Introduction to Aerospace Propulsion

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Lecture No - 7

In this lecture ...

- Solve problems related to calculation of
  - Work done (displacement work)

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Problem 1

- A hydraulic cylinder has a piston of cross sectional area 25 cm^2 and a fluid pressure of 2 MPa. If the piston is moved 0.25 m, how much work is done?

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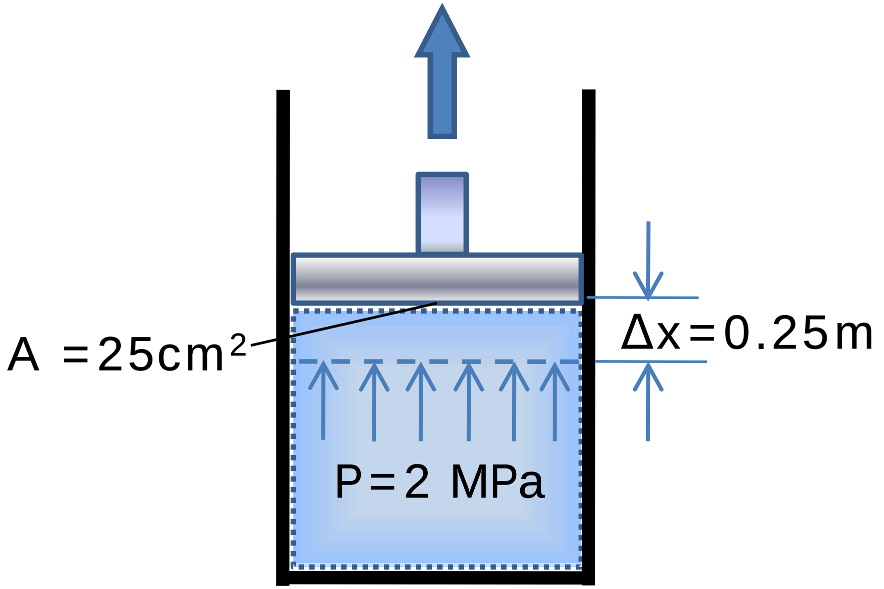
Solution: Problem 1



Assumption: The above process is in quasi-equilibrium.

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Solution: Problem 1

- The work is a force with a displacement and force is constant: $F = PA$

$$W = \int F\,dx = \int P\,A\,dx = PA\Delta x$$

$$= 2000\,\text{kPa x } 25\text{x}\,10^{-4}\,\text{m}^2\text{ x } 0.25\,\text{m} = 1.25\,\text{kJ}$$

Work done to move the piston = 1.25 kJ

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Problem 2

- A piston cylinder has 1.5 kg of air at 300 K and 150 kPa. It is now heated up in a two step process. First constant volume to 1000 K (state 2) then followed by a constant pressure process to 1500 K (state 3). Find the final volume and the work in the process.

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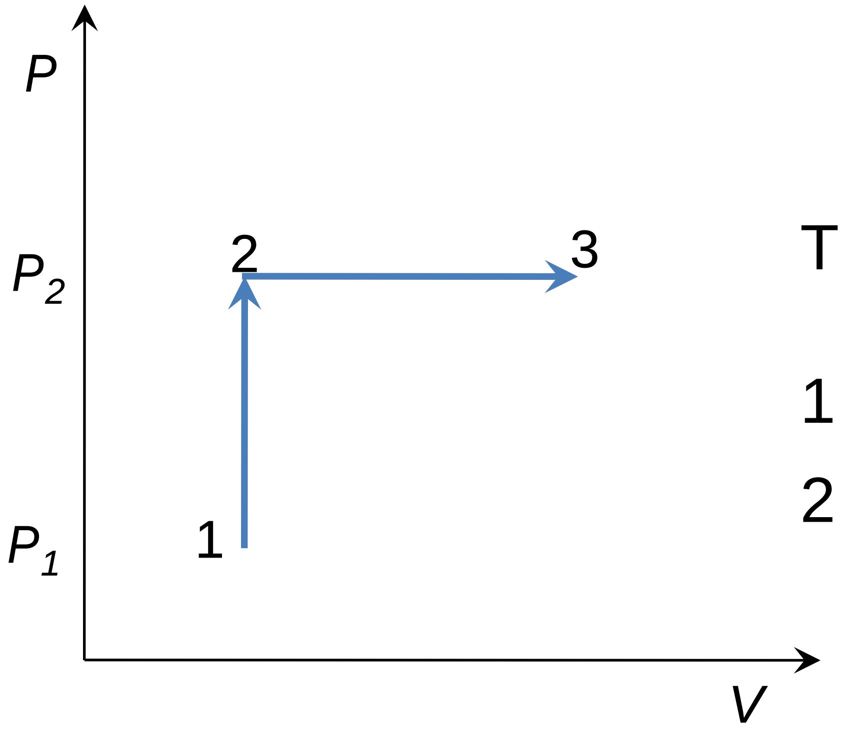
Solution: Problem 2



The two processes are:

1 → 2: Constant volume $V_2 = V_1$

2 → 3: Constant pressure $P_3 = P_2$

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Solution: Problem 2

- We use ideal gas approximation for air.

- State 1: Since T_1, P_1, m (mass), R (gas constant) are known,

$$V_1 = mRT_1/P_1$$

$$= 1.5 \times 0.287 \times 300/150 = 0.861\ m^3$$

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Solution: Problem 2

- State 2: Since $V_2 = V_1$

$P_2 = P_1 \, (T_2/T_1)$

$= 150 \times 1000/300 = 500$ kPa

- State 3: $P_3 = P_2$

$V_3 = V_2 \, (T_3/T_2)$

$= 0.861 \times 1500/1000 = 1.2915 \text{ m}^3$

Hence, the final volume $V_3 = 1.2915 \text{ m}^3$

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Solution: Problem 2

- Work done during the process, W_{1-3}

$$W_{1-3} = W_{1-2} + W_{2-3}$$

- Work done during process 1-2, $W_{1-2} = 0$ as this is a constant volume process.

- Hence $W_{1-3} = W_{2-3} = P_3(V_3 - V_2) = P_2(V_3 - V_2)$

$$= 500(1.2915 - 0.861) = 215.3 \text{ kJ}$$

- The work done in the process = 215.3 kJ

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Problem 3

- A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

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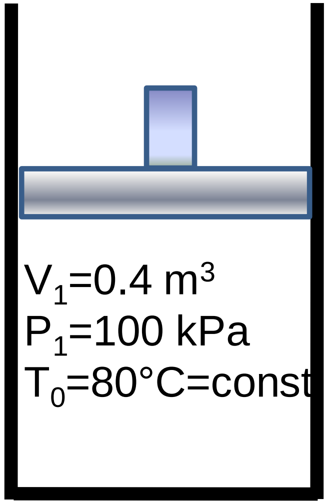
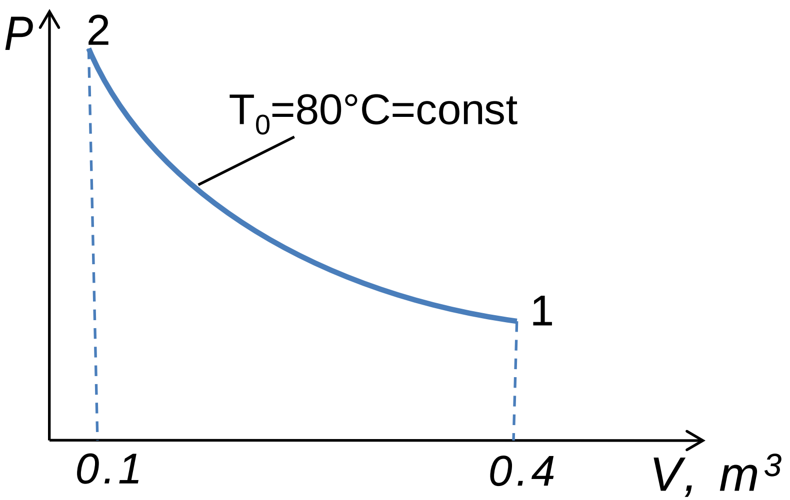
Solution: Problem 3





Assumptions:

- The compression process is quasi-equilibrium.
- At specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values.

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Solution: Problem 3

- For an ideal gas at constant temperature T_0

$$PV = mRT_0 = C \text{ or, } P = \frac{C}{V}, \text{where } C \text{ is a constant.}$$

$$\text{Work}, W = \int_1^2 P dV = \int_1^2 \frac{C}{V} dV = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1}$$

$P_1 V_1$ can be replaced by $P_2 V_2$ or mRT_0.

Also, V_2 / V_1 can be replaced by P_1 / P_2 as $P_1 V_1 = P_2 V_2$

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Solution: Problem 3

Substituting the numerical values,

$$W = (100\,kPa)(0.4\,m^3)\left(\ln\frac{0.1}{0.4}\right) = -55.5\,\text{kJ}$$

The work done during the process is -55.5 KJ. The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

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Problem 4

- A gas in a piston cylinder assembly undergoes an expansion process where $PV^{1.5}$ = constant. The initial pressure is 3 bar, the initial volume is 0.1 m^3 and the final volume is 0.2 m^3. Determine the work done for this process.

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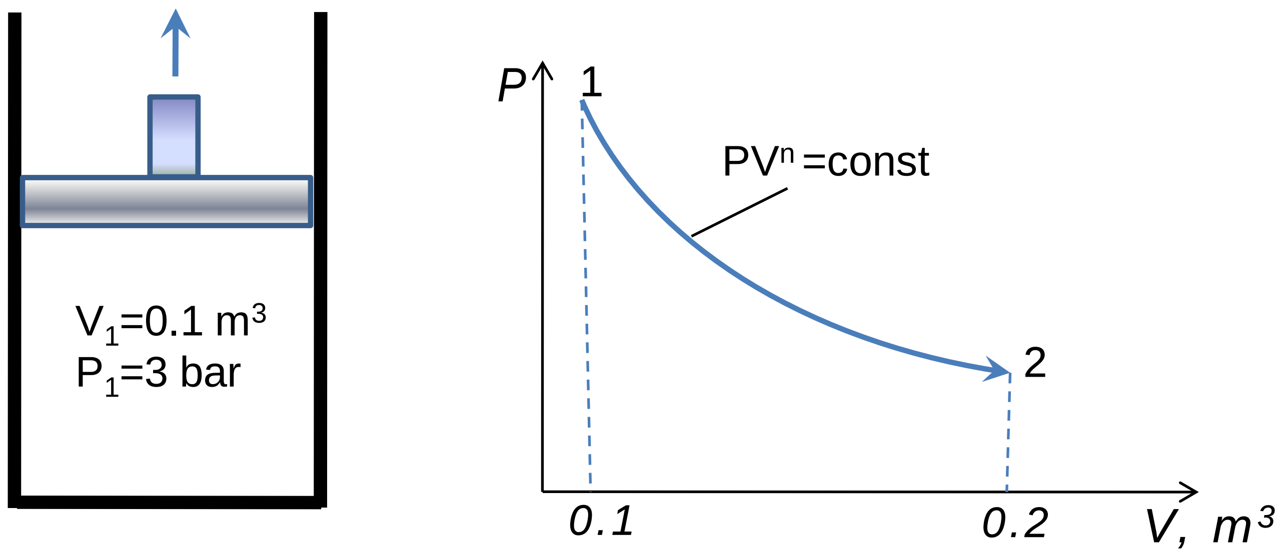
Solution: Problem 4



- Assumptions: (a) The gas is in a closed system. (b) The expansion is a polytropic process. (c) *PdV* is the only work mode.

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Solution: Problem 4

$$W_{1-2} = \int_1^2 pdV = \int_1^2 CV^{-n} dV$$

$$= C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

The pressure at state 2 can be found using

$$P_2V_2^{\,n} = P_1V_1^{\,n}$$

$$or, P_2 = P_1\left(\frac{V_1}{V_2}\right)^n = (3\text{x}10^5\,Pa)\left(\frac{0.1}{0.2}\right)^{1.5} = 1.06\text{x}10^5\,Pa$$

$$\text{Hence, W} = \left(\frac{(1.06\text{x}10^5\,Pa)(0.2m^3) - (3\text{x}10^5\,Pa)(0.1m^3)}{1-1.5}\right) = +17.6\,\text{kJ}$$

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Solution: Problem 4

- Therefore the work done during this process: +17.6 kJ

- If in the above problem, n=1.0, the net work done comes out to be +20.79 kJ
- If n=0, the work done will be + 30 kJ

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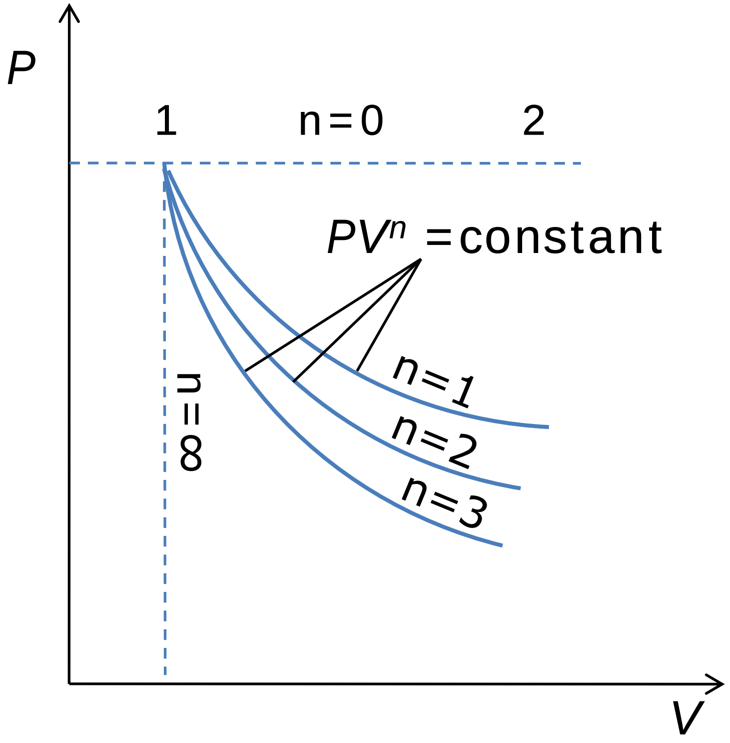
Solution: Problem 4



- $W_{n=1.5}$ = +17.6 kJ
- $W_{n=1.0}$ = +20.79 kJ
- $W_{n=0.0}$ = +30 kJ

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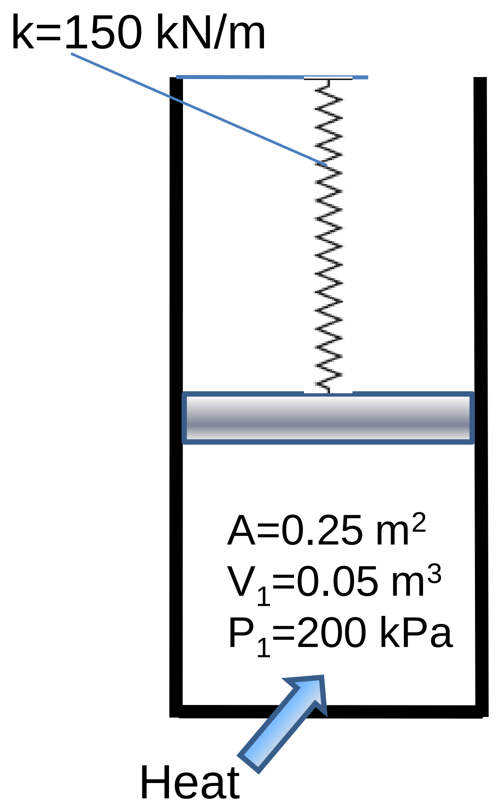
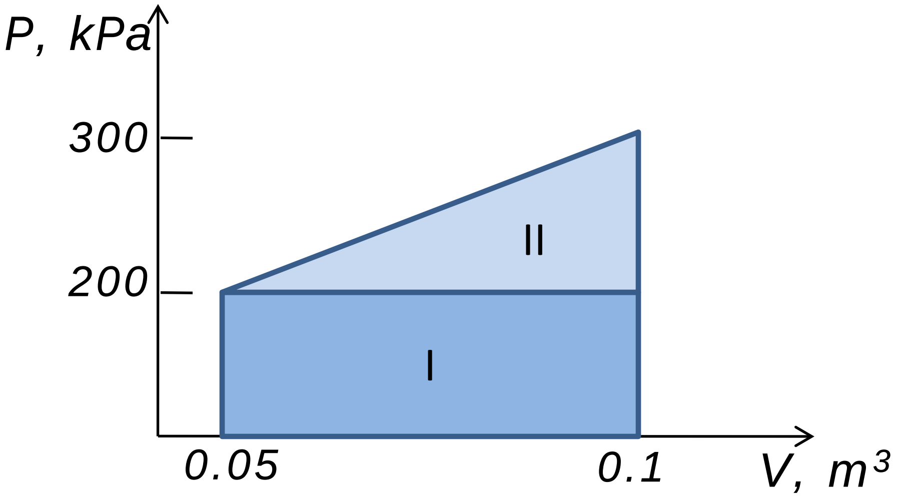
Problem 5

- A piston–cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine
- (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.

Solution: Problem 5





Assumptions: (a) The expansion process is quasi-equilibrium. (b) The spring is linear in the range of interest.

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Solution: problem 5

- The enclosed volume at the final state is

$$V_2 = 2V_1 = 2V_1 = (2)(0.05\ m^3) = 0.1\ m^3$$

- Then the displacement of the piston (and of the spring) becomes

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05)m^3}{0.25m^2} = 0.2m$$

- The force applied by the linear spring at the final state is

$$F = kx = (150\ kN/m)(0.2\ m) = 30\ kN$$

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Solution: problem 5

- The additional pressure applied by the spring on the gas at this state is

$$P = \frac{F}{A} = \frac{30kN}{0.25m^2} = 120\,kPa$$

- Without the spring, the pressure of the gas would remain constant at 200 kPa while the piston is rising. But under the effect of the spring, the pressure rises linearly from 200 kPa to

  200 + 120 = 320 kPa at the final state.

- The final pressure in the cylinder = 320 kPa

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Solution: problem 5

- From the P-V diagram, it is clear that the work done during the process is the area under the process (a trapezoid in this case).

$$W = \text{area} = \frac{(200+320)kPa}{2}(0.1-0.05)m^3 = 13\,kJ$$

- The total work done by the gas is 13 kJ

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Solution: problem 5

- The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring.

$$W_{spring} = \frac{1}{2}\left[(320-200)kPa\right]\left(0.05m^3\right) = 3\,kJ$$

- The fraction of this work done against the spring to compress it is 3 kJ

$$\text{Note}: W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2) = 3\,kJ, \text{where, } x_2 = 0.2m, x_1 = 0m$$

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Exercise problem # 1

- A fluid contained in a horizontal cylinder is continuously agitated using a stirrer passing through the cylinder cover. The cylinder diameter is 0.40 m. During the stirring process lasting 10 minutes, the piston slowly moves out a distance of 0.485 m. The net work done by the fluid during the process is 2 kJ. The speed of the electric motor driving the stirrer is 840 rpm. Determine the torque in the shaft and the power output of the motor.
- Ans: 0.08 Nm, 6.92 W

Exercise problem # 2

- Consider a two-part process with an expansion from 0.1 to 0.2 m^3 at a constant pressure of 150 kPa followed by an expansion from 0.2 to 0.4 m^3 with a linearly rising pressure from 150 kPa ending at 300 kPa. Show the process in a PV diagram and find the boundary work.
- Ans: 60 kJ

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Exercise problem # 3

- A piston/cylinder contains water at 500°C, 3 MPa. It is cooled in a polytropic process to 200°C, 1 MPa. Find the polytropic exponent and the specific work in the process.
- Ans: 1.919, 155.2 kJ

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Exercise problem # 4

- Consider a gas enclosed in a piston-cylinder assembly as the system. The gas is initially at a pressure of 500 kPa and occupies a volume of 0.2 m^3. The gas is taken to the final state where the pressure is 100 kPa by the following two different processes. Calculate the work done by the gas in each case:
- (a) volume of the gas is inversely proportional to pressure (Ans: 160.94 kJ)
- (b) the process follows $PV^{\gamma} = Const.$ where, $\gamma = 1.4$

(Ans: 92.15 kJ)

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In the next lecture ...

- First law of thermodynamics for closed systems
  - Energy balance
  - Energy change for a system
  - Energy transfer mechanisms
  - First law for a cycle
  - First law for a system undergoing change of state

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