Jet Aircraft Propulsion

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Lect-11

In this lecture...

- Tutorial
  - Solve problems involving real cycle analysis

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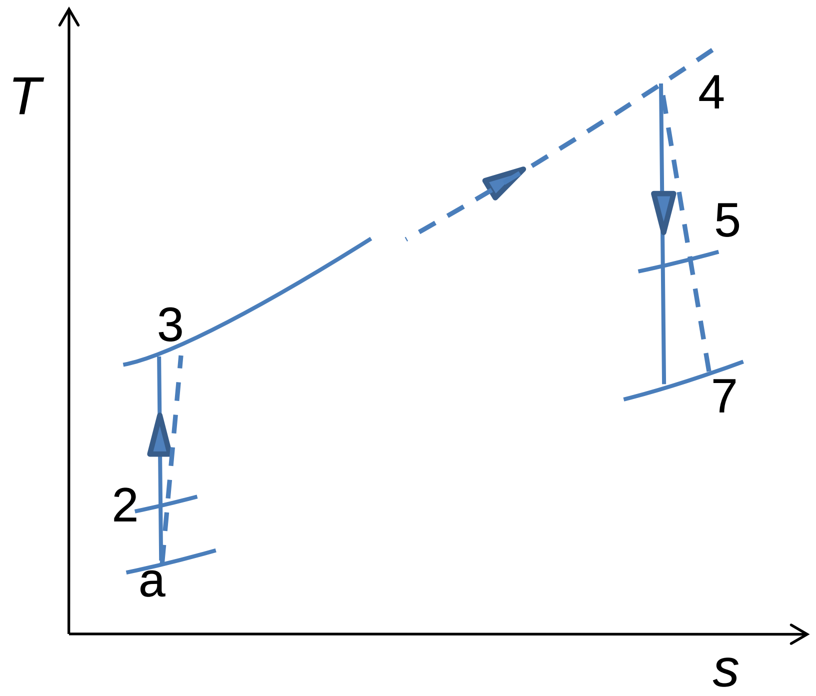
Problem # 1

- An aircraft using a simple turbojet engine, flies at Mach 0.8 where the ambient temperature and pressure are 223.3 K and 0.265 bar, respectively. The compressor pressure ratio is 8.0 and the turbine inlet temperature is 1200 K. The isentropic efficiencies of: compressor=0.87, turbine=0.90, intake=0.93, nozzle=0.95, mechanical=0.99, combustor=0.98. The pressure loss in the combustor=4% of compressor delivery pressure. Determine the thrust and SFC.

Problem # 1



Real turbojet cycle (without afterburning) on a T-s diagram

Solution: Problem # 1

- For the given ambient conditions and the Mach number, the flight speed is V=239.6 m/s
- Intake exit stagnation temperature and pressure

$$T_{02} = T_a + \frac{V^2}{2c_P} = 223.3 + \frac{239.6^2}{2\times 1005} = 251.9\,K$$

$$\frac{P_{02}}{P_a} = \left[1 + \eta_d\left(\frac{T_{02}}{T_a} - 1\right)\right]^{\gamma/(\gamma-1)} = 1.482$$

$$or,\ P_{02} = 0.2650\times 1.482 = 0.393\,bar$$

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Solution: Problem # 1

- The compressor exit conditions are determined as follows:

Compressor exit pressure is

$$P_{03} = \pi_c P_{02} = 8.0 \times 0.393 = 3.144\ bar$$

$$T_{03} = T_{02}\left\{\frac{1}{\eta_C}\left[\pi_c^{\ (\gamma-1)/\gamma} - 1\right] + 1\right\}$$

$$= 251.9\left\{\frac{1}{0.87}\left[8^{(1.4-1)/1.4} - 1\right] + 1\right\}$$

$$= 486.8\ K$$

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Solution: Problem # 1

- Combustion chamber:

$$h_{04} = h_{03} + \eta_b f\dot{Q}_f$$

$$c_{pg}T_{04} = c_{pa}T_{03} + \eta_b f\dot{Q}_f$$

$$or,\ f = \frac{c_{pg}T_{04}/c_{pa}T_{03} - 1}{\eta_b\dot{Q}_f/c_{pa}T_{03} - c_{pg}T_{04}/c_{pa}T_{03}}$$

$$\text{Substituting all the values, f} = 0.0198$$

$$\text{Also, } P_{04} = \pi_b P_{03} = 0.96 \times 3.144 = 3.018 bar$$

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Solution: Problem # 1

- Turbine: Since the turbine produces work to drive the compressor, $W_{turbine}$ = $W_{compressor}$

$$\eta_m(\dot{m}+\dot{m}_f)c_{pg}(T_{04}-T_{05})=\dot{m}c_{pa}(T_{03}-T_{02})$$

$$T_{05}=c_{pg}T_{04}-c_{pa}(T_{03}-T_{02})/\eta_m(1+f)$$

$$=1147\times1200-1005(486.8-251.9)/0.99(1+0.0198)$$

$$=992.3\,K$$

Similarly,

$$P_{05}=P_{04}\left[1-\frac{1}{\eta_t}(1-T_{05}/T_{04})\right]^{\gamma/(\gamma-1)}=1.284\ bar$$

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Solution: Problem # 1

- Nozzle: We shall first check for nozzle choking.

The nozzle pressure ratio is : $\dfrac{P_{05}}{P_a} = \dfrac{1.284}{0.265} = 4.845$

The critical pressure ratio is

$$\frac{P_{05}}{P_c} = \frac{1}{\left[1-\dfrac{1}{\eta_n}\left(\dfrac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/(\gamma-1)}} = \frac{1}{\left[1-\dfrac{1}{0.95}\left(\dfrac{1.33-1}{1.33+1}\right)\right]^{1.33/(1.33-1)}}$$

$$=1.914$$

Since $P_{05}/P_a > P_{05}/P_c$, the nozzle is choking.

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Solution: Problem # 1

- Therefore the nozzle exit conditions are fixed by the critical parameters.

$$T_7 = T_c = \left(\frac{2}{\gamma+1}\right)T_{05} = 850.7K$$

$$P_7 = P_c = P_{05}\left(\frac{1}{P_{05}/P_c}\right) = 0.671bar$$

$$\rho_7 = P_7 / RT_7 = 0.275\,kg/m^3$$

$$V_{ex} = \sqrt{\gamma RT_7} = 570.5m/s$$

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Solution: Problem # 1

$$\frac{A_e}{\dot{m}} = \frac{1}{\rho_7 V_{ex}} = 0.006374\ m^2 s / kg$$

$$\therefore \text{Specific thrust is, } F_n = (1+f)\left(V_{ex} - V\right) + \frac{A_e}{\dot{m}}(P_c - P_a)$$

$$= 596.25 Ns / kg$$

$$\text{SFC} = \frac{f}{F_n} = \frac{0.0198}{596.25} = 3.32 \times 10^{-5}\ kg / sN$$

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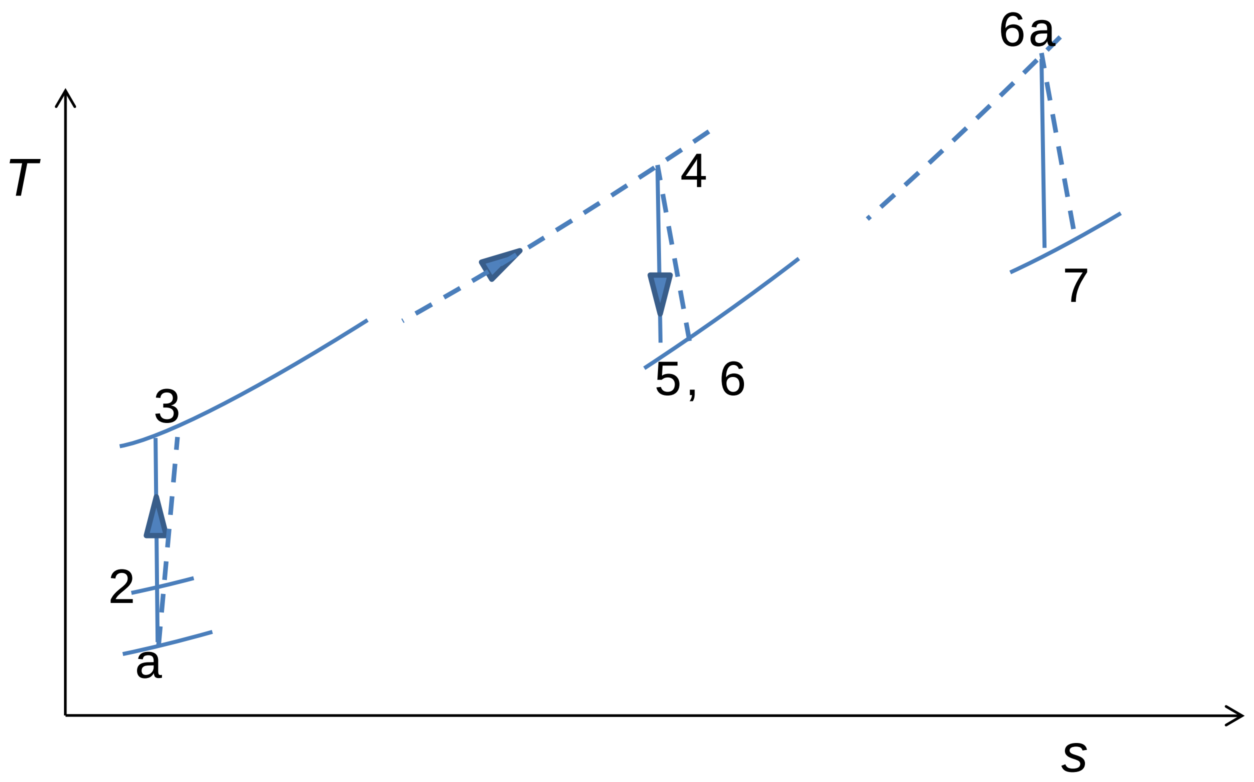
Problem # 2

- Determine the thrust and SFC in the above problem if the engine operates with an afterburner. The nozzle inlet temperature in this case is limited to 1800 K. All other parameters and operating conditions remain unchanged.

Solution: Problem # 2



Real turbojet cycle (with afterburning) on a T-s diagram

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Solution: Problem # 2

- Since all other operating conditions and parameters remain unchanged, the cycle analysis up to the turbine exit is exactly the same as discussed for the previous problem.
- The nozzle will be choking and the exit conditions will need to be calculated.
- Besides this the fuel flow rate in the afterburner too needs to be determined.
- The total fuel flow rate will be the sum of the fuel in the main combustor and that of the afterburner.

Solution: Problem # 2

- To calculate fuel flow rate in the afterburner,

$$h_{06} = h_{05} + \eta_b f_2 \dot{Q}_f$$

$$or,\ f_2 = \frac{c_{pg}T_{06}/c_{pg}T_{05} - 1}{\eta_b \dot{Q}_f / c_{pg}T_{05} - c_{pg}T_{06}/c_{pg}T_{05}}$$

$$\text{Substituting all the values, } \mathrm{f}_2 = 0.02256$$

$$\therefore\ f = f_1 + f_2 = 0.0198 + 0.02256 = 0.04236$$

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Solution: Problem # 2

- At the nozzle exit,

$$T_7 = T_c = \left(\frac{2}{\gamma+1}\right) T_{06} = 1545.06K$$

$$P_7 = P_c = P_{05}\left(\frac{1}{P_{05}/P_c}\right) = 0.671 bar$$

$$\rho_7 = P_7 / RT_7 = 0.151 kg/m^3$$

$$V_{ex} = \sqrt{\gamma R T_7} = 787.9\ m/s$$

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Solution: Problem # 2

$$\frac{A_e}{\dot{m}} = \frac{1}{\rho_7 V_{ex}} = 0.0084\ m^2 s / kg$$

$$\therefore \text{Specific thrust is, } F_n = (1+f)\left(V_{ex} - V\right) + \frac{A_e}{\dot{m}}(P_c - P_a)$$

$$= 912.56 Ns / kg$$

$$\text{SFC} = \frac{f}{F_n} = \frac{0.04236}{912.56} = 4.64 \times 10^{-5}\ kg / sN$$

- Afterburning therefore leads to substantial thrust augmentation (about 35%). But this is accompanied by about 28% increase in SFC.

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Problem # 3

- A twin spool un-mixed turbofan engine has the fan driven by the LP turbine and the compressor by the HP turbine. The overall pressure ratio is 25 and the fan pressure ratio is 1.65. The engine has a bypass ratio of 5.0 and a turbine inlet temperature of 1550 K. The fan, turbine and compressor have polytropic efficiencies of 0.90. The nozzle efficiency is 0.95 and the mechanical efficiency for each spool is 0.99. The combustor pressure loss is 1.5 bar and the total air mass flow is 215 kg/s. Find the thrust under sea level static conditions, where ambient pressure and temperature are 1 bar and 288 K.

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Solution: Problem # 3

- Under static conditions, $T_{01}=T_a$ and $P_{01}=P_a$.

$$T_{02'} = T_{01}(\pi_f)^{(\gamma-1)/\eta_{poly,fan}\gamma} = 337.6\,K$$

Since the overall pressure ratio is 25,

$$\frac{P_{03}}{P_{02}} = \frac{25}{1.65} = 15.15$$

$$T_{03} = T_{02}\left(\frac{P_{03}}{P_{02}}\right)^{(\gamma-1)/\eta_{poly,comp}\gamma} = 800.1\,K$$

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Solution: Problem # 3

The cold nozzle pressure ratio is the fan pressure ratio $=1.65$

The critical pressure ratio is

$$\frac{P_{02'}}{P_c}=\frac{1}{\left[1-\frac{1}{\eta_n}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/(\gamma-1)}}=\frac{1}{\left[1-\frac{1}{0.95}\left(\frac{1.4-1}{1.4+1}\right)\right]^{1.4/(1.4-1)}}$$

$$=1.965$$

Therefore the nozzle is not choking.

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Solution: Problem # 3

The secondary nozzle exhaust velocity, $V_{ex,f}$

$$V_{ex,f} = \sqrt{2c_p \eta_n T_{02'} \left[1 - \left(P_a / P_{02'}\right)^{(\gamma-1)/\gamma}\right]}$$

$$= \sqrt{2 \times 1005 \times 0.95 \times 337.6 \left[1 - \left(1/1.965\right)^{(1.4-1)/1.4}\right]}$$

$$= 293.2 \ m/s$$

Since the bypass ratio is 5, the cold mass flow is

$$\dot{m}_C = \frac{\dot{m}B}{B+1} = 179.2 \ kg/s$$

Therefore the thrust developed by the secondary nozzle is

$$F_{n,\sec} = \dot{m}_C V_{ex,f} = 52.532 kN$$

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Solution: Problem # 3

For the HP turbine,

$$T_{04} - T_{05'} = \frac{c_{pa}}{\eta_m c_{pg}}(T_{03} - T_{02})$$

$$\therefore T_{05'} = 1550 - \frac{1005}{0.99 \times 1147}(800.1 - 337.6) = 1141\ K$$

For the LP rotor,

$$T_{05'} - T_{05} = (B+1)\frac{c_{pa}}{\eta_m c_{pg}}(T_{02'} - T_{01})$$

$$\therefore T_{05} = 877.8\ K$$

Solution: Problem # 3

$$\frac{P_{04}}{P_{05'}} = \left(\frac{T_{04}}{T_{05'}}\right)^{\eta_{poly,turb}\gamma/(\gamma-1)} = 3.902$$

$$\frac{P_{05'}}{P_{05}} = \left(\frac{T_{05'}}{T_{05}}\right)^{\eta_{poly,turb}\gamma/(\gamma-1)} = 3.208$$

$$P_{04} = P_{03} - \Delta P_b = 25.0 \times 1.0 - 1.50 = 23.5\ bar$$

$$P_{05} = \frac{P_{04}}{(P_{04}/P_{05'})(P_{05'}/P_{05})} = 1.878\ bar$$

The hot nozzle pressure ratio is

$$P_{05}/P_a = 1.878$$

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Solution: Problem # 3

The critical pressure ratio is

$$\frac{P_{05}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/(\gamma-1)}} = \frac{1}{\left[1 - \frac{1}{0.95}\left(\frac{1.33-1}{1.33+1}\right)\right]^{1.33/(1.33-1)}}$$

$$= 1.914$$

Therefore the nozzle is not choking.

The primary nozzle exhaust velocity, V_{ex}

$$V_{ex} = \sqrt{2c_p \eta_n T_{05}\left[1 - \left(P_a / P_{05}\right)^{(\gamma-1)/\gamma}\right]} = 528.3\ m/s$$

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Solution: Problem # 3

Mass flow rate through the hot nozzle is

$$\dot{m}_h = \frac{\dot{m}}{B+1} = 35.83\,kg/s$$

$$F_{n,\,primary} = 35.83 \times 528.3 = 18.931\,kN$$

The total thrust is thus,

$$F_n = F_{n,\,primary} + F_{n,\sec} = 71.5\,kN$$

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Problem # 4

- An aircraft operating on a turboprop engine flies at 200 m/s while ingesting a primary mass flow of 20 kg/s. The propeller of the engine having an efficiency of 0.8, generates a thrust of 10000 N, while the jet thrust is 2000N. The power turbine and nozzle have efficiencies of 0.88 and 0.92 respectively. If we remove the power turbine and the nozzle, what would be the thrust developed by the engine while operating under the same conditions?

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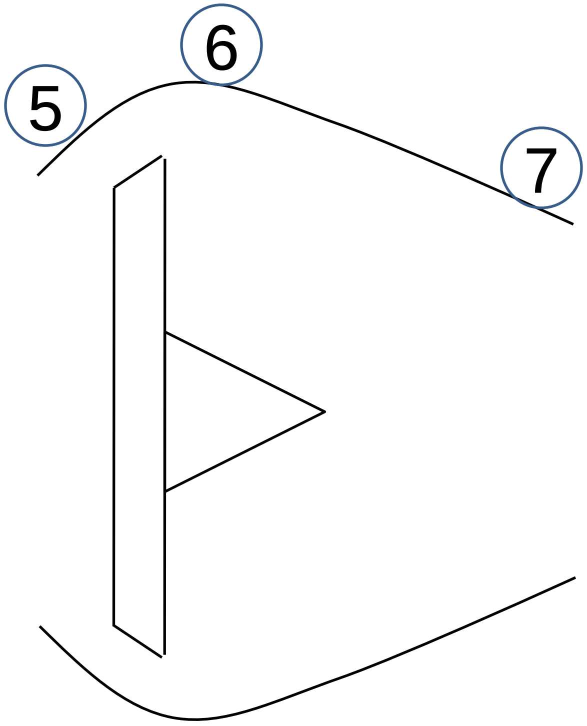
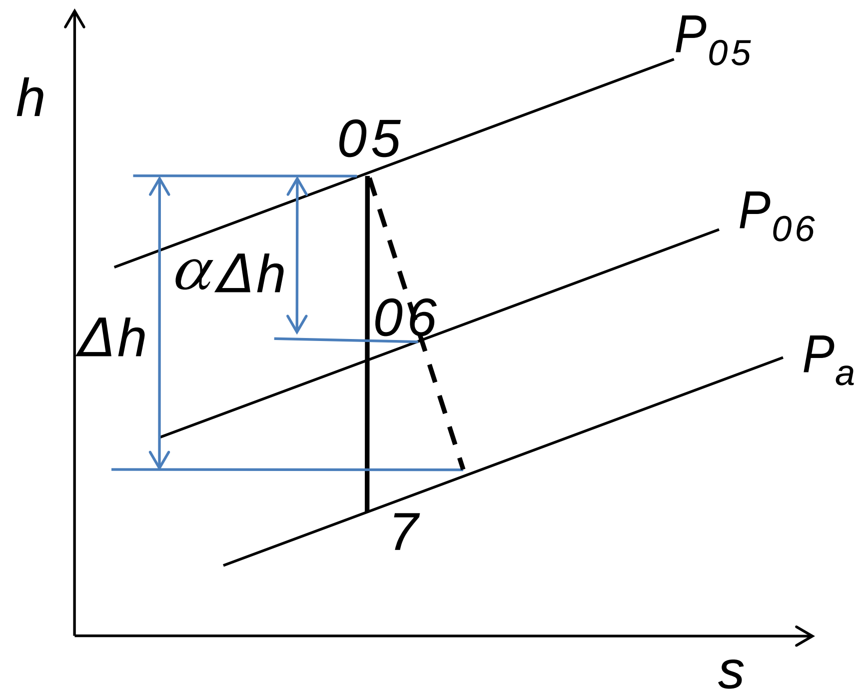
Solution: Problem # 4





Enthalpy-entropy diagram for power turbine-exhaust nozzle analysis

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Solution: Problem # 4

- We know that the thrust power developed by the propeller is given by,

$$F_{n,pr} \times V = \eta_{pr} \times \eta_{pr} \times \alpha \Delta h \times \dot{m}$$

$$\therefore \alpha \Delta h = \frac{10000 \times 200}{0.8 \times 0.88 \times 20} = 142045.45 \ J/kg$$

The nozzle thrust is

$$F_{n,nozzle} = \dot{m}(V_{ex} - V)$$

$$2000 = 20(V_{ex} - 200) \ \ or, V_{ex} = 300 \, m/s$$

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Solution: Problem # 4

$$V_{ex} = \sqrt{2\eta_n(1-\alpha)\Delta h}$$

$$300^2 = 2\times 0.92\times(1-\alpha)\Delta h$$

$$\therefore \Delta h = 190958.49\ J/kg$$

With the power turbine and the propeller removed, the entire Δh drop occurs through the nozzle.

$$\therefore V_{ex} = \sqrt{2\eta_n\Delta h} = \sqrt{2\times 0.92\times 190958.49}$$

$$= 592.76\ m/s$$

$$\therefore \text{Thrust} = 20(592.76 - 200) = 7855.18\ N$$

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Exercise Problem # 1

- A simple turbojet is operating with a compressor pressure ratio of 8.0, a turbine inlet temperature of 1200 K and a mass flow rate of 15 kg/s, when the aircraft is flying at 260 m/s at and altitude of 7000m. Assuming the following component efficiencies, calculate the nozzle area required, the net thrust and the SFC: polytropic efficiencies of turbine and compressor: 0.87, intake and nozzle efficiency: 0.95, Mechanical efficiency: 0.99, combustion efficiency: 0.97, combustor pressure loss: 6% of compressor delivery pressure.
- Ans: 0.0713 m^3, 7896 N, 0.126 kg/h N

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Exercise Problem # 2

- The gases at the turbine exit (given in problem #1) are reheated to 2000 K and the combustion pressure loss is 3% of the pressure at the outlet from the turbine. Calculate the percentage increase in the nozzle area required if the mass flow rate is to remain unchanged and also the percentage increase in the net thrust.
- Ans: 48.3 % and 64.5 %

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Exercise Problem # 3

- The following data apply to a twin-spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate hot and cold nozzles are used. Overall pressure ratio: 19.0, Fan pressure ratio: 1.65, By pass ratio: 3.0, Turbine inlet temperature: 1300 K, Combustor pressure loss: 1.25 bar, Total air mass flow: 115 kg/s. It is required to find out the thrust under sea level static conditions where the ambient pressure and temperature are 1.0 bar and 288 K. Assume fan, compressor and turbine efficiencies as 0.90 and that of each of the nozzle as 0.95.
- Ans: 47.6 kN

Exercise Problem # 4

- A turboprop is operating under the following conditions: Flight speed at standard sea level: 0 m/s; Airflow entering the compressor: 1 kg/s; Compressor pressure ratio: 12; Efficiencies: Diffuser: 100 %, Compressor: 87 %, Turbine to drive the compressor: 89 %, Turbine to drive the propeller: 89 %, Nozzle: 100 %, Turbine inlet temperature: 1400 K, Stagnation pressure leaving the second turbine: 4.6 bar. Take into account the mass of fuel added. Calculate:
- (a)the horse power delivered to the propeller
- (b)the thrust developed by the gases passing through the engine.
- Ans: 632 kW, 875 N

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