Introduction to Aerospace Propulsion

Prof. Bhaskar Roy, Prof. A M Pradeep
Department of Aerospace Engineering,
IIT Bombay

Lecture No - 22-B

In this lecture ...

- Shock Waves and Expansion
  - Normal Shocks
  - Oblique Shocks
  - Prandtl–Meyer Expansion Waves
- Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)
  - Property Relations for Rayleigh Flow
- Duct flow with friction without heat transfer (Fanno flow)

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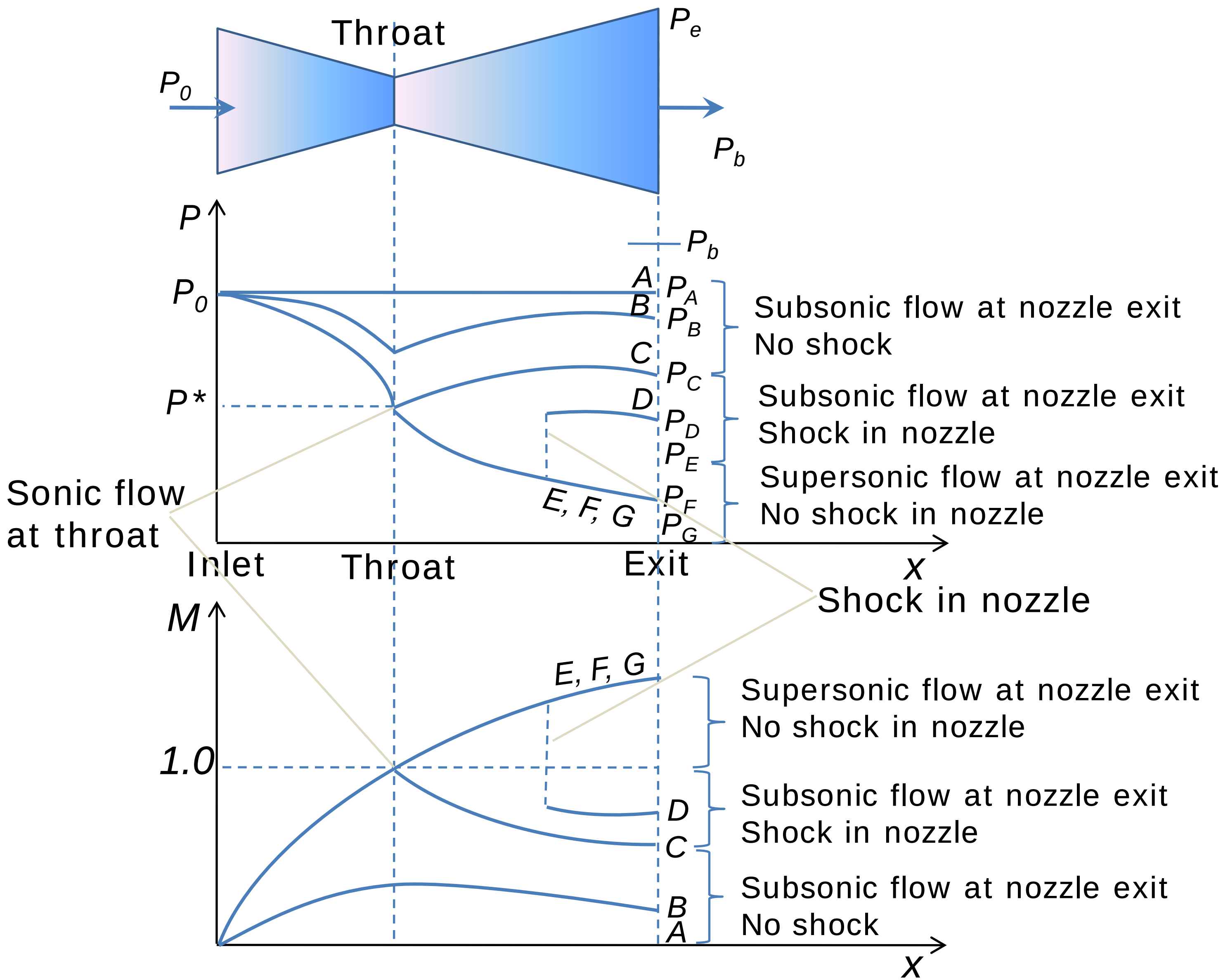
Shock waves

- Sound waves are caused by infinitesimally small pressure disturbances, and travel through a medium at the speed of sound.
- Under certain flow conditions, abrupt changes in fluid properties occur across a very thin section: shock wave.
- Shock waves are characteristic of supersonic flows, that is when the fluid velocity is greater than the speed of sound.
- Flow across a shock is highly irreversible and cannot be approximated as isentropic.



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Normal shocks

- Shock waves that occur in a plane normal to the direction of flow: Normal shocks.
- A supersonic flow across a normal shock becomes subsonic.
- Conservation of energy principle requires that the enthalpy remains constant across the shock.

$$h_{01} = h_{02}$$

- For an ideal gas, $h = h(T)$ and thus

$$T_{01} = T_{02}$$

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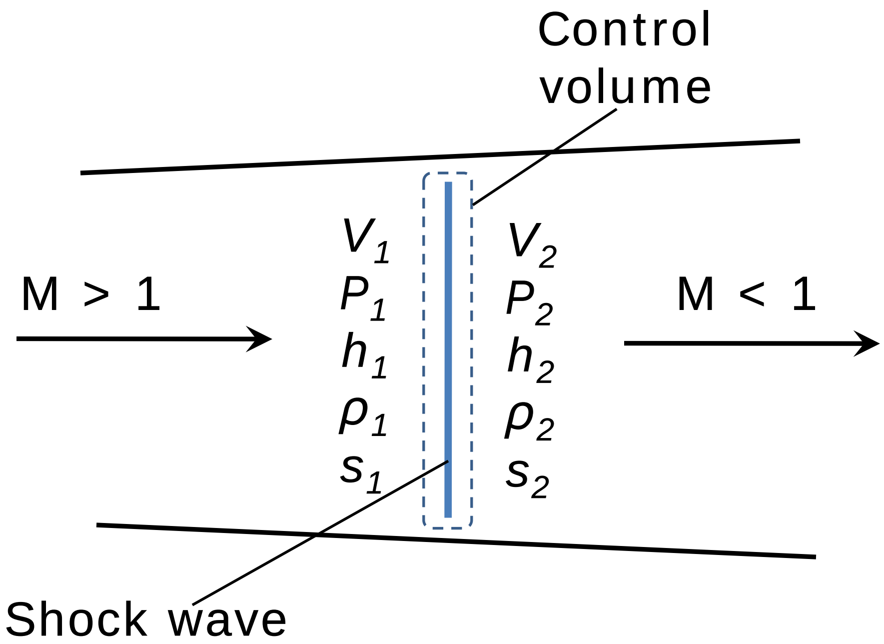
Normal shocks



Flow across a normal shock wave

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Normal shocks

- Across the normal shock we apply the governing equations of fluid motion:
- *Mass:* $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$
- *Energy:* $h_{01} = h_{02}$
- *Momentum:* $A(P_1 - P_2) = \dot{m}\,(V_2 - V_1)$
- *Entropy:* $s_2 - s_1 \geq 0$
- If we combine mass and energy equations and plot them on h-s diagram: Fanno line
- Similarly combining mass and momentum gives: Rayleigh line

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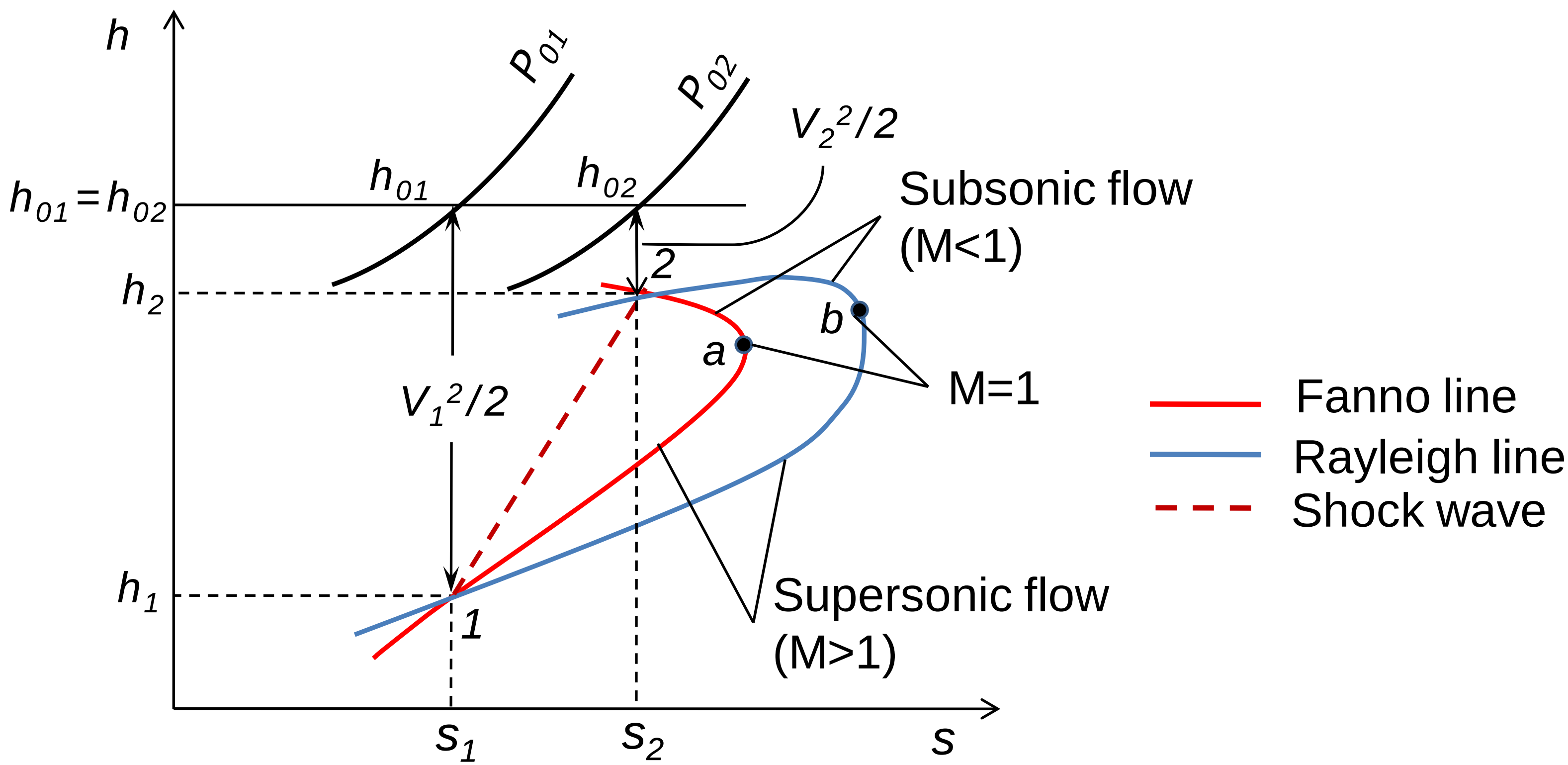
Normal shocks



The *h*-*s* diagram for flow across a normal shock.

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Normal shocks

- To derive expressions before and after the shock

$$\frac{T_{01}}{T_1} = 1 + \left(\frac{\gamma - 1}{2}\right) M_1^2 \quad \text{and} \quad \frac{T_{02}}{T_2} = 1 + \left(\frac{\gamma - 1}{2}\right) M_2^2$$

Since $T_{01} = T_{02}$, and simplifying,

$$\frac{P_2}{P_1} = \frac{M_1 \sqrt{1 + M_1^2 (\gamma - 1)/2}}{M_2 \sqrt{1 + M_2^2 (\gamma - 1)/2}}$$

- This is the Fanno line equation for an ideal gas with constant specific heats.

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Normal shocks

- Similarly if we combine and simplify the mass and momentum equations, we can get an equation for Rayleigh line.

$$M_2^2 = \frac{M_1^2 + 2/(\gamma - 1)}{2M_1^2\gamma/(\gamma - 1) - 1}$$

- This represents the intersections of the Fanno and Rayleigh lines
- This equation relates the Mach number upstream of the shock with that downstream of the shock.

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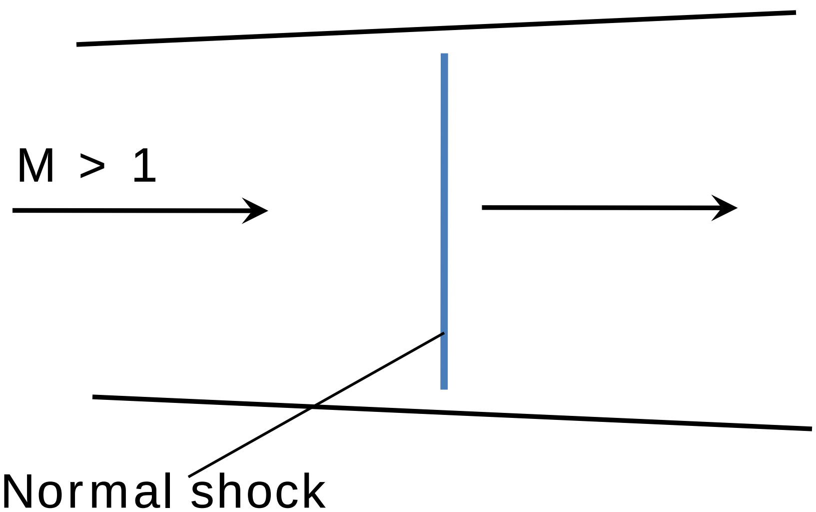
Normal shocks



P increases
P_0 decreases
V decreases
M decreases
T increases
T_0 remains constant
s increases

Variation of flow properties across a normal shock.

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Oblique shocks

- Shock waves that are inclined to the flow at an angle: oblique shocks.
- In a supersonic flow, information about obstacles cannot flow upstream and the flow takes an abrupt turn when it hits the obstacle.
- This abrupt turning takes place through shock waves.
- The angle though which the fluid turns: deflection angle or turning angle, θ.
- The inclination of the shock: shock angle or wave angle, β.

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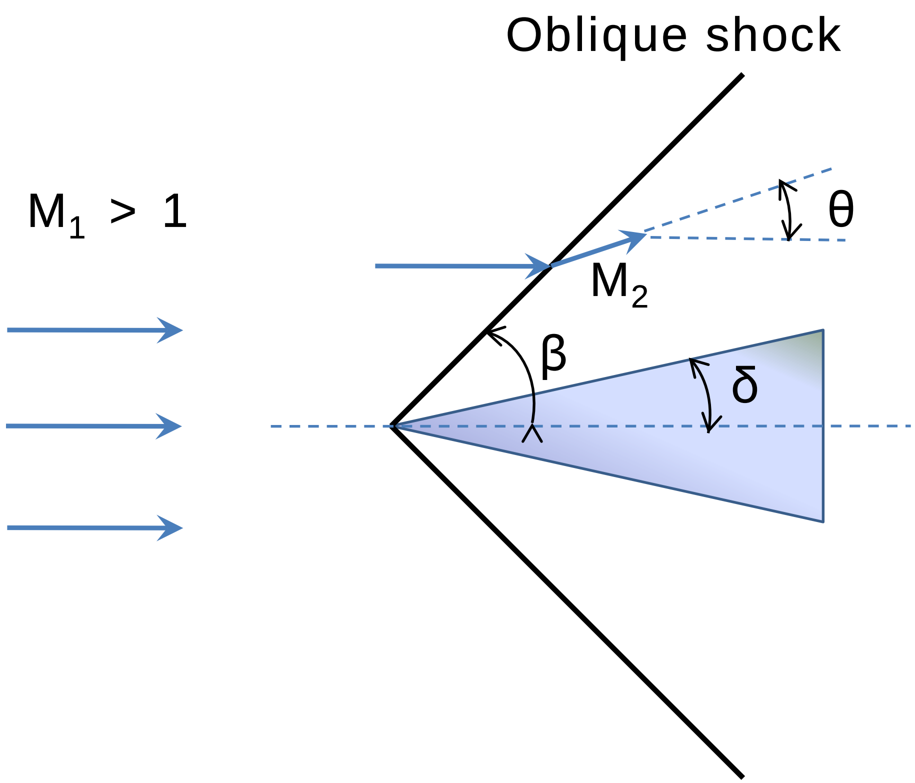
Oblique shocks



Flow across an oblique shock

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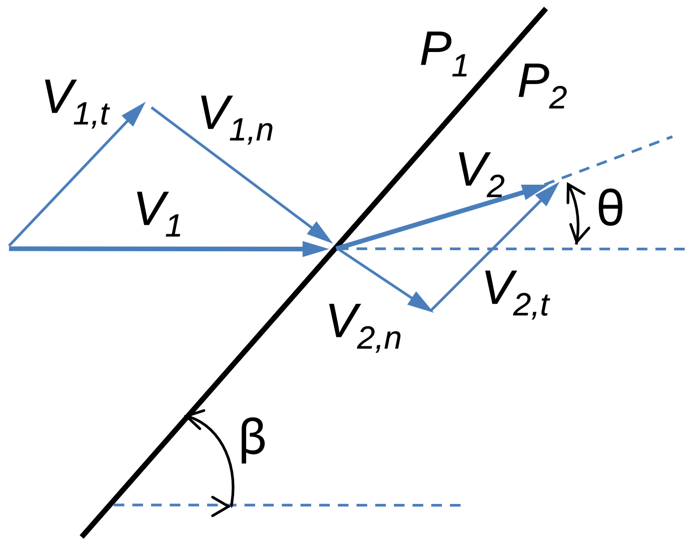
Oblique shocks

- Oblique shocks are possible only in supersonic flows.
- However, the flow downstream of the shock can be either supersonic, sonic or subsonic, depending upon the upstream Mach number and the turning angle.
- To analyse an oblique shock, we decompose the velocity vectors upstream and downstream of the shock into normal and tangential components.

Oblique shocks



Across the shock, the tangential component of velocity does not change, $V_{1,t} = V_{2,t}$

Velocity vectors through an oblique shock of shock angle β and deflection angle θ.

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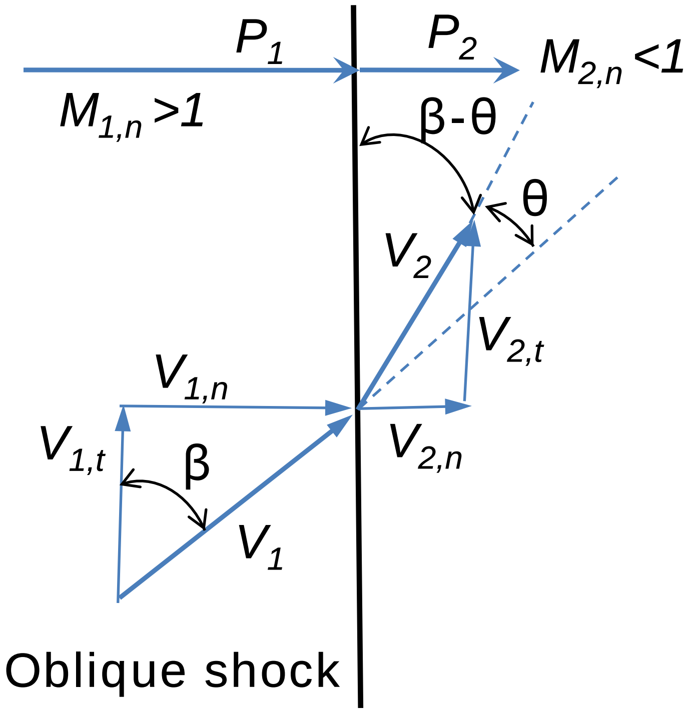
Oblique shocks

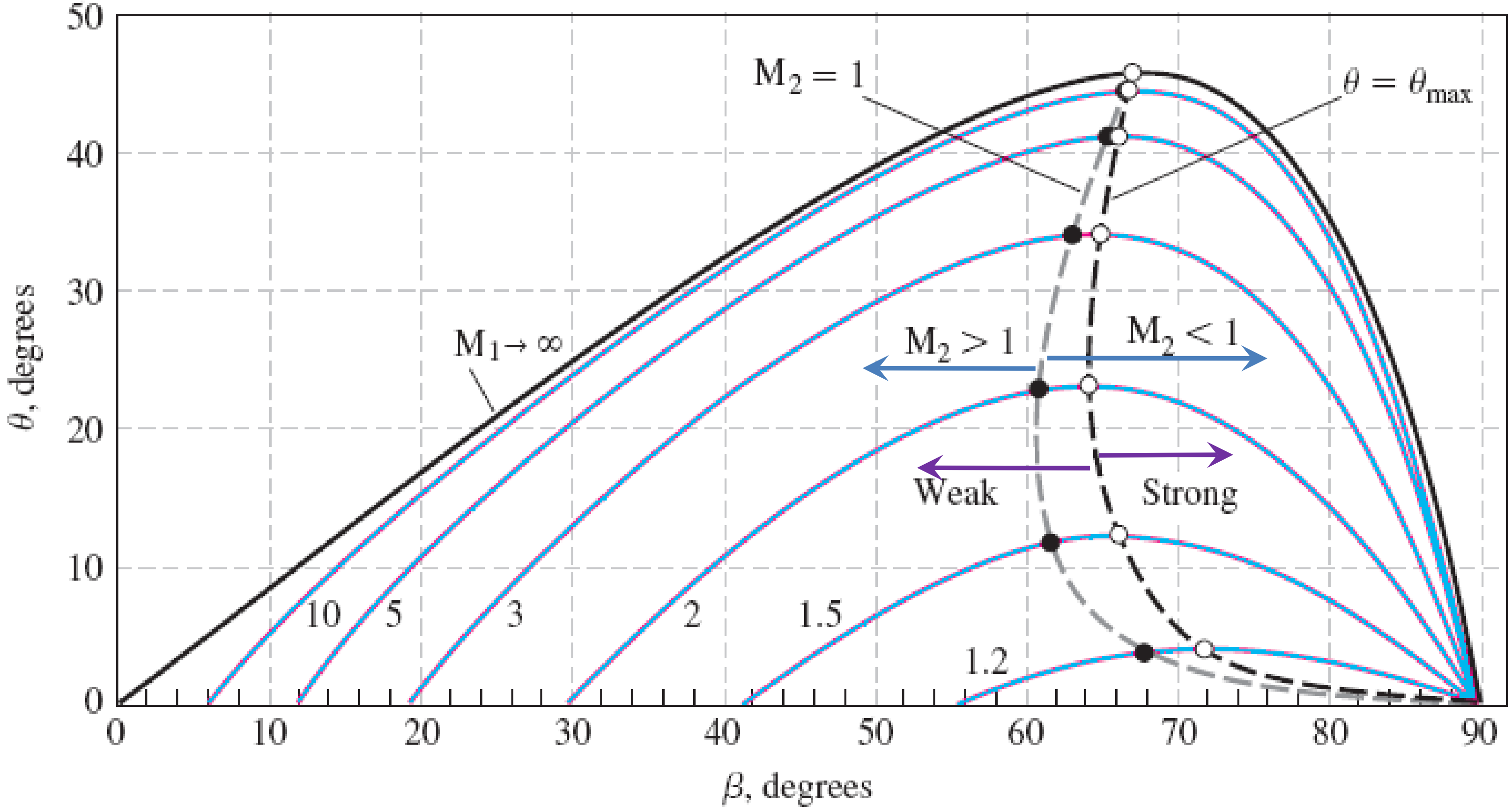


$$M_{1,n} = M_1 \sin\beta \quad \text{and} \quad M_{2,n} = M_2 \sin(\beta - \theta)$$

$$\text{Where, } M_{1,n} = V_{1,n} / c_1 \quad \text{and} \quad M_{2,n} = V_{2,n} / c_2$$

If we use normal components of velocity, all the equations, tables etc. For a normal shock can be used for an oblique shock as well.

Oblique shocks



θ–β–M relationship

Oblique shocks

- There are two possible values of β for $\theta<\theta_{max}$.
- $\theta=\theta_{max}$ line: Weak oblique shocks occur to the left of this line, while strong oblique shocks are to the right of this line.
- $M=1$ line: Supersonic flow to the left and subsonic flow to the right of this line.
- For a given value of upstream Mach number, there are two shock angles.
- $\beta=\beta_{min}$ represents the weakest possible oblique shock at that Mach number, which is called a Mach wave.

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Prandtl-Meyer expansion waves

- An expanding supersonic flow, for eg, on a two-dimensional wedge, does not result in a shock wave.
- There are infinite Mach waves forming an expansion fan.
- These waves are called Prandtl-Meyer expansion waves.
- The Mach number downstream of the expansion increases $(M_2 > M_1)$, while pressure, density, and temperature decrease.

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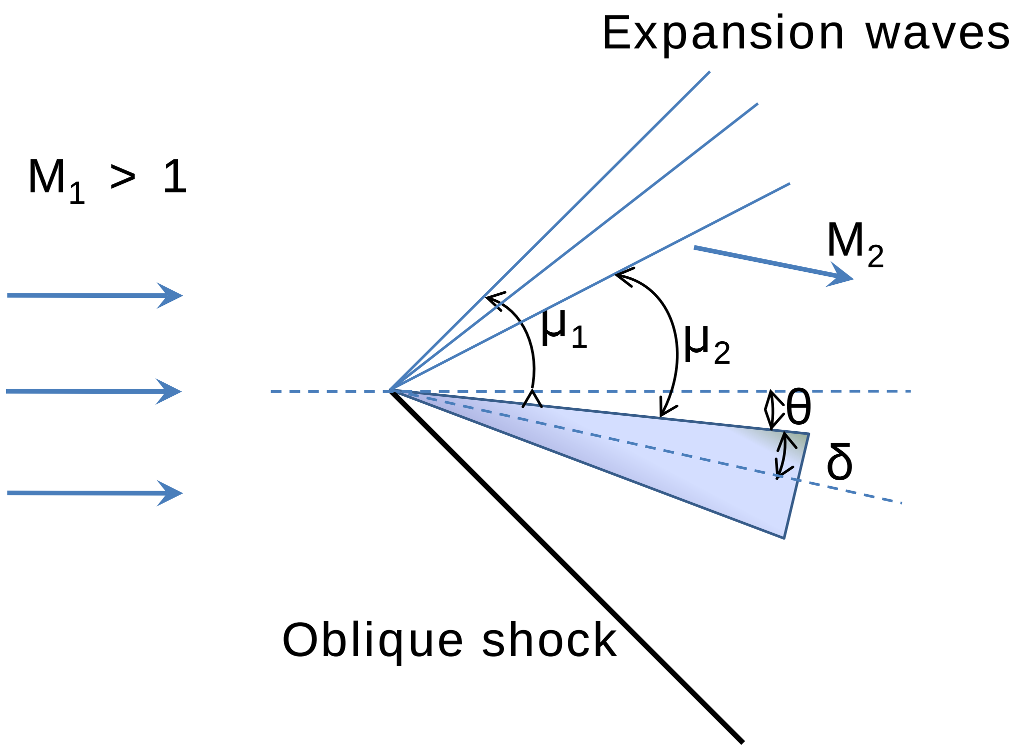
Prandtl-Meyer expansion waves



Flow across an oblique shock

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Prandtl-Meyer expansion waves

- Prandtl–Meyer expansion waves are inclined at the local Mach angle μ.
- The Mach angle of the first expansion wave

$$\mu_1 = \sin^{-1}(1/M_1)$$

- Similarly, $\mu_2 = \sin^{-1}(1/M_2)$
- Turning angle across an expansion fan is

$$\theta = \nu(M_2) - \nu(M_1)$$

- $\nu(M)$ is called the Prandtl–Meyer function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1}\left[\sqrt{\frac{\gamma+1}{\gamma-1}(M^2-1)}\right] - \tan^{-1}(\sqrt{M^2-1})$$

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Duct flow with heat transfer and negligible friction

- Many engineering problems involve compressible flow which involve chemical reactions like combustion, may involve heat transfer across the system boundaries.
- Such problems are usually analysed by modelling combustion as a heat gain process.
- The changes in chemical composition are neglected.

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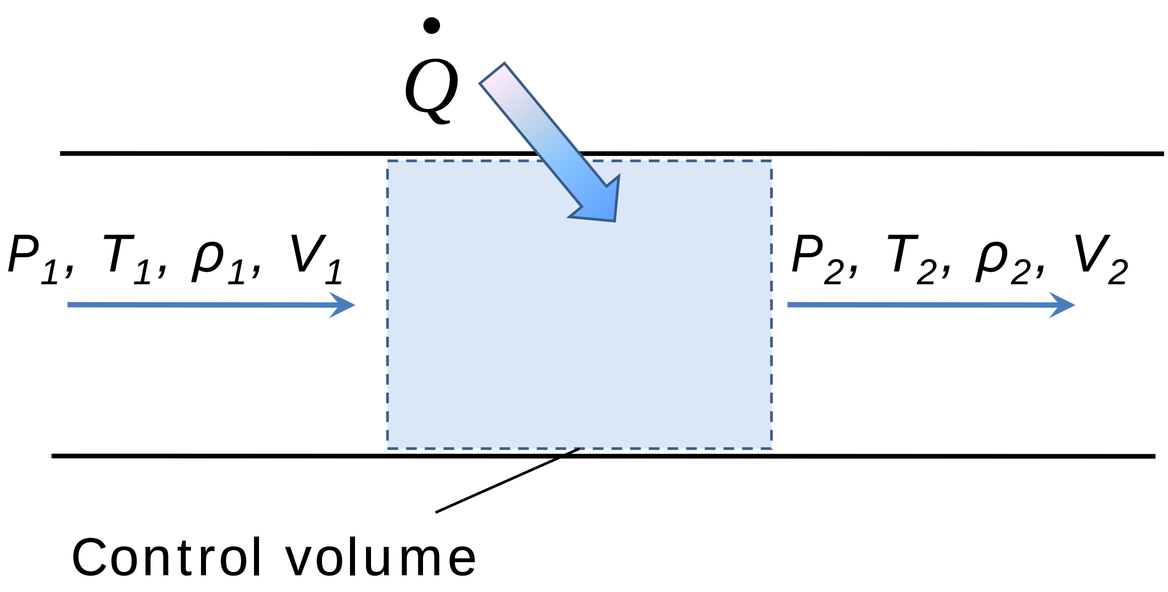
Duct flow with heat transfer and negligible friction

- 1-D flow of an ideal gas with constant specific heats through a duct of constant area with heat transfer and negligible friction: Rayleigh flows.



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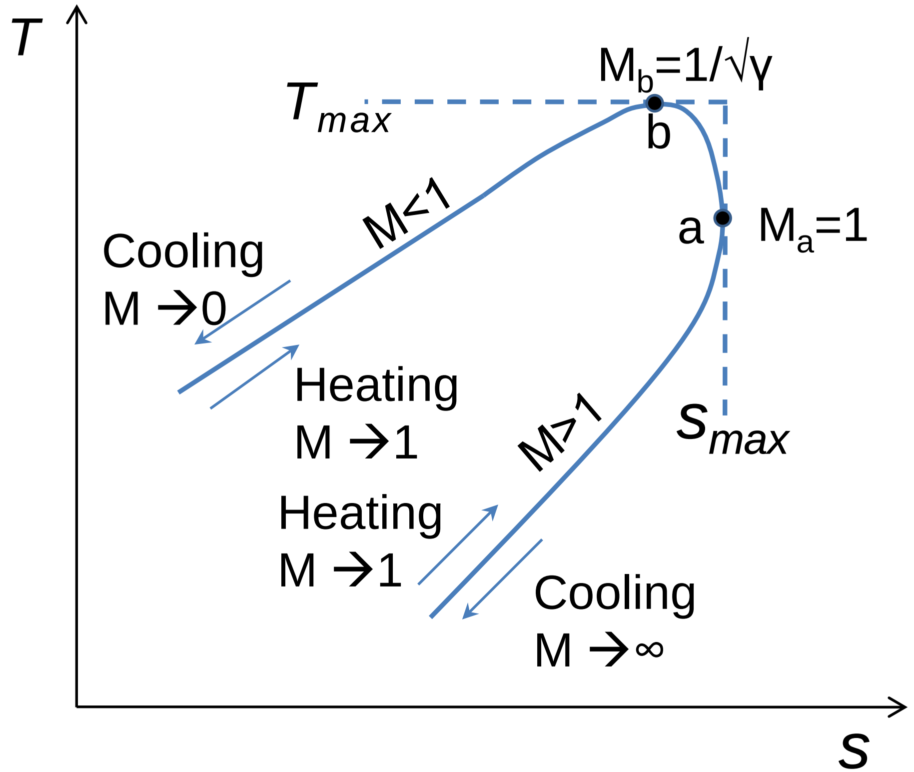
Duct flow with heat transfer and negligible friction

- For a gas whose inlet properties P_1, T_1, ρ_1, V_1 and s_1 are known, the exit properties can be calculated from the five governing equations:
- Mass, momentum, energy, entropy and equation of state.
- The Rayleigh flow on T-s diagram is called the Rayleigh line.
- The Rayleigh line is the locus of all physically attainable downstream states corresponding to an initial state.

Duct flow with heat transfer and negligible friction



T-s diagram for Rayleigh flow

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Duct flow with heat transfer and negligible friction

- The Mach number is *M=1* at point *a*, which is the point of maximum entropy.
- The states on the upper arm of the Rayleigh line above point *a* are subsonic, and the states on the lower arm below point *a* are supersonic.
- Heating increases the Mach number for subsonic flow, but decreases it for supersonic flow.
- Mach number approaches unity in both cases during heating.

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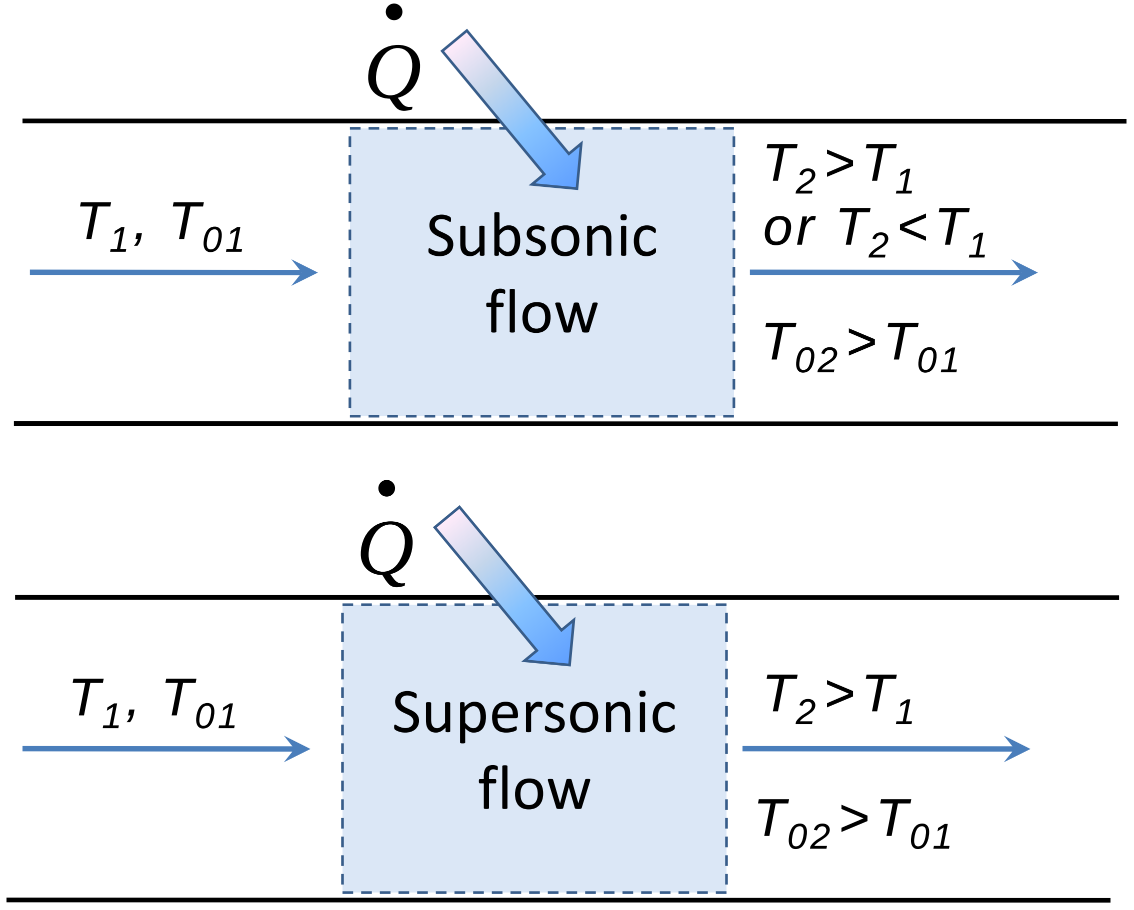
Duct flow with heat transfer and negligible friction



Duct flow with friction and negligible heat transfer

- An adiabatic flow with friction of an ideal gas with constant specific heats: Fanno flow.
- Fanno line represents the states obtained by solving the mass and energy equations.
- For adiabatic flow, the entropy must increase in the flow direction.
- Mach number of a subsonic flow increases due to friction.
- In a supersonic flow, frictions acts to decrease the Mach number.

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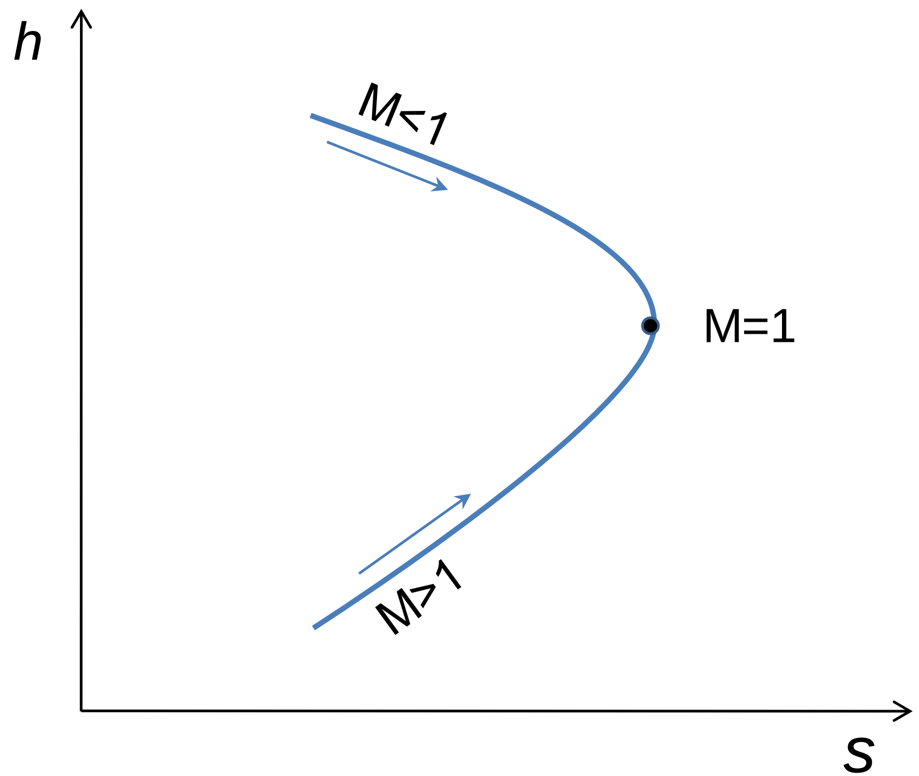
Duct flow with friction and negligible heat transfer



h-s diagram for Fanno flow

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Duct flow with friction and negligible heat transfer

- The point where $M=1$ is called choking point.
- If we consider a flow on the upper half of the Fanno line, a subsonic flow accelerates (due to friction) and reaches a maximum Mach number of one when the flow chokes.
- Similarly, a supersonic flow decelerates (due to friction) and in the limiting case, reaches a Mach number of one.

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