Introduction to Aerospace Propulsion

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Lecture No - 38

Rocket Theories

----------------------- Continued

and

Nozzle theory fundamentals

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Various Liquid Propellants and their typical Characteristics

Prop	Ox/F ratio	Thrust Vac (kN)	Thrust SL (kN)	I_{sp} Vac s	I_{sp} SL s	P_c bar	C_F Vac	V^* m/s	
Lox / LH_2	5.2	1075	813	431	310	105	1.87	2380	small
Lox / LH_2	6.0	2323	1853	455	363	204	1.91	2410	big
Lox / Ker	2.77	7893	6880	358	265	70	1.82	1810	big
Lox / Ker	2.25	1043	934	295	263	48	1.60	1820	small

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Solid Rocket Performance Parameters

Propellant weight flow rate,

$$\dot{W} = A_G . b_r . \rho_G$$

where, A_G = Area of grain burning surface

b_r = Burning rate (linear) ; ρ_G= Density of the grain

Burning rate of a propellant grain may be given as

$$b_r = a . p_{cc}^n$$

b_r is in cm/sec

where p_{cc}= Combustion chamber pressure, and, *a* and *n* are burn or combustion indexes for the grain

alternately $$b_r = x + y . p_{cc}^n$$

X and Y are burn constants

0.2<n<0.8, always n<1.0

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Now :

Propeller mass burnt =
mass increase in the comb. chamber + gas flow in nozzle

$$\dot{W} = A_G.b_r.\rho_G = \frac{d}{dt}\left(\rho_{cc}.v_{cc}\right) + A_t.P_{cc}.\sqrt{\frac{\gamma_g}{R.T_{cc}}\left(\frac{2}{\gamma_g+1}\right)^{\frac{\gamma_g+1}{\gamma_g-1}}}$$

where A_t is the nozzle throat area

Now if mass variation inside the combustion chamber is considered zero then,

$$\frac{d}{dt}\left(\rho_{cc}.v_{cc}\right) = 0$$

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Hence,

$$\frac{A_G}{A_t} = \frac{P_{cc}}{\rho_G . r} \cdot \sqrt{\frac{\gamma_g}{R.T_{cc}} \left(\frac{2}{\gamma_g + 1} \right)^{\frac{\gamma_g + 1}{\gamma_g - 1}}} = \frac{P_{cc}^{\;1-n}}{\rho_G . a} \cdot \sqrt{\frac{\gamma_g}{R.T_{cc}} \left(\frac{2}{\gamma_g + 1} \right)^{\frac{\gamma_g + 1}{\gamma_g - 1}}}$$

Simplified expression $\frac{A_G}{A_t} = P_{cc}^{1-n}$ or, $\boxed{P_{cc} = \left(\frac{A_G}{A_t} \right)^{\frac{1}{1-n}}}$

This expression means that if n is large, variation of burning surface A_G will have large effects on the chamber pressure and on the propellant burning rate. Thus, n should be low.

Burning rate and Erosion :

Simplified reduced order model $b_r = a.P_{cc}^n$ and is not fully representative of various physical factors.

Burning Rate = *f* (Chemical composition, geometrical shape, initial temperature, fabrication process, radiation, gas velocity on the surfaces, burning time)

The combined effect of all these factors involving physical and chemical interactions need to be taken into account.

Erosive burning is the term used to indicate that the burning rate of a solid propellant is affected by the flow of high velocity gases parallel to the burning surface. It is more pronounced at the beginning.

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Following general points are relevant to the solid propellant grain design and rocket performance:

- The combustion pressure is not uniform along the length of the chamber with the fastest burning rate near the front end.
- Because of various pressure losses, actual chamber pressure at the nozzle entry is less than the theoretically computed value.
- The pressure and burning rate at any of one station will <u>vary with time of burning </u>as cross-sectional area increases.

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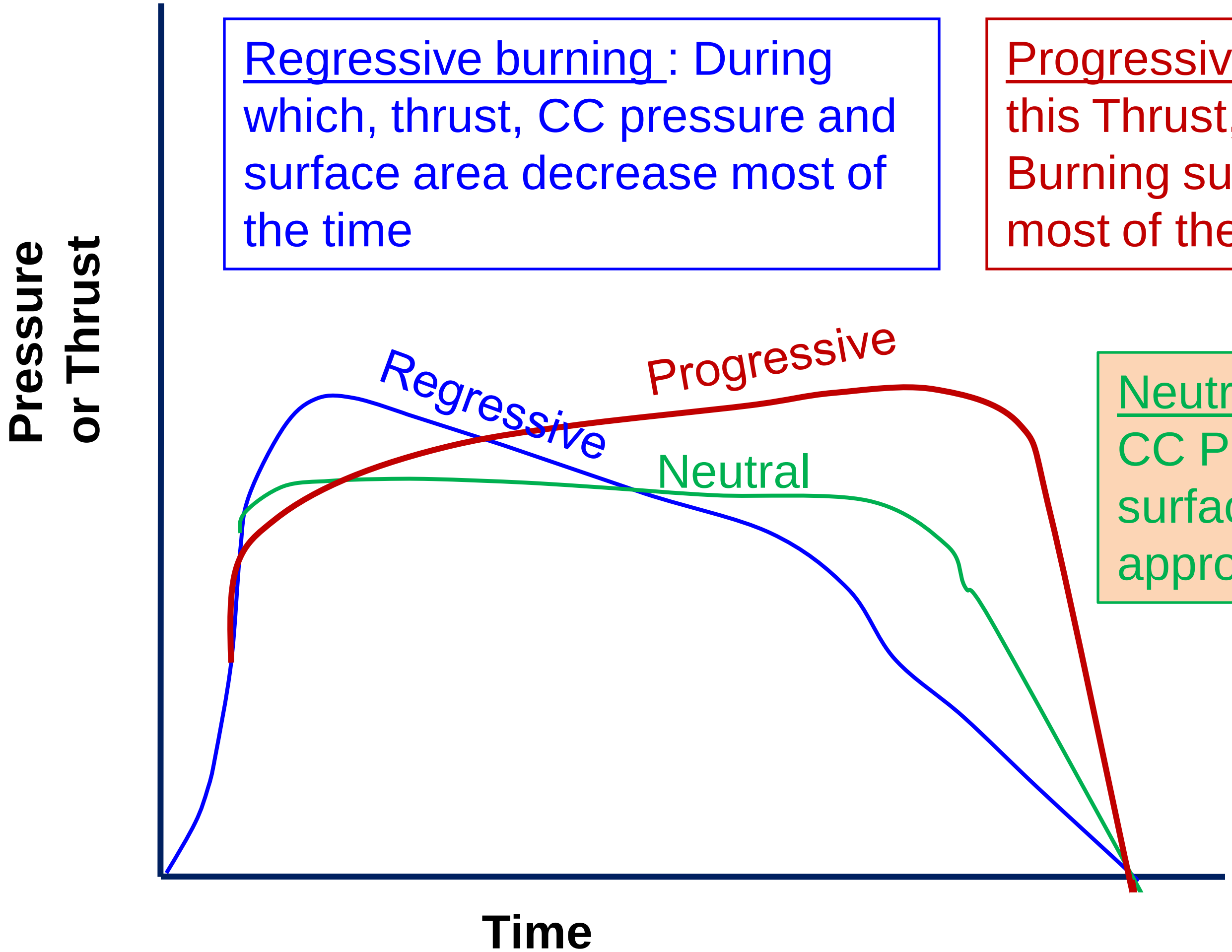
Burning characteristics of Solid Grain

Regressive burning : During which, thrust, CC pressure and surface area decrease most of the time

Progressive burning : During this Thrust, CC pressure and Burning surface area increase most of the time



Neutral Burning : Thrust, CC Pressure and Burning surface area remain approx constant

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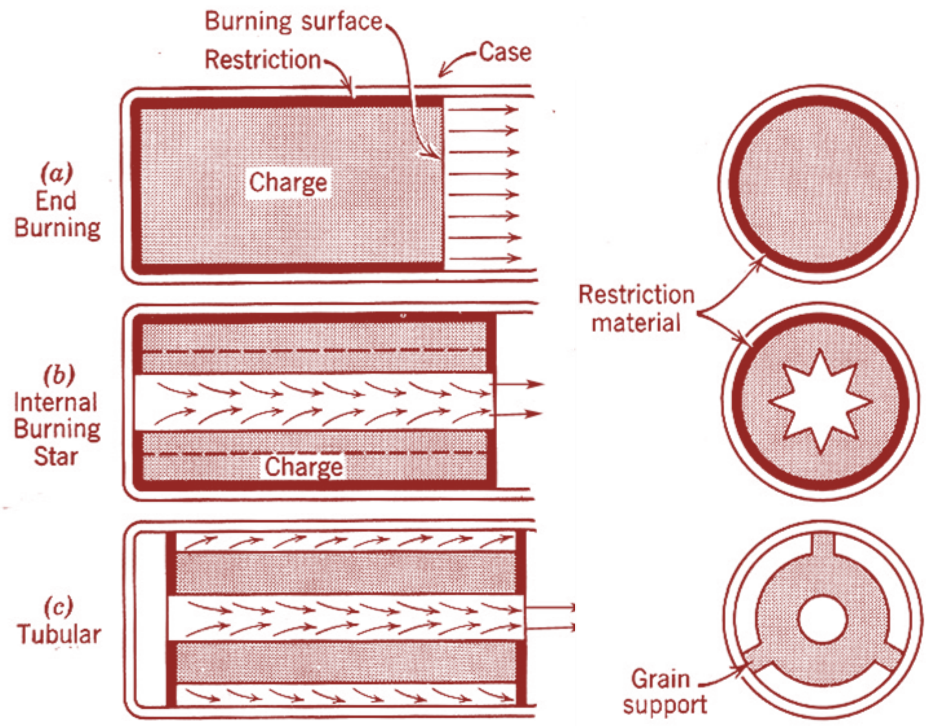
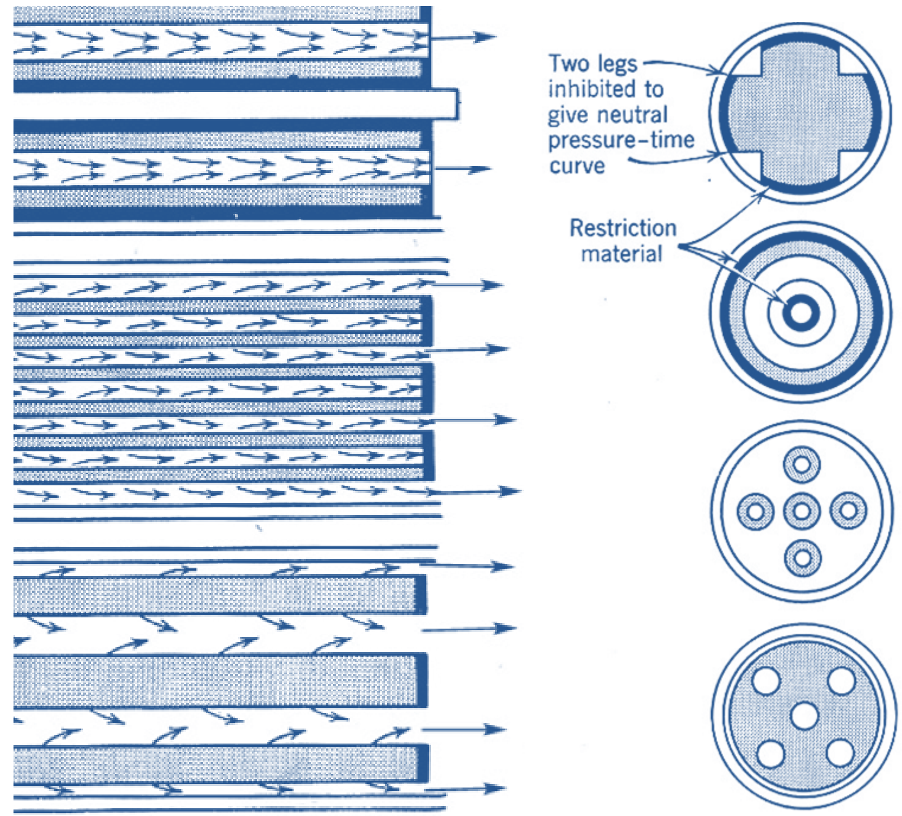
Solid Propellant grains





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Rocket Nozzles

- Most rocket nozzles operate with pressure ratios above 25 or 30, or upto 100, and hence, all are convergent–divergent types
- Thus the condition at the nozzle throat is critical at all times of the operation of the nozzle.
- Since this criticality decides the mass flow through the nozzle and hence the thrust produced, the geometry of the nozzle must be such as to promote required amount of mass flow through the nozzle at all operating conditions.
- The nozzles are generally fixed geometry type.
- There are some nozzles which can be swiveled to produce change in direction of the thrust produced

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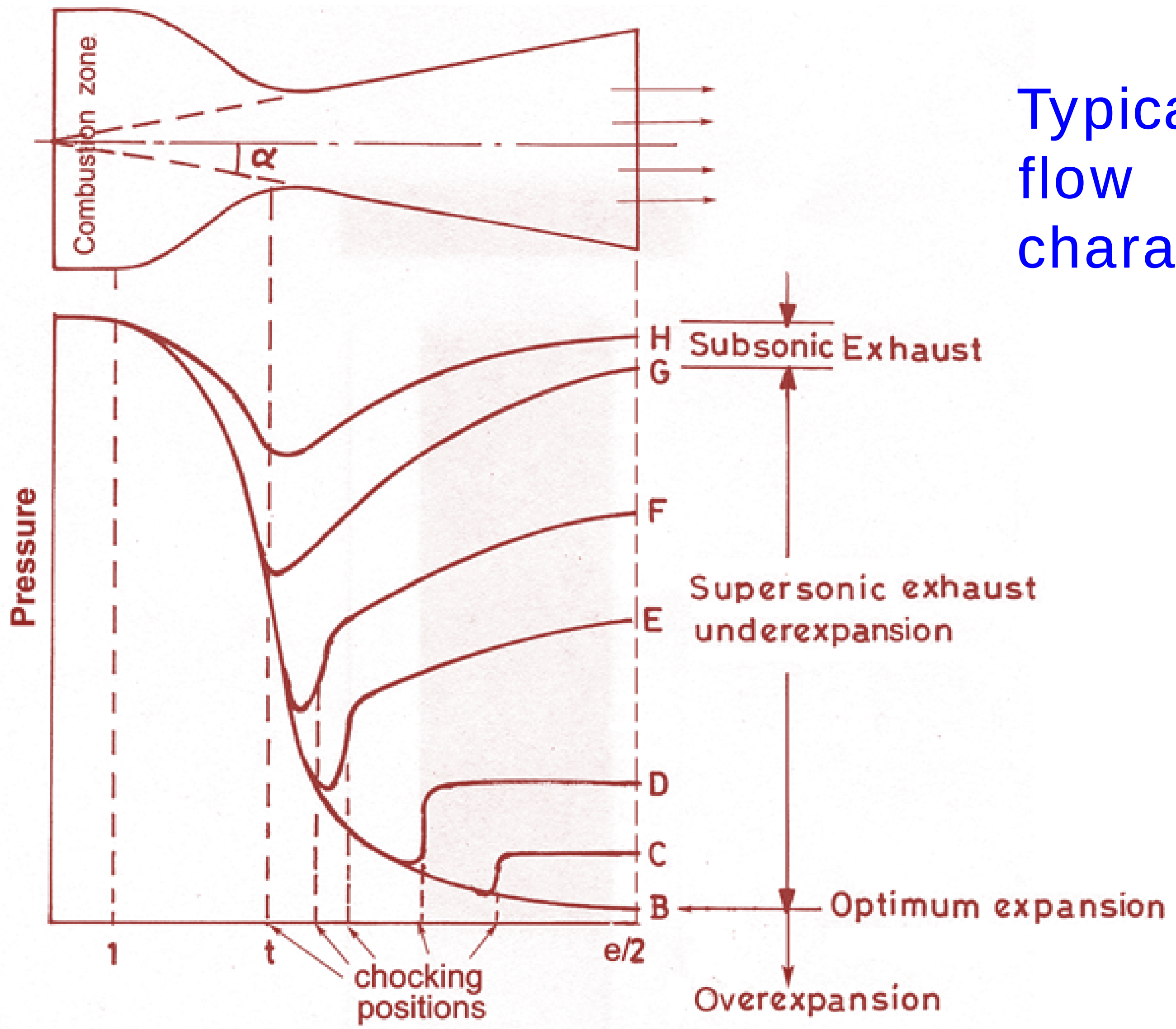
Typical nozzle flow characteristics



Rocket Nozzles

- The effect of underexpansion is reduction in the exhaust velocity and therefore lowering of exit kinetic energy and lowering of thrust production.
- Overexpansion produces separation inside the nozzle, as the flow completes the expansion process when it is still inside the nozzle, and often experiences a separation thereafter
- The direction of thrust produced is not altered by the flow separation in the nozzle, <u>if the flow separates symmetrically</u> over the cross section around the nozzle surface.
- Separation occurs when the ambient pressure is 2.5 to 3.5 times the nozzle inside wall pressure.

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Rocket Nozzles

- A nozzle, is often designed for full expansion at a high altitude, and is likely to give higher than the ideal thrust at sea level (where ambient pressure is high).
- The characteristic velocity V^* of the rocket, (lect-37), is independent of the nozzle shape and is dependant on the fuel and oxidizer characteristics, combustion chamber design and the thermodynamic parameters after combustion.
- However, the definition implicitly assumes fully expanded ideal nozzle flow.

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Rocket Nozzles

- Most nozzles are of circular cross section, but of various shapes .
- Diameter of the subsonic part of nozzle is governed by (i) volume for combustion (liquid rocket), (ii) size of the grain (solid propellant rocket), (iii) size and arrangements of injectors (liquid propellant).
- The slope of the convergent nozzle is not important
- The length and the shape of the nozzle is primarily decided by the throat area and the exit area and the velocity variation within it.
- Thus the "design of a rocket nozzle" essentially means determining the length and shape of the divergent part of the nozzle.

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Rocket Nozzles

• The selection of a suitable divergence shape (configuration and angle of divergence) is made with following criteria:

i) Large divergence angles make the nozzle short – hence give low friction loss.
ii) Small exit diameter gives low aerodynamic drag of the vehicle, but increases nozzle length & surface area and hence weight of the rocket.

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Rocket Nozzles

iii) Large divergence angle near the nozzle exit produces radial component of the flow , which is a not a thrust producing flow component and is, thus, a loss.

iv) Large divergence may also produce separation and related loses near the exit.

v) Long nozzle (with low divergence angle) is difficult to incorporate in the rocket body

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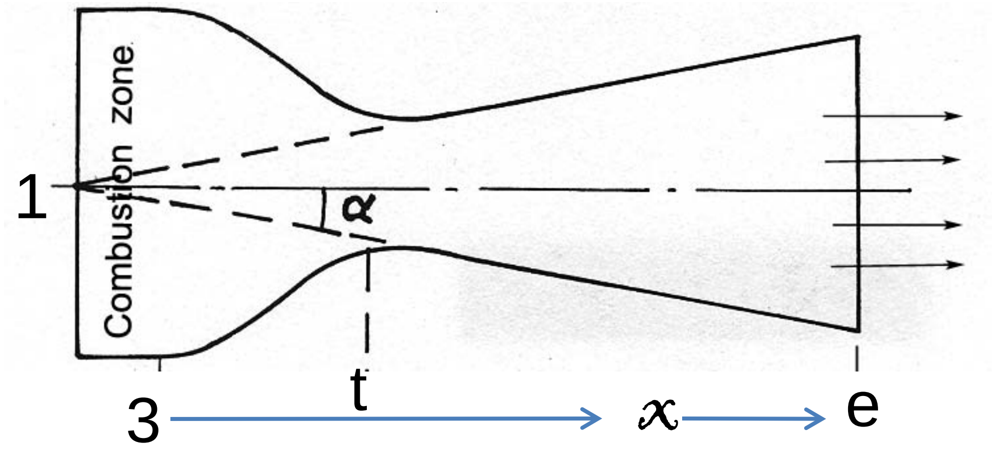
Rocket Nozzles



From Isentropic flow equations :

Pressure ratio across the **convergent part of the nozzle** is

$$\frac{p_t}{p_1} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}}$$

Temperature ratio across the **convergent part of the nozzle**

$$= \frac{2}{\gamma + 1}$$

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Rocket Nozzles

Velocity anywhere in the convergent nozzle

$$V_x = \sqrt{2.c_p.(T_{03} - T_x) + V_1^{\,2}} = \sqrt{\frac{2.\gamma.R.(T_{03} - T_x)}{\gamma - 1} + V_1^{\,2}}$$

$$= \sqrt{\frac{2.\gamma.R.T_{03}.\left(1 - \left(\frac{P_x}{P_{03}}\right)^{\frac{\gamma-1}{\gamma}}\right)}{\gamma - 1} + V_1^{\,2}}$$

Rocket Nozzles

Velocity at the throat

$$V_t = \sqrt{\gamma.R.T_t} = \sqrt{\frac{2\gamma}{\gamma+1}.R.T_{cc}}$$

Mass Flow,

$$\dot{m} = A_t.V_t.\rho_t = A_t.p_{cc}.\gamma.\frac{\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}{\sqrt{\gamma.R.T_{cc}}}$$

Rocket Nozzles

In the divergent part of the nozzle

Area Ratio

$$\frac{A_t}{A_x} = \frac{V_x . v_t}{V_t . v_x} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma+1}} . \left(\frac{p_x}{p_t}\right)^{\frac{1}{\gamma}} . \sqrt{\frac{\gamma+1}{\gamma+1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Velocity ratio

$$\frac{V_x}{V_t} = \frac{\gamma+1}{\gamma-1} . \left[1-\left(\frac{p_x}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Rocket Nozzles

Thrust $$F = A_t.V_t.\rho_t.V_e + (p_e - p_a).A_e$$

$$F = A_t.p_{cc}.\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_{cc}}\right)^{\frac{\gamma}{\gamma-1}}\right]} + (p_e - p_a).A_e$$

Thrust Co-efficient

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_{cc}}\right)^{\frac{\gamma}{\gamma-1}}\right]} + \left(\frac{p_e - p_a}{p_{cc}}\right).\frac{A_e}{A_t}$$

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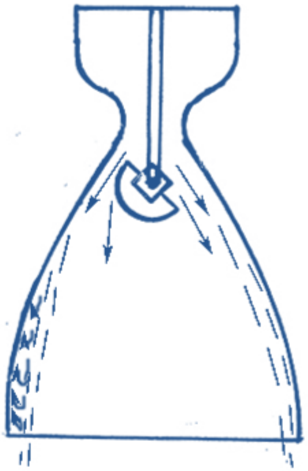
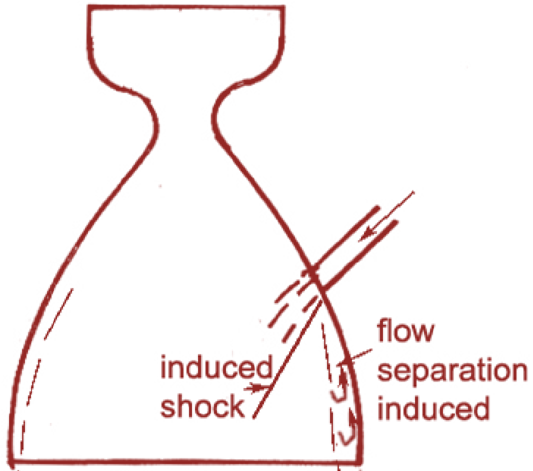
Rocket Nozzles

Movable Nozzle with a Center plug



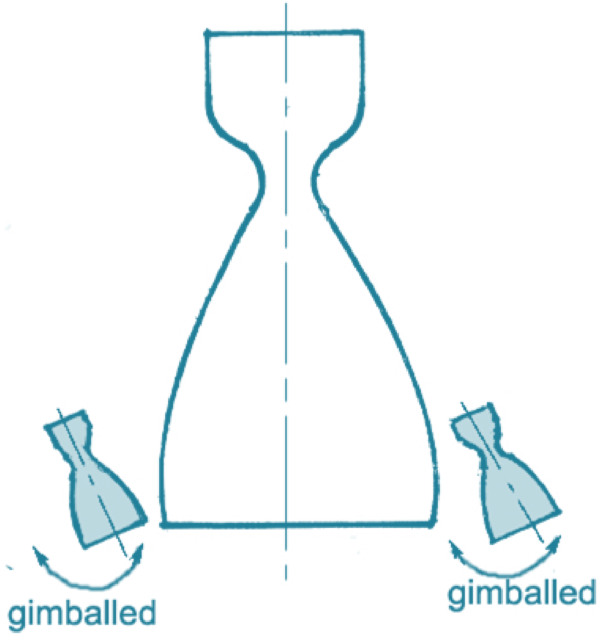
Assymetric nozzle flow

Side Injection



Assymetric flow due to injection induced shocks on one side

Small thrust control chambers



Saturn Rocket Launch