Introduction to Aerospace Propulsion

Prof. Bhaskar Roy, Prof. A M Pradeep
Department of Aerospace Engineering,
IIT Bombay

Lecture No - 29

Propeller Theories

1) Momentum Theory
2) Blade Element Theory

- The first one does not use the blade shape for design or analysis at all.
- The second set of theories use the propeller blade shapes made up of stacked airfoils.
- Both the theories are used, to design and predict the propeller performance, using the fundamental parameters defined earlier.

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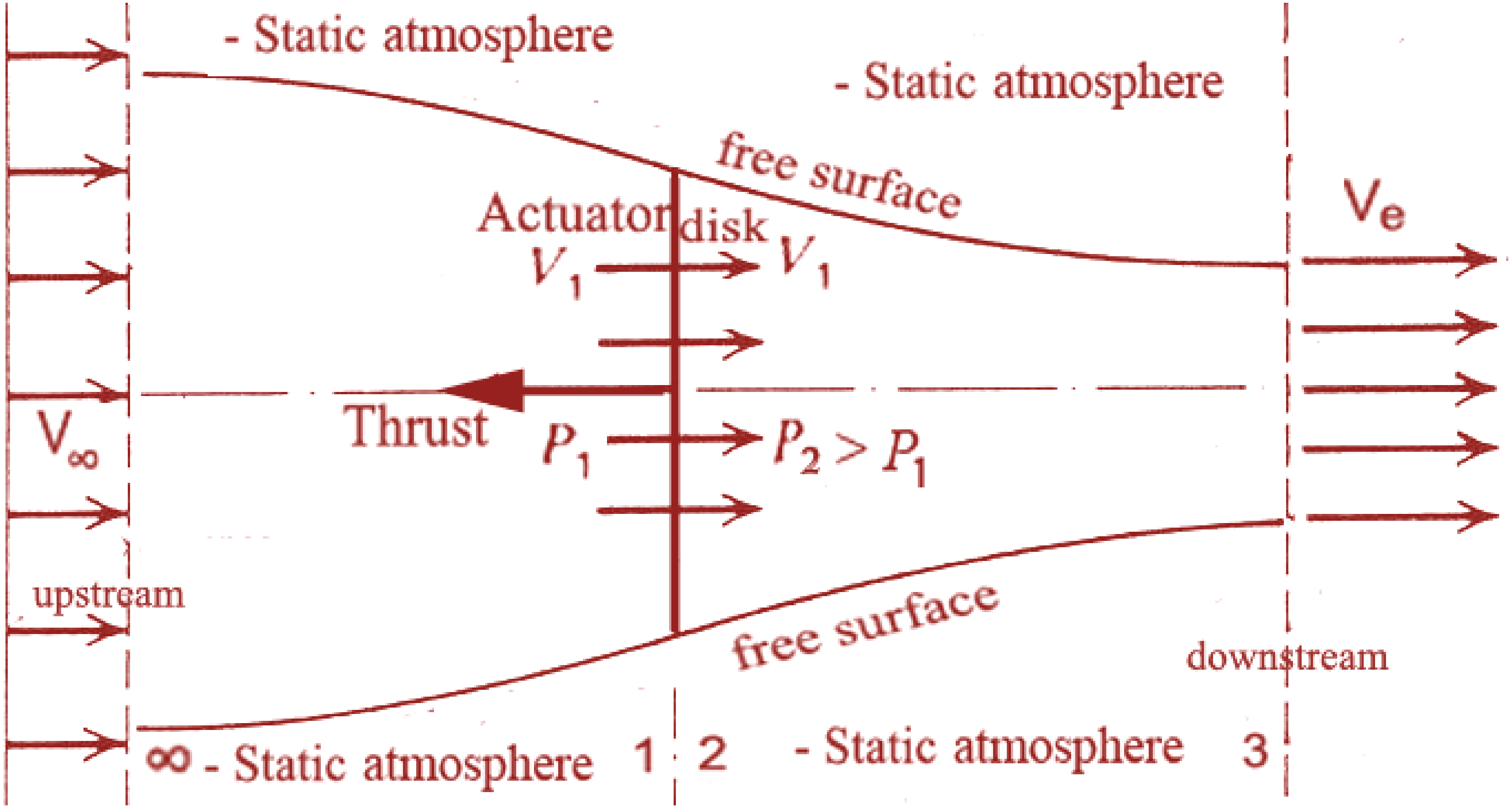
Momentum Theory for Propeller
(or Actuator Disk Theory)



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Assumptions for *conceptual modeling* of a propeller (Fig.)

1) The propeller is assumed to be replaced by an '*actuator disk*', a flow energizer.

2) The 'disk' is assumed to be of very small thickness and is a continuous and 100% porous body of no mass, with a projected frontal area '**A**' (*swept area*) equal to the annulus of the rotating propeller blades.

3) There is no 'resistance' (i.e. drag) of the air passing through the 'actuator disk', (since there are no propeller blades)

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Assumptions for *modeling* of a propeller

4) The axial velocity, V_1 through the 'disk' is uniform over the 'actuation' area and is considered to be smooth across the disk i.e. no abrupt changes are 'experienced'.

5) The received energy manifests itself in the working medium (i.e. air) finally in the form of differential pressure (**$p_2 - p_1$**), a jump change across the actuator disk, uniformly distributed across the disk surface.

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Assumptions for *modeling* of a propeller

6) The fluid medium, air, is a assumed to be a perfect incompressible fluid. Flow is assumed 'irrotational' in front of and behind the disk, but not through it.

and

7) The static pressures far from the disk, i.e. far upstream and far downstream, are both assumed equal to the atmospheric pressure. The corresponding velocities are independent values, to be determined separately.

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Momentum Theory for Propeller

The mass flow through the disk from continuity, is $\dot{m} = \rho.A.V$

The thrust produced by the disk from Newton's II and III laws (change in momentum in air) resulting in reaction force, thrust.

$$\boldsymbol{T} = \dot{m}\,.\partial V \quad = \quad \boldsymbol{\rho.A.V.(V_e - V_\infty)}$$

From simple fluid statics, thrust is produced by the differential static pressure on either side of the disk ,multiplied by its surface area (swept area)

$$\boldsymbol{T = A\ (P_2 - P_1)}$$

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Momentum Theory for Propeller

Applying Bernoulli's equation on either side of the disk, but not through it, gives *[note : Bernoulli's theory is not valid if any energy is added within the flow domain.]*

$$P_\infty + \tfrac{1}{2}\rho V_\infty^{\,2} = P_1 + \tfrac{1}{2}\rho V_1^{2} \quad \text{--upstream}$$
$$P_2 + \tfrac{1}{2}\rho V_2^{\,2} = P_\infty + \tfrac{1}{2}\rho V_e^{2} \quad \text{--downstream}$$

Using, $V_1 = V_2 = constant\ through\ the\ disk$,

$$P_2 - P_1 = \tfrac{1}{2}.\rho.(V_e^2 - V_\infty^2)$$

From above equations $\boldsymbol{V_1 = \tfrac{1}{2}.(V_e + V_\infty)}$

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Momentum Theory for Propeller

- This simple analysis shows that the *air flow velocity through the actuator disk is the mean of the velocities far upstream and far downstream of the propeller*.
- This simple conclusion drawn out of the simplified flow model permits design, analysis, and even experimental verification of the propeller performance rather quickly. Thus, thrust $T = \frac{1}{2} . \rho . (V_e^2 - V_\infty^2) . A$

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Momentum Theory for Propeller

The velocity at the disk comes out to be the *free stream axial velocity, V_∞ plus induced (axial) velocity (v)*, whereas, the *far downstream velocity is equal to the free stream velocity plus two times the induced velocity, v.*

$$V_1 = V_\infty + v \; ; \quad \text{and} \quad V_e = V_\infty + 2.v$$

Therefore , $T = \rho\, A\, (V_\infty + v)\, 2.v = 2.\dot{m}.v$

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Momentum Theory for Propeller

From the equation the *induced velocity*, *v*, can be found as,

$$v = \frac{[- V_\infty + \sqrt{\{V_\infty^2 - (2T / \rho.A)\}}]}{2}$$

For a static thrust, where the propeller is not in forward motion (at take off), V_∞ = zero,

$$v = \sqrt{\frac{T}{2\rho.A}}$$

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Momentum Theory for Propeller

So, power input needed for static thrust production (at Take off),

Power at T.O. $P_{in} = T^{3/2}\sqrt{2\rho A}$

Where A is the swept area of the propeller
And, ρ is the air density

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Momentum Theory for Propeller

The ideal efficiency can be calculated by using classical definition of efficiency ,

$$\eta_p = P_{out} / P_{in}.$$

Power output needs to be equal to thrust generated by the disk multiplied by velocity of the actuator disk through the air medium (i.e. flight velocity of the aircraft). The power input is the thrust generated by the disk multiplied by the airflow velocity through the disk at the disk plane,

$$P_{out} = T\,V_\infty \quad \text{and} \quad P_{in} = T\,V_1$$

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Momentum Theory for Propeller

Therefore, $\eta_i = P_{out} / P_{in}$

$$= T.V_\infty / T.V_1$$

$$= V_\infty / [½(V_\infty + V_e)]$$

$$= 2V_\infty / (V_e + V_\infty)$$

Therefore, $\boldsymbol{\eta_i = 1 / [1 + (v / V_\infty)]}$

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Momentum Theory for Propeller

- The efficiency estimated using momentum theory is referred to as "*induced efficiency*".

- The induced efficiency is zero for zero forward velocity and approaches 1.0 as induced velocity, *v*, tends towards zero.

- The induced efficiency reaches a maxima but does not show any fall with increasing J

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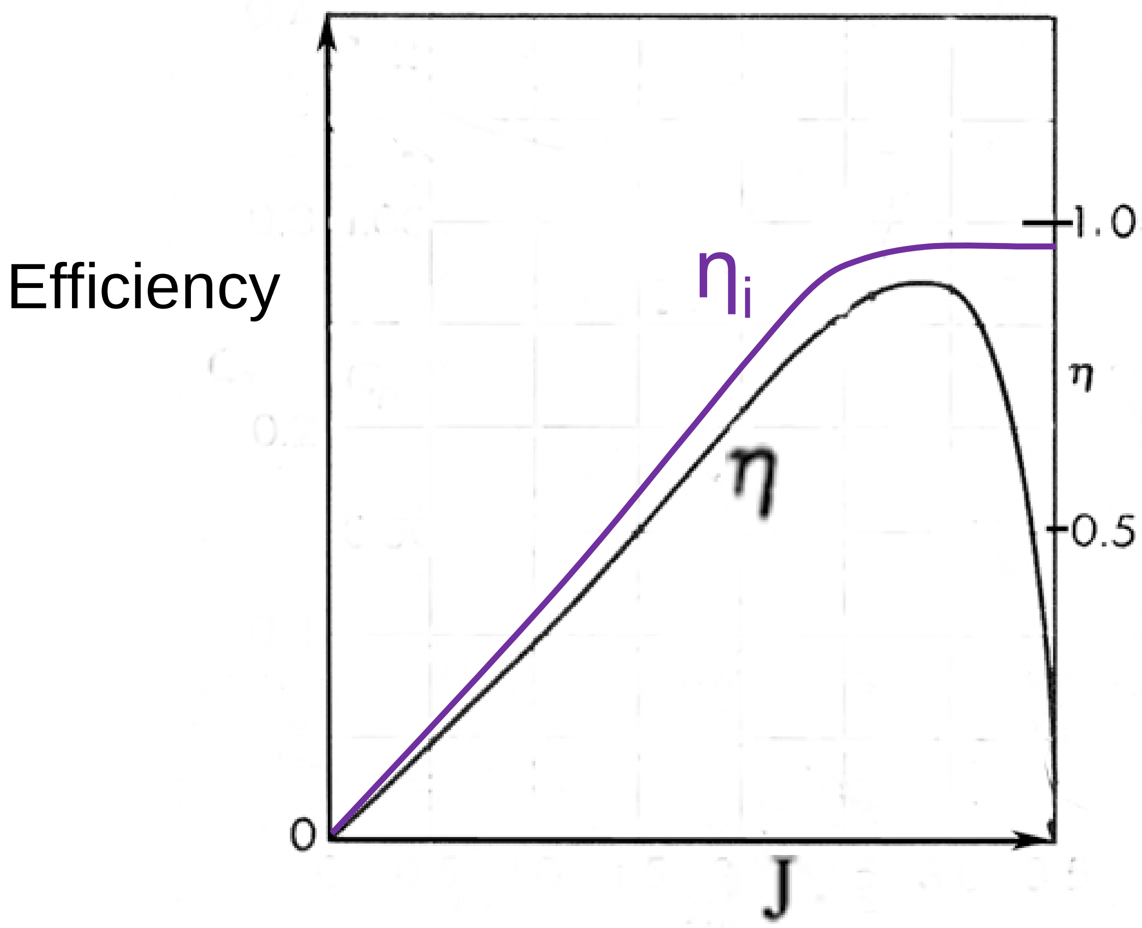
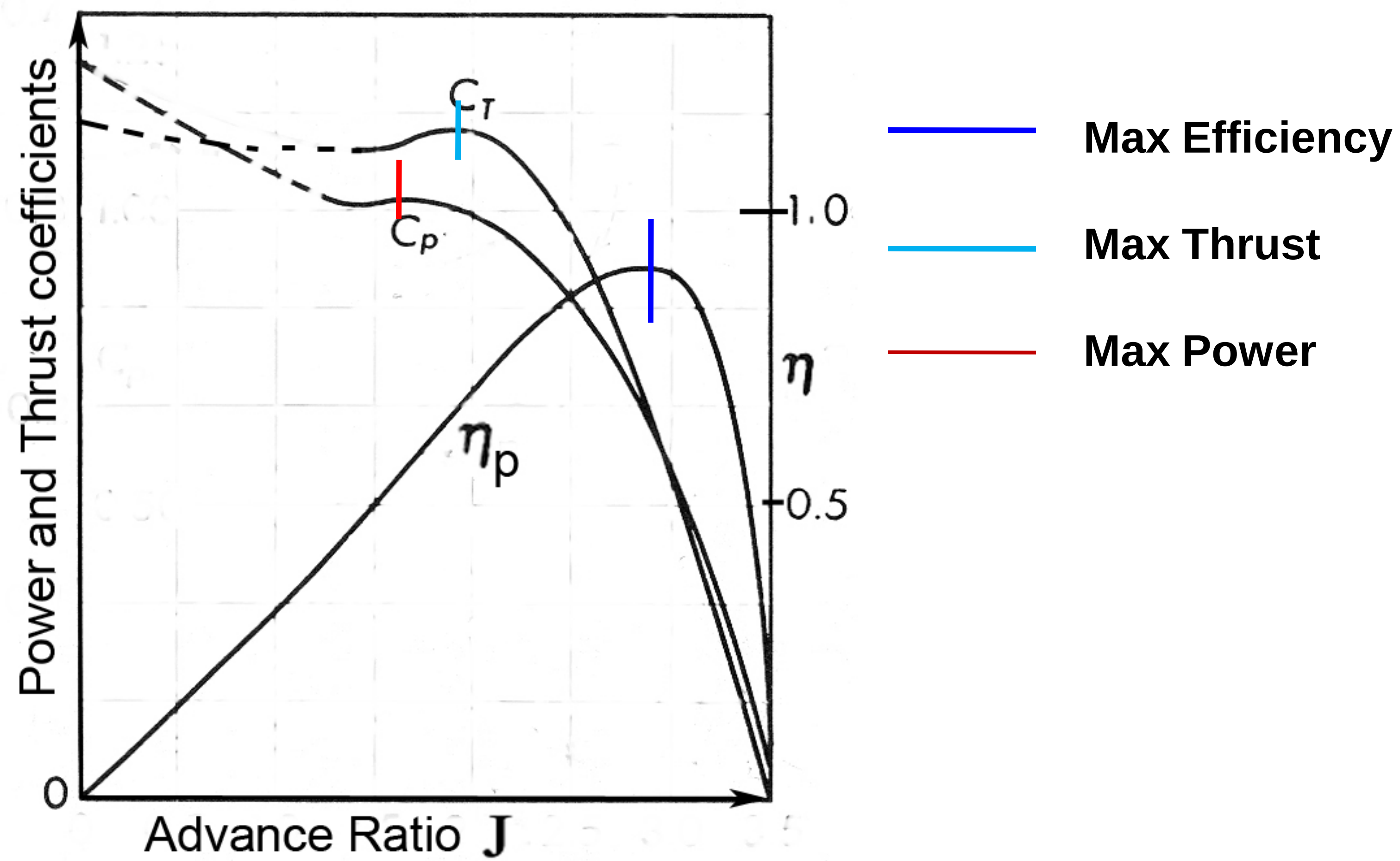
Momentum Theory for Propeller



Propeller Characteristics



Momentum Theory for Propeller

- Induced efficiency cannot be realized as the energy lost in the rotational motion acquired by the flow in passing through the propeller.
- Losses due to non uniform thrust loading over the blade length.
- Blade interference losses due the interaction of flows over the neighbouring blades.
- Propeller profile drag losses, incurred over all the blade surfaces , *and*
- Changes in flow properties due to effect of compressibility, which are not accounted for.

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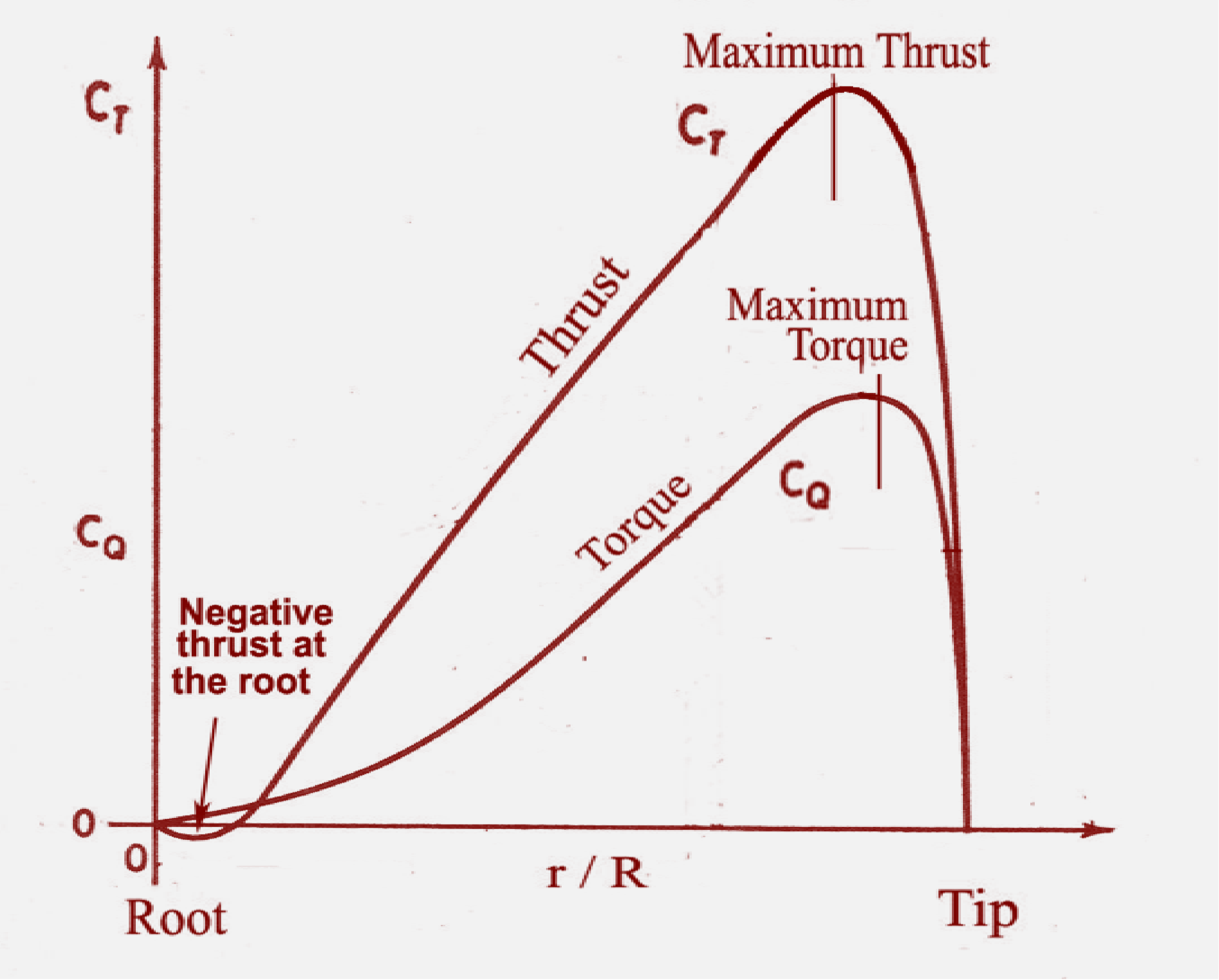
Propeller thrust and power distribution in the disk



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Propeller Blade element Theory

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