Introduction to Aerospace Propulsion

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Lecture No - 34

In this lecture ...

- Solve problems
  - Ideal cycle analysis of air breathing engines

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Problem # 1

- The following data apply to a turbojet flying at an altitude where the ambient conditions are 0.458 bar and 248 K.
- Speed of the aircraft: 805 km/h
- Compressor pressure ratio: 4:1
- Turbine inlet temperature: 1100 K
- Nozzle outlet area 0.0935 m^2
- Heat of reaction of the fuel: 43 MJ/kg

Find the thrust and TSFC assuming c_p as 1.005 kJ/kgK and γ as 1.4

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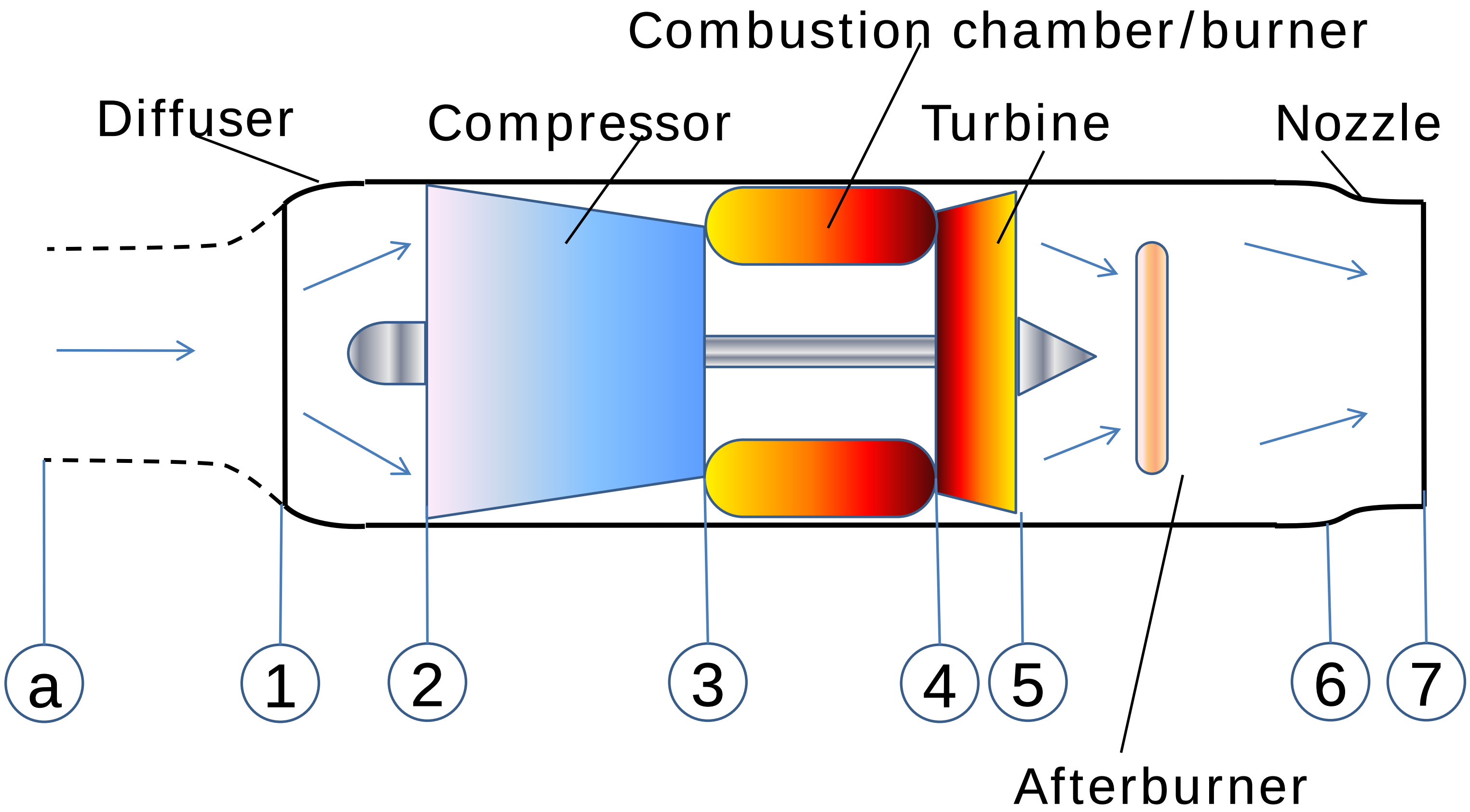
Ideal cycle for jet engines



Schematic of a turbojet engine and station numbering scheme

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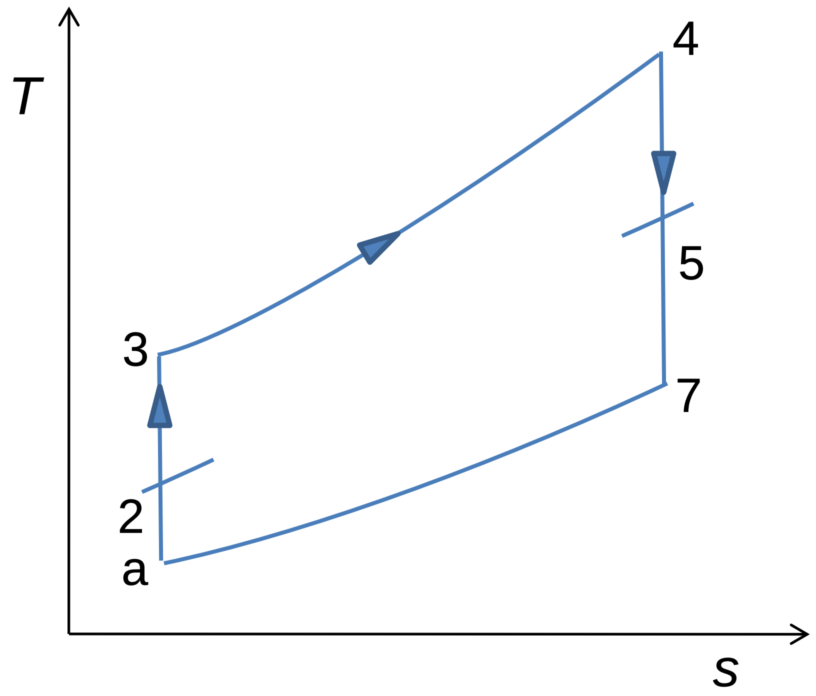
Ideal cycle for jet engines



Ideal turbojet cycle (without afterburning) on a T-s diagram

Solution: Problem # 1

- Speed of the aircraft = 805x1000/3600=223.6 m/s
- Mach number = 223.6/√(γRT)
  = 223.6/ √(1.4x287x248)
  = 0.708
- Intake:

$$T_{02} = T_a\left(1+\frac{\gamma-1}{2}M^2\right) = 248\left(1+\frac{1.4-1}{2}0.708^2\right) = 272.86\ \mathrm{K}$$

$$P_{02} = P_a\left(\frac{T_{02}}{T_a}\right)^{\gamma/(\gamma-1)} = 0.458(272.86/248)^{1.4/(1.4-1)} = 0.639\ \mathrm{bar}$$

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Solution: Problem # 1

- Compressor:

$$P_{03} = \pi_c P_{02} = 4 \times 0.639 = 2.556 \text{ bar}$$

$$T_{03} = T_{02}\left(\pi_c\right)^{(\gamma-1)/\gamma} = 272.86(4)^{(1.4-1)/1.4} = 405.63\,\text{K}$$

- Combustion chamber: From energy balance,

$$h_{04} = h_{03} + fQ_R$$

$$or,\ f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}}$$

$$= \frac{1100/405.63 - 1}{(43 \times 10^6/1005 \times 405.63) - 1100/405.63} = 0.017$$

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Solution: Problem # 1

- Turbine: Since the turbine produces work to drive the compressor, $W_{turbine} = W_{compressor}$

$$\dot{m}_t c_p (T_{04} - T_{05}) = \dot{m}_a c_p (T_{03} - T_{02})$$

$$T_{05} = T_{04} - (T_{03} - T_{02})/(1 + f)$$

$$= 1100 - (405.63 - 272.86)/(1 + 0.017) = 969.45\,\mathrm{K}$$

$$\text{Hence, } P_{05} = P_{04}\left(\frac{T_{05}}{T_{04}}\right)^{\gamma/(\gamma-1)} = 2.556(969.45/1100)^{1.4/(1.4-1)}$$

$$= 1.642 \text{ bar}$$

Solution: Problem # 1

- Nozzle: we first check for choking of the nozzle.
- The nozzle pressure ratio is $P_{05}/P_a = 1.642/0.458 = 3.58$
- The critical pressure ratio is

$$\frac{P_{05}}{P^*} = \left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)} = \left(\frac{1.4+1}{2}\right)^{1.4/(1.4-1)} = 1.893$$

- Therefore the nozzle is choking.
- The nozzle exit conditions will be determined by the critical properties.

Solution: Problem # 1

$$T_7 = T^* = \left(\frac{2}{\gamma+1}\right) T_{05} = \frac{2}{1.4+1} 969.5 = 807.92\,K$$

$$P_7 = P^* = P_{05}\left(\frac{1}{P_{04}/P^*}\right) = \frac{1.642}{1.893} = 0.867$$

$$\rho_7 = P_7 / RT_7 = 0.867 \times 10^5 /(287 \times 807.92) = 0.374\,\mathrm{kg/m^3}$$

$$\text{Therefore, } \mathrm{u_e} = \sqrt{\gamma R T_7} = \sqrt{1.4 \times 287 \times 807.92} = 569.75\ \mathrm{m/s}$$

$$\text{The mass flow rate is, } \dot{m} = \rho_7 A_7 u_e = 19.92\,\mathrm{kg/s}$$

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Solution: Problem # 1

The thrust developed is $\Im = \dot{m}\left[(1+f)u_e - u\right] + A_7(P^* - P_a)$

$$= 19.92\left[(1+0.017)569.75 - 223.6\right]$$

$$+ 0.0935(0.867 - 0.458)\times 10^5$$

$$= 10.912\ \text{kN}$$

Fuel flow rate, $\dot{m}_f = f \times \dot{m}_a = 0.017 \times 19.92 = 0.3387\ \text{kg/s}$

Therefore, $\text{TSFC} = \dot{m}_f / \Im = 3.1\times 10^{-5}\,\text{kg/Ns} = 0.111\ \text{kg/N h}$

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Problem # 2

- The following data apply to a twin spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate hot and cold nozzles are used.
- Overall pressure ratio: 19.0
- Fan pressure ratio: 1.65
- Bypass ratio: 3.0
- Turbine inlet temperature: 1300 K
- Air mass flow: 115 kg/s
- Find the sea level static thrust and TSFC if the ambient pressure and temperature are 1 bar and 288 K. Heat of reaction of the fuel: 43 MJ/kg

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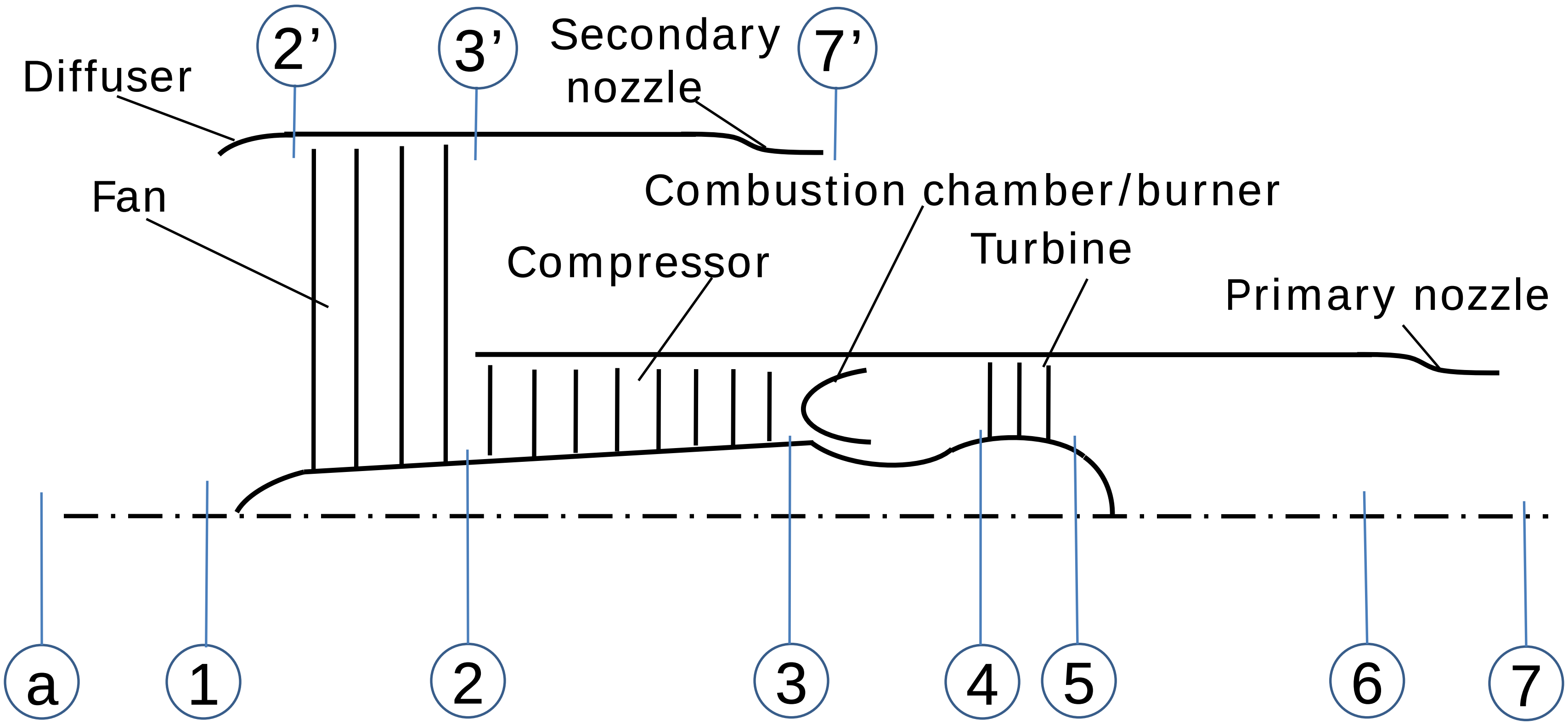
Ideal turbofan engine



Schematic of an unmixed turbofan engine and station numbering scheme

Solution: Problem # 2

- Since we are required to find the static thrust, the Mach number is zero.
- Intake:

$$T_{02'} = T_a\left(1+\frac{\gamma-1}{2}M^2\right) = 288\ \mathrm{K}$$

$$P_{02'} = P_a\left(\frac{T_{02'}}{T_a}\right)^{\gamma/(\gamma-1)} = 1\,\mathrm{bar}$$

- Fan: Fan pressure ratio is known: $\pi_f = P_{03'}/P_{02'}$

$$P_{03'} = \pi_f P_{02'} = 1.65\ \mathrm{bar}$$

$$T_{03'} = T_{02'}(\pi_f)^{(\gamma-1)/\gamma} = 288(1.65)^{(1.4-1)/1.4} = 332.35\,\mathrm{K}$$

Solution: Problem # 2

- Compressor:

$$\pi_c = \text{Overall pressure ratio}/1.65 = 19/1.65 = 11.515$$

$$P_{03} = \pi_c P_{02} = 11.5151 \times 1.65 = 19.0 \text{ bar}$$

$$T_{03} = T_{02}\left(\pi_c\right)^{(\gamma-1)/\gamma} = 332.35 \times (11.515)^{(1.4-1)/1.4} = 668.53 \text{ K}$$

- Combustion chamber: From energy balance,

$$f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}}$$

$$= \frac{1300/668.53 - 1}{(43 \times 10^6/1005 \times 668.53) - 1300/668.53} = 0.01522$$

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Solution: Problem # 2

- High pressure turbine:

$$\dot{m}_t c_p (T_{04} - T_{05'}) = \dot{m}_{aH} c_p (T_{03} - T_{02})$$

Here, $T_{05'}$ is the temperature at the HPT exit.

$$\therefore T_{05'} = T_{04} - (T_{03} - T_{02})/(1 + f)$$

$$= 1300 - (668.53 - 332.53)/(1 + 0.01522) = 969.04\ \text{K}$$

$$\text{Hence, } P_{05'} = P_{04}\left(\frac{T_{05'}}{T_{04}}\right)^{\gamma/(\gamma-1)} = 19\left(\frac{969.04}{1300}\right)^{1.4/(1.4-1)} = 6.79\ \text{bar}$$

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Solution: Problem # 2

- Low pressure turbine:

$$\dot{m}_t c_p (T_{05'} - T_{05}) = \dot{m}_{aC} c_p (T_{03'} - T_{02'})$$

Here, $T_{05'}$ is the temperature at the HPT exit/LPT inlet.

$$\therefore \; T_{05} = T_{05'} - \mathrm{B}(T_{03'} - T_{02'})/(1+f), \text{where, } \mathrm{B} = \frac{\dot{m}_{aC}}{\dot{m}_{aH}}$$

$$= 969.04 - 3\times(332.35 - 288)/(1+0.01522) = 837.98 \text{ K}$$

$$\text{And, } P_{05} = P_{05'}\left(\frac{T_{05}}{T_{05'}}\right)^{\gamma/(\gamma-1)} = 6.79\left(\frac{837.98}{969.04}\right)^{1.4/(1.4-1)} = 4.08 \text{ bar}$$

Solution: Problem # 2

- Primary nozzle: we first check for choking of the nozzle.
- The nozzle pressure ratio is $P_{05}/P_a = 4.08/1 = 4.08$ bar
- The critical pressure ratio is

$$\frac{P_{05}}{P^*} = \left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)} = \left(\frac{1.4+1}{2}\right)^{1.4/(1.4-1)} = 1.893$$

- Therefore the nozzle is choking.
- The nozzle exit conditions will be determined by the critical properties.

Solution: Problem # 2

$$T_7 = T^* = \left(\frac{2}{\gamma+1}\right) T_{05} = \frac{2}{1.4+1} 837.98 = 698.32\,\text{K}$$

$$P_7 = P^* = P_{05}\left(\frac{1}{P_{05}/P^*}\right) = \frac{4.08}{1.893} = 2.155\,\text{bar}$$

$$\text{Therefore, } u_e = \sqrt{\gamma R T_7} = \sqrt{1.4 \times 287 \times 698.32} = 529.7 \text{ m/s}$$

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Solution: Problem # 2

- Secondary nozzle:
- The nozzle pressure ratio is $P_{03'}/P_a = 1.65/1 = 1.65$ bar
- The critical pressure ratio is

$$\frac{P_{05}}{P^*} = \left[\frac{\gamma+1}{2}\right]^{\gamma/(\gamma-1)} = \left(\frac{1.4+1}{2}\right)^{1.4/(1.4-1)} = 1.893$$

- Therefore the nozzle is not choking.

$$\therefore u_{ef} = \sqrt{2c_p T_{03'}\left[1-\left(P_a/P_{03'}\right)^{(\gamma-1)/\gamma}\right]}$$

$$= \sqrt{2\times 1005\times 332.35\left[1-(1/1.65)^{(1.4-1)/1.4}\right]} = 298.52$$

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Solution: Problem # 2

- Thrust,

$$\Im = \dot{m}_{aH}\left[(1+f)u_e - u\right] + \mathrm{B}\dot{m}_{aH}(u_{ef} - u)$$

assuming $(P_e - P_a)A_e$ to be negligible.

$$\dot{m}_{aC}/\dot{m}_{aH} = 3.0,\ \ \dot{m}_{aH} + \dot{m}_{aC} = 115 \text{ kg/s}$$

$$\therefore \dot{m}_{aH} = 115/4 = 28.75 \text{ kg/s}$$

$$\Im = 28.75[(1+0.01522)\times 529.7 - 0]$$

$$+3\times 28.75(298.52 - 0)$$

$$= 40.74\,\text{kN}$$

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Solution: Problem # 2

- Exercise: calculate the thrust by factoring the pressure thrust term as well. Hint: you can calculate the exit area from mass flow, density and exhaust velocity.

- TSFC,

$$\text{Fuel flow rate, } \dot{m}_f = f \times \dot{m}_a = 0.01522 \times 28.75 = 0.4376 \text{ kg/s}$$

$$\text{Therefore, TSFC} = \dot{m}_f / \Im = 1.075 \times 10^{-5} \text{kg/Ns} = 0.0388 \text{ kg/N h}$$

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Problem # 3

- A helicopter using a turboshaft engine is flying at 300 km/h at an altitude where the ambient temperature is 5^{o}C. Determine the specific power output and thermal efficiency. The specifications of the engine are: compressor pressure ratio=9.0, turbine inlet temperature = 800^{o}C.

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Problem # 3

- For a turboshaft engine, there is no nozzle thrust.
- u=300x1000/3600= 83.33 m/s
- T_a=278 K
- Therefore, Mach number M=83.33/√(1.4x287x278) =0.25
- Intake:

$$T_{02} = T_a\left(1+\frac{\gamma-1}{2}M^2\right) = 278\left(1+\frac{1.4\text{-}1}{2}0.25^2\right) = 281.48\,\mathrm{K}$$

$$P_{02} = P_a\left(\frac{T_{02}}{T_a}\right)^{\gamma/(\gamma-1)} = 0.8\left(\frac{281.48}{278}\right)^{1.4/(1.4-1)} = 0.835\ \text{bar}$$

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Problem # 3

- Compressor:

$$P_{03} = \pi_c P_{02} = 9.0 \times 0.835 = 7.52 \text{ bar}$$

$$T_{03} = T_{02}\left(\pi_c\right)^{(\gamma-1)/\gamma} = 281.48 \times (9.0)^{(1.4-1)/1.4} = 527.67 \text{ K}$$

Specific work required to drive the compressor,

$$W_c = c_p (T_{03} - T_{02}) = 1.005(527.67 - 281.48) = 247.42 \text{ kJ/kg}$$

- Combustor:

$$f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}}$$

$$= \frac{1073/527.67 - 1}{(43 \times 10^6/1005 \times 527.67) - 1073/527.67} = 0.013$$

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Problem # 3

- Turbine:

$$\frac{P_{04}}{P_{05}}=\frac{P_{03}}{P_a}=\frac{P_{03}}{P_{02}}\frac{P_{02}}{P_a}=9\times\frac{0.835}{0.8}=9.394$$

$$\frac{T_{04}}{T_{05}}=\left(\frac{P_{04}}{P_{05}}\right)^{(\gamma-1)/\gamma}=9.394^{(1.4-1)/1.4}=1.897$$

$$T_{05}=565.63K$$

Work done by the turbine, $W_t=(1+f)c_p(T_{04}-T_{05})$

$$=(1+0.013)\times1.005\times(1073-565.63)$$

$$=516.54\text{ kJ/kg}$$

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Problem # 3

- Specific work output, $W_{net} = W_t - W_c$
  $= 516.54 - 247.42$
  $= 269.12$ kJ/kg

- Thermal efficiency: W_{net}/Q_{in}
- $Q_{in} = c_p(T_{04} - T_{03}) = 1.005(1073 - 527.67)$
  $= 548.05$ kJ/kg
- Therefore, thermal efficiency $= 269.12/548.05$
  $= 0.49$ or 49%

Exercise Problem # 1

- A turbojet engine inducts 51 kg of air per second and propels an aircraft with a uniform flight speed of 912 km/h. The enthalpy change for the nozzle is 200 kJ/kg. The fuel-air ratio is 0.0119 and the heating value of the fuel is 42 MJ/kg. Determine the thermal efficiency, TSFC, propulsive power.
- Ans: 0.34, 0.1034 kg/Nh, 8012 kW.

Exercise Problem # 2

- A twin spool mixed turbofan engine operates with an overall pressure ratio of 18. The fan operates with a pressure ratio is 1.5 and the bypass ratio is 5.0. The turbine inlet temperature is 1200 K. If the engine is operating at a Mach number of 0.75 at an altitude where the ambient temperature and pressure are 240 K and 0.5 bar.
- Determine the thrust and the SFC.
- Ans: 74 kN, 0.027 kg/N h

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Exercise Problem # 3

- An aircraft using a turboprop engine is flying at 800 km/h at an altitude where the ambient conditions are 0.567 bar and -20°C. Compressor pressure ratio is 8.0 and the turbine inlet temperature is 1100 K. Assuming that the turboprop does not generate any nozzle thrust, determine the specific power output and the thermal efficiency.
- Ans: 311 kJ/kg, 0.44

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