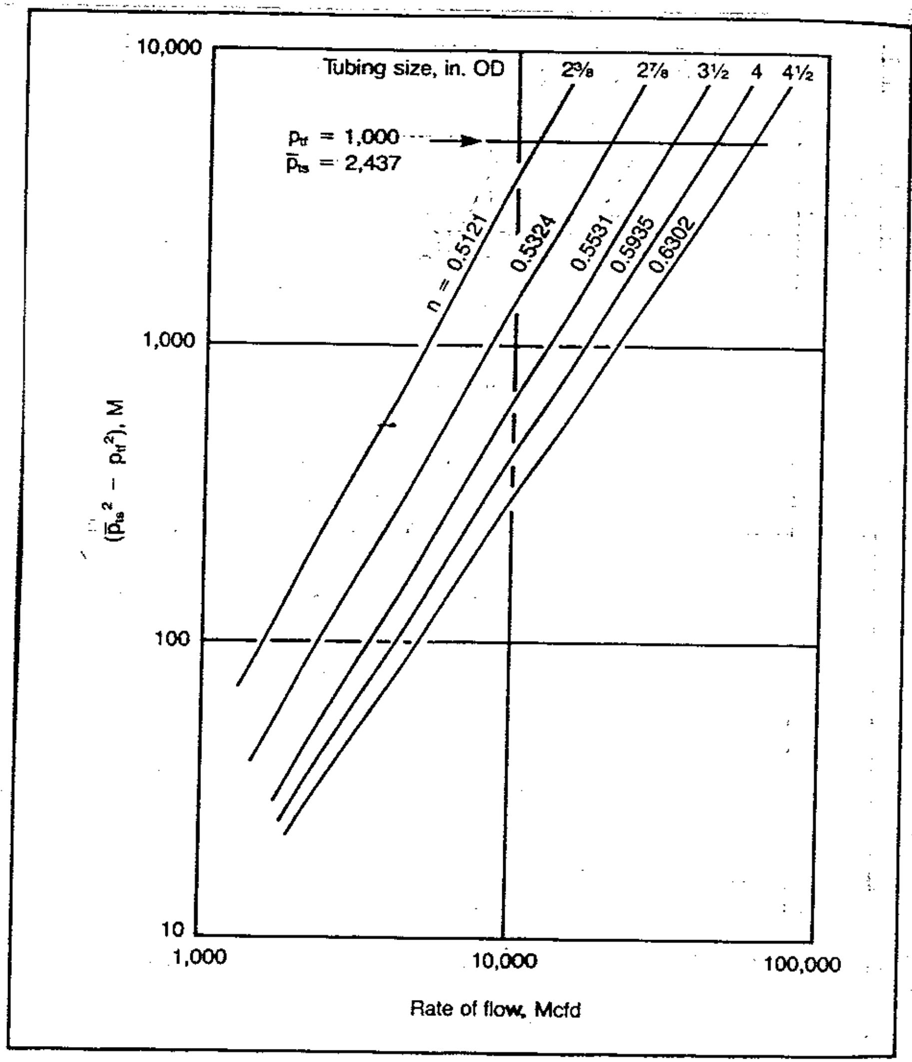

Figure 11–2 *Effect of tubing size on availability of gas at wellhead for a hypothetical well.*

As experience is gained in developing a reservoir and the market possibilities for the gas have been explored, it is probably necessary to make a complete analysis to determine the proper tubing size for only one well in the reservoir. Studying Figure 11–3 shows that the choice of size for average gas demands of 5,000 to 10,000 Mcfd is limited to 2⅜, 2⅞, or 3½-in. tubing. Only an exceptional circumstance, such as high peak demands for gas, justifies the 3½-in. tubing.

Table 11–3 *Flow rates for hypothetical well through various sizes of tubing.*

Tubing Size (OD, in.)	$\bar{p}_{ts}$ = 2,437 p_{tf} = 1,000 *Flow Rate (Mcfd)*	$\bar{p}_{ts}$ = 1,500 p_{tf} = 1,000 *Flow Rate (Mcfd)*	$\bar{p}_{ts}$ = 1,200 p_{tf} = 1,000 *Flow Rate (Mcfd)*
2⅜	11,500	5,700	3,300
2⅞	19,200	9,200	5,300
3½	30,100	14,100	7,900
4	42,400	18,800	10,100
4½	56,700	23,900	12,400

SIZING TUBING TO LIFT LIQUIDS AT VARIOUS FLOW RATES

If meeting the market demand for gas is the only requirement, tubing for gas wells can be sized by installing the largest tubing the casing and workover procedures will accommodate. But the tubing should also be sized to lift liquids from the well. Practically all gas wells produce some liquids by either or both of two mechanisms. Liquids, either hydrocarbon or water, may enter the wellbore along with the gas, or they may condense out of the gas as it moves up the tubing string. If the velocity is too low to carry the liquids up the tubing, they will move down the tubing against the flow of the gas and accumulate in the wellbore.

The necessity for keeping liquids moving out of the wellbore and flowstring is often neglected through oversight and failure to realize the problems that arise when a gas well is allowed to accumulate liquids in the wellbore. The obvious reasons for liquid removal are that the column of liquid imposes an additional back pressure on the formation with adverse effects on productivity and the possibility that enough liquid to kill the well will accumulate. This happens often in low-pressure wells that produce large quantities of formation water. A more subtle reason for keeping liquids from the wellbore is that liquids standing in a wellbore opposite the producing formation tend to increase the liquid saturation in the vicinity of the wellbore and thereby decrease its permeability to gas. The net result is damage to the productive capacity of the well.

Experience with the low-pressure wells in the West Panhandle and

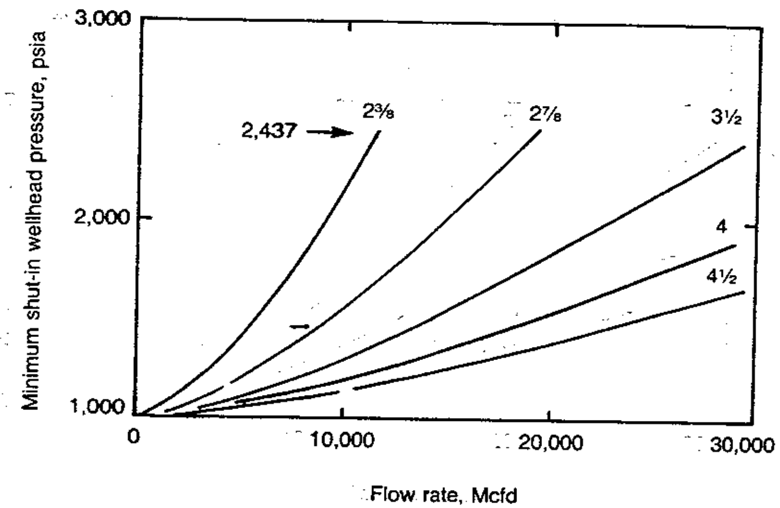

Figure 11–3 *Minimum shut-in wellhead pressure required to produce indicated flow rates against 1,000-psia back pressure through various sizes of tubing.*

Hugoton fields has shown that a velocity of 5 to 10 ft/sec is necessary to consistently remove hydrocarbon liquids, and a velocity of 10 to 20 ft/sec is required for water. The problem was described by Duggan, who estimated that a velocity of 5 ft/sec in tubing is required to remove hydrocarbon liquids.[1] The problem was analyzed theoretically by Turner, Hubbard and Dukler.[2] They presented theoretical equations for calculating the minimum velocity for removing hydrocarbon condensate and water from the flow strings of gas wells. The equations indicate that the minimum velocity is dependent upon the interfacial tension between the liquid and the gas raised to the 0.25 power and the difference between liquid density and gas density to the 0.25 power and is inversely proportional to the square root of the gas density. After an adjustment to improve agreement with experimental data on actual gas wells and substitution of average values for the densities of water and condensate, Turner et al. presented the following equations:

$$u_g(water) = \frac{5.62(67 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \tag{11–2}$$

$$u_g(condensate) = \frac{4.02(45 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \tag{11–3}$$

where 0.0031p represents the density of the gas. This term in equations 11–2 and 11–3 can be replaced with equation 2–18 of Chapter 2, giving the density of the gas phase:

$$\rho_g = 2.6988 \frac{p\gamma_g}{zT} \tag{2–18}$$

However, most applications to gas well problems do not require the more-accurate gas densities calculated by equation 2–18.

It is of interest to determine the impact of equations 11–2 and 11–3 on gas well operation as illustrated in Figure 11–4 in which the minimum velocity required to lift condensate and water are shown as functions of pressure. Figure 11–4 shows that the lower the pressure, the larger the velocity required to lift liquids. This is unfortunate because low-pressure wells are the most likely wells to have liquids in the wellbore and are the least capable of producing the higher velocities. Eventually, it is necessary to install mechanical pumps in low-pressure wells to keep them free of liquids.

The velocity calculated from equations 11–2 and 11–3 can be converted to flow rates using equation 5–12, which has been rearranged as follows:

$$q_m = 0.01673 \frac{pd^2u}{zT} \tag{11–4}$$

or:

$$q_m = 3.067 \frac{pAu}{zT} \tag{11–5}$$

where:

A = cross-sectional area sq ft

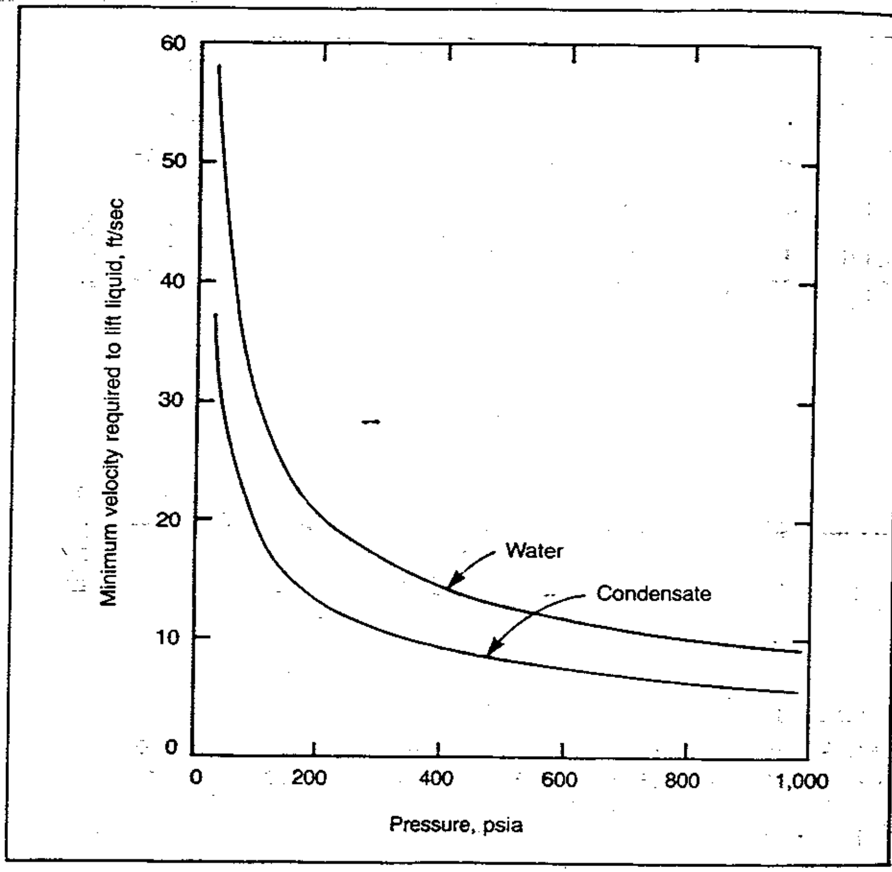

Figure 11–4 *Minimum gas velocity required to lift hydrocarbon liquid and water in a gas well.*

Returning to Figure 11–3, the region on the flow capacity curves where the velocity of the gas at 1,000 psia is not enough to lift liquids from the well can be calculated. At 1,000 psia, the minimum velocities required are 5.81 ft/sec for hydrocarbon condensate and 9.02 ft/sec for water. Substitution into equation 11–4 gives the minimum rates of flow required for lifting water and condensate in the various sizes of tubing (see Table 11–4).

Note: The minimum rates of flow given in Table 11–4 are independent of the capacity of a well to produce gas. Thus, a well completed with $4\frac{1}{2}$-in. tubing as casing but without a smaller flow string installed would require

Table 11–4 *Minimum rates of flow required to lift water and condensate in various sizes of tubing at 1,000 psia.*

$$\gamma_g = 0.625$$
$$t = 130°F$$
$$p = 1{,}000 \text{ psia}$$

Tubing		*Minimum Rate of Flow, MMcfd*	
Nominal	*ID (in.)*	*Water* $u = 9.02$ *ft/sec*	*Condensate* $u = 5.81$ *ft/sec*
$2\frac{3}{8}$	1.995	1.14	0.73
$2\frac{7}{8}$	2.441	1.71	1.10
$3\frac{1}{2}$	2.992	2.56	1.65
4	3.476	3.46	2.23
$4\frac{1}{2}$	4.082	4.77	3.07

a rate of flow of 4.77 MMcfd at a pressure of 1,000 psia, a temperature of 130°F, and a specific gravity of 0.625 to keep the wellbore free of water. At lesser rates of flow, water would accumulate in the wellbore.

Equally important to the proper sizing of the tubing is the placement of the bottom of the tubing string. In order to effectively lift water from the productive interval in a well, it is necessary to set the tubing near the bottom of the productive zone. A low-capacity well with $2\frac{3}{8}$-in. tubing set several hundred feet above the productive zone in $5\frac{1}{2}$-in. casing very likely will load up with liquids. Here, the minimum rate of flow would be determined by the velocity required to lift liquids in the $5\frac{1}{2}$-in. casing.

Turning attention to the low-pressure gas wells now being completed in shallow producing areas, some of the problems in lifting liquids from low-pressure wells can be identified. At current sale prices, a well producing 100 Mcfd at a wellhead pressure of 200 psia produces sufficient income to justify operation. However, an examination of the results of calculations given in Table 11–5 shows that water probably could not be lifted through 1-in. nominal tubing. If production is impeded by the accumulation of water in the wellbore, the only remedy remaining is to install a mechanical pump. Indeed, removing liquids with mechanical pumps from small-capacity gas wells is becoming a common method of operation.

All of the foregoing discussion is premised on the assumption that the well produces very moderate amounts of liquid. A large-enough quantity of water can kill almost any gas well no matter how large its capacity.

Table 11–5 *Minimum rates of flow required to lift water and condensate in various sizes of tubing at 200 psia.*

$\gamma_g = 0.625$
$t = 130°F$
$p = 200$ psia

Tubing		Minimum Rate of Flow, MMcfd	
Nominal	ID (in.)	Water $u = 20.37$ ft/sec	Condensate $u = 13.18$ ft/sec
1	1.049	0.13	0.08
1¼	1.380	0.22	0.15
1½	1.610	0.31	0.20
2⅜	1.995	0.47	0.30
2⅞	2.441	0.70	0.45
3½	2.992	1.06	0.68
4	3.476	1.42	0.92
4½	4.082	1.96	1.27

PROBLEMS

1. A small-capacity gas well with a depth of 600 ft and a shut-in pressure at the bottom of the hole of 260 psia has an open-flow capacity of 750 Mcfd and an exponent of 0.85. Estimate the largest size of tubing and rate of flow that will still lift water at the initial bottom-hole pressure of 260 psia when the pressure has been reduced to 150 psia.

2. Calculate a table similar to Tables 11–4 and 11–5 for a well operating at a pressure of 3,000 psia and 200°F at the wellhead.

REFERENCES

1. Duggan, J.O. "Estimating Flow Rate Required to Keep Gas Wells Unloaded." *J. Pet. Tech.* (December 1961), p. 1173.

2. Turner, R.G., M.G. Hubbard, and A.E. Dukler. "Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells." *J. Pet. Tech. Trans.* AIME, 246, (November 1969) p. 1475.

12 Remedial Measures for Gas Wells

This chapter is concerned with maintaining the performance characteristics of gas wells as opposed to methods for increasing the productive capacity or the stimulation of wells such as fracture treatments, acidization, etc. Stimulation methods will not be considered since the technical literature on stimulation methods is voluminous and is considered beyond the present purposes. However, we will consider those occurrences that more often than not happen slowly and gradually damage the performance of a well. For this purpose it is necessary to distinguish between the capability to produce gas and the performance characteristics of the well.

The capability of a well to produce gas decreases as the shut-in pressure decreases but the performance characteristic can remain constant. A very good example of this concept is illustrated by the isochronal test data on a gas well shown in Figure 7–4. The figure shows the results of early isochronal tests along with the results of 72-hour one-point tests taken over a period of eight years. Although the shut-in reservoir pressure decreased from 441.6 to 336.1 psia over the total period of nine years (eight years for the annual tests) the positions of the data points for the 72-hour one-point tests were all on the same isochronal curve for the 72-hour test within experimental error. If we define the performance characteristic as the coefficient in the isochronal or stabilized back pressure equation, the definition is:

$$C(t) = \frac{(\bar{p}_{Ri}{}^2 - p_{wf}{}^2)^n}{q_k} \tag{12–1}$$

For a stabilized condition the time dependent indication in equation 12–1 would be dropped, and the resultant equation would be analogous to the concept of the productivity index often used in discussing the producing characteristics of oil wells. In summary, the capability of the gas well of Figure 7–4 decreased over the period of nine years, but the performance characteristic as measured by the coefficient $C(t)$ remained constant at an average value of 52.0. Nothing occurred in the time interval of nine years that harmed the performance of the well.

Equation 12–1 describes the performance of the well at the face of the productive formation and is appropriate for reservoir problems. Since our primary concern is the capability of the well to deliver gas at the wellhead, all further discussion will be about performance at the wellhead. The following equation will apply:

$$G(t) = \frac{(\bar{p}_{ts}^{\,2} - p_{tf}^{\,2})^n}{q_k} \tag{12–2}$$

Thus, the wellhead performance is affected by the equipment installed in the well. In other words, undersized tubing can interfere with meeting the market demand for gas as much as an obstruction in the perforations of a well having properly sized tubing.

Although many things can happen to gas wells to cause damage, and these difficulties have been known for many years, those concerned with the remedies have written very little for the technical literature. The difficulties of removing liquids—especially water—were recognized in the 1920s.[1] Later the use of gas well tests to solve production problems for gas wells was discussed by Rawlins and Schellhardt.[2] Since then the literature on two-phase flow in wells has become extensive. A review of the problems besetting the performance of gas wells and possible remedies was published by Smith.[3]

RECORDS FOR DIAGNOSIS OF WELL PROBLEMS

Records showing a history of shut-in pressures and performance for a gas well are essential for the proper diagnosis and the planning of appropriate remedial measures for well problems. A form that has proved convenient for keeping such records is given on Figure 12–1. An adequate test file for a gas well should consist of the following:

1. A plot (usually the cover sheet) of $(\bar{p}_{ts}^{\,2} - p_{tf}^{\,2})$ against q_k on 3X3-inch cycle log-log coordinate paper showing results of

Figure 12–1 *Summary of gas well performance data.*

Operator ____________ Lease ____________ Well No. ____________

Field & Formation ____________

Date	SIP, psia		WP, psia		Flow thru	q_k Mcfd	$p_{ts}^2 - p_{tf}^2$	γ_g	API	GOR	Remarks
	CSG.	TBG.	CSG.	TBG.							

Note: Remarks column should show duration of shut-in, duration of flow and other pertinent information.

all types of tests (multipoint, isochronal, and one-point) taken on the well. An example is given on Figure 7–4.

2. A tabulation as suggested on Figure 12–1 showing in chronological order all measured wellhead or subsurface pressures and a summary of each flow test on the well.
3. A summary tabulation and plot of all shut-in-pressure data and the corresponding cumulative production for the well.

Although maintaining such test files may appear time consuming and expensive, it has been the experience of the author that good test files and an alert attitude on the part of the engineering staff can increase significantly the income from a group of gas wells. One actual example was a gas well producing at capacity into a low-pressure line. The installation of larger inside diameter wellhead fittings down stream of the master tubing valve increased production by about 500 Mcfd. Nothing had happened to damage the reservoir around the well, but there had been excessive pressure loss across the wellhead fittings. However, the excessive pressure loss had not been serious until the demand for gas could not be met by the capability of the well to deliver gas through the wellhead equipment.

CAUSES OF DETERIORATION IN PERFORMANCE

A listing and discussion of the causes of deterioration has been given by Smith.[3] These were liquids, cavings and sand in the wellbore, casing and tubing leaks, foreign objects, salt deposition, hydrates, and a decrease in the permeability of the reservoir around the wellbore. Also, undersized equipment such as tubing, subsurface equipment, and wellhead equipment can restrict the capacity of the well to deliver gas. The causes of deterioration in performance can happen quickly or they can grow gradually over a long period of time. The proper sizing of tubing and wellhead equipment can best be handled at the completion of the well. The sizing of tubing was discussed in Chapter 11.

The pinpointing of the cause for the deterioration of the performance of a gas well is more of an art than a science. Experience and alertness to nuances in the data from the well can be invaluable in the diagnosis of the troubles besetting a well.

For example, a gas well in New Mexico was selected for investigation because it had not made its allowable production for a period of about six months. After much argument about stimulating the well, it was decided to have an experienced engineer test the well. In shutting in the well to

measure the shut-in pressure, it was noticed that the master tubing valve was partially closed. Upon opening the valve fully (as it should have been in the first place) the well started producing in excess of its allowable production. Sometimes the solutions to production problems are very simple.

If a well is suspected of having performance troubles, the steps to diagnose the problem are:

1. Review production history with emphasis on gas and liquid production.
2. Review performance file and decide type of tests, if any, to be run.
3. Decide on remedial measures, make cost estimate, and estimate economies for the proposed remedial work.

Testing

If it appears testing is necessary, a current shut-in pressure should be obtained, then the well should be started to flow at a predetermined rate and allowed to flow undisturbed for at least 72 hours. Test data should be taken as recommended for a one-point test in Chapter 7. Particular attention should be given to production of water, condensate, or oil and to indications that liquids may be accumulating in the wellbore and flow string.

Analysis of Test Results

The first decision to be made is to determine whether the measured wellhead pressure is abnormally low. This can be done best by comparison to previously measured pressures or more accurately by calculating $(\bar{p}_R/z)$ opposite the producing formation and comparison with the previously established relationship between $(\bar{p}_R/z)$ and cumulative production for the well (see Fig. 3–1).

If the shut-in pressure is abnormally low, the most probable cause is liquid standing in the wellbore of flow string. Liquid levels in the flow string can be found by echometer measurements, the use of subsurface pressure gauges to determine pressure gradients, and by the use of small bailers or floats on a wire line. Subsurface pressure gauges have the added advantage in that the pressure can be measured opposite or very near the productive formation. Thus, it can be determined whether the reservoir pressure is abnormally low. Also, subsurface guages, bailers, or floats will determine whether there is an obstruction in the tubing or flow string.

If both the wellhead and reservoir pressures should appear abnormally low without an apparent reason, then a gas leak through a hole in the tubing or casing may be the culprit. The position of the leak usually can be determined with subsurface temperature measurements, sound intensity measurements, or flow indicators.

The next step in the analysis is to calculate the rate of flow and the value of $(\bar{p}_{ts}^2 - p_{tf}^2)$ and adding the results of the test to the performance plot for the well as shown in Figure 12–2. As an example, the circle to the left of the line represents the test data. All of the data shown on Figure 12–2 are for a hypothetical well. If the assumption is made that at the time of the test, the well had a shut-in pressure at the wellhead of 1,755 psia and had the capacity to produce 1,300 Mcfd against a wellhead pressure of 1,000

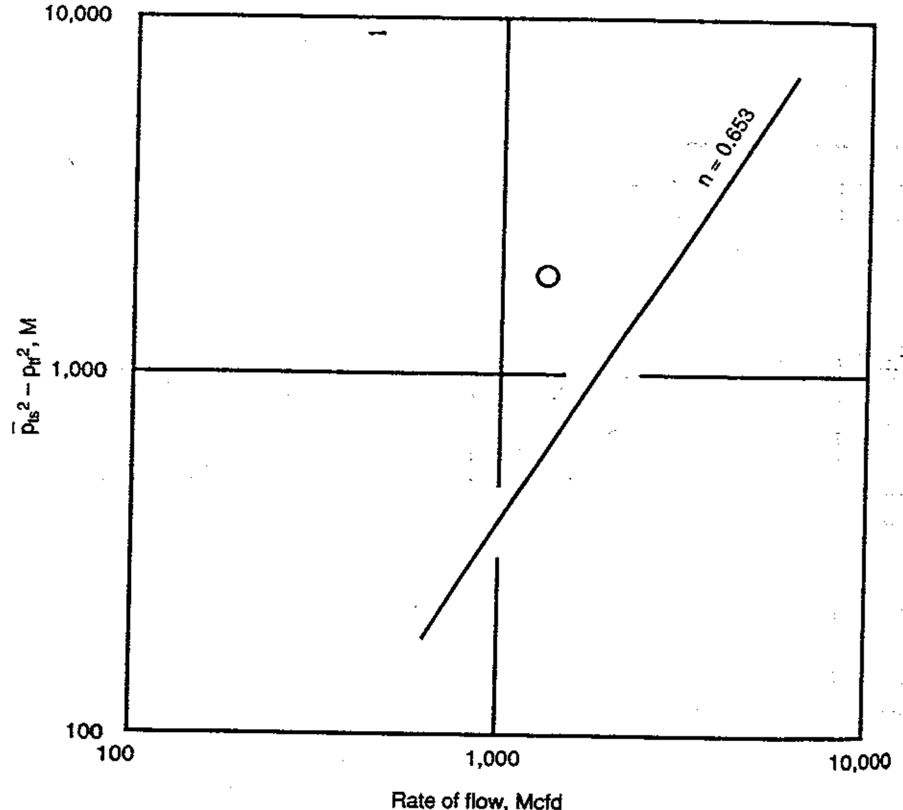

Figure 12–2 *Wellhead performance curve showing deterioration in performance.*

psia, the performance is represented by the circle. If the line represents previous performance, the well should have been capable of producing about 2,800 Mcfd under the same pressure conditions. Thus, we can conclude the performance at the time of the test was 46% of its normal performance. As long as the market demand for gas from the well is more than 1,300 Mcfd, the owner of the well is suffering a loss in income.

Since most performance difficulties for gas wells are caused by the accumulation of liquid in the wellbore and flow string, we are justified in exploring this possibility. If the temperature in the flow string is about 130°F and the specific gravity (molar mass ratio) is about 0.625, we can conclude data given in Table 11–4 are applicable to the problem at hand. Table 11–4 indicates that 2⅜-in. tubing (I.D. = 1.995 in.) should lift water consistently to the surface. If the tubing size in the well is larger than 2⅜-in., then it is probable that water has accumulated in the wellbore. If the tubing is 2⅜-in. we must look elsewhere for the cause of the difficulty. The depth of the bottom of the tubing must be compared with the lowest point of entry of the gas into the wellbore. If the bottom of the tubing is an appreciable distance above the casing perforations, liquid could accumulate in the wellbore below the bottom of the tubing and still not be lifted out of the well by the 2⅜-in. tubing.

Although the foregoing discussion is based on hypothetical conditions in an imaginary well, it has been included to illustrate the type of analysis necessary to determine the causes of a decrease in well performance. If water entry is suspected as the cause, then the physical presence of water in the well should be confirmed by further testing or observation of water production from the well. This is necessary because any number of conditions could have caused the same kind of behavior. For example, deposition of salt in the perforations or flow string could have been the cause. In this case a wash of the perforations and flow string with fresh water or acids would restore the performance. No hard and fast rules can be given for identification of the cause of the trouble.

Liquids

The most common cause for deterioration of performance in gas wells is the accumulation of liquids (water, oil, or condensate) in the wellbore. Liquids enter wells by flowing into the well from the gas reservoir with the gas, by entry from other zones through leaks in the casing or cement, or by condensation from the gas phase as the gas moves up the flow string. Both hydrocarbon condensate and fresh water can condense from gas if favorable conditions of composition, pressure, and temperature exist in the well.

The removal of liquids from gas wells has been a problem since the start of the industry. In the 1920s, siphon strings—tubing strings with orifices to admit gas from the annular space between the casing and tubing—were used extensively.[1] Later, gas-lift valves set to open under predetermined pressure conditions were used in place of orifices. However, siphon strings may waste gas. Plunger lifts have been tried but in the absence of hydrocarbons or oils with lubricating properties their use was less than satisfactory. Foaming agents greatly facilitated the lifting of water from wells, but the difficulties of breaking the foams and keeping the foams out of pipelines and gasoline plants proved insurmountable.[4] As the value of natural gas increased in recent years the use of mechanical pumps to lift water has become more economical. (Refer to Chapter 11 which includes a section about removing liquids from a wellbore.)

Sometimes it is possible to prevent water entry into wells by plug back operations. This requires careful identification of the source of the water as coming from below the gas bearing zone. Often plug back operations are successful only for a short time, especially if there are no permeability barriers between the water and gas zones.

Salt water that is shown to be foreign to the gas zone—by analyses or fresh water in excessive amounts—is indicative of casing leaks or faulty cement around the casing. However, a knowledge of the composition of the water native to the gas zone is necessary.

In the early days of the natural gas industry the most direct way to remove water or other liquids from a gas well was by blowing the well to the atmosphere. However, blowing a well was wasteful of gas and dangerous to personnel, equipment, the reservoir, and surface property.

The importance of keeping a gas wellbore free of liquids cannot be overemphasized. Water or oil standing in a wellbore opposite the productive formation for any appreciable length of time will increase the liquid saturation in the formation around the well. This will result in a decrease in the relative permeability to gas and result in a deterioration of well performance. After this type of damage to a well, remedial measures become highly expensive and it may be necessary to redrill the well. The author knows of an 18,000-ft. well that had to be redrilled because of these reasons.

Cavings and Sand

Open-hole completions are generally prone to accumulate cavings. As the cavings encroach on the productive zone, the performance of the well is damaged. If the cavings contain shale that is sensitive to the fresh water that condenses out of the gas as it moves up the well, the resulting muck

can be extremely harmful to the performance of the well. The presence of cavings can be determined by comparison of the measured depth to the drilled depth and to the top and bottom of the productive zone. Remedial measures consist of cleaning out, the possibility of installing liners, and the use of acid washes whenever the cavings are soluble in acid.

Sand from unconsolidated formations may move through perforations into the wellbore and into the tubing of a well. Here again, remedial measures consist of cleaning out, and the installation of a liner may be necessary to prevent reoccurrence.

Hydrates

Although hydrate formation may be a common occurrence in surface equipment during cold weather, hydrates can form in the flow strings of gas wells in cold climates. The author has seen an accumulation of hydrates completely plug the tubing of a gas well in Wyoming. The formation of hydrates in flow strings may by prevented by installation of bottom-hole chokes, down-hole heating equipment, and the down-hole injection of chemicals such as alcohols and glycols.

One strategy to alleviate the effect of hydrate formation is to design well and surface equipment so as to reduce the chances of accumulation—allowing the hydrates to flow out of the well. This requires the elimination, as far as possible of abrupt changes in diameter and in the direction of flow.

Deposition of Salts

Salts such as sodium chloride or other chemical compounds may be deposited in perforations, in the wellbore, or in the flow strings of gas wells. Sodium chloride and other water soluble salts often can be removed by water and acid washes, thereby eliminating the replacement of the flow string. Thick heavy crude oil in some cases may enter the wellbore with the gas, form emulsions with the water present, and cause serious problems. Cleaning the wellbore and equipment with kerosene may be effective in restoring the productivity of the well.

Leaks in Tubing and Casing

Tubing leaks defeat the purpose of the tubing. They may prevent water removal from the well and subject the casing to excessive pressure. Leaks in the casing can cause migration of gas into other formations with a resultant loss of reserves. Casing leaks in low-pressure wells may allow water to enter the well and can even kill wells. Leaks in tubing and casing should be repaired.

Foreign Objects

Care should be taken in the completion of wells to remove all foreign objects such as swab rubbers, gaskets, small pieces of metal, etc., from the wellbore and flow string. These objects can move up the flow string and will usually lodge on the upstream side of chokes, causing no end of trouble.

SUMMARY

Many things can happen to gas wells to cause a decrease in the performance characteristics of the well. Remedial measures require identification of the cause, planning specific remedial work, and estimation of the economics for the proposed work. Since many causes have the same net effect, all available information must be reviewed and new test information obtained (if necessary) and carefully evaluated to pinpoint the cause of the difficulty. Only then can appropriate remedial measures be designed.

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13 Gas Sales Contracts

A gas sales contract is a legal document between a seller who owns, controls, or has the right to sell and dispose of the gas to be produced from certain oil and gas leaseholds and a buyer who is engaged or is to be engaged in buying and selling gas in wholesale quantities. Usually, the buyer is engaged also in transporting gas in a pipeline system. As such, the gas sales contract is primarily the concern of attorneys and accountants, but the natural gas engineer makes important contributions in the successful administration of a gas sales contract. Therefore, it is not the intent of this chapter to outline a model gas sales contract but to bring out those areas of the contract that merit the attention of the natural gas engineer. Although gas sales contracts may vary in content, the contents have enough in common for the principal features to be discussed in general terms.

Lands or Leaseholds Dedicated

Contracts are usually specific as to the leaseholds dedicated to the performance of a certain contract. Such information determines the economic factors that are important in evaluating the repair or the redrilling of old wells or a proposal to drill new wells on dedicated acreage. If the acreage is not dedicated, the factors for determining the economics of a new well can be estimated on the basis of an anticipated contract.

Quantity and Rate of Take

The basic provisions of a contract are the quantity of gas to be sold, the rate at which the gas is to be taken, the quality of the gas, and the price. The provisions vary from contract to contract with the quantity of gas to be taken over a period of time, usually a year, set proportional to the mutually agreed recoverable reserve for the leases or to a specific statement of an average quantity of gas per day. The daily rate at which the gas is taken may vary from day to day, but the buyer may be required to take an average daily quantity of gas over a period of time, usually a year. Such clauses may vary from a daily, fixed rate to 67% of yearly quantity taken in six specific winter months with the remaining 33% of the yearly quantity taken during the six summer months. In case the seller's wells cannot meet the rate requirements of the contract, alternative rate requirements become controlling.

In areas or fields in which a regulatory body sets allowable production for the wells, the allowables become the controlling factors for rates of production. Under these conditions, the buyers usually have the responsibility for making nominations for gas takes. Contracts may specify how such nominations of quantities of gas are to be determined.

In any event, the engineer responsible for the performance of the wells should have detailed knowledge as to how the contractual or allowable rates of production are determined.

Quality of Gas

Contracts are usually specific as to the quality of the gas. Rigid limits may be placed on dust, liquids, hydrogen sulfide, and total sulfur contents. The seller may be required to dehydrate the gas so that the water vapor content may be no more than 7 lb/MMcf or some similar water vapor content. Also, a minimum heating value for the gas may be specified. If the heating value should fall below the minimum, the buyer may have the option of refusing to take the gas.

Quality requirements have a strong influence on the type and cost of surface equipment installed at the seller's expense at the well. Therefore, these requirements cannot be ignored by those responsible for approving cost estimates for well completions.

Price

Contracts contain various price provisions ranging from a fixed, stated price to an acceptance of the maximum lawful price established in section 109 of the Natural Gas Policy Act of 1978 (NGPA) computed in accordance with subsection (b) of section 109 of the NGPA. The contract may contain provisions for an increase in sales price, provided the gas becomes qualified under the NGPA. In determining future prices, the engineer is well advised to seek competent assistance.

Natural gas is sold either by volume, usually in units of 1,000 cu ft (Mcf) at measurement pressure and temperature bases, or by the heating value in units of 1 million British thermal units (Btu). If sold by its heating value, the determination of the value becomes as important as the accuracy of the volume measurement.

Many contracts and possibly most future contracts relate or will relate the sales price of the gas to its heating value. Thus, the method for determining the heating value and how often the heating value is to be determined become important in determining income from the properties. A knowledge of the procedures and requirements as set out in the contract is necessary for proper management of the properties. Also, it should be realized that the compositions of gases from many gas reservoirs change with time depending upon reservoir conditions. This usually occurs as a result of evaporation of immobile reservoir hydrocarbon liquids into the gas as the reservoir pressure is lowered. The trend can become significant if reservoir pressures are reduced below 100 psia.

Definitions

Most contracts set out the definitions for the terms under which the gas is sold. The definitions may include gross heating value, British thermal units, day, contract year, measurement bases, and other terms important to an understanding of the contract. It is possible that a contract might define the standard cubic foot of gas for measurement to be 14.65 psia and 60°F, the standard cubic foot for pricing purposes to be 14.73 psia and 60°F, and for the gross heating value the standard cubic foot to be at a pressure of 30 in. of mercury at 32°F and a temperature of 60°F. The only safe way through the thicket of definitions is through the sales contract.

Point of Delivery

One of the most important pieces of information for the field engineer is the point of delivery because the seller is usually responsible for the equipment up to that point. The title to the gas passes from the seller to the buyer at the point of delivery. Also, the contract may state that the seller shall deliver gas at the varying pressures maintained at the delivery point, which means that the seller must furnish the compression facilities that may be necessary to deliver the gas.

Gas Measurement

The gas measurement section of the contract usually states that the orifice meters shall be installed, operated, and gas volumes computed in accordance with the recommendations prescribed in *"Orifice Metering of Natural Gas,"* American National Standard, ANSI/API 2530, First Edition, Gas Measurement Committee Report No. 3 dated September 1984 of the American Gas Association or such subsequent changes as are mutually agreed upon. The contract may specify the barometric pressure to be used in calculating absolute pressures and may adopt constant values for the Reynolds number factor, F_r, the expansion factor, Y, the supercompressibility factor, F_{pv}, and the manometer factor, F_m (see equation 4–5). Other factors may be specified unless they are measured.

Also, the gas measurement provisions may set specific times for calibrating the meter, checking the orifice plate, measuring specific gravity, determining heating value, and setting procedures for adjusting errors in gas measurement. All of these are important considerations for the seller.

SUMMARY

The sale of natural gas differs from that of crude oil in several ways. Gas cannot be stored on the lease except by leaving it unproduced. Therefore, the seller must have his wells ready to produce when there is a market for the gas. The price paid for gas is subject to complicated regulation at present writing, and the price may vary from well to well even though the wells are in the same reservoir. As a result, the engineer involved in the management and operation of gas wells must have a working knowledge of the sales contracts that govern the sale of gas. The pertinent features of the contracts that influence engineering decisions have been identified.

APPENDIX A

Compressibility Factors for Natural Gas

Table A–1 *Pseudocritical properties of hydrocarbon gases,* p_{pc} *and* T_{pc}.*

γ_g	p_{pc}	T_{pc}	γ_g	p_{pc}	T_{pc}
0.55	673	336	0.85	664	441
0.56	673	341	0.86	664	444
0.57	672	346	0.87	663	448
0.58	672	350	0.88	663	451
0.59	672	354	0.89	662	454
0.60	671	358	0.90	662	457
0.61	671	362	0.91	662	461
0.62	671	365	0.92	662	464
0.63	670	368	0.93	661	467
0.64	670	372	0.94	661	471
0.65	670	375	0.95	660	474
0.66	670	378	0.96	660	477
0.67	669	382	0.97	659	481
0.68	669	385	0.98	659	484
0.69	669	388	0.99	659	487
0.70	668	392	1.00	658	491
0.71	668	395	1.01	658	494
0.72	668	398	1.02	657	497
0.73	668	401	1.03	656	500
0.74	667	405	1.04	656	504
0.75	667	408	1.05	655	507
0.76	667	411	1.06	655	510
0.77	666	415	1.07	654	514
0.78	666	418	1.08	654	517
0.79	666	421	1.09	653	520
0.80	665	424	1.10	652	524
0.81	665	428	1.11	652	527
0.82	665	431	1.12	651	530
0.83	665	434	1.13	651	534
0.84	664	438	1.14	650	537

Do not interpolate; values are inclusive to the next higher value.

* Used by permission of the Pacific Energy Association (formerly the California Natural Gasoline Association), taken from Bulletin No. TS–461, 1947. Data are for the average of the natural gas mixtures occuring in the western states—primarily California.

Table A–2 *Correction for pseudocritical properties of hydrocarbon gases for carbon dioxide, nitrogen, and hydrogen sulfide.**

If positive, add to values in Table A–1. If negative, subtract from values in Table A–1.

Volume Percent of CO_2 or N_2 in Gas	*Carbon Dioxide, CO_2*		*Nitrogen, N_2*		*Hydrogen sulfide, H_2S*	
	p_{pc}	T_{pc}	p_{pc}	T_{pc}	p_{pc}	T_{pc}
1	+ 4	− 1	− 1	− 3	+ 2	−2
2	+ 8	− 3	− 3	− 6	+ 4	−2
3	+ 12	− 5	− 5	− 9	+ 7	−3
4	+ 17	− 7	− 7	−11	+10	−3
5	+ 21	− 9	− 9	−14	+15	−4
6	+ 25	−11	−11	−17		
7	+ 30	−12	−12	−20		
8	+ 34	−14	−14	−22		
9	+ 39	−16	−16	−25		
10	+ 44	−17	−17	−28		
11	+ 48	−19	−19	−30		
12	+ 53	−21	−21	−33		
13	+ 57	−22	−22	−36		
14	+ 61	−24	−24	−39		
15	+ 66	−26	−26	−42		
16	+ 70	−27	−27	−44		
17	+ 74	−29	−29	−47		
18	+ 79	−31	−31	−50		
19	+ 83	−32	−32	−52		
20	+ 87	−34	−34	−55		
21	+ 92	−36	−36	−58		
22	+ 96	−37	−37	−60		
23	+100	−39	−39	−63		
24	+104	−41	−41	−66		
25	+109	−42	−42	−68		
26	+113	−44	−44	−71		
27	+117	−46	−46	−74		
28	+122	−47	−47	−77		
29	+126	−49	−49	−79		
30	+130	−51	−51	−82		
31	+134	−52	−52	−85		
32	+139	−54	−54	−87		
33	+143	−56	−56	−90		
34	+147	−57	−57	−93		
35	+152	−59	−59	−95		
36	+156	−61	−61	−98		

* Values for carbon dioxide and nitrogen are based on data from the California Natural Gasoline Association Bulletin No. TS–461. Used by permision of the Interstate Oil Compact Commission. Values for hydrogen sulfide are supplied by the author.

Table A–3 *Compressibility factors for natural gas at various pseudoreduced pressures, p_{pr}, and pseudoreduced temperatures, T_{pr}.**

Pseudo-reduced Pressure, p_{pr}	*Pseudoreduced Temperature, T_{pr}*									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
0.20	0.938	0.948	0.953	0.959	0.964	0.969	0.971	0.973	0.978	0.979
0.25	0.922	0.934	0.941	0.949	0.955	0.960	0.964	0.968	0.972	0.974
0.30	0.905	0.920	0.928	0.938	0.946	0.951	0.956	0.962	0.965	0.969
0.35	0.887	0.905	0.914	0.927	0.937	0.943	0.949	0.956	0.959	0.964
0.40	0.868	0.889	0.900	0.916	0.928	0.934	0.942	0.949	0.953	0.959
0.45	0.849	0.872	0.887	0.905	0.919	0.926	0.934	0.943	0.948	0.954
0.50	0.829	0.855	0.874	0.894	0.909	0.918	0.926	0.937	0.942	0.949
0.55	0.810	0.839	0.862	0.883	0.900	0.909	0.919	0.930	0.937	0.944
0.60	0.790	0.822	0.849	0.872	0.890	0.900	0.912	0.923	0.931	0.939
0.65	0.769	0.804	0.835	0.861	0.881	0.893	0.905	0.917	0.926	0.935
0.70	0.747	0.785	0.820	0.850	0.871	0.885	0.898	0.911	0.921	0.930
0.75	0.722	0.767	0.806	0.839	0.861	0.878	0.890	0.905	0.916	0.925
0.80	0.698	0.748	0.791	0.827	0.851	0.870	0.883	0.899	0.910	0.920
0.85	0.672	0.729	0.776	0.815	0.842	0.861	0.876	0.893	0.905	0.916
0.90	0.645	0.710	0.761	0.802	0.832	0.852	0.870	0.887	0.899	0.911

* Table A–3, in its entirety, was taken from F.H. Poettmann and P.G. Carpenter, "The Multiphase Flow of Gas, Oil, and Water through Vertical Flow Strings with Application to the Design of Gas-Lift Installations," *API Drilling and Production Practice,* pp. 279–91 (1952). Use herein was granted by permission of the American Petroleum Institute.

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
0.95	0.618	0.690	0.748	0.791	0.823	0.845	0.863	0.881	0.895	0.906
1.00	0.590	0.670	0.734	0.780	0.813	0.838	0.856	0.875	0.890	0.901
1.05	0.555	0.647	0.719	0.768	0.803	0.829	0.849	0.869	0.885	0.897
1.10	0.520	0.624	0.703	0.755	0.793	0.820	0.842	0.862	0.879	0.893
1.15	0.482	0.602	0.688	0.743	0.783	0.812	0.836	0.856	0.874	0.889
1.20	0.444	0.580	0.672	0.731	0.773	0.804	0.829	0.850	0.869	0.884
1.25	0.398	0.555	0.656	0.719	0.763	0.797	0.823	0.845	0.864	0.880
1.30	0.351	0.530	0.640	0.707	0.753	0.789	0.817	0.839	0.859	0.876
1.35	0.307	0.505	0.623	0.695	0.743	0.781	0.810	0.834	0.855	0.872
1.40	0.263	0.480	0.605	0.682	0.732	0.772	0.803	0.828	0.850	0.868
1.45	0.259	0.453	0.588	0.670	0.721	0.765	0.797	0.824	0.846	0.864
1.50	0.254	0.425	0.570	0.657	0.710	0.757	0.791	0.817	0.841	0.859
1.55	0.253	0.409	0.556	0.646	0.701	0.750	0.785	0.811	0.837	0.855
1.60	0.251	0.393	0.541	0.634	0.691	0.742	0.779	0.805	0.832	0.851
1.65	0.251	0.386	0.529	0.623	0.682	0.735	0.773	0.800	0.828	0.847
1.70	0.252	0.378	0.517	0.612	0.672	0.728	0.767	0.795	0.823	0.843
1.75	0.256	0.374	0.507	0.602	0.664	0.720	0.761	0.790	0.819	0.840
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.80	0.260	0.370	0.497	0.592	0.656	0.712	0.755	0.784	0.815	0.837
1.85	0.265	0.370	0.489	0.582	0.648	0.706	0.750	0.779	0.811	0.834
1.90	0.270	0.369	0.480	0.572	0.640	0.699	0.744	0.774	0.806	0.830
1.95	0.275	0.369	0.474	0.564	0.634	0.693	0.738	0.771	0.803	0.827
2.00	0.280	0.370	0.467	0.555	0.627	0.687	0.732	0.767	0.799	0.824
2.05	0.286	0.371	0.462	0.547	0.620	0.681	0.727	0.762	0.795	0.821
2.10	0.292	0.372	0.457	0.539	0.612	0.674	0.721	0.757	0.791	0.818
2.15	0.298	0.374	0.454	0.534	0.607	0.669	0.716	0.753	0.788	0.815
2.20	0.305	0.376	0.450	0.528	0.602	0.663	0.711	0.748	0.784	0.811
2.25	0.311	0.378	0.449	0.525	0.598	0.658	0.707	0.745	0.781	0.808
2.30	0.318	0.380	0.448	0.522	0.593	0.652	0.702	0.741	0.778	0.805
2.35	0.324	0.383	0.448	0.521	0.590	0.649	0.699	0.737	0.775	0.803
2.40	0.330	0.387	0.449	0.520	0.586	0.645	0.695	0.733	0.771	0.800
2.45	0.336	0.390	0.450	0.520	0.583	0.642	0.692	0.731	0.768	0.798
2.50	0.343	0.394	0.451	0.519	0.581	0.638	0.688	0.728	0.765	0.796
2.55	0.349	0.398	0.454	0.519	0.581	0.636	0.685	0.725	0.763	0.793
2.60	0.356	0.402	0.458	0.520	0.580	0.633	0.682	0.722	0.760	0.791
2.65	0.362	0.406	0.460	0.521	0.580	0.631	0.680	0.720	0.758	0.789
2.70	0.369	0.410	0.463	0.522	0.579	0.629	0.678	0.718	0.756	0.787
2.75	0.375	0.414	0.467	0.524	0.579	0.628	0.676	0.716	0.754	0.785
2.80	0.381	0.419	0.472	0.526	0.579	0.627	0.673	0.713	0.752	0.783
2.85	0.387	0.424	0.476	0.528	0.579	0.626	0.672	0.712	0.750	0.781
2.90	0.394	0.429	0.481	0.530	0.580	0.625	0.670	0.710	0.748	0.779

Table A–3, continued

Pseudo-reduced Pressure, P_{pr}	*Pseudoreduced Temperature,* T_{pr}									
	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
0.20	0.984	0.987	0.990	0.991	0.993	0.995	0.998	0.999	1.000	1.001
0.25	0.980	0.984	0.988	0.990	0.992	0.995	0.998	0.999	1.000	1.001
0.30	0.976	0.981	0.985	0.988	0.990	0.994	0.997	0.999	1.000	1.001
0.35	0.972	0.978	0.983	0.986	0.989	0.994	0.997	0.999	1.000	1.001
0.40	0.968	0.975	0.980	0.983	0.987	0.992	0.996	0.998	1.000	1.002
0.45	0.964	0.972	0.978	0.981	0.985	0.991	0.995	0.998	1.000	1.002
0.50	0.960	0.969	0.976	0.979	0.983	0.990	0.994	0.998	1.000	1.002
0.55	0.956	0.966	0.974	0.978	0.982	0.989	0.994	0.998	1.000	1.002
0.60	0.952	0.963	0.971	0.976	0.981	0.988	0.993	0.997	1.000	1.003
0.65	0.949	0.961	0.969	0.974	0.980	0.987	0.993	0.997	1.000	1.003
0.70	0.945	0.958	0.967	0.972	0.978	0.986	0.992	0.997	1.001	1.004
0.75	0.942	0.955	0.965	0.971	0.977	0.985	0.992	0.997	1.001	1.004
0.80	0.938	0.952	0.962	0.969	0.975	0.984	0.991	0.997	1.001	1.005
0.85	0.935	0.950	0.960	0.967	0.974	0.983	0.991	0.997	1.001	1.005
0.90	0.931	0.947	0.958	0.965	0.972	0.983	0.990	0.996	1.001	1.006
0.95	0.927	0.944	0.956	0.963	0.971	0.982	0.990	0.996	1.001	1.006
1.00	0.923	0.941	0.953	0.961	0.970	0.981	0.989	0.996	1.001	1.007
1.05	0.920	0.939	0.951	0.960	0.969	0.980	0.989	0.996	1.001	1.007
1.10	0.917	0.936	0.949	0.958	0.967	0.979	0.988	0.996	1.002	1.008
1.15	0.913	0.933	0.947	0.957	0.966	0.979	0.988	0.996	1.002	1.008
1.20	0.909	0.930	0.945	0.955	0.964	0.978	0.987	0.996	1.002	1.008
1.25	0.906	0.928	0.943	0.954	0.963	0.978	0.987	0.996	1.002	1.008
1.30	0.901	0.925	0.941	0.952	0.962	0.977	0.987	0.996	1.002	1.009
1.35	0.898	0.923	0.940	0.951	0.961	0.976	0.986	0.996	1.002	1.009
1.40	0.895	0.920	0.938	0.949	0.960	0.975	0.986	0.996	1.002	1.010
1.45	0.892	0.918	0.936	0.947	0.959	0.975	0.986	0.996	1.002	1.010
1.50	0.889	0.915	0.934	0.946	0.957	0.974	0.985	0.995	1.003	1.010
1.55	0.886	0.913	0.932	0.945	0.956	0.973	0.985	0.995	1.003	1.010
1.60	0.882	0.910	0.930	0.943	0.954	0.972	0.984	0.995	1.003	1.011
1.65	0.880	0.908	0.929	0.942	0.953	0.972	0.984	0.995	1.003	1.011
1.70	0.877	0.905	0.927	0.941	0.952	0.971	0.983	0.995	1.004	1.012
1.75	0.874	0.903	0.925	0.940	0.951	0.971	0.983	0.995	1.004	1.012
1.80	0.871	0.900	0.923	0.938	0.950	0.970	0.983	0.995	1.004	1.012
1.85	0.869	0.899	0.922	0.937	0.949	0.970	0.983	0.995	1.004	1.012
1.90	0.866	0.897	0.921	0.936	0.948	0.969	0.982	0.995	1.005	1.013
1.95	0.864	0.895	0.920	0.935	0.948	0.969	0.982	0.995	1.005	1.013
2.00	0.861	0.893	0.918	0.933	0.947	0.968	0.982	0.995	1.006	1.014
2.05	0.859	0.891	0.917	0.932	0.946	0.968	0.982	0.995	1.006	1.014
2.10	0.856	0.889	0.915	0.931	0.945	0.967	0.981	0.995	1.007	1.015
2.15	0.854	0.887	0.914	0.930	0.945	0.967	0.981	0.995	1.007	1.015
2.20	0.851	0.885	0.912	0.929	0.944	0.966	0.981	0.995	1.007	1.016
2.25	0.850	0.884	0.911	0.928	0.943	0.966	0.981	0.995	1.007	1.016
2.30	0.848	0.882	0.909	0.927	0.942	0.965	0.980	0.995	1.008	1.017
2.35	0.846	0.881	0.908	0.927	0.942	0.965	0.980	0.995	1.008	1.017
2.40	0.843	0.879	0.907	0.926	0.941	0.964	0.980	0.995	1.008	1.018
2.45	0.842	0.878	0.906	0.925	0.941	0.964	0.980	0.995	1.008	1.018
2.50	0.840	0.876	0.904	0.924	0.941	0.963	0.980	0.995	1.009	1.019
2.55	0.839	0.875	0.903	0.923	0.941	0.963	0.980	0.995	1.009	1.019
2.60	0.837	0.873	0.902	0.922	0.940	0.963	0.980	0.995	1.009	1.020
2.65	0.835	0.872	0.901	0.922	0.940	0.963	0.980	0.995	1.009	1.020
2.70	0.833	0.871	0.900	0.921	0.939	0.962	0.980	0.996	1.010	1.021
2.75	0.832	0.870	0.900	0.921	0.939	0.962	0.980	0.996	1.010	1.021
2.80	0.830	0.869	0.899	0.920	0.938	0.962	0.980	0.966	1.010	1.022
2.85	0.829	0.868	0.898	0.920	0.938	0.962	0.980	0.996	1.010	1.022
2.90	0.828	0.867	0.897	0.919	0.938	0.962	0.980	0.997	1.011	1.023

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
2.95	0.401	0.434	0.485	0.532	0.580	0.625	0.670	0.709	0.747	0.778
3.00	0.407	0.440	0.489	0.534	0.581	0.624	0.669	0.707	0.745	0.777
3.05	0.413	0.446	0.494	0.537	0.582	0.624	0.669	0.706	0.744	0.776
3.10	0.420	0.452	0.499	0.540	0.584	0.625	0.668	0.705	0.742	0.775
3.15	0.426	0.457	0.503	0.543	0.586	0.625	0.668	0.704	0.742	0.774
3.20	0.432	0.463	0.507	0.546	0.588	0.626	0.668	0.703	0.741	0.773
3.25	0.438	0.469	0.511	0.548	0.590	0.627	0.668	0.701	0.741	0.773
3.30	0.445	0.475	0.516	0.551	0.592	0.628	0.669	0.702	0.740	0.772
3.35	0.451	0.481	0.520	0.555	0.594	0.629	0.669	0.702	0.740	0.772
3.40	0.458	0.488	0.525	0.559	0.597	0.631	0.670	0.703	0.739	0.771
3.45	0.464	0.494	0.530	0.562	0.599	0.632	0.670	0.703	0.739	0.771
3.50	0.471	0.500	0.535	0.566	0.602	0.633	0.670	0.704	0.739	0.771
3.55	0.477	0.505	0.539	0.569	0.605	0.635	0.672	0.705	0.739	0.771
3.60	0.483	0.511	0.544	0.572	0.608	0.637	0.673	0.706	0.740	0.771
3.65	0.490	0.517	0.548	0.577	0.611	0.639	0.675	0.707	0.740	0.771
3.70	0.497	0.523	0.553	0.582	0.614	0.641	0.677	0.708	0.741	0.772
3.75	0.503	0.529	0.558	0.586	0.617	0.642	0.678	0.709	0.741	0.772
3.80	0.509	0.535	0.563	0.590	0.620	0.644	0.680	0.710	0.742	0.773
3.85	0.515	0.541	0.568	0.594	0.623	0.646	0.681	0.711	0.742	0.773
3.90	0.522	0.547	0.573	0.599	0.627	0.649	0.683	0.713	0.743	0.774
3.95	0.528	0.552	0.577	0.603	0.630	0.651	0.685	0.714	0.744	0.775
4.00	0.534	0.558	0.582	0.608	0.633	0.653	0.687	0.716	0.746	0.776
4.05	0.540	0.564	0.587	0.612	0.636	0.656	0.689	0.717	0.747	0.777
4.10	0.547	0.570	0.592	0.616	0.640	0.659	0.692	0.719	0.749	0.779
4.15	0.553	0.575	0.596	0.620	0.644	0.662	0.695	0.720	0.750	0.780
4.20	0.560	0.581	0.601	0.624	0.648	0.665	0.697	0.722	0.752	0.781
4.25	0.566	0.587	0.606	0.628	0.651	0.668	0.699	0.724	0.753	0.782
4.30	0.572	0.593	0.611	0.632	0.655	0.671	0.701	0.726	0.755	0.784
4.35	0.579	0.598	0.616	0.636	0.658	0.674	0.703	0.728	0.757	0.785
4.40	0.586	0.604	0.621	0.641	0.662	0.678	0.706	0.730	0.759	0.787
4.45	0.592	0.610	0.626	0.645	0.666	0.680	0.709	0.732	0.761	0.788
4.50	0.599	0.616	0.631	0.650	0.670	0.683	0.712	0.734	0.763	0.790
4.55	0.605	0.621	0.636	0.654	0.673	0.687	0.715	0.736	0.765	0.792
4.60	0.612	0.627	0.641	0.659	0.677	0.691	0.718	0.739	0.768	0.794
4.65	0.617	0.633	0.646	0.663	0.681	0.694	0.720	0.741	0.770	0.796
4.70	0.623	0.639	0.651	0.668	0.685	0.698	0.723	0.743	0.772	0.798
4.75	0.630	0.644	0.655	0.672	0.689	0.701	0.726	0.746	0.774	0.800
4.80	0.637	0.650	0.662	0.677	0.693	0.705	0.729	0.749	0.777	0.802
4.85	0.643	0.655	0.667	0.681	0.697	0.708	0.732	0.751	0.779	0.804
4.90	0.650	0.661	0.672	0.686	0.701	0.712	0.736	0.753	0.782	0.806
4.95	0.656	0.666	0.677	0.690	0.705	0.716	0.739	0.756	0.784	0.808
5.00	0.663	0.672	0.683	0.694	0.709	0.720	0.742	0.760	0.787	0.810
5.05	0.669	0.677	0.688	0.698	0.713	0.723	0.745	0.763	0.789	0.812
5.10	0.675	0.683	0.693	0.703	0.717	0.727	0.749	0.766	0.792	0.814
5.15	0.681	0.689	0.698	0.708	0.721	0.731	0.752	0.769	0.795	0.817
5.20	0.688	0.695	0.703	0.713	0.725	0.735	0.755	0.772	0.798	0.820
5.25	0.694	0.700	0.708	0.717	0.729	0.739	0.758	0.775	0.800	0.822
5.30	0.701	0.706	0.713	0.722	0.734	0.743	0.762	0.778	0.803	0.825
5.35	0.707	0.712	0.718	0.726	0.738	0.747	0.765	0.781	0.806	0.827
5.40	0.713	0.718	0.723	0.731	0.742	0.751	0.769	0.785	0.809	0.830
5.45	0.720	0.723	0.728	0.735	0.746	0.755	0.772	0.788	0.812	0.832
5.50	0.727	0.729	0.733	0.740	0.751	0.759	0.776	0.792	0.815	0.835
5.55	0.733	0.734	0.738	0.745	0.755	0.763	0.779	0.795	0.818	0.837
5.60	0.739	0.740	0.744	0.750	0.760	0.768	0.783	0.799	0.821	0.840
5.65	0.745	0.745	0.749	0.755	0.764	0.772	0.787	0.802	0.824	0.843

Table A–3, continued

Pseudoreduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
2.95	0.827	0.866	0.897	0.919	0.938	0.962	0.980	0.997	1.011	1.023
3.00	0.825	0.864	0.896	0.918	0.938	0.962	0.981	0.997	1.012	1.024
3.05	0.823	0.863	0.896	0.918	0.938	0.962	0.981	0.997	1.012	1.024
3.10	0.822	0.862	0.895	0.917	0.937	0.962	0.981	0.998	1.013	1.025
3.15	0.822	0.862	0.895	0.917	0.937	0.962	0.981	0.998	1.013	1.025
3.20	0.821	0.861	0.894	0.917	0.937	0.962	0.982	0.998	1.014	1.026
3.25	0.820	0.860	0.894	0.917	0.937	0.962	0.982	0.998	1.014	1.026
3.30	0.819	0.859	0.893	0.916	0.937	0.962	0.982	0.999	1.015	1.027
3.35	0.819	0.859	0.893	0.916	0.937	0.962	0.982	0.999	1.015	1.027
3.40	0.818	0.858	0.892	0.916	0.937	0.963	0.983	1.000	1.016	1.028
3.45	0.818	0.858	0.892	0.916	0.937	0.963	0.983	1.000	1.016	1.029
3.50	0.817	0.858	0.891	0.915	0.937	0.963	0.984	1.000	1.017	1.030
3.55	0.817	0.858	0.891	0.915	0.937	0.963	0.984	1.000	1.017	1.030
3.60	0.816	0.857	0.890	0.915	0.937	0.964	0.985	1.001	1.018	1.031
3.65	0.816	0.857	0.890	0.915	0.937	0.964	0.985	1.001	1.019	1.031
3.70	0.816	0.856	0.890	0.916	0.937	0.965	0.986	1.002	1.019	1.032
3.75	0.816	0.856	0.890	0.916	0.937	0.965	0.986	1.002	1.019	1.032
3.80	0.816	0.855	0.891	0.916	0.937	0.966	0.987	1.003	1.020	1.033
3.85	0.816	0.855	0.891	0.916	0.937	0.966	0.988	1.004	1.021	1.033
3.90	0.817	0.855	0.891	0.917	0.938	0.967	0.989	1.005	1.022	1.034
3.95	0.817	0.855	0.891	0.917	0.938	0.967	0.989	1.006	1.022	1.034
4.00	0.818	0.856	0.892	0.917	0.939	0.968	0.990	1.007	1.023	1.035
4.05	0.819	0.856	0.892	0.917	0.939	0.968	0.991	1.008	1.023	1.035
4.10	0.820	0.857	0.893	0.918	0.940	0.969	0.992	1.009	1.024	1.036
4.15	0.821	0.857	0.894	0.918	0.940	0.970	0.993	1.010	1.025	1.036
4.20	0.822	0.859	0.895	0.919	0.941	0.971	0.994	1.011	1.026	1.037
4.25	0.823	0.859	0.896	0.919	0.941	0.972	0.995	1.012	1.026	1.037
4.30	0.824	0.861	0.897	0.920	0.942	0.973	0.996	1.013	1.027	1.038
4.35	0.825	0.861	0.897	0.921	0.942	0.973	0.997	1.013	1.028	1.039
4.40	0.826	0.862	0.898	0.922	0.943	0.974	0.998	1.014	1.029	1.040
4.45	0.827	0.862	0.899	0.922	0.943	0.975	0.999	1.015	1.029	1.040
4.50	0.829	0.864	0.900	0.923	0.944	0.977	1.000	1.016	1.030	1.041
4.55	0.830	0.865	0.901	0.924	0.945	0.977	1.001	1.017	1.031	1.042
4.60	0.832	0.867	0.902	0.925	0.946	0.978	1.002	1.018	1.032	1.043
4.65	0.833	0.868	0.903	0.926	0.947	0.979	1.003	1.019	1.033	1.043
4.70	0.835	0.869	0.904	0.927	0.948	0.981	1.004	1.020	1.034	1.044
4.75	0.836	0.870	0.905	0.928	0.949	0.982	1.005	1.021	1.035	1.045
4.80	0.838	0.872	0.907	0.929	0.950	0.983	1.007	1.022	1.036	1.046
4.85	0.840	0.873	0.908	0.930	0.951	0.984	1.008	1.023	1.037	1.047
4.90	0.842	0.875	0.910	0.932	0.952	0.985	1.009	1.024	1.038	1.048
4.95	0.844	0.877	0.911	0.933	0.953	0.986	1.010	1.025	1.038	1.048
5.00	0.846	0.879	0.912	0.934	0.954	0.988	1.011	1.027	1.039	1.049
5.05	0.848	0.880	0.914	0.935	0.955	0.989	1.012	1.028	1.040	1.050
5.10	0.850	0.882	0.916	0.937	0.957	0.990	1.013	1.029	1.041	1.051
5.15	0.851	0.884	0.917	0.938	0.958	0.991	1.014	1.030	1.042	1.051
5.20	0.853	0.886	0.919	0.940	0.960	0.992	1.016	1.031	1.043	1.052
5.25	0.855	0.887	0.920	0.941	0.961	0.993	1.017	1.032	1.044	1.052
5.30	0.858	0.889	0.921	0.943	0.963	0.995	1.018	1.033	1.045	1.053
5.35	0.860	0.891	0.923	0.944	0.964	0.996	1.019	1.034	1.046	1.053
5.40	0.862	0.893	0.925	0.946	0.966	0.998	1.021	1.035	1.047	1.055
5.45	0.865	0.895	0.926	0.947	0.967	0.999	1.022	1.036	1.048	1.056
5.50	0.868	0.897	0.928	0.949	0.969	1.000	1.023	1.038	1.049	1.057
5.55	0.870	0.899	0.929	0.950	0.970	1.001	1.024	1.039	1.050	1.058
5.60	0.872	0.900	0.931	0.952	0.972	1.003	1.026	1.040	1.051	1.058
5.65	0.874	0.902	0.933	0.953	0.974	1.004	1.027	1.041	1.052	1.059

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
5.70	0.751	0.751	0.754	0.760	0.769	0.777	0.791	0.806	0.827	0.846
5.75	0.757	0.756	0.759	0.764	0.773	0.781	0.795	0.809	0.830	0.848
5.80	0.763	0.762	0.764	0.769	0.778	0.785	0.799	0.813	0.833	0.851
5.85	0.769	0.768	0.770	0.774	0.782	0.789	0.803	0.817	0.836	0.854
5.90	0.775	0.774	0.777	0.779	0.787	0.794	0.807	0.821	0.839	0.857
5.95	0.781	0.779	0.782	0.784	0.792	0.798	0.810	0.824	0.842	0.860
6.00	0.788	0.785	0.787	0.790	0.797	0.803	0.814	0.828	0.845	0.863
6.05	0.794	0.790	0.792	0.795	0.801	0.807	0.818	0.831	0.848	0.865
6.10	0.800	0.796	0.797	0.800	0.805	0.812	0.822	0.835	0.852	0.868
6.15	0.805	0.801	0.802	0.805	0.809	0.816	0.826	0.838	0.855	0.870
6.20	0.811	0.807	0.808	0.810	0.814	0.820	0.830	0.842	0.858	0.873
6.25	0.816	0.812	0.813	0.815	0.818	0.824	0.834	0.846	0.861	0.876
6.30	0.822	0.818	0.818	0.820	0.823	0.829	0.839	0.850	0.864	0.880
6.35	0.829	0.824	0.823	0.825	0.828	0.833	0.843	0.853	0.867	0.883
6.40	0.835	0.830	0.829	0.830	0.833	0.838	0.848	0.857	0.871	0.886
6.45	0.841	0.835	0.834	0.835	0.837	0.842	0.852	0.861	0.874	0.889
6.50	0.848	0.841	0.839	0.840	0.842	0.847	0.856	0.865	0.878	0.892
6.55	0.853	0.846	0.844	0.845	0.846	0.851	0.860	0.868	0.881	0.895
6.60	0.858	0.852	0.850	0.851	0.851	0.855	0.864	0.871	0.884	0.898
6.65	0.864	0.857	0.855	0.856	0.856	0.859	0.868	0.875	0.888	0.900
6.70	0.870	0.863	0.860	0.861	0.861	0.863	0.872	0.880	0.892	0.903
6.75	0.875	0.868	0.865	0.866	0.865	0.867	0.876	0.884	0.895	0.906
6.80	0.881	0.874	0.871	0.872	0.870	0.872	0.880	0.888	0.899	0.910
6.85	0.886	0.879	0.876	0.877	0.875	0.877	0.884	0.891	0.902	0.913
6.90	0.892	0.885	0.881	0.881	0.881	0.881	0.888	0.895	0.906	0.917
6.95	0.897	0.890	0.886	0.885	0.885	0.885	0.892	0.898	0.909	0.919
7.00	0.903	0.896	0.892	0.890	0.890	0.890	0.896	0.902	0.913	0.922
7.05	0.909	0.901	0.897	0.895	0.894	0.897	0.899	0.906	0.916	0.925
7.10	0.915	0.907	0.903	0.900	0.899	0.899	0.903	0.910	0.920	0.929
7.15	0.921	0.912	0.908	0.905	0.903	0.903	0.907	0.914	0.923	0.932
7.20	0.928	0.918	0.913	0.910	0.908	0.907	0.911	0.918	0.927	0.935
7.25	0.933	0.923	0.918	0.915	0.912	0.910	0.915	0.921	0.930	0.938
7.30	0.938	0.929	0.923	0.920	0.917	0.914	0.919	0.925	0.934	0.942
7.35	0.943	0.934	0.928	0.924	0.921	0.918	0.923	0.928	0.937	0.946
7.40	0.949	0.940	0.933	0.929	0.926	0.923	0.927	0.932	0.941	0.950
7.45	0.955	0.945	0.938	0.934	0.930	0.927	0.931	0.936	0.944	0.953
7.50	0.961	0.951	0.943	0.939	0.935	0.932	0.935	0.940	0.948	0.956
7.55	0.966	0.956	0.948	0.944	0.939	0.936	0.939	0.944	0.951	0.959
7.60	0.972	0.961	0.953	0.949	0.944	0.941	0.943	0.949	0.955	0.962
7.65	0.987	0.966	0.958	0.954	0.949	0.945	0.947	0.952	0.958	0.965
7.70	0.983	0.972	0.963	0.959	0.954	0.950	0.952	0.955	0.962	0.969
7.75	0.989	0.977	0.968	0.963	0.958	0.954	0.956	0.959	0.965	0.972
7.80	0.995	0.983	0.973	0.968	0.962	0.959	0.960	0.963	0.969	0.976
7.85	1.000	0.988	0.978	0.973	0.966	0.963	0.964	0.967	0.973	0.979
7.90	1.005	0.993	0.984	0.978	0.970	0.968	0.968	0.972	0.977	0.982
7.95	1.011	0.999	0.989	0.983	0.974	0.972	0.972	0.976	0.980	0.986
8.00	1.017	1.005	0.995	0.988	0.979	0.977	0.976	0.980	0.984	0.990
8.05	1.022	1.010	1.000	0.993	0.983	0.982	0.980	0.984	0.988	0.994
8.10	1.027	1.016	1.006	0.998	0.987	0.987	0.984	0.989	0.992	0.998
8.15	1.032	1.021	1.011	1.002	0.991	0.991	0.988	0.992	0.996	1.000
8.20	1.038	1.026	1.016	1.007	0.996	0.996	0.992	0.996	1.000	1.003
8.25	1.044	1.031	1.021	1.012	1.000	1.000	0.996	0.999	1.004	1.007
8.30	1.050	1.037	1.026	1.017	1.005	1.004	1.001	1.003	1.008	1.011
8.35	1.055	1.042	1.031	1.021	1.010	1.008	1.005	1.007	1.011	1.014
8.40	1.060	1.048	1.036	1.026	1.015	1.012	1.010	1.012	1.015	1.018

Table A–3, continued

Pseudo-reduced Pressure, P_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
5.70	0.877	0.905	0.935	0.955	0.976	1.006	1.028	1.042	1.053	1.060
5.75	0.879	0.907	0.937	0.957	0.977	1.007	1.029	1.043	1.054	1.061
5.80	0.882	0.909	0.939	0.959	0.979	1.008	1.031	1.045	1.055	1.062
5.85	0.884	0.911	0.940	0.961	0.980	1.009	1.032	1.046	1.056	1.063
5.90	0.887	0.913	0.942	0.963	0.982	1.011	1.033	1.048	1.057	1.064
5.95	0.889	0.915	0.944	0.965	0.984	1.012	1.034	1.049	1.058	1.065
6.00	0.892	0.918	0.946	0.967	0.986	1.013	1.036	1.050	1.059	1.066
6.05	0.894	0.920	0.948	0.968	0.987	1.015	1.037	1.051	1.060	1.067
6.10	0.897	0.922	0.950	0.970	0.989	1.017	1.038	1.052	1.061	1.068
6.15	0.899	0.924	0.952	0.971	0.991	1.018	1.039	1.053	1.062	1.069
6.20	0.902	0.927	0.954	0.973	0.993	1.019	1.041	1.055	1.063	1.070
6.25	0.904	0.930	0.956	0.975	0.995	1.020	1.042	1.056	1.064	1.071
6.30	0.907	0.931	0.958	0.977	0.997	1.022	1.043	1.057	1.065	1.072
6.35	0.909	0.932	0.960	0.979	0.998	1.023	1.045	1.058	1.066	1.072
6.40	0.912	0.936	0.962	0.981	1.000	1.025	1.047	1.059	1.067	1.073
6.45	0.915	0.938	0.964	0.983	1.001	1.026	1.048	1.060	1.068	1.074
6.50	0.918	0.941	0.966	0.985	1.003	1.028	1.049	1.062	1.069	1.075
6.55	0.920	0.943	0.968	0.987	1.005	1.029	1.050	1.063	1.070	1.076
6.60	0.923	0.945	0.970	0.989	1.007	1.031	1.052	1.064	1.072	1.077
6.65	0.925	0.947	0.972	0.991	1.008	1.032	1.053	1.065	1.073	1.078
6.70	0.928	0.950	0.974	0.993	1.010	1.034	1.054	1.067	1.074	1.080
6.75	0.930	0.952	0.976	0.995	1.011	1.036	1.055	1.068	1.075	1.081
6.80	0.933	0.955	0.979	0.997	1.013	1.038	1.057	1.069	1.077	1.082
6.85	0.936	0.957	0.981	0.998	1.015	1.040	1.058	1.070	1.078	1.083
6.90	0.939	0.960	0.983	1.000	1.017	1.041	1.059	1.071	1.079	1.084

Pseudo-reduced Pressure, P_{pr}	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
6.95	0.941	0.962	0.985	1.002	1.018	1.042	1.060	1.072	1.080	1.085
7.00	0.944	0.965	0.988	1.004	1.020	1.044	1.062	1.073	1.081	1.086
7.05	0.947	0.967	0.990	1.006	1.022	1.045	1.063	1.075	1.082	1.087
7.10	0.950	0.970	0.992	1.008	1.024	1.047	1.065	1.077	1.083	1.088
7.15	0.953	0.972	0.994	1.010	1.026	1.048	1.066	1.078	1.084	1.089
7.20	0.956	0.975	0.997	1.012	1.028	1.050	1.067	1.079	1.086	1.091
7.25	0.959	0.977	0.999	1.014	1.029	1.051	1.068	1.080	1.087	1.092
7.30	0.962	0.980	1.001	1.017	1.031	1.053	1.070	1.081	1.088	1.093
7.35	0.965	0.982	1.003	1.019	1.033	1.055	1.071	1.082	1.089	1.094
7.40	0.968	0.985	1.005	1.021	1.035	1.057	1.073	1.084	1.091	1.095
7.45	0.970	0.988	1.007	1.023	1.037	1.058	1.074	1.085	1.092	1.096
7.50	0.973	0.991	1.010	1.025	1.039	1.060	1.076	1.087	1.093	1.098
7.55	0.976	0.993	1.012	1.027	1.041	1.061	1.077	1.088	1.094	1.099
7.60	0.979	0.996	1.015	1.029	1.043	1.063	1.079	1.089	1.096	1.100
7.65	0.981	0.998	1.017	1.031	1.045	1.065	1.080	1.090	1.097	1.101
7.70	0.984	1.001	1.019	1.033	1.047	1.067	1.082	1.092	1.098	1.102
7.75	0.987	1.003	1.021	1.035	1.049	1.068	1.083	1.093	1.099	1.103
7.80	0.990	1.006	1.024	1.038	1.051	1.070	1.084	1.095	1.100	1.104
7.85	0.993	1.009	1.026	1.040	1.053	1.071	1.085	1.096	1.101	1.105
7.90	0.997	1.012	1.029	1.042	1.055	1.073	1.087	1.098	1.103	1.107
7.95	0.999	1.015	1.031	1.044	1.057	1.075	1.088	1.099	1.104	1.108
8.00	1.001	1.018	1.033	1.047	1.059	1.078	1.090	1.100	1.106	1.110
8.05	1.004	1.020	1.035	1.049	1.060	1.079	1.091	1.101	1.107	1.111
8.10	1.008	1.022	1.038	1.051	1.062	1.081	1.093	1.102	1.109	1.112
8.15	1.011	1.025	1.040	1.053	1.064	1.082	1.095	1.103	1.110	1.113
8.20	1.014	1.028	1.043	1.055	1.066	1.084	1.097	1.105	1.111	1.115
8.25	1.017	1.030	1.045	1.057	1.068	1.086	1.098	1.106	1.112	1.116
8.30	1.021	1.033	1.048	1.060	1.070	1.088	1.100	1.108	1.113	1.118
8.35	1.024	1.036	1.050	1.062	1.072	1.089	1.101	1.109	1.115	1.119
8.40	1.027	1.039	1.052	1.064	1.074	1.091	1.102	1.111	1.117	1.120

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
8.45	1.065	1.053	1.041	1.030	1.019	1.016	1.014	1.016	1.019	1.021
8.50	1.070	1.058	1.046	1.035	1.024	1.021	1.019	1.020	1.023	1.025
8.55	1.075	1.063	1.051	1.040	1.029	1.025	1.023	1.024	1.026	1.028
8.60	1.081	1.068	1.056	1.045	1.034	1.030	1.028	1.028	1.030	1.031
8.65	1.086	1.073	1.061	1.049	1.039	1.034	1.032	1.032	1.034	1.035
8.70	1.092	1.078	1.066	1.054	1.044	1.039	1.037	1.036	1.038	1.039
8.75	1.097	1.083	1.070	1.058	1.049	1.043	1.041	1.040	1.041	1.043
8.80	1.102	1.088	1.075	1.063	1.054	1.048	1.045	1.044	1.045	1.047
8.85	1.108	1.094	1.080	1.068	1.058	1.052	1.049	1.048	1.048	1.049
8.90	1.113	1.099	1.085	1.073	1.063	1.056	1.053	1.052	1.052	1.052
8.95	1.118	1.103	1.090	1.077	1.067	1.060	1.057	1.056	1.056	1.056
9.00	1.124	1.108	1.095	1.082	1.072	1.064	1.061	1.060	1.060	1.060
9.05	1.129	1.113	1.100	1.086	1.076	1.068	1.065	1.064	1.064	1.064
9.10	1.135	1.118	1.105	1.091	1.081	1.073	1.069	1.068	1.068	1.068
9.15	1.140	1.123	1.109	1.096	1.085	1.077	1.073	1.071	1.071	1.071
9.20	1.146	1.128	1.114	1.101	1.090	1.082	1.078	1.075	1.075	1.074
9.25	1.151	1.133	1.119	1.105	1.094	1.086	1.082	1.079	1.078	1.077
9.30	1.157	1.138	1.124	1.110	1.099	1.091	1.086	1.083	1.082	1.081
9.35	1.162	1.143	1.129	1.115	1.104	1.095	1.090	1.087	1.086	1.084
9.40	1.167	1.148	1.134	1.120	1.109	1.100	1.095	1.092	1.090	1.088
9.45	1.172	1.153	1.139	1.124	1.113	1.104	1.099	1.096	1.094	1.092
9.50	1.178	1.159	1.144	1.129	1.118	1.109	1.103	1.100	1.098	1.096
9.55	1.182	1.164	1.149	1.134	1.122	1.113	1.107	1.104	1.101	1.099
9.60	1.189	1.169	1.154	1.139	1.127	1.118	1.111	1.108	1.104	1.102
9.65	1.194	1.174	1.158	1.143	1.132	1.122	1.115	1.112	1.108	1.105
9.70	1.200	1.179	1.163	1.148	1.137	1.127	1.120	1.116	1.112	1.109
9.75	1.205	1.184	1.168	1.153	1.141	1.131	1.124	1.120	1.115	1.113
9.80	1.210	1.189	1.173	1.158	1.146	1.135	1.128	1.124	1.119	1.117
9.85	1.215	1.194	1.178	1.162	1.150	1.139	1.132	1.128	1.123	1.120
9.90	1.221	1.200	1.183	1.167	1.155	1.143	1.137	1.132	1.127	1.123
9.95	1.226	1.205	1.187	1.171	1.159	1.147	1.141	1.136	1.130	1.126
10.00	1.231	1.210	1.192	1.176	1.164	1.152	1.145	1.140	1.134	1.130
10.05	1.236	1.215	1.197	1.180	1.168	1.156	1.149	1.144	1.138	1.134
10.10	1.242	1.220	1.202	1.185	1.173	1.160	1.153	1.148	1.142	1.138
10.15	1.247	1.225	1.206	1.190	1.177	1.164	1.157	1.152	1.145	1.141
10.20	1.252	1.230	1.211	1.195	1.181	1.169	1.162	1.155	1.149	1.145
10.25	1.257	1.235	1.216	1.199	1.185	1.174	1.166	1.159	1.153	1.148
10.30	1.263	1.240	1.221	1.204	1.190	1.179	1.170	1.163	1.157	1.152
10.35	1.268	1.245	1.226	1.208	1.194	1.183	1.174	1.167	1.160	1.156
10.40	1.274	1.250	1.231	1.213	1.199	1.188	1.178	1.171	1.164	1.160
10.45	1.279	1.255	1.236	1.218	1.203	1.192	1.182	1.175	1.168	1.164
10.50	1.285	1.260	1.241	1.223	1.208	1.196	1.186	1.179	1.172	1.168
10.55	1.290	1.265	1.246	1.227	1.212	1.200	1.190	1.183	1.176	1.171
10.60	1.296	1.270	1.251	1.232	1.217	1.204	1.195	1.187	1.180	1.174
10.65	1.301	1.275	1.256	1.236	1.221	1.208	1.199	1.191	1.183	1.177
10.70	1.307	1.280	1.261	1.241	1.226	1.213	1.203	1.195	1.187	1.181
10.75	1.312	1.285	1.265	1.246	1.231	1.217	1.207	1.198	1.191	1.185
10.80	1.318	1.291	1.270	1.251	1.236	1.222	1.211	1.202	1.195	1.189
10.85	1.323	1.296	1.275	1.255	1.240	1.226	1.215	1.206	1.198	1.193
10.90	1.329	1.301	1.280	1.260	1.245	1.230	1.219	1.210	1.202	1.197
10.95	1.334	1.306	1.284	1.265	1.249	1.235	1.223	1.214	1.206	1.200
11.00	1.339	1.311	1.289	1.270	1.254	1.240	1.228	1.218	1.210	1.204
11.05	1.344	1.316	1.294	1.275	1.258	1.244	1.232	1.221	1.213	1.208
11.10	1.350	1.321	1.299	1.280	1.263	1.249	1.236	1.225	1.217	1.211
11.15	1.355	1.326	1.303	1.284	1.268	1.253	1.240	1.229	1.221	1.214

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
11.20	1.360	1.331	1.308	1.289	1.273	1.257	1.245	1.233	1.225	1.218
11.25	1.365	1.336	1.313	1.294	1.277	1.261	1.249	1.237	1.228	1.221
11.30	1.370	1.341	1.318	1.299	1.282	1.265	1.253	1.241	1.232	1.225
11.35	1.375	1.346	1.323	1.303	1.286	1.269	1.257	1.245	1.236	1.228
11.40	1.381	1.351	1.328	1.308	1.291	1.273	1.261	1.249	1.240	1.232
11.45	1.386	1.356	1.333	1.313	1.295	1.277	1.265	1.253	1.243	1.236
11.50	1.391	1.362	1.338	1.318	1.300	1.282	1.269	1.257	1.247	1.240
11.55	1.396	1.367	1.343	1.322	1.304	1.286	1.273	1.260	1.251	1.244
11.60	1.402	1.372	1.348	1.327	1.309	1.291	1.277	1.264	1.255	1.248
11.65	1.407	1.377	1.353	1.331	1.313	1.295	1.281	1.268	1.258	1.251
11.70	1.412	1.382	1.358	1.336	1.317	1.300	1.286	1.272	1.262	1.255
11.75	1.417	1.387	1.363	1.340	1.321	1.305	1.288	1.276	1.265	1.259
11.80	1.423	1.392	1.368	1.345	1.326	1.309	1.294	1.280	1.269	1.262
11.85	1.428	1.397	1.373	1.350	1.330	1.313	1.298	1.284	1.273	1.266
11.90	1.433	1.402	1.378	1.355	1.335	1.318	1.302	1.288	1.277	1.270
11.95	1.438	1.407	1.383	1.359	1.339	1.322	1.306	1.292	1.280	1.274
12.00	1.444	1.413	1.388	1.364	1.344	1.327	1.310	1.296	1.284	1.278
12.05	1.449	1.418	1.393	1.368	1.348	1.331	1.314	1.299	1.288	1.281
12.10	1.454	1.423	1.398	1.373	1.353	1.335	1.318	1.303	1.292	1.285
12.15	1.459	1.428	1.403	1.378	1.357	1.339	1.322	1.307	1.296	1.288
12.20	1.465	1.433	1.408	1.383	1.362	1.344	1.326	1.311	1.300	1.292
12.25	1.470	1.438	1.413	1.388	1.366	1.348	1.330	1.315	1.304	1.295
12.30	1.475	1.443	1.418	1.392	1.371	1.353	1.334	1.319	1.307	1.299
12.35	1.480	1.448	1.423	1.397	1.375	1.357	1.338	1.323	1.311	1.303
12.40	1.486	1.453	1.428	1.402	1.380	1.362	1.342	1.327	1.315	1.307
12.45	1.491	1.458	1.433	1.407	1.384	1.366	1.346	1.330	1.319	1.310
12.50	1.496	1.464	1.438	1.411	1.389	1.370	1.350	1.334	1.323	1.313
12.55	1.501	1.469	1.443	1.416	1.394	1.374	1.354	1.338	1.326	1.317
12.60	1.507	1.474	1.448	1.420	1.399	1.379	1.358	1.342	1.330	1.321
12.65	1.512	1.479	1.453	1.425	1.404	1.383	1.362	1.346	1.334	1.325
12.70	1.517	1.484	1.458	1.430	1.409	1.388	1.367	1.350	1.337	1.329
12.75	1.522	1.489	1.463	1.435	1.413	1.392	1.371	1.354	1.341	1.332
12.80	1.527	1.495	1.468	1.439	1.418	1.397	1.375	1.358	1.345	1.336
12.85	1.532	1.500	1.473	1.444	1.422	1.401	1.379	1.362	1.348	1.339
12.90	1.537	1.505	1.477	1.449	1.427	1.405	1.383	1.365	1.352	1.342
12.95	1.542	1.510	1.482	1.454	1.431	1.409	1.387	1.369	1.356	1.346
13.00	1.548	1.515	1.487	1.458	1.436	1.414	1.391	1.374	1.360	1.350
13.05	1.553	1.520	1.492	1.463	1.440	1.418	1.395	1.378	1.364	1.354
13.10	1.558	1.525	1.497	1.468	1.445	1.422	1.399	1.381	1.367	1.358
13.15	1.563	1.530	1.502	1.473	1.449	1.426	1.403	1.385	1.371	1.361
13.20	1.568	1.535	1.507	1.477	1.453	1.431	1.408	1.389	1.374	1.364
13.25	1.573	1.540	1.512	1.482	1.458	1.435	1.412	1.393	1.378	1.368
13.30	1.578	1.546	1.517	1.486	1.463	1.440	1.416	1.398	1.382	1.372
13.35	1.583	1.551	1.521	1.491	1.467	1.444	1.420	1.401	1.385	1.375
13.40	1.589	1.556	1.526	1.495	1.472	1.449	1.424	1.404	1.389	1.379
13.45	1.594	1.561	1.531	1.500	1.476	1.453	1.428	1.408	1.393	1.382
13.50	1.599	1.566	1.536	1.504	1.481	1.457	1.432	1.412	1.397	1.386
13.55	1.604	1.571	1.541	1.509	1.485	1.461	1.436	1.416	1.400	1.389
13.60	1.609	1.577	1.546	1.514	1.490	1.466	1.440	1.420	1.404	1.393
13.65	1.614	1.582	1.551	1.518	1.494	1.470	1.444	1.424	1.408	1.396
13.70	1.619	1.587	1.556	1.523	1.499	1.475	1.448	1.428	1.412	1.400
13.75	1.624	1.592	1.561	1.527	1.503	1.479	1.452	1.432	1.416	1.404
13.80	1.630	1.597	1.566	1.532	1.508	1.483	1.456	1.436	1.419	1.408
13.85	1.635	1.602	1.571	1.537	1.513	1.487	1.460	1.439	1.423	1.411
13.90	1.640	1.608	1.575	1.542	1.517	1.492	1.465	1.443	1.426	1.414

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
11.20	1.210	1.205	1.202	1.204	1.204	1.205	1.205	1.205	1.205	1.205
11.25	1.213	1.208	1.205	1.206	1.206	1.207	1.207	1.207	1.207	1.207
11.30	1.217	1.211	1.208	1.209	1.209	1.209	1.209	1.209	1.209	1.209
11.35	1.220	1.214	1.211	1.211	1.211	1.211	1.211	1.211	1.210	1.210
11.40	1.224	1.217	1.214	1.214	1.214	1.214	1.214	1.213	1.212	1.212
11.45	1.227	1.220	1.217	1.216	1.216	1.216	1.216	1.215	1.214	1.213
11.50	1.231	1.224	1.220	1.219	1.219	1.219	1.218	1.217	1.216	1.215
11.55	1.234	1.227	1.223	1.222	1.221	1.221	1.220	1.219	1.218	1.217
11.60	1.238	1.230	1.226	1.225	1.224	1.223	1.222	1.221	1.220	1.219
11.65	1.241	1.233	1.229	1.227	1.226	1.225	1.223	1.222	1.221	1.220
11.70	1.245	1.236	1.232	1.230	1.229	1.227	1.225	1.224	1.223	1.222
11.75	1.248	1.239	1.235	1.233	1.232	1.229	1.227	1.226	1.225	1.224
11.80	1.252	1.243	1.238	1.236	1.234	1.231	1.229	1.228	1.227	1.226
11.85	1.255	1.246	1.240	1.238	1.236	1.233	1.231	1.229	1.228	1.227
11.90	1.259	1.250	1.243	1.241	1.239	1.236	1.233	1.231	1.230	1.229
11.95	1.262	1.253	1.246	1.244	1.241	1.238	1.235	1.233	1.231	1.230
12.00	1.266	1.256	1.249	1.247	1.244	1.240	1.237	1.235	1.233	1.232
12.05	1.269	1.259	1.252	1.249	1.246	1.242	1.238	1.236	1.235	[illegible]
12.10	1.272	1.262	1.255	1.251	1.249	1.244	1.240	1.238	1.237	[illegible]
12.15	1.275	1.265	1.258	1.254	1.251	1.246	1.242	1.240	1.238	1.237
12.20	1.279	1.268	1.261	1.257	1.254	1.248	1.244	1.242	1.240	1.239
12.25	1.283	1.271	1.264	1.259	1.256	1.250	1.246	1.244	1.242	1.241
12.30	1.287	1.274	1.267	1.262	1.259	1.252	1.248	1.246	1.244	1.243
12.35	1.290	1.277	1.270	1.265	1.261	1.254	1.250	1.247	1.246	1.245
12.40	1.293	1.281	1.273	1.268	1.264	1.256	1.252	1.249	1.248	1.247
12.45	1.296	1.284	1.276	1.270	1.266	1.258	1.254	1.250	1.249	1.248
12.50	1.300	1.288	1.279	1.273	1.269	1.261	1.256	1.252	1.251	1.250
12.55	1.303	1.290	1.282	1.275	1.271	1.263	1.258	1.254	1.252	1.251
12.60	1.307	1.293	1.285	1.278	1.273	1.265	1.260	1.256	1.254	1.253
12.65	1.310	1.297	1.288	1.280	1.276	1.267	1.261	1.257	1.256	1.255
12.70	1.314	1.300	1.291	1.283	1.279	1.269	1.263	1.259	1.258	1.257
12.75	1.317	1.303	1.294	1.286	1.281	1.271	1.265	1.261	1.259	1.258
12.80	1.321	1.306	1.297	1.289	1.283	1.273	1.267	1.263	1.261	1.260
12.85	1.324	1.309	1.299	1.291	1.286	1.275	1.269	1.265	1.263	1.261
12.90	1.328	1.312	1.302	1.294	1.289	1.277	1.271	1.267	1.265	1.263
12.95	1.331	1.315	1.305	1.297	1.291	1.279	1.272	1.268	1.266	[illegible]
13.00	1.335	1.319	1.308	1.300	1.293	1.281	1.274	1.270	1.268	[illegible]
13.05	1.338	1.321	1.310	1.302	1.295	1.283	1.276	1.271	1.269	1.268
13.10	1.341	1.324	1.313	1.304	1.298	1.286	1.278	1.273	1.271	1.270
13.15	1.345	1.327	1.316	1.307	1.300	1.288	1.280	1.275	1.273	1.271
13.20	1.349	1.331	1.320	1.310	1.303	1.290	1.282	1.277	1.275	1.273
13.25	1.352	1.334	1.322	1.313	1.305	1.292	1.284	1.278	1.276	1.275
13.30	1.355	1.337	1.325	1.316	1.308	1.294	1.286	1.280	1.278	1.277
13.35	1.358	1.340	1.328	1.318	1.310	1.296	1.287	1.281	1.279	1.278
13.40	1.362	1.343	1.331	1.321	1.313	1.298	1.289	1.283	1.281	1.280
13.45	1.365	1.346	1.334	1.323	1.315	1.300	1.291	1.285	1.283	1.281
13.50	1.369	1.349	1.337	1.326	1.317	1.302	1.293	1.287	1.285	1.283
13.55	1.372	1.352	1.340	1.328	1.319	1.304	1.295	1.289	1.287	1.285
13.60	1.376	1.355	1.343	1.331	1.322	1.306	1.297	1.291	1.289	1.287
13.65	1.379	1.358	1.346	1.334	1.324	1.308	1.298	1.292	1.290	1.288
13.70	1.382	1.361	1.349	1.337	1.327	1.310	1.300	1.294	1.292	1.290
13.75	1.385	1.364	1.351	1.339	1.329	1.312	1.301	1.296	1.293	1.291
13.80	1.389	1.368	1.354	1.342	1.332	1.314	1.303	1.298	1.295	1.293
13.85	1.393	1.371	1.357	1.345	1.334	1.316	1.305	1.300	1.297	1.295
13.90	1.397	1.374	1.360	1.348	1.337	1.318	1.307	1.301	1.299	1.297

Table A–3, continued

Pseudo-reduced Pressure, p_{pr}	Pseudoreduced Temperature, T_{pr}									
	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
13.95	1.645	1.613	1.580	1.546	1.522	1.496	1.469	1.447	1.430	1.417
14.00	1.650	1.618	1.585	1.550	1.526	1.500	1.473	1.451	1.434	1.421
14.05	1.655	1.623	1.590	1.555	1.531	1.504	1.478	1.455	1.437	1.425
14.10	1.661	1.628	1.595	1.560	1.535	1.509	1.482	1.459	1.441	1.429
14.15	1.666	1.633	1.600	1.565	1.539	1.513	1.486	1.463	1.445	1.432
14.20	1.671	1.639	1.605	1.570	1.544	1.518	1.490	1.467	1.449	1.436
14.25	1.676	1.644	1.609	1.575	1.548	1.522	1.494	1.471	1.452	1.439
14.30	1.681	1.649	1.614	1.580	1.553	1.527	1.498	1.475	1.456	1.443
14.35	1.686	1.650	1.619	1.584	1.557	1.531	1.502	1.478	1.460	1.446
14.40	1.692	1.659	1.624	1.589	1.561	1.536	1.506	1.482	1.464	1.450
14.45	1.697	1.664	1.629	1.594	1.565	1.540	1.510	1.486	1.468	1.454
14.50	1.702	1.669	1.634	1.598	1.570	1.544	1.515	1.490	1.472	1.458
14.55	1.707	1.674	1.639	1.603	1.575	1.548	1.520	1.494	1.476	1.461
14.60	1.712	1.679	1.643	1.608	1.580	1.552	1.524	1.498	1.480	1.465
14.65	1.717	1.684	1.648	1.613	1.584	1.556	1.528	1.502	1.484	1.468
14.70	1.722	1.690	1.653	1.617	1.589	1.561	1.532	1.505	1.488	1.472
14.75	1.727	1.695	1.658	1.622	1.594	1.565	1.536	1.509	1.491	1.475
14.80	1.733	1.700	1.663	1.627	1.598	1.570	1.540	1.513	1.495	1.479
14.85	1.738	1.705	1.668	1.632	1.603	1.574	1.544	1.517	1.498	1.483
14.90	1.743	1.710	1.673	1.636	1.607	1.579	1.549	1.521	1.502	1.487
14.95	1.748	1.715	1.678	1.641	1.612	1.583	1.553	1.525	1.505	1.490
15.00	1.753	1.720	1.682	1.645	1.616	1.588	1.558	1.529	1.508	1.493

	1.60	1.70	1.80	1.90	2.00	2.20	2.40	2.60	2.80	3.00
13.95	1.399	1.377	1.363	1.350	1.339	1.320	1.309	1.302	1.300	1.298
14.00	1.402	1.380	1.366	1.353	1.341	1.322	1.311	1.304	1.302	1.300
14.05	1.405	1.383	1.369	1.356	1.343	1.324	1.313	1.306	1.303	1.301
14.10	1.409	1.386	1.372	1.358	1.346	1.326	1.315	1.308	1.305	1.303
14.15	1.412	1.389	1.375	1.360	1.348	1.328	1.317	1.309	1.307	1.305
14.20	1.416	1.392	1.378	1.363	1.351	1.330	1.319	1.311	1.309	1.307
14.25	1.419	1.395	1.381	1.366	1.353	1.332	1.320	1.313	1.310	1.308
14.30	1.423	1.398	1.384	1.369	1.356	1.334	1.322	1.315	1.312	1.310
14.35	1.426	1.401	1.387	1.371	1.358	1.336	1.323	1.316	1.313	1.311
14.40	1.430	1.404	1.390	1.374	1.360	1.338	1.325	1.318	1.315	1.313
14.45	1.433	1.407	1.393	1.376	1.362	1.340	1.327	1.320	1.317	1.315
14.50	1.437	1.411	1.396	1.379	1.365	1.342	1.329	1.322	1.319	1.317
14.55	1.440	1.414	1.398	1.382	1.367	1.344	1.331	1.324	1.321	1.318
14.60	1.443	1.417	1.400	1.385	1.370	1.346	1.333	1.326	1.323	1.320
14.65	1.446	1.420	1.403	1.387	1.372	1.348	1.334	1.327	1.324	1.321
14.70	1.450	1.423	1.407	1.390	1.375	1.350	1.336	1.329	1.326	1.323
14.75	1.453	1.426	1.410	1.392	1.377	1.352	1.338	1.330	1.327	1.325
14.80	1.457	1.429	1.413	1.395	1.380	1.354	1.340	1.332	1.329	1.327
14.85	1.460	1.432	1.416	1.398	1.382	1.356	1.341	1.334	1.331	1.328
14.90	1.463	1.436	1.419	1.401	1.385	1.358	1.343	1.336	1.333	1.330
14.95	1.466	1.439	1.421	1.403	1.387	1.360	1.345	1.338	1.334	1.331
15.00	1.470	1.442	1.424	1.406	1.390	1.362	1.347	1.340	1.336	1.333

APPENDIX B

Gas Measurement Factors

Table B–1 *Basic orifice factors, F_b, for flange taps, Mcfd/$\sqrt{h_w p_m}$.**

p_b = 14.65 psia T_m = 520°R or 60°F
T_b = 520°R or 60°F γ_g = 1.000
All other factors = 1.000

Nominal Diameter (in.)	2			3				4	
ASA Schedule	160	80	40	xxHvy	160	80	40	xxHvy	160
Inside Diameter (in.)	1.689	1.939	2.067	2.300	2.626	2.900	3.068	3.152	3.438
Orifice Diameter (in.)			†				†		
0.250	0.3064	0.3066	0.3067	0.3068	0.3068	0.3067	0.3066	0.3065	0.3064
0.375	0.6871	0.6863	0.6860	0.6856	0.6852	0.6849	0.6848	0.6847	0.6845
0.500	1.225	1.221	1.219	1.217	1.215	1.214	1.214	1.213	1.213
0.625	1.933	1.919	1.914	1.908	1.902	1.899	1.897	1.897	1.895
0.750	2.826	2.790	2.779	2.764	2.751	2.744	2.740	2.739	2.735
0.875	3.932	3.851	3.824	3.792	3.765	3.750	3.744	3.741	3.733
1.000	5.303	5.127	5.073	5.006	4.951	4.924	4.912	4.907	4.893
1.125	7.022	6.665	6.557	6.428	6.324	6.273	6.251	6.242	6.217
1.250	9.310	8.533	8.329	8.087	7.901	7.810	7.771	7.755	7.713
1.375	—	10.82	10.46	10.03	9.705	9.551	9.486	9.459	9.388
1.500	—	—	13.09	12.33	11.78	11.52	11.41	11.37	11.25
1.625	—	—	—	15.06	14.16	13.75	13.58	13.51	13.33
1.750	—	—	—	—	16.92	16.28	16.01	15.90	15.63
1.875	—	—	—	—	20.15	19.16	18.75	18.59	18.18
2.000	—	—	—	—	—	22.46	21.86	21.62	21.01
2.125	—	—	—	—	—	26.33	25.40	25.05	24.17
2.250	—	—	—	—	—	—	29.52	28.96	27.70
2.375	—	—	—	—	—	—	—	—	31.65
2.500	—	—	—	—	—	—	—	—	36.16

† Use if only the nominal diameter is known.

Table B–1, continued

Nominal Diameter (in.)	4		6				8		
ASA Schedule	80	40	xxHvy	160	80	40	80	40	30
Inside Diameter (in.)	3.826	4.026	4.897	5.189	5.761	6.065	7.625	7.981	8.071
Orifice Diameter (in.)		†				†			
0.250	0.3062	0.3061	—	—	—	—	—	—	—
0.375	0.6842	0.6841	—	—	—	—	—	—	—
0.500	1.212	1.212	1.211	1.211	1.211	1.211	—	—	—
0.625	1.893	1.892	1.890	1.890	1.889	1.889	—	—	—
0.750	2.731	2.729	2.724	2.723	2.721	2.720	—	—	—
0.875	3.726	3.723	3.713	3.711	3.707	3.706	3.700	3.700	3.700
1.000	4.879	4.874	4.859	4.855	4.850	4.847	4.838	4.836	4.836
1.125	6.194	6.186	6.161	6.156	6.147	6.143	6.129	6.127	6.126
1.250	7.675	7.661	7.622	7.613	7.600	7.595	7.575	7.572	7.571
1.375	9.326	9.303	9.242	9.230	9.211	9.203	9.176	9.172	9.171
1.500	11.16	11.12	11.03	11.01	10.98	10.97	10.93	10.93	10.93
1.625	13.17	13.12	12.98	12.95	12.91	12.90	12.84	12.84	12.84
1.750	15.39	15.31	15.10	15.06	15.01	14.98	14.91	14.90	14.90
1.875	17.83	17.71	17.40	17.34	17.26	17.23	17.14	17.13	17.13
2.000	20.50	20.32	19.88	19.80	19.69	19.65	19.53	19.51	19.51
2.125	23.43	23.18	22.56	22.45	22.30	22.24	22.07	22.05	22.05
2.250	26.66	26.30	25.44	25.29	25.09	25.01	24.79	24.76	24.75
2.375	30.22	29.72	28.55	28.34	28.06	27.95	27.66	27.62	27.61
2.500	34.15	33.48	31.88	31.60	31.22	31.09	30.70	30.65	30.64
2.625	38.50	37.60	35.45	35.09	34.59	34.41	33.92	33.85	33.84
2.750	43.37	42.15	39.31	38.82	38.18	37.94	37.30	37.22	37.21
2.875	—	47.19	43.46	42.82	41.99	41.69	40.87	40.77	40.74
3.000	—	52.97	47.94	47.12	46.04	45.66	44.61	44.49	44.46
3.125	—	—	52.78	51.72	50.35	49.86	48.54	48.39	48.36
3.250	—	—	58.02	56.68	54.94	54.32	52.67	52.48	52.44

Table B–1, continued

Nominal Diameter (in.)	4		6				8		
ASA Schedule	80	40	xxHvy	160	80	40	80	40	30
Inside Diameter (in.)	3.826	4.026 †	4.897	5.189	5.761	6.065 †	7.625	7.981	8.071
Orifice Diameter (in.)									
3.375	—	—	63.70	62.01	59.83	59.05	56.99	56.76	56.71
3.500	—	—	69.87	67.77	65.04	64.04	64.07	61.24	61.17
3.625	—	—	76.76	73.97	70.60	69.40	66.28	65.92	65.85
3.750	—	—	—	80.73	76.55	75.08	71.25	70.82	70.73
3.875	—	—	—	88.27	82.91	81.12	76.46	75.94	75.83
4.000	—	—	—	—	89.73	87.56	81.91	81.29	81.15
4.250	—	—	—	—	105.01	101.8	93.62	92.72	92.53
4.500	—	—	—	—	—	118.3	106.5	105.2	105.0
4.750	—	—	—	—	—	—	120.7	118.9	118.5
5.000	—	—	—	—	—	—	136.3	134.0	133.4
5.250	—	—	—	—	—	—	153.7	150.5	149.8
5.500	—	—	—	—	—	—	173.0	168.7	167.8
5.750	—	—	—	—	—	—	—	189.0	187.7
6.000	—	—	—	—	—	—	—	—	210.1

* Adapted by the Interstate Oil Compact Commission from *American Gas Association Gas Measurement Committee Report No. 3.* Used by permission of the Interstate Oil Compact Commission.

† Use if only the nominal diameter is known.

Table B–2 *Basic orifice factors, F_b, for pipe taps, Mcfd/$\sqrt{h_w p_m}$.**

p_b = *14.65 psia* T_m = *520°R or 60°F*
T_b = *520°R or 60°F* γ_g = *1.000*
All other factors = 1.000

Nominal Diameter (in.)	2			3				4	
ASA Schedule	160	80	40	xxHvy	160	80	40	xxHvy	160
Inside Diameter (in.)	1.689	1.939	2.067 †	2.300	2.626	2.900	3.068 †	3.152	3.438
Orifice Diameter (in.)									
0.250	0.3101	0.3092	0.3089	0.3085	0.3080	0.3078	0.3076	0.3076	0.3074
0.375	0.7085	0.7022	0.6999	0.6970	0.6943	0.6928	0.6921	0.6918	0.6910
0.500	1.296	1.275	1.266	1.255	1.245	1.239	1.237	1.235	1.232
0.625	2.105	2.049	2.029	2.001	2.974	1.958	1.951	1.948	1.939
0.750	3.191	3.061	3.016	2.955	2.897	2.864	2.848	2.840	2.820
0.875	4.651	4.368	4.273	4.149	4.036	3.972	3.941	3.928	3.889
1.000	6.647	6.060	5.871	5.630	5.419	5.303	5.249	5.226	5.159
1.125	9.458	8.277	7.915	7.467	7.090	6.889	6.797	6.756	6.645
1.250	—	11.24	10.57	9.762	9.106	8.770	8.618	8.554	8.375
1.375	—	—	14.09	12.66	11.55	11.00	10.76	10.65	10.37
1.500	—	—	—	16.39	14.54	13.65	13.27	13.11	12.68
1.625	—	—	—	—	18.23	16.83	16.24	15.99	15.34
1.750	—	—	—	—	22.85	20.67	19.77	19.40	18.43
1.875	—	—	—	—	—	25.35	23.99	23.44	22.01
2.000	—	—	—	—	—	31.15	29.09	28.28	26.20
2.125	—	—	—	—	—	—	35.36	34.15	31.12
2.250	—	—	—	—	—	—	—	—	36.97
2.375	—	—	—	—	—	—	—	—	43.99

† Use if only the nominal diameter is known.

Table B–2, continued

Nominal Diameter (in.)	4		6				8		
ASA Schedule	*80*	*40*	*xxHvy*	*160*	*80*	*40*	*86*	*40*	*30*
Inside Diameter (in.)	*3.826*	*4.026*	*4.897* †	*5.189*	*5.761*	*6.065* †	*7.625*	*7.981*	*8.071* †
Orifice Diameter (in.)									
0.250	0.3071	0.3070	—	—	—	—	—	—	—
0.375	0.6901	0.6898	—	—	—	—	—	—	—
0.500	1.229	1.228	1.224	1.224	1.222	1.222	—	—	—
0.625	1.930	1.927	1.917	1.915	1.912	1.910	—	—	—
0.750	2.801	2.793	2.771	2.766	2.759	2.756	—	—	—
0.875	3.851	3.836	3.791	3.782	3.768	3.762	3.743	3.740	3.739
1.000	5.093	5.066	4.986	4.969	4.943	4.933	4.899	4.894	4.893
1.125	6.537	6.494	6.364	6.335	6.291	6.274	6.217	6.209	6.207
1.250	8.202	8.134	7.933	7.888	7.819	7.791	7.701	7.687	7.685
1.375	10.11	10.00	9.703	9.636	9.534	9.492	9.354	9.334	9.330
1.500	12.28	12.12	11.68	11.59	11.44	11.38	11.18	11.16	11.15
1.625	14.75	14.52	13.89	13.76	13.56	13.47	13.19	13.15	13.14
1.750	17.56	17.23	16.35	16.16	15.88	15.77	15.38	15.33	15.32
1.875	20.76	20.30	19.06	18.81	18.43	18.28	17.77	17.69	17.68
2.000	24.41	23.77	22.07	21.73	21.22	21.02	20.35	20.25	20.23
2.125	28.60	27.71	25.39	24.93	24.27	24.00	23.14	23.01	22.98
2.250	33.43	32.20	29.07	28.46	27.69	27.24	26.13	25.98	25.9[illegible]
2.375	39.02	37.34	33.14	32.34	31.21	30.76	29.35	29.15	29.1[illegible]
2.500	45.55	43.25	37.66	36.61	35.15	34.57	32.80	32.55	32.49
2.625	53.24	50.10	42.67	41.32	39.44	38.71	36.49	36.18	36.11
2.750	—	58.09	48.26	46.52	44.12	43.20	40.43	40.05	39.97
2.875	—	—	54.50	52.27	49.23	48.08	44.64	44.18	44.07
3.000	—	—	61.50	58.65	54.81	53.37	49.13	48.57	48.44
3.125	—	—	69.38	65.75	60.91	59.13	53.93	53.24	53.09
3.250	—	—	78.30	73.67	67.61	65.40	59.04	58.22	58.03
3.375	—	—	88.46	82.55	74.98	72.23	64.49	63.51	63.28
3.500	—	—	—	92.56	83.09	79.71	70.31	69.13	68.87
3.625	—	—	—	103.9	92.05	87.90	76.52	75.12	74.80
3.750	—	—	—	—	102.0	96.89	83.16	81.48	81.11
3.875	—	—	—	—	113.1	106.8	90.25	88.27	87.82
4.000	—	—	—	—	125.4	117.7	97.84	95.49	94.96
4.250	—	—	—	—	—	—	114.7	111.4	110.7
4.500	—	—	—	—	—	—	134.0	129.6	128.6
4.750	—	—	—	—	—	—	156.5	150.4	149.0
5.000	—	—	—	—	—	—	182.7	174.3	172.5
5.250	—	—	—	—	—	—	213.6	202.1	19[illegible]
5.500	—	—	—	—	—	—	—	234.7	23[illegible]

* Adapted by the Interstate Oil Compact Commission from *American Gas Association Gas Measurement Committee Report No. 3.* Used by permission of the Interstate Oil Compact Commission.

† Use if only the nominal diameter is known.

Table B–3 ***Basic critical flow prover factors, F_p, Mcfd/psia.****

p_b = *14.65 psia*
T_b = *520°R or 60°F*
T_m = *520°R or 60°F*
γ_g = *1.000*

2-in. Prover		*4-in. Prover*	
Orifice Diameter (in.)	*Factor, F_p*	*Orifice Diameter (in.)*	*Factor, F_p*
1/16	0.06569	1/4	1.074
3/32	0.1446	3/8	2.414
1/8	0.2716	1/2	4.319
3/16	0.6237	5/8	6.729
7/32	0.8608	3/4	9.643
1/4	1.115	7/8	13.11
5/16	1.714	1	17.08
3/8	2.439	1 1/8	21.52
7/16	3.495	1 1/4	26.57
1/2	4.388	1 3/8	31.99
5/8	6.638	1 1/2	38.12
3/4	9.694	1 3/4	52.07
7/8	13.33	2	68.80
1	17.53	2 1/4	88.19
1 1/8	22.45	2 1/2	110.6
1 1/4	28.34	2 3/4	136.9
1 3/8	34.82	3	168.3
1 1/2	43.19		

* Adapted from *Bureau of Mines Monograph 7* by the Interstate Oil Compact Commission. Used by permission of the Interstate Oil Compact Commission.

Table B–4 ***Flowing temperature factors, F_t.****

$$F_t = \sqrt{\frac{520}{T_m}}$$

Observed Temperature, °F	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	1.063	1.062	1.061	1.060	1.059	1.057	1.056	1.055	1.054	1.053
10	1.052	1.051	1.050	1.049	1.047	1.046	1.045	1.044	1.043	1.042
20	1.041	1.040	1.039	1.038	1.037	1.036	1.034	1.033	1.032	1.031
30	1.030	1.029	1.028	1.027	1.026	1.025	1.024	1.023	1.022	1.021
40	1.020	1.019	1.018	1.017	1.016	1.015	1.014	1.013	1.012	1.011
50	1.010	1.009	1.008	1.007	1.006	1.005	1.004	1.003	1.002	1.001
60	1.000	0.9990	0.9981	0.9971	0.9962	0.9952	0.9943	0.9933	0.9924	0.9915
70	0.9905	0.9896	0.9887	0.9877	0.9868	0.9859	0.9850	0.9840	0.9831	0.9822
80	0.9813	0.9804	0.9795	0.9786	0.9777	0.9768	0.8759	0.9750	0.9741	0.9732
90	0.9723	0.9715	0.9706	0.9697	0.9688	0.9680	0.9671	0.9662	0.9653	0.9645
100	0.9636	0.9628	0.9619	0.9610	0.9602	0.9594	0.9585	0.9577	0.9568	0.9560
110	0.9551	0.9543	0.9535	0.9526	0.9518	0.9510	0.9501	0.9493	0.9485	0.9477
120	0.9469	0.9460	0.9452	0.9444	0.9436	0.9428	0.9420	0.9412	0.9404	0.9396
130	0.9388	0.9380	0.9372	0.9364	0.9356	0.9349	0.9341	0.9333	0.9325	0.9317
140	0.9309	0.9302	0.9294	0.9286	0.9279	0.9271	0.9263	0.9256	0.9248	0.9240

* Used by permission of the Interstate Oil Compact Commission.

Table B–4, continued

Observed Temperature, °F	0	1	2	3	4	5	6	7	8	9
150	0.9233	0.9225	0.9217	0.9210	0.9202	0.9195	0.9187	0.9180	0.9173	0.9165
160	0.9158	0.9150	0.9143	0.9135	0.9128	0.9121	0.9114	0.9106	0.9099	0.9092
170	0.9085	0.9077	0.9071	0.9063	0.9055	0.9048	0.9042	0.9035	0.9028	0.9020
180	0.9014	0.9007	0.9000	0.8992	0.8985	0.8979	0.8972	0.8965	0.8958	0.8951
190	0.8944	0.8937	0.8931	0.8923	0.8916	0.8910	0.8903	0.8896	0.8889	0.8882
200	0.8876	0.8870	0.8863	0.8856	0.8849	0.8843	0.8836	0.8830	0.8823	0.8816
210	0.8810	0.8803	0.8797	0.8790	0.8784	0.8777	0.8770	0.8764	0.8758	0.8751
220	0.8745	0.8738	0.8732	0.8725	0.8719	0.8713	0.8706	0.8700	0.8694	0.8687
230	0.8681	0.8675	0.8668	0.8662	0.8656	0.8650	0.8644	0.8637	0.8631	0.8625
240	0.8619	0.8613	0.8606	0.8600	0.8594	0.8588	0.8582	0.8576	0.8570	0.8564

Table B–5 *Specific gravity factors, F_g.**

$$F_g = \sqrt{\frac{1}{\gamma_g}}$$

Specific Gravity γ_g	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.550	1.348	1.347	1.346	1.345	1.344	1.342	1.341	1.340	1.339	1.338
0.560	1.336	1.335	1.334	1.333	1.332	1.330	1.329	1.328	1.327	1.326
0.570	1.325	1.323	1.322	1.321	1.320	1.319	1.318	1.316	1.315	1.314
0.580	1.313	1.312	1.311	1.310	1.309	1.307	1.306	1.305	1.304	1.303
0.590	1.302	1.301	1.300	1.299	1.298	1.296	1.295	1.294	1.293	1.292
0.600	1.291	1.290	1.289	1.288	1.287	1.286	1.285	1.284	1.282	1.281
0.610	1.280	1.279	1.278	1.277	1.276	1.275	1.274	1.273	1.272	1.271
0.620	1.270	1.269	1.268	1.267	1.266	1.265	1.264	1.263	1.262	1.261
0.630	1.260	1.259	1.258	1.257	1.256	1.255	1.254	1.253	1.252	1.251
0.640	1.250	1.249	1.248	1.247	1.246	1.245	1.244	1.243	1.242	1.241
0.650	1.240	1.239	1.238	1.237	1.237	1.236	1.235	1.234	1.233	1.232
0.660	1.231	1.230	1.229	1.228	1.227	1.226	1.225	1.224	1.224	1.223
0.670	1.222	1.221	1.220	1.219	1.218	1.217	1.216	1.215	1.214	1.214
0.680	1.213	1.212	1.211	1.210	1.209	1.208	1.207	1.206	1.206	1.205
0.690	1.204	1.203	1.202	1.201	1.200	1.200	1.199	1.198	1.197	1.196
0.700	1.195	1.194	1.194	1.193	1.192	1.191	1.190	1.189	1.188	1.188
0.710	1.187	1.186	1.185	1.184	1.183	1.183	1.182	1.181	1.180	1.179
0.720	1.179	1.178	1.177	1.176	1.175	1.174	1.174	1.173	1.172	1.171
0.730	1.170	1.170	1.169	1.168	1.167	1.166	1.166	1.165	1.164	1.163
0.740	1.162	1.162	1.161	1.160	1.159	1.159	1.158	1.157	1.156	1.155

Table B–5, continued

Specific Gravity γ_g	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.750	1.155	1.154	1.153	1.152	1.152	1.151	1.150	1.149	1.149	1.148
0.760	1.147	1.146	1.146	1.145	1.144	1.143	1.143	1.142	1.141	1.140
0.770	1.140	1.139	1.138	1.137	1.137	1.136	1.135	1.134	1.134	1.133
0.780	1.132	1.132	1.131	1.130	1.129	1.129	1.128	1.127	1.127	1.126
0.790	1.125	1.124	1.124	1.123	1.122	1.122	1.121	1.120	1.119	1.119
0.800	1.118	1.117	1.117	1.116	1.115	1.115	1.114	1.113	1.112	1.112
0.810	1.111	1.110	1.110	1.109	1.108	1.108	1.107	1.106	1.106	1.105
0.820	1.104	1.104	1.103	1.102	1.102	1.101	1.100	1.100	1.099	1.098
0.830	1.098	1.097	1.096	1.096	1.095	1.094	1.094	1.093	1.092	1.092
0.840	1.091	1.090	1.090	1.089	1.089	1.088	1.087	1.087	1.086	1.085
0.850	1.085	1.084	1.083	1.083	1.082	1.081	1.081	1.080	1.080	1.079
0.860	1.078	1.078	1.077	1.076	1.076	1.075	1.075	1.074	1.073	1.073
0.870	1.072	1.072	1.071	1.070	1.070	1.069	1.068	1.068	1.067	1.067
0.880	1.066	1.065	1.065	1.064	1.064	1.063	1.062	1.062	1.061	1.061
0.890	1.606	1.059	1.059	1.058	1.058	1.057	1.056	1.056	1.055	1.055
0.900	1.054	1.054	1.053	1.052	1.052	1.051	1.051	1.050	1.049	1.049
0.910	1.048	1.048	1.047	1.047	1.046	1.045	1.045	1.044	1.044	1.043
0.920	1.043	1.042	1.041	1.041	1.040	1.040	1.039	1.039	1.038	1.038
0.930	1.037	1.036	1.036	1.035	1.035	1.034	1.034	1.033	1.033	1.032
0.940	1.031	1.031	1.030	1.030	1.029	1.029	1.028	1.028	1.027	1.027
0.950	1.026	1.025	1.025	1.024	1.024	1.023	1.023	1.022	1.022	1.021
0.960	1.021	1.020	1.020	1.019	1.019	1.018	1.017	1.017	1.016	1.016
0.970	1.015	1.015	1.014	1.014	1.013	1.013	1.012	1.012	1.011	1.011
0.980	1.010	1.010	1.009	1.009	1.008	1.008	1.007	1.007	1.006	1.006
0.990	1.005	1.005	1.004	1.004	1.003	1.003	1.002	1.002	1.001	1.001

* Used by permission of the Interstate Oil Compact Commission.

APPENDIX C

SI Metric Conversion Factors

in.	× 2.54*	E +01	=	mm
in.	× 2.54*	E +00	=	cm
ft	× 3.048*	E −01	=	m
mile	× 1.609 344*	E +00	=	km
sq ft	× 9.290 304*	E −02	=	m^2
acre	× 4.046 873	E +03	=	m^2 (1)
acre	× 4.046 856	E −01	=	ha
cu ft	× 2.831 685	E −02	=	m^3
bbl (42 U.S. gal)	× 1.589 873	E −01	=	m^3
acre-ft	× 1.233 489	E +03	=	m^3 (1)
lbm	× 4.535 924	E −01	=	kg
Btu/ft^3	× 3.725 895	E +01	=	kJ/m^3
°R	× 5/9		=	°K
°F	(°F − 32)/1.8		=	°C
°F/1000 ft	× 1.822 689	E +02	=	mK/m
in. Hg (32°F)	× 3.386 38	E +00	=	kPa
in. Hg (60°F)	× 3.376 85	E +00	=	kPa
psi	× 6.894 757	E +00	=	kPa
atm	× 1.013 25	E +02	=	kPa
bar	× 1.0*	E +02	=	kPa
psi/ft	× 2.262 059	E +01	=	kPa/m
lbm/ft^3	× 1.601 846	E +01	=	kg/m^3
API	141.5/(131.5 + °API)		=	g/cm^3
ft^3/lbm	× 6.242 796	E −02	=	m^3/kg
bbl/acre-ft	× 1.288 923	E −04	=	m^3/m^3
darcy	× 9.869 233	E −01	=	μm^2
millidarcy	× 9.869 233	E −04	=	μm^2
bbl/D	× 1.589 873	E −01	=	m^3/d
ft^3/D	× 2.831 685	E −02	=	m^3/d
ft/s	× 3.048*	E −01	=	m/s
cp	× 1.0*	E −03	=	Pa.s

* Conversion factor is exact.

(1) Based on the U.S. Survey foot which is 1200/3937 m. These conversion factors were taken from "The SI Metric System of Units and SPE Metric Standard."

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APPENDIX D

Solutions to Problems

Note: All compressibility factors used in the solutions to the problems at the end of the chapters were calculated by computer using the method of Hall and Yarborough, *Oil & Gas Journal,* 71, No. 25 (June 18, 1973), p. 82 and *Oil & Gas Journal,* 72 (February 18, 1974), p. 86.

Specific solutions to one or two problems are not given in the main text but the problems are included so that the reader can use personal initiative.

Chapter 2, Problem 1 (Assume ideal gas laws apply.)

Moles CO_2 = 2/44.010 = 0.0454

$$\text{Total moles} = n = \frac{pV}{zRT} = \frac{(30.0)(40.0)}{(1.0)(10.732)(530)} = 0.2110$$

Moles air = 0.2110 − 0.0454 = 0.1656

Moles N_2 = (0.80)(0.1656) = 0.1325; Moles O_2 = (0.20)(0.1656) = 0.0331

	Moles	*Mole %*	*Vol %*	*MM**	*Wt*	*Wt%*	*Partial press. (psia)*
N_2	0.1325	62.79	62.79	28.013	17.59	54.83	18.84
O_2	0.0331	15.69	15.69	31.9988	5.02	15.65	4.71
CO_2	0.0454	21.52	21.52	44.010	9.47	29.52	6.45
Total	0.2110	100.00	100.00		32.08	100.00	30.00

* Molar mass or molecular weight

γ_g = 32.08/28.964 = 1.108

$$\rho = 2.6988\,\frac{(20.0)(1.108)}{(1.0)(530)} = 0.113 \text{ lb/cu ft. (20.0 psia \& 70°F)}$$

$$\rho = 2.6988\,\frac{(14.7)(1.108)}{(1.0)(520)} = 0.0845 \text{ lb/cu ft. (14.7 psia \& 60°F)}$$

$$\rho = 2.6988\,\frac{(14.7)(1.108)}{(1.0)(492)} = 0.0893 \text{ lb/cu ft. (14.7 psia \& 32°F)}$$

Chapter 2, Problem 2 (Assume ideal gas laws apply.)

Component	*Mole %*	*Molar mass*	*Mass, lb*	
Methane	58.7	16.043	9.42	
Ethane	16.5	30.070	4.96	
Propane	9.9	44.097	4.37	
Butanes	5.0	58.123	2.91	
Pentanes +	3.5	75.000*	2.63	
Hydrogen sulfide	6.4	34.08	2.18	
Total	100.0		26.47	* Estimated

Component	*(1) Mole Fraction*	*(2) Gross Heating Value**	=	*(1) × (2)*	
Methane	0.587	1010.0		592.9	
Ethane	0.165	1769.8		292.0	
Propane	0.099	2516.2		249.1	
Butanes	0.050	3260**		163.0	
Pentanes +	0.035	4100**		143.5	
Hydrogen sulfide	0.064	637.1		40.8	
Total	1.000			1481	** Estimated

* Gross heating value (dry) = 1481 Btu per ideal cubic foot at 14.696 psia and 60°F.
Note the influence of the estimated gross heating values on the calculated gross heating values.

Chapter 2, Problem 3

Press., psia	*Compressibility Factors Temperature* 100°F	150°F	200°F*
500	0.9501	0.9646	0.9749
1,000	0.9076	0.9356	0.9552
1,500	0.8756	0.9146	0.9420
2,000	0.8573	0.9034	0.9361
2,500	0.8543	0.9028	0.9380
3,000	0.8656	0.9123	0.9475

Chapter 2, Problem 4

Press., psia	*Compressibility Factors Temperature* 100°F	150°F	200°F
500	0.9341	0.9522	0.9651
1,000	0.8752	0.9110	0.9360
1,500	0.8285	0.8791	0.9142
2,000	0.7998	0.8592	0.9013
2,500	0.7921	0.8532	0.8982
3,000	0.8037	0.8606	0.9048

Chapter 2, Problem 5

Compressibility, c_g = 3.578 E−04
Formation volume factor, B_g = 6.196 E − 03

Chapter 2, Problem 6

Viscosity = 0.01849 *Pa.s*

Chapter 2, Problem 7

Natural Gas Properties from Composition
Conditions : 60°F, 14.73 psia

DRY ANALYSIS

Methane	= 0.88000	Hydrogen	= 0.00000
Ethane	= 0.05760	Helium	= 0.00120
Propane	= 0.01760	Water	= 0.00000
i-Butane	= 0.00280	Carbon monoxide	= 0.00000
n-Butane	= 0.00830	Nitrogen	= 0.02200
i-Pentane	= 0.00130	Oxygen	= 0.00000
n-Pentane	= 0.00210	Hydrogen sulfide	= 0.00000
Hexanes	= 0.00270	Argon	= 0.00000
Heptanes	= 0.00230	Carbon dioxide	= 0.00210
Octanes	= 0.00000		
Nonanes	= 0.00000		
Decanes	= 0.00000		

Molar mass = 18.693
Molar mass ratio = 0.64542

Relative density	=	0.64690	0.64692 *
Compressibility factor	=	0.99732	0.99732 *
Gross heating value, Btu/lb	=	22540.7	
Gross heating value, Btu/ideal cf	=	1112.9	1115.9 *
Gross heating value, Btu/real cf	=	1115.9	

* Indicates computations by hand procedures

WATER SATURATED ANALYSIS

Methane	= 0.86468	Hydrogen	= 0.00000
Ethane	= 0.05660	Helium	= 0.00118
Propane	= 0.01729	Water	= 0.01740
i-Butane	= 0.00275	Carbon monoxide	= 0.00000
n-Butane	= 0.00816	Nitrogen	= 0.02162
i-Pentane	= 0.00128	Oxygen	= 0.00000
n-Pentane	= 0.00206	Hydrogen sulfide	= 0.00000
Hexanes	= 0.00265	Argon	= 0.00000
Heptanes	= 0.00226	Carbon dioxide	= 0.00206
Octanes	= 0.00000		
Nonanes	= 0.00000		
Decanes	= 0.00000		

Molar mass	=	18.681	
Molar mass ratio	=	0.64501	
Relative density	=	0.64653	0.64673 *
Compressibility factor	=	0.99727	0.99697 *
Gross heating value, Btu/lb	=	22180.2	
Gross heating value, Btu/ideal cf	=	1094.4	
Gross heating value, Btu/real cf	=	1097.4	1097.7 *

* Indicates computations by hand procedures

Chapter 3, Problem 1

$P_{pc} = 657$; $T_{pc} = 388$, temperature = 152°F

$\overline{p}_R$	z	$\overline{p}_R/z$	$(\overline{p}_R/z)_1 - (\overline{p}_R/z)_2$	*Gas produced (MMcf)*
2066.5	0.8127	2542.9	0	0
1997.1	0.8145	2451.8	91.1	50
1782.2	0.8228	2166.0	376.9	479
1728.4	0.8255	2093.9	449.0	597
1647.3	0.8299	1985.0	557.9	722
1597.6	0.8329	1918.2	624.7	837
1533.1	0.8370	1831.7	711.2	945

G_i = 3,480 MMcf $\overline{p}_R/z$ vs. G_p, cartesian coordinates

G_i = 3,700 MMcf $(\overline{p}_R/z)_1 - (\overline{p}_R/z)_2$ vs. G_p, log-log coordinates

G_i = 3,750 MMcf $\overline{p}_R$ vs. G_p, cartesian coordinates

Chapter 3, Problem 2

$p_{pc} = 671$, $T_{pc} = 362$

$\overline{p}_R$	z	$\overline{p}_R/z$	$(\overline{p}_R/z)_1 - (\overline{p}_R/z)_2$	*Gas produced (MMcf)*
2080	0.8075	2575.8	0	0
1885	0.8142	2315.1	260.7	6900
1620	0.8283	1955.9	619.9	14000
1205	0.8605	1400.3	1175.5	23700
888	0.8921	995.5	1580.3	31000
645	0.9193	701.6	1874.2	36200

G_i = 50,000 MMcf

Chapter 4, Problem 1

Conversion of Bureau of Mines coefficients for Table B–3

$$F_p = C\frac{(14.4)(1)}{(14.65)(\sqrt{520})} = C(0.043105)$$

Chapter 5, Problem 1

When k = 0.0006 in., equation 5–10 becomes:

$$\sqrt{1/f} = 4 \log d + 15.1602$$

The coefficient of friction (f) is:

$$f = 1/(\sqrt{1/f})^2$$

and

$$F_r = \left(\frac{2.6665\, f}{d^5}\right)^{0.5}$$

If the inside pipe diameter is 1.049 in.

$$\sqrt{1/f} = 15.243$$

$$f = 0.004304,\ \sqrt{f} = 0.065605$$

$$F_r = 0.09505$$

The minimum Reynolds number, N_{Re}, at the values given for $\sqrt{1/f}$ and f is the Reynolds number which satisfies equation 5–9. Equation 5–9 can be rearranged to give

$$\log N_{Re} = \frac{(\sqrt{1/f} + 0.6)}{4} - \log \sqrt{f}$$

Thus

$$\log N_{Re} = \frac{15.243 + 0.6}{4} - \log 0.065606; \; N_{Re} = 139{,}000$$

The values thus calculated are given in the first five lines of Table 5–2.

Chapter 5, Problem 2

By equation 5–12, the average velocity = 11.76 ft/sec.

By equation 5–13, the Reynolds number, N_{Re} = 919,000.

Chapter 5, Problem 3

The flowing pressure at a depth of 8,248 feet is 1,758.7 psia. Nine steps were used in the calculations.

Chapter 5, Problem 4

The shut-in pressure at a depth 8,248 feet is 2,460.4 psia when the wellhead pressure is 2,048.0 psia.

Chapter 5, Problem 5

The pressure at a depth of 3,500 feet was calculated to be 1,441.1 psia.

Chapter 5, Problem 6

In problem 2, the fraction of the energy converted to kinetic energy is 0.0002618/2,747 = 0.000095.

In problem 5, the fraction is 0.001967/2.704 = 0.00073.

The kinetic energy term could have been omitted in both problems.

Chapter 5, Problem 7

H	L	d	t_n	p_n	p_n^2/1000	MMcfd
0	0	10.192	65.0	314.0	98.60	10.000
−1772	89777	10.192	65.0	313.0	97.97	10.000
258	113624	10.192	65.0	331.1	109.62	10.000
0	0	10.192	65.0	314.0	98.60	7.500
−1772	89777	10.192	65.0	308.3	95.08	7.500
258	113624	10.192	65.0	325.0	105.60	7.500
0	0	10.192	65.0	314.0	98.60	5.000
−1772	89777	10.192	65.0	304.8	92.89	5.000
258	113624	10.192	65.0	320.2	102.56	5.000

C = 1,976 with $(p_1^2 - p_2^2)$ expressed in thousands and q_k in Mcfd. n = 0.679.

Note that in the above calculations, the elevation term, H, is cumulative. However, the calculations were made for each interval separately.

Chapter 5, Problem 8

C = 1523 with the difference in pressures squared expressed in thousands and q in Mcfd.

n = 0.538.

Chapter 6, Problem 1

Permeability = 1.96 darcies.

Chapter 6, Problem 2

New rate of flow = 303.7 Mcfd.

Chapter 6, Problem 3

Average permeability = 195.1 *md*.

Chapter 7, Problem 1

Where q is in cu ft/day and $(p_e^2 - p_w^2)$ is in psi squared.
C = 73.2 and n = 1.00.

When q_k is in Mcfd $(p_e^2 - p_w^2)$ is in thousands
$C = 73.2$ and $n = 1.00$, also.

Chapter 7, Problem 2

$q_k = 24.2\ (p_e^2 - p_w^2)^{1.00}$; damage $= 24.2/73.2 = 0.33$

Chapter 7, Problem 3

Undamaged case, the pressure drop from $r = 2.0$ ft to wellbore is 22.7% of total pressure drop.

Damaged case, pressure drop from $r = 2.0$ ft to wellbore is 75.3% of total pressure drop.

Chapter 7, Problem 4

$q_k = 3.38\ (\overline{p}_R^2 - p_{wf}^2)^{0.759}$

Chapter 7, Problem 5

$q_k = 0.96\ (\overline{p}_R^2 - p_{wf}^2)^{0.777}$ and $q_k = 112$ Mcfd into line at 50 psia.

Chapter 7, Problem 6

$\gamma_{gf} = 0.668$

Chapter 7, Problem 7

At $(\overline{p}_r^2 - p_{wf}^2) = 10.69$

Time of Flow, (hr)	q_k, *(Mcfd)*
0.1	4,180
0.2	3,570
0.5	2,810
1.0	2,350
3.0	2,040
24.0	1,550

Chapter 7, Problem 8

Time of Flow, (hr)	q_k, *(Mcfd)*
72.0	1,380
240.0	1,240

Chapter 7, Problem 9

See Figure 1, next page.

Chapter 7, Problem 10

Time (hr)	*Open Flow (Mcfd)*
0.1	40,300
0.2	34,400
0.5	27,100
1.0	22,600
3.0	19,700
24.0	14,900
72.0	13,300
240.0	12,000

Chapter 8, Problem 1

The time required for the pressure disturbance to reach the boundary at a radius of 2,640 feet is about 25.4 days. The time for the rate of flow to reach 400 Mcfd is about 214 days and about 4.5 percent of the original gas in place would have been produced.

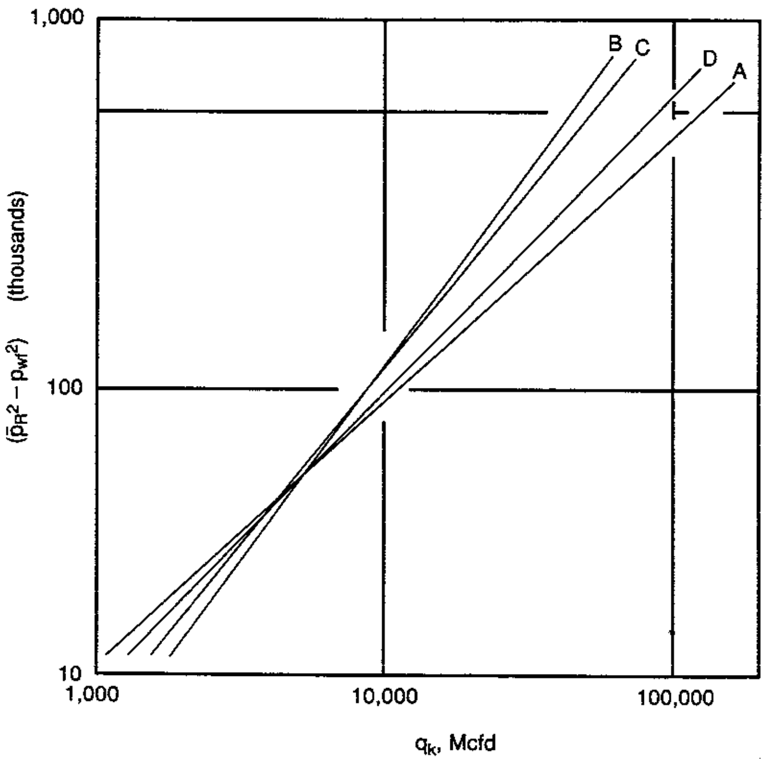
Chapter 8, Problem 2

Time (Years)	t_D	q_D	*Production Rate,* q_k *(Mcfd)*
0.5	3.75 E+03	0.218	491
1.0	7.51 E+03	0.201	452*
2.0	1.50 E+04	0.165	371
3.0	2.25 E+04	0.132	297
4.0	3.00 E+04	0.107	241
5.0	3.75 E+04	0.089	200

Time (Years)	t_D	q_D	Production Rate, q_k (Mcfd)
6.0	4.51 E+04	0.074	167
7.0	5.26 E+04	0.062	140
8.0	6.01 E+04	0.052	117
9.0	6.76 E+04	0.045	101
10.0	7.51 E+04	0.0415	93
15.0	1.13 E+05	0.0150	34
20.0	1.50 E+05	0.0068	15
25.0	1.88 E+05	0.0033	7

* Stabilization

Chapter 7, Problem 9, Figure 1



Chapter 8, Problem 3

Time, (Years)	t_D	q_D	Production Rate, q_k (Mcfd)
0.5	2.44 E+02	0.304	684
1.0	4.88 E+02	0.277	623
2.0	9.76 E+02	0.250	563
3.0	1.46 E+03	0.240	540
4.0	1.95 E+03	0.232	522
4.30	2.10 E+03	0.230	518*
5.0	2.44 E+03	0.221	497
6.0	2.93 E+03	0.209	470
7.0	3.42 E+03	0.195	439
8.0	3.90 E+03	0.183	412
9.0	4.39 E+03	0.171	385
10.0	4.88 E+03	0.159	358
15.0	7.32 E+03	0.119	268
20.0	9.76 E+03	0.089	200
25.0	1.22 E+04	0.065	146

* Stabilization

Note: It was assumed that same back pressure was held on the well as was for the example in the text.

Chapter 9, Problem 1

a. The well will no longer be capable of delivering 1,000 Mcfd against a back pressure of 915 psia when the shut-in wellhead pressure decreases to 983.3 psia. If the material balance is based on wellhead pressures and compressibility factors are neglected, 732,500 Mcf will be produced over a period of 2.01 years before the delivery rate falls below 1,000 Mcfd. If reservoir pressures are calculated and the compressibility factors are considered, the produced volume will be 787,100 Mcf over a period of 2.16 years.

b.

Time, (Months)	A_x	x	$\bar{p}_{ts}$	Average Mcfd
0	2.0871	1.0746	983.3	1,000
1	2.1707	1.0521	962.7	904
2	2.2539	1.0352	947.2	680
3	2.3372	1.0224	935.5	513
4	2.4204	1.0133	927.2	364
5	2.5035	1.0069	921.3	259
6	2.5869	1.0031	917.8	154

c.

Time, Years	Gas Produced, (Mcfd)	Time, Years	Gas Produced, (Mcfd)
1	844	6	196
2	604	7	154
3	443	8	122
4	332	9	98
5	253	10	80

Cumulated production for 10 years = 1,141,000 Mcf.

Production rates are average for the year indicated.

Chapter 9, Problem 2

Production forecasts

Year	Against 425 psia, (Mcf)	Against 850 psia, (Mcf)	Production Increase, (Mcf)
1	549,900	524,900	25,000
2	513,400	487,600	25,800
3	477,500	450,900	26,600
4	442,500	414,900	27,600
5	408,100	379,500	28,600
Total	2,391,400	2,257,800	133,600

Chapter 9, Problem 3

The theoretical time for the production rate and the deliverability to be the same is 1.546 years. This assumes that the deliverability rate is a continuous function with time. In actual practice, deliverability is determined once a year and that deliverability holds for the following year.

Chapter 10, Problem 1

Initial gas in place = 3,071,000 Mcf.

Gas produced = (2,458,000/3,071,000) × 100 = 80.0%

Chapter 10, Problem 2

Initial gas in place = 304,600 Mcf.

Gas produced = (250,900/304,600) × 100 = 82.4%

Chapter 10, Problem 3

Example 1, Figure 10–8: 28 months

Example 2, Figure 10–9: 25 months.

Example 3, Figure 10–12: (a) 30 months.
(b) 60 months.

Example 4, Figure 10–14: about 6 years.

However, all are judgement decisions.

Chapter 11, Problem 1

The largest size of tubing at the rate of flow of 531 Mcfd that will lift water is 2-in. (ID = 1.995 in.).

Chapter 11, Problem 2

Minimum rates of flow required to lift water and condensate in various sizes of tubing at 1,000 psia.

$\gamma_g = 0.625$

$t = 200°F$

$p = 3{,}000$ psia

Tubing Nominal	ID, in.	Minimum Rate of Flow, MMcfd: Water u = 5.08 ft/sec	Condensate u = 3.22 ft/sec
$2\frac{3}{8}$	1.995	1.71	1.08
$2\frac{7}{8}$	2.441	2.55	1.62
$3\frac{1}{2}$	2.992	3.84	2.43
4	3.476	5.18	3.28
$4\frac{1}{2}$	4.082	7.14	4.53

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