Introduction to Aerospace Propulsion

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Lecture No - 32

In this lecture ...

- Ideal gas turbine cycles
- Thrust and efficiency
- The thrust equation
- Other engine performance parameters
- Ideal cycle for jet engines
  - Turbojet engine
  - Turbojet engine with afterburning

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Gas turbine cycles

- Gas turbine engines operate on Brayton cycles.
- Ideal Brayton cycle is a closed cycle, whereas gas turbines operate in the open cycle mode.
- Ideal cycle assumes that there are no irreversibilities in the processes, air behaves like an ideal gas with constant specific heats, and that there are no frictional losses.

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Thrust and efficiency

- We will now derive expressions for thrust and efficiency of air-breathing engines from the momentum and energy equations.
- We shall consider a generalized thrust producing device with a single inlet and single exhaust.
- We assume that the thrust and conditions at all points within the control volume do not change with time.

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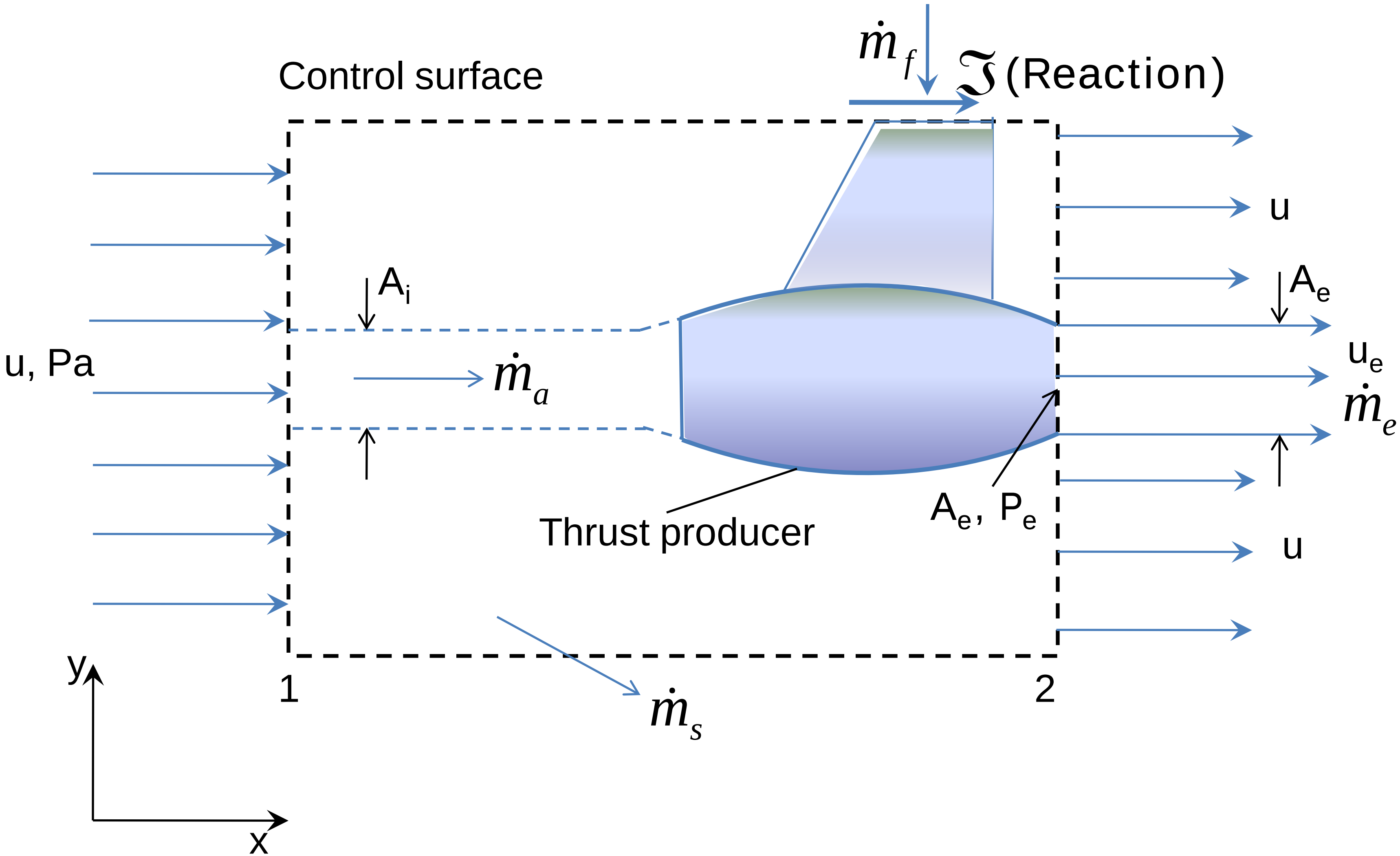
The thrust equation



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The thrust equation

- The reaction to the thrust, $\Im$, is transmitted to the support. The engine thrust is thus the vector summation of all forces on the internal and external surfaces of the engine.
- Therefore, $$\sum \vec{F} = \int_{CS} \vec{u}\,\rho(\vec{u}.\vec{n})dA$$
- Considering the components of force and the momentum flux in the x-direction only,

$$\sum F_x = \int_{CS} u_x\,\rho(\vec{u}.\vec{n})dA$$

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The thrust equation

- The pressure and velocity can be assumed to be constant over the entire control surface, except over the exhaust area, A_e.
- The net pressure force acting on this control volume is $(P_a - P_e)A_e$.
- The only other force acting on the control volume is the reaction to the thrust, $\Im$.
- Adding up the forces in the x-direction,

$$\sum F_x = (P_a - P_e)A_e + \Im$$

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The thrust equation

- The mass flow that enters the capture area, A_i, is $\dot{m}_a = \rho u A_i$
- Similarly, the mass flow crossing the exhaust area A_e, is, $\dot{m}_e = \rho_e u_e A_e$
- Also, $\dot{m}_e = \dot{m}_i + \dot{m}_f$

  Or, $\dot{m}_f = \rho_e u_e A_e - \rho u A_i$
- Continuity equation for the CV gives,

$$\rho_e u_e A_e + \rho u (A - A_e) + \dot{m}_s - \dot{m}_f - \rho u A = 0$$

$$\text{Rearranging, } \dot{m}_s = \dot{m}_f + \rho u A_e - \rho_e u_e A_e$$

$$\text{Which is, } \dot{m}_s = \rho u (A_e - A_i)$$

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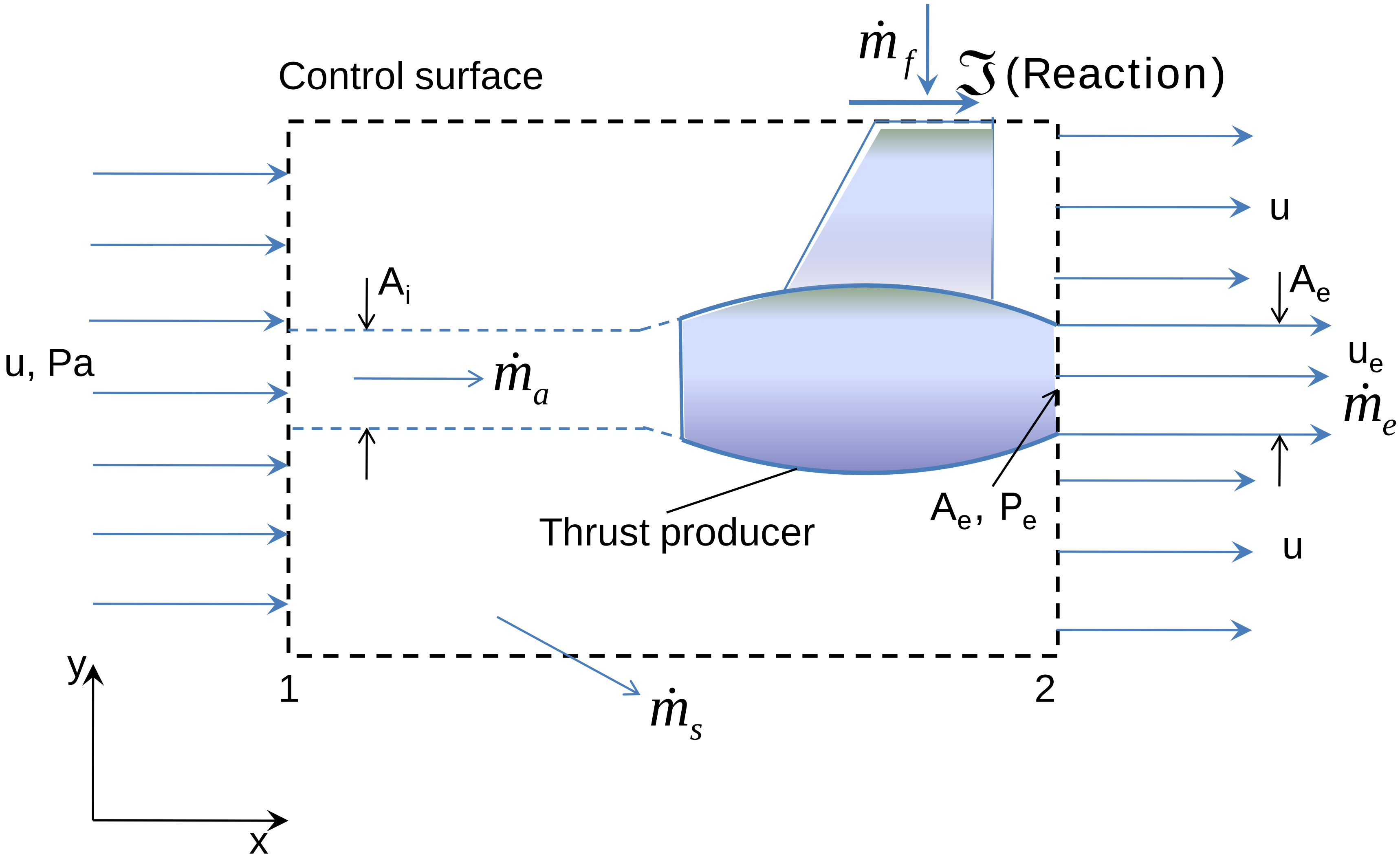
The thrust equation



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The thrust equation

- From the momentum balance across the CV,

$$\int_{CS} u_x \rho(\vec{u}.\vec{n})dA = \dot{m}_e u_e + \dot{m}_s u + \rho u(A - A_e)u - \dot{m}_a u - \rho u(A - A_i)u$$

- This is the net outward flux of x-momentum.
- This equation reduces to

$$\int_{CS} u_x \rho(\vec{u}.\vec{n})dA = \dot{m}_e u_e - \dot{m}_a u$$

- From the force balance equation, we have,

$$\Im = \dot{m}_e u_e - \dot{m}_a u + (P_e - P_a)A_e$$

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The thrust equation

- If we define fuel-air ratio, $f = \dot{m}_f / \dot{m}_a$

$$\Im = \dot{m}_a\left[(1+f)u_e - u\right] + (P_e - P_a)A_e$$

- This is the generalised thrust equation for air-breathing engines.
- The term $(P_e – P_a)A_e$ is not zero only if the exhaust jet is supersonic and the nozzle does not expand the exhaust jet to ambient pressure.
- However if $P_a \ll P_e$, it can be substantial contribution.

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Engine performance parameters

- The engine performance is described by different efficiency definitions, thrust and the fuel consumption.
- The efficiency definitions that we shall now be discussing are applicable to an engine with a single propellant stream (turbojets or ramjets).
- For other types of jet engines (turbofan, turboprop) the equations need to be appropriately modified.

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Engine performance parameters

- Propulsion efficiency: The ratio of thrust power to the rate of production of propellant kinetic energy.

$$\eta_P = \frac{\Im u}{\dot{m}_a\left[(1+f)(u_e^2/2) - u^2/2\right]}$$

- If we assume that f«1 and the pressure thrust term is negligible,

$$\eta_P = \frac{(u_e - u)u}{u_e^2/2 - u^2/2} = \frac{2u/u_e}{1+u/u_e}$$

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Engine performance parameters

- Thermal efficiency: The ratio of the rate of production of propellant kinetic energy to the total energy consumption rate

$$\eta_{th} = \frac{\dot{m}_a\left[(1+f)(u_e^2/2) - u^2/2\right]}{\dot{m}_f Q_R} = \frac{\left[(1+f)(u_e^2/2) - u^2/2\right]}{f Q_R}$$

where, Q_R, is the heat of reaction of the fuel.

- For a turboprop or turboshaft engine, the output is largely shaft power. In this case,

$$\eta_{th} = \frac{P_s}{\dot{m}_f Q_R}$$ where, P_s, is the shaft power output of the engine.

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Engine performance parameters

- Overall efficiency: The product of thermal efficiency and propulsion efficiency.

$$\eta_o = \eta_p \eta_{th}$$

- In the case of aircraft that generate thrust using propellers,

$$\eta_o = \eta_{pr} \eta_{th}$$

Where, η_{pr} is the propeller efficiency.

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Engine performance parameters

- Thrust specific fuel consumption, TSFC

$$TSFC = \frac{\dot{m}_f}{\Im} \approx \frac{\dot{m}_f}{\dot{m}_a\left[(1+f)u_e - u\right]}$$

- For turbine engines that produce shaft power, brake specific fuel consumption, BSFC

$$BSFC = \frac{\dot{m}_f}{P_s}$$

- For engine (like turboprop) that produce both, equivalent brake specific fuel consumption,

$$EBSFC = \frac{\dot{m}_f}{P_{es}} = \frac{\dot{m}_f}{P_s + \Im u}$$

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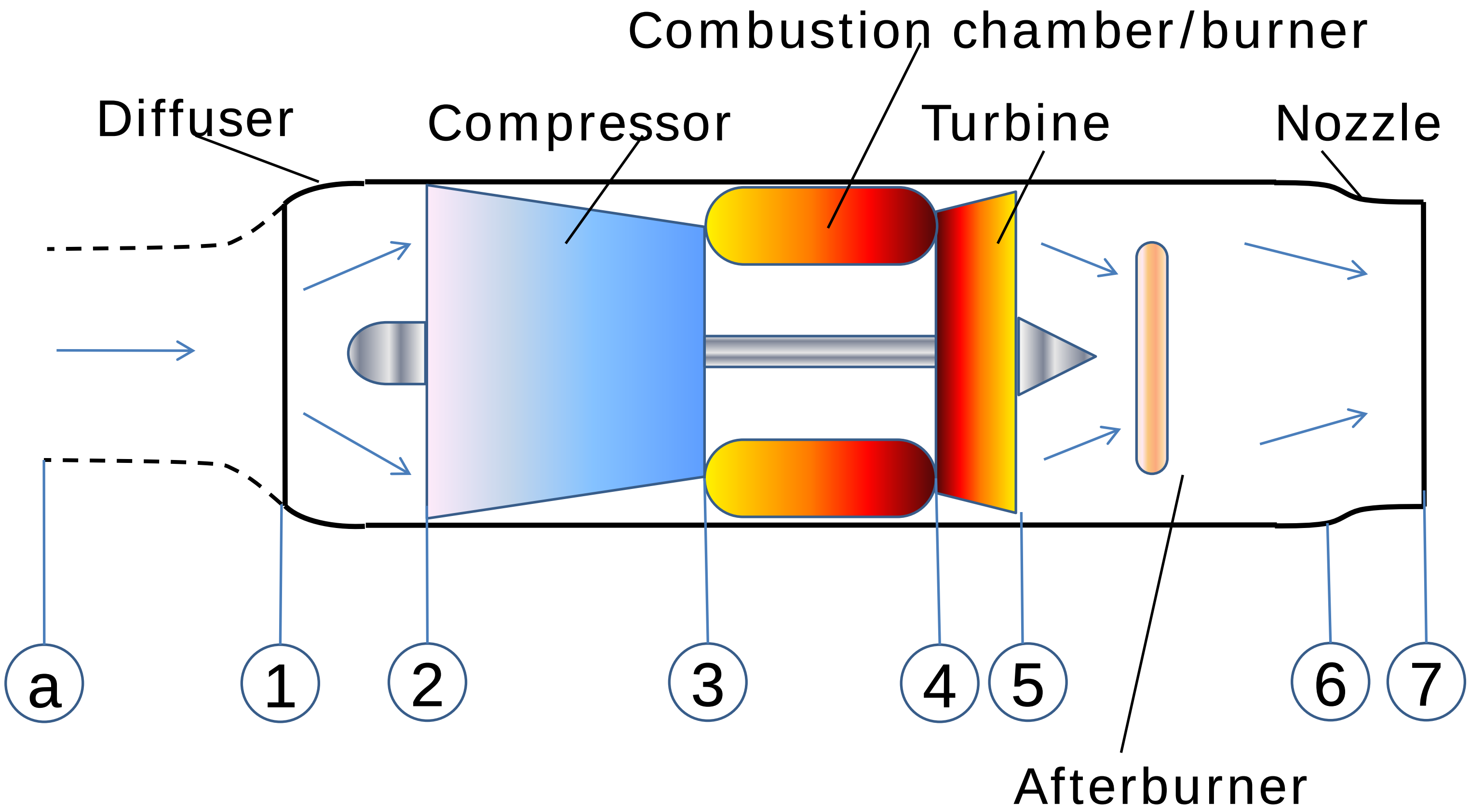
Ideal cycle for jet engines

- All air-breathing jet engines operate on the Brayton cycle (open cycle mode).
- The most basic form of a jet engine is a turbojet engine.
- Some of the parameters of a jet engine cycle are usually design parameters and hence often fixed *a priori*: eg. compressor pressure ratio, turbine inlet temperature etc.
- Cycle analysis involves determining the performance parameters of the cycle with the known design parameters.

Ideal cycle for jet engines



Schematic of a turbojet engine and station numbering scheme

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Ideal cycle for jet engines

- The different processes in a turbojet cycle are the following:
- a-1: Air from far upstream is brought to the air intake (diffuser) with some acceleration/deceleration
- 1-2: Air is decelerated as is passes through the diffuser
- 2-3: Air is compressed in a compressor (axial or centrifugal)
- 3-4 The air is heated using a combustion chamber/burner

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Ideal cycle for jet engines

- 4-5: The air is expanded in a turbine to obtain power to drive the compressor
- 5-6: The air may or may not be further heated in an afterburner by adding further fuel
- 6-7: The air is accelerated and exhausted through the nozzle.

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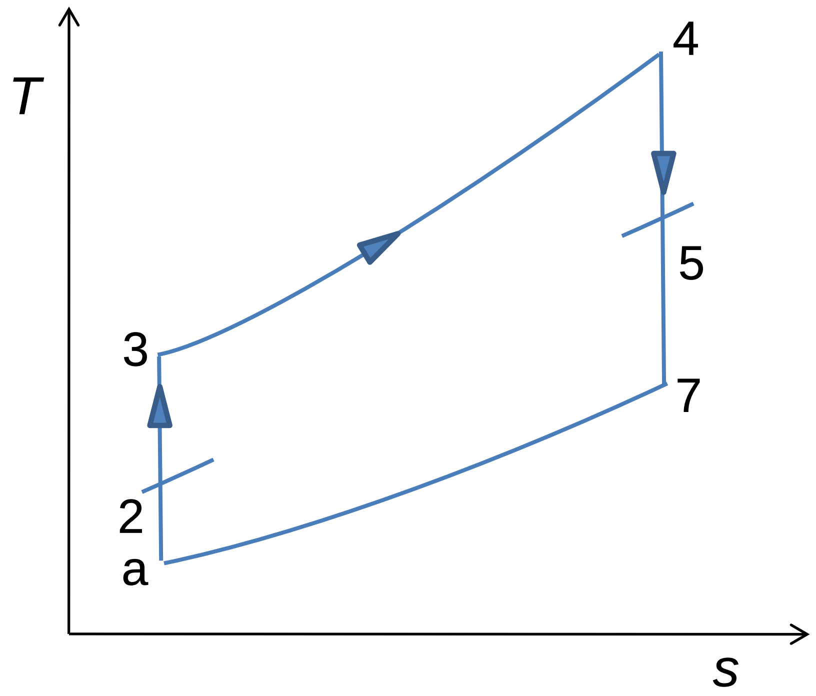
Ideal cycle for jet engines



Ideal turbojet cycle (without afterburning) on a T-s diagram

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Ideal cycle for jet engines

- For cycle analysis we shall take up each component and determine the exit conditions based on known inlet parameters.
- Intake: Ambient pressure, temperature and Mach number are known, P_a, T_a and M
- Intake exit stagnation temperature and pressure are determined from the isentropic relations:

$$T_{02} = T_a\left(1+\frac{\gamma-1}{2}M^2\right)$$

$$P_{02} = P_a\left(\frac{T_{02}}{T_a}\right)^{\gamma/(\gamma-1)}$$

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Ideal cycle for jet engines

- Compressor: Let the known compressor pressure ratio be denoted as π_c

$$P_{03} = \pi_c P_{02}$$

$$T_{03} = T_{02}\left(\pi_c\right)^{(\gamma-1)/\gamma}$$

- Combustion chamber: From energy balance,

$$h_{04} = h_{03} + fQ_R$$

$$or,\ f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}}$$

- Hence, we can determine the fuel-air ratio.

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Ideal cycle for jet engines

- Turbine: Since the turbine produces work to drive the compressor, $W_{turbine} = W_{compressor}$

$$\dot{m}_t c_p (T_{04} - T_{05}) = \dot{m}_a c_p (T_{03} - T_{02})$$

$$or,\ (1+f)(T_{04} - T_{05}) = (T_{03} - T_{02})$$

$$T_{05} = T_{04} - (T_{03} - T_{02})/(1+f)$$

$$\text{Hence, } P_{05} = P_{04}\left(\frac{T_{05}}{T_{04}}\right)^{\gamma/(\gamma-1)}$$

$$\text{For an ideal combustion chamber, } P_{04} = P_{03}$$

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Ideal cycle for jet engines

- Nozzle: With no afterburner, $T_{06}=T_{05}$, $P_{06}=P_{05}$

Therefore, the nozzle exit kinetic energy,

$$\frac{u_e^2}{2} = h_{07} - h_7$$

Since, $h_{07} = h_{06}$

$$u_e = \sqrt{2c_p T_{06}\left[1-\left(P_a / P_{06}\right)^{(\gamma-1)/\gamma}\right]}$$

- Thrust, TSFC and efficiencies can now be determined using the formulae derived earlier.

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Ideal cycle for jet engines

- Thrust, $\Im = \dot{m}_a\left[(1+f)u_e - u\right] + (P_e - P_a)A_e$

  If $(P_e - P_a)A_e$ is negligible,

  $\Im = \dot{m}_a\left[(1+f)u_e - u\right]$

- $TSFC = \dfrac{\dot{m}_f}{\Im} \approx \dfrac{\dot{m}_f}{\dot{m}_a\left[(1+f)u_e - u\right]}$

- Propulsion efficiency, $\eta_P = \dfrac{\Im u}{\dot{m}_a\left[(1+f)(u_e^2/2) - u^2/2\right]}$

- Thermal efficiency, $\eta_{th} = \dfrac{\left[(1+f)(u_e^2/2) - u^2/2\right]}{fQ_R}$

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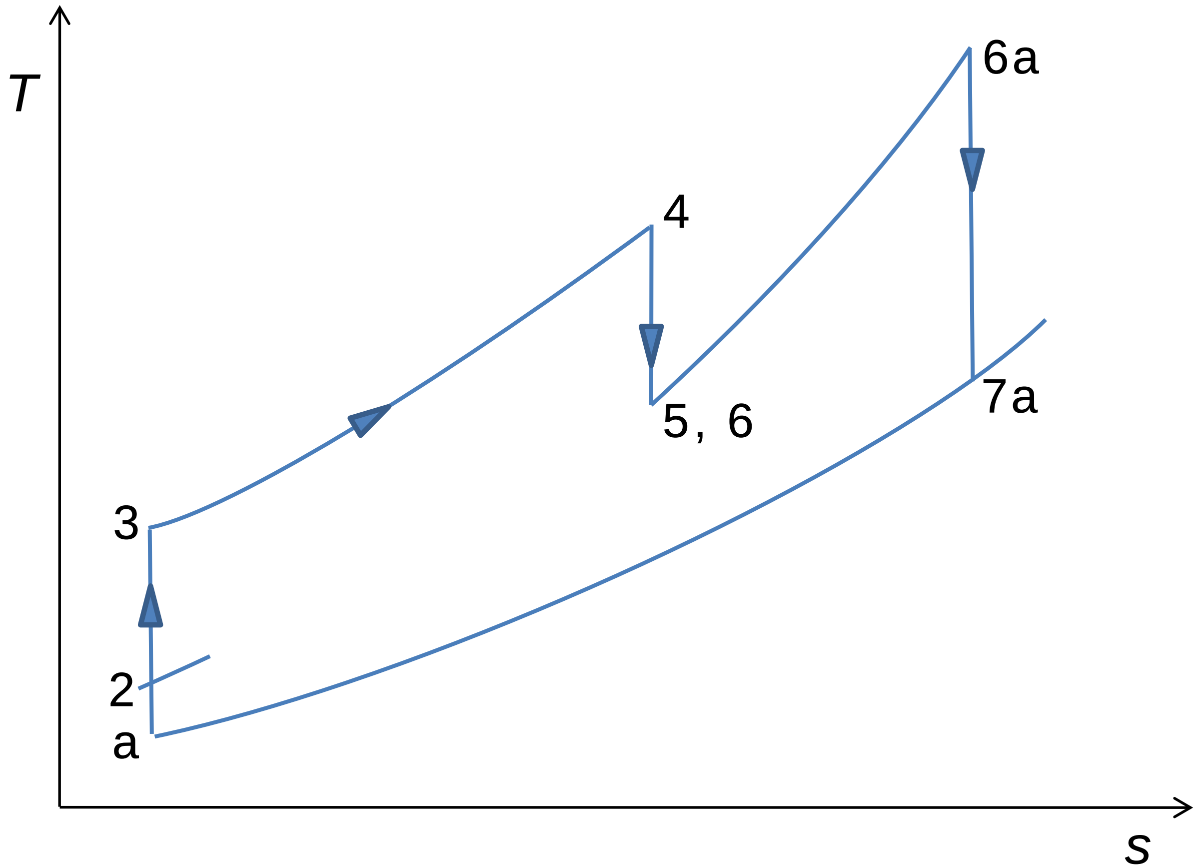
Ideal cycle for jet engines



Ideal turbojet cycle with afterburning on a *T*-*s* diagram

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Ideal cycle for jet engines

- Afterburning: used when the aircraft needs a substantial increment in thrust. For eg. to accelerate to and cruise at supersonic speeds.
- Since the air-fuel ratio in gas turbine engines are much greater than the stoichiometric values, there is sufficient amount of air available for combustion at the turbine exit.
- There are no rotating components like a turbine in the afterburner, the temperatures can be taken to much higher values than that at turbine entry.

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Ideal cycle for jet engines

- For calculating the fuel flow rate required to achieve a temperature of T_{6a}, we carry out an energy balance,

$$h_{06a} = h_{05} + f_2 Q_R$$

$$or, \; f_2 = \frac{T_{06a}/T_{05} - 1}{Q_R / c_p T_{05} - T_{06}/T_{05}}$$

Where, $f = f_1 + f_2$, f is the total fuel flow ratio, f_1 is the fuel flow ratio in the main combustor.

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In the next lecture ...

- Ideal cycle for jet engines
  - Turbofan engine
  - Different configurations of turbofan engines
  - Turboprop engines
  - Turboshaft engines
  - Ramjets

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