# Jet Aircraft Propulsion

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**Lecture** 12

# Thermodynamics of Compressors

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• The main purpose of thermodynamic analysis and calculation of  a compressor/ turbine of a gas turbine engine, is to obtain a reasonable prediction of the work to be supplied, and the efficiency with which this work may be expected to be performed.

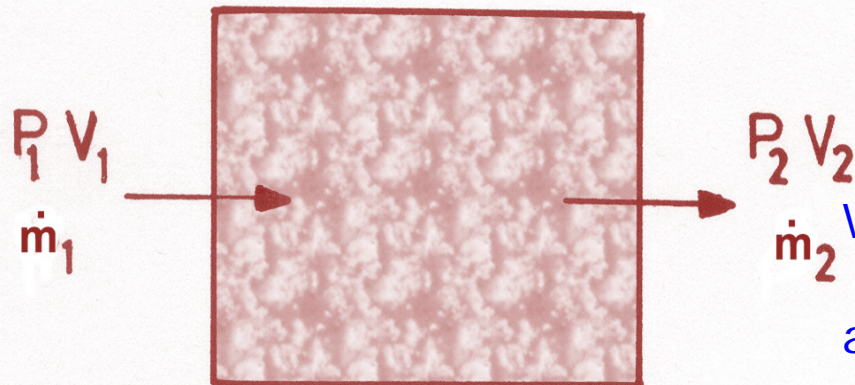• This allows an optimized cycle design, which precedes the detailed design of the engine.

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## Work in a closed system



Closed Systems
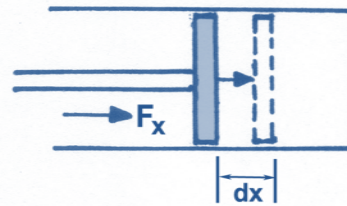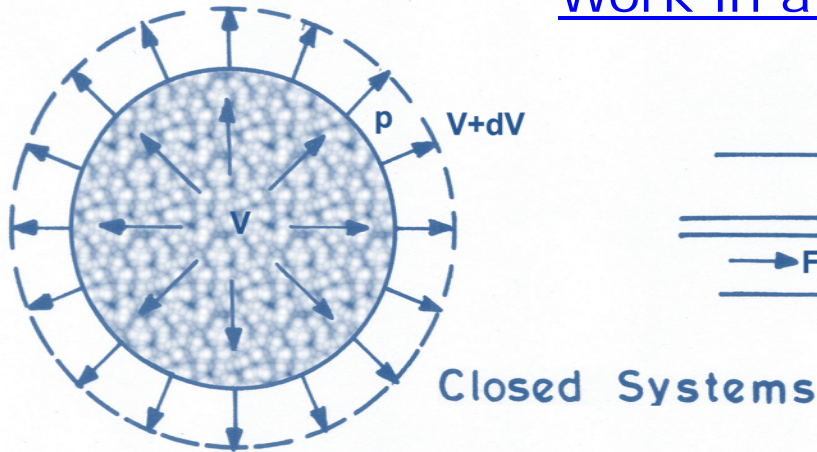
### Work done by piston

$$\partial W = F_x dx$$

### Work done by gas expansion

$$\partial W = p.dV$$

Open Systems



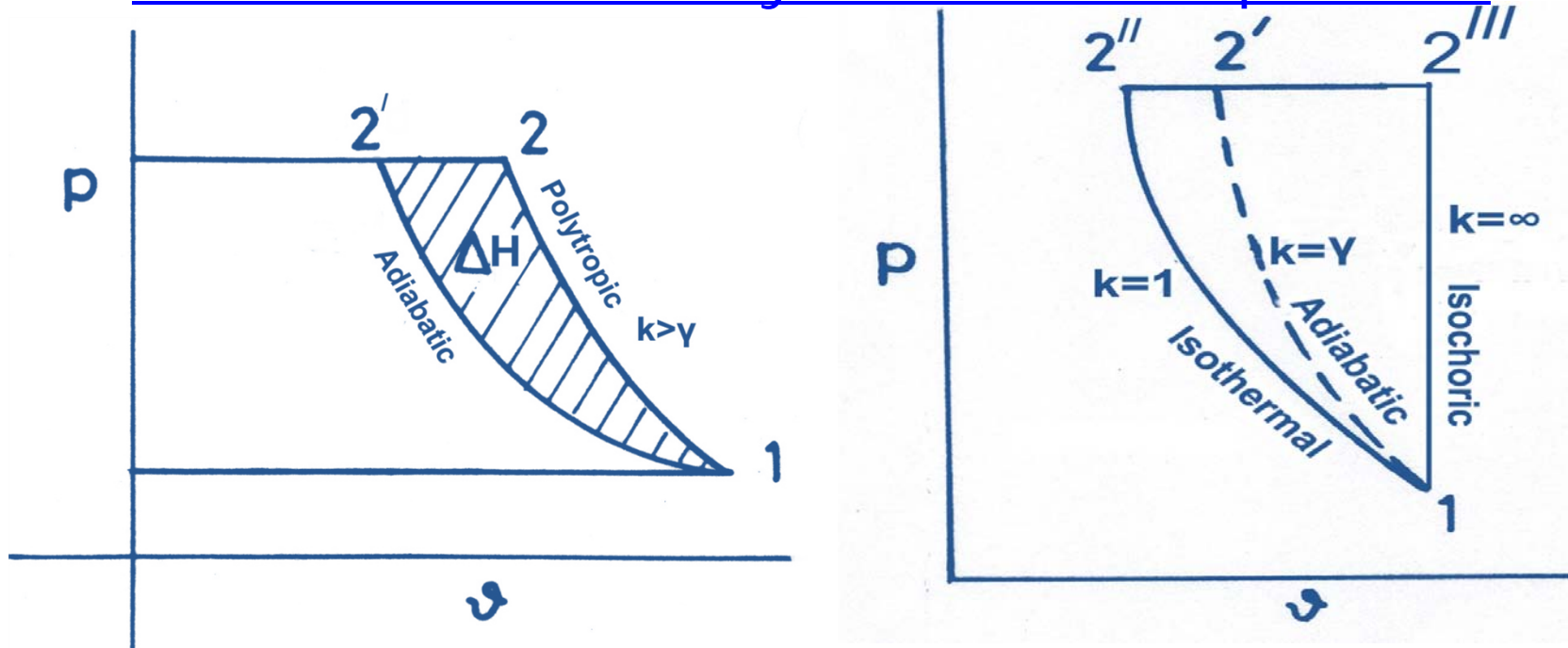$P_1 V_1$
$\dot{m}_1$
$P_2 V_2$
$\dot{m}_2$

### Work done by an open compression

$$\partial W = p_1 v_1 - p_2 v_2 + \int\limits_{V_1}^{V_2} p.dv$$

Where, $p_1 v_1$ = work done for air entry
$p_2 v_2$ = work done for air exhaust
and, the 3rd term is the work done for compression.

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## Ideal / Real thermodynamics of compression



i)   Adiabatic (process 1-2$'$)
ii)  Isothermal (process 1-2$''$),
iii) isochoric (Process 1-2$'''$),
iv) polytropic (1-2)

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If $p_1$, $v_1$, $T_1$ are the inlet (initial) conditions with mass flow $\dot{m}_1$ and $p_2$, $v_2$, $T_2$ are the outlet (final) conditions with mass flow $\dot{m}_2$,

Work done by the system is given by

$$\int_1^2 \partial W = -\int_1^2 v.dp = -\int_1^2 \frac{dp}{\rho}$$

where $v$ is specific volume and $\rho$ is the density of the gas

In a real compressor, the flow is quasi-static, i.e. $\dot{m}_1 \neq \dot{m}_2$ there are some loss of heat and some unused energy that is let out at the outlet.

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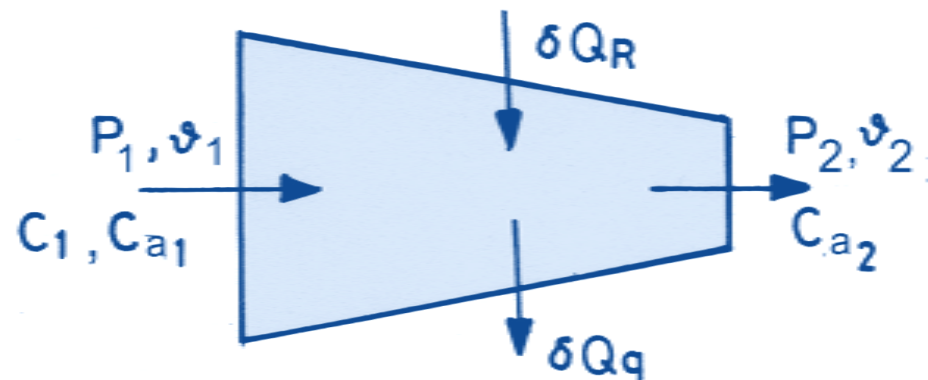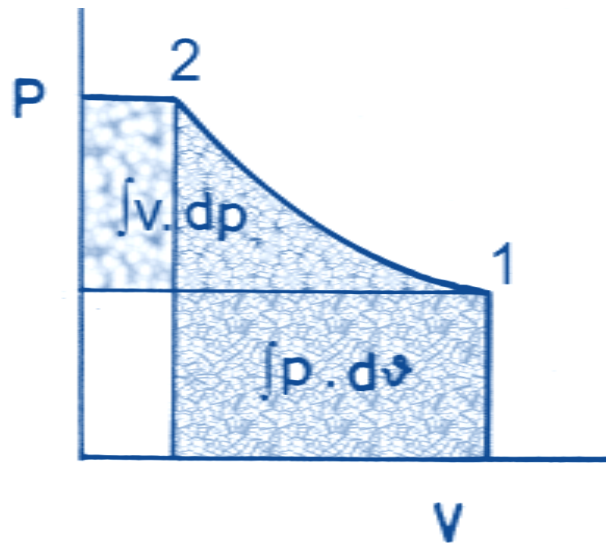
Heat added to the system is given by  $\partial Q = \partial Q_R - \partial Q_q$

Where $Q_R$ is the heat added to the fluid due by friction and $Q_q$ is the heat lost to the surrounding

Now,  $\partial Q = c_r.dT$   where $c_r$ is the specific heat of the fluid for any __real__ situation

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Work done in any <u>real process </u>may be split up
in to work done in <u>two ideal processes</u>

Energy added to the fluid $\partial Q = c_p.dT - v.dp$

Energy taken from fluid $\partial Q = c_v.dT + p.dv$

For an isentropic process, work done $\delta Q = c_n . dT$
And then the net energy transaction being zero,
$c_p.dt = v.dp$ ; and $c_v .dt = - p.dv$
Then <u>isentropic index </u>is normally defined as :

$$\gamma = c_p/c_v = v.dp/-p.dv$$

where $c_p$ specific heat at constant pressure, and $c_v$
specific heat at constant volume, for the air or gas

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## Similarly the polytropic index is defined for a real process

$$k = \frac{-v.dp}{p.dv} = \frac{c_n - c_p}{c_n - c_v} = \frac{\gamma\left(1 - \dfrac{dQ}{c_p.dT}\right)}{1 - \gamma.\dfrac{dQ}{c_p.dT}}$$

and $c_n$ is the specific heat for an isentropic process

Thus, If

1) $\partial Q = \pm\partial Q_R + \partial Q_q > 0$ , then $k > \gamma$ .

2) $\partial Q = 0$ , i.e. the process is isentropic

3) If the process is isothermal ($dT=0$) then $k = 1$ for ideal gas.

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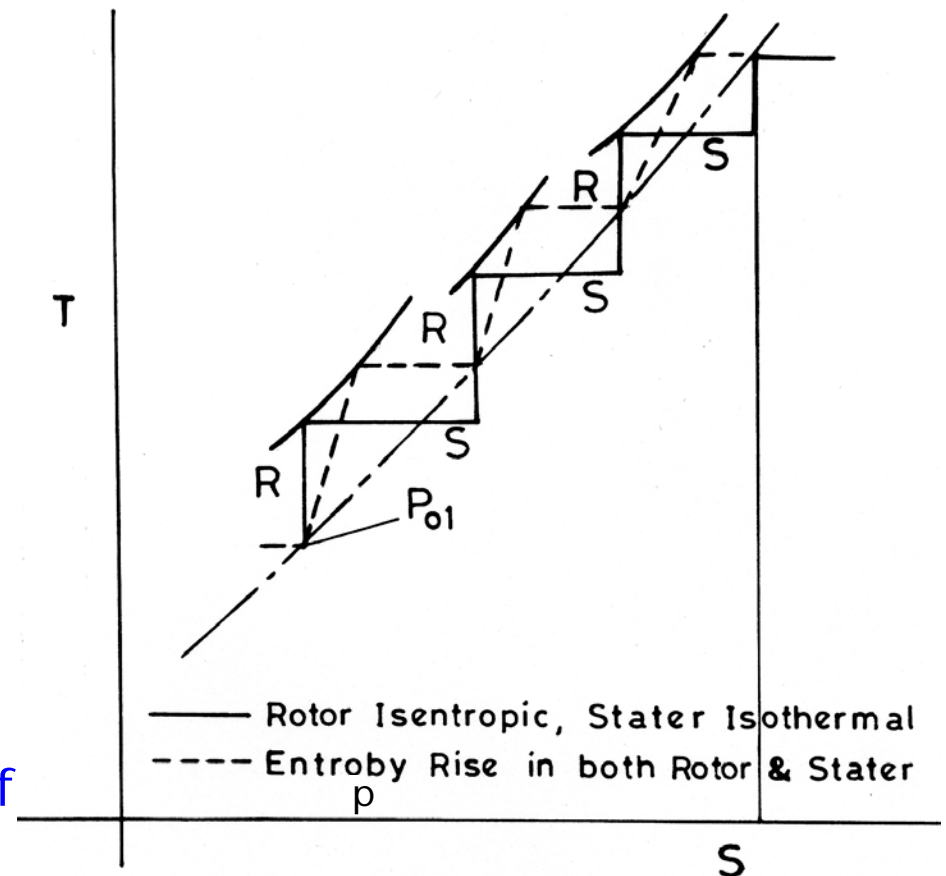# Enthalpy/ Temperature - Entropy diagrams
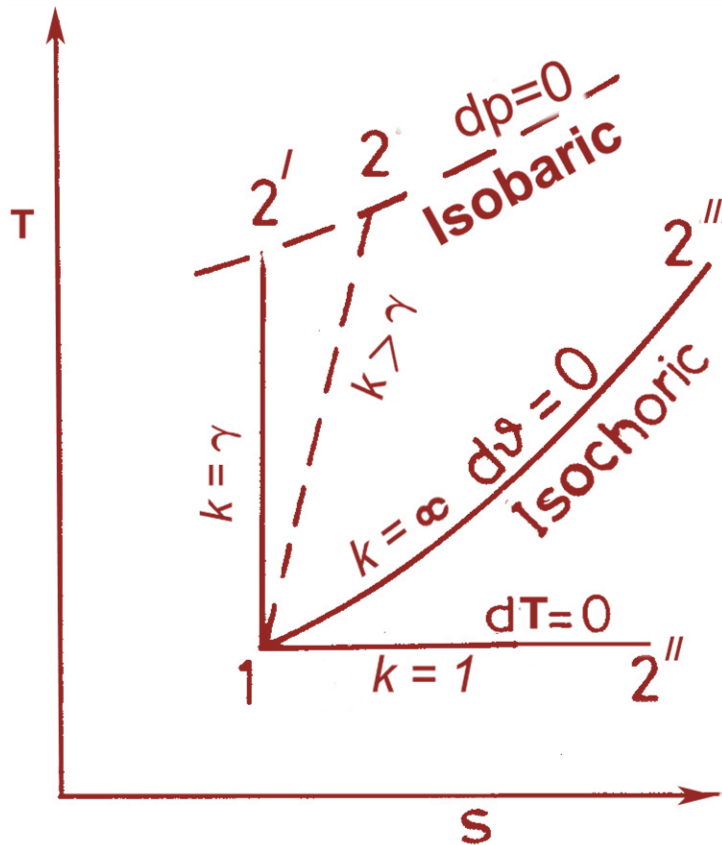


• The enthalpy(or temperature) - entropy diagram depicting the compression/expansion processes as the working components form the basis of matching them and with other components of the engine.

• Since the working is accomplished in open volume processes , in continuous flow of high speed air/gas (real gas), we often adopt adiabatic+reversible (i.e. isentropic) process as the ideal process and any departure from the ideal is shown as "**Isentropic efficiency**" of the processes.

• The thermodynamic depiction of the row by row compression through rotor and stator brings out that actual thermodynamic path taken by air.

• It also shows that flow at the rotor exit with high kinetic energy is still to be converted to static pressure through diffusion.

• Normally the exit kinetic energy of a compressor is of the same order as the entry kinetic energy and the entire work input is expected to be converted to pressure.



T

R

S

R

S

R

S

$P_{o1}$

——— Rotor Isentropic, Stater Isothermal
– – – – Entroby Rise in both Rotor & Stater

p

S

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High efficiency of the compressor and the turbine allows the flows through them to conform closely to the Joule-Brayton cycle.

Thermodynamic efficiencies are shown as :

$$\eta_c = \frac{h_2'' - h_0}{h_2 - h_0} = \frac{C_p(T_2'' - T_0)}{C_p(T_2 - T_0)} = \frac{T_2''/T_0 - 1}{T_2/T_0 - 1}$$

$$\eta_c = \frac{\left(p_2/p_0\right)^{\frac{\gamma-1}{\gamma}} - 1}{T_2/T_0 - 1}$$

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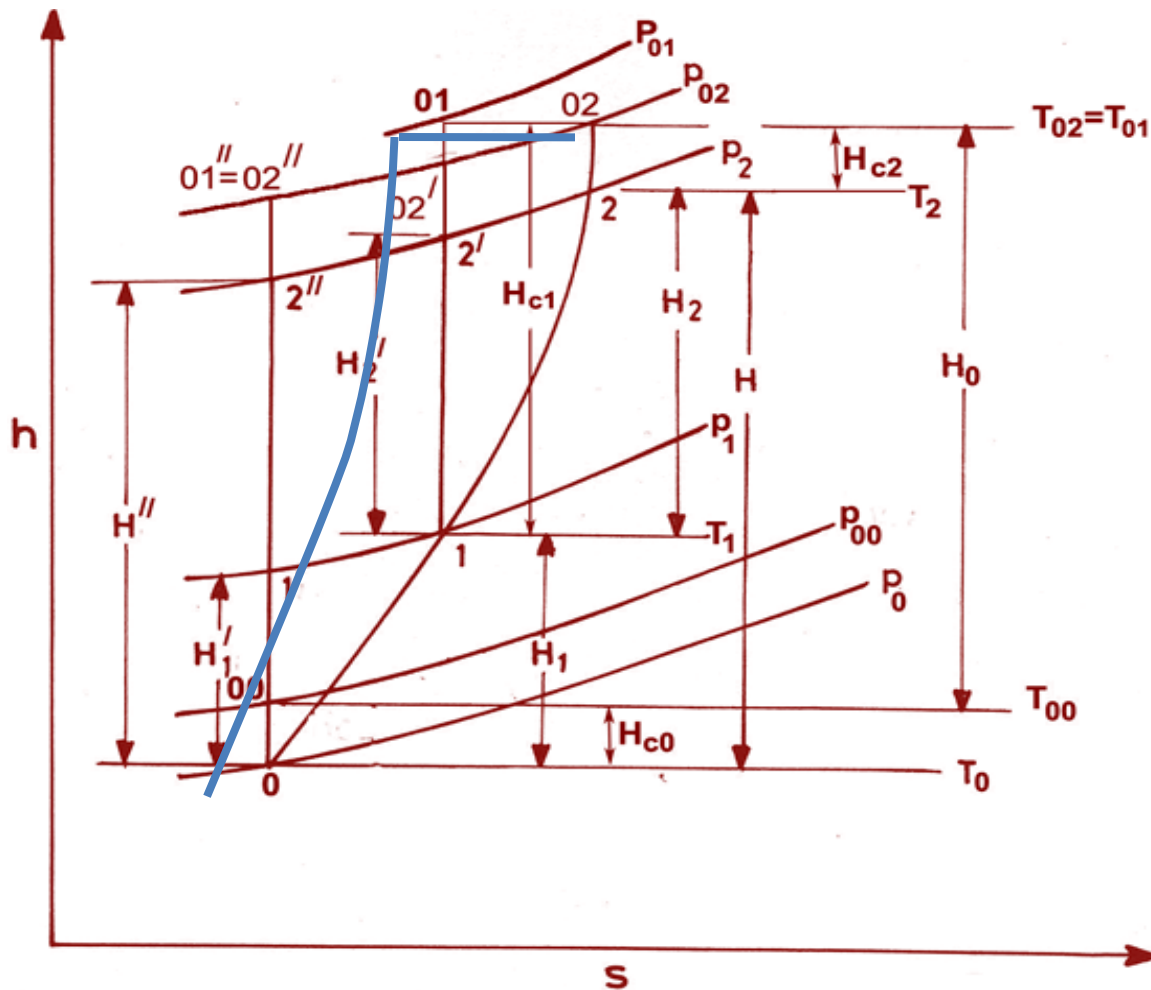
$H^{//}$= reversible adiabatic enthalpy exchange in a stage.

$H_1^{/}$= Isentropic enthalpy exchange in rotor (compressor) in stage.

$H_2^{/}$= Isentropic enthalpy exchange in stator (compressor)

$H^{/}$ = Ideal enthalpy exchange in a stage

$H_1$ , $H_2$ , H= Real static enthalpy exchange in rotor, stator and  stage

$H_0$ = Total work done in the stage.

$H_{c0}$ , $H_{c1}$ , $H_{c2}$ = Kinetic energies at stations

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In the case of reversible compression work done can be calculated with the help of two mean indices for a finite process as:

First Mean Index:

$$k_1 = \frac{\int\limits_1^2 -vdp}{\int\limits_1^2 pdv}$$

Second Mean Index:

$$k_2 = \frac{\ln\left(\dfrac{p_2}{p_1}\right)}{\ln\left(\dfrac{v_2}{v_1}\right)}, \quad \text{i.e} \quad p_1.v_1^{k_2} = p_2.v_2^{k_2}$$

It is customary to assume some average value of the index as constant ($k_2 = k_1 = k$) in design computation.

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Using the _Total Head based Specific Heat_ _for_ real process, $C_{0r}$

$$c_{0r} = c_r \frac{\ln \dfrac{T_2}{T_1}}{\ln \dfrac{T_{02}}{T_{01}}}$$

$$\boxed{\Delta s = c_r . \ln T_3/T_1 = c_{0r} \ln T_{03}/T_{01}}$$

Introducing $k_0$ as the _specific heat ratio_ _based on total conditions_ of the real (polytropic) process

$$\frac{k_0/\gamma - 1}{k_0} = \frac{k/\gamma - 1}{k} \frac{\ln \dfrac{T_2}{T_1}}{\ln \dfrac{T_{02}}{T_{01}}}$$

If, $M_1 = M_3$, $k_0 = k$ and If, $M_1 > M_3$, $k_0 > k$ .
For example if $k = 1.5$, the value of $k_0$ will vary above 1.5 for $M_1 > M_3$

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## Effect of $k$ on compression efficiency

| $k$ | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 |
|---|---|---|---|---|---|
| $\eta_{c_{poly}}$ | 1.0 | 0.858 | 0.764 | 0.695 | 0.644 |

## Effect of $k_0$ on compression efficiency

| $\pi_C^*$ | 1 | 1.2 | 1.5 | 2 | 4 | 6 | 10 |
|---|---|---|---|---|---|---|---|
| $\eta_{C-poly}^* \big/ \eta_{C_{ad}}^*$ for $k^*=1.5$ | 1 | 0.992 | 0.99 | 0.985 | 0.984 | 0.95 | 0.94 |
| $\eta_{C_{poly}}^* \big/ \eta_{C_{ad}}^*$ for $k^*=1.8$ | 1 | 0.991 | 0.971 | 0.945 | 0.888 | 0.85 | 0.81 |

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# Compressor with heat Transfer

- The amounts of work done by the compression process with heat transfer are given by the areas under the curves of the respective constant pressure lines
- For example if we consider *a polytropic process with no net heat transfer (process 1-2)* the enthalpy at the end and at the beginning of the process are given as :

$$\text{Final enthalpy, } H_{Tc} = C_p \left( T_c - 0 \right) = \Delta 274$$

$$\text{Initial enthalpy, } H_{Ta} = C_p \left( T_a - 0 \right) = \Delta 103$$

both the areas are considered to be triangles

- Neglecting change in kinetic energy, i.e. $C_2 = C_1$ and assuming that, for small change in thermodynamic status, the constant pressure lines are linear and parallel to each other, the areas Δ103 and Δ576 are considered equal to each other.
- Then, total enthalpy change, $H_0 =$ area 25642

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• Thus it can be shown that for a cooled polytropic process (1-2$^{//}$,  or 1-2$^{///}$), work input necessary, for same amount of compression, will be less than the polytropic and adiabatic process respectively.

• If  $\partial Q_R \pm \partial Q_q > 0$   ,  then   $k < \gamma$          .

• This thermodynamic possibility has given rise to *cooled or inter-cooled compressor*, where cooling is resorted to at the beginning of compression (especially on a hot day, by water or water-alcohol mixture injection) or by cooling in between the two stages or spools of compressors.

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Next Class :

Thermodynamics
of
Turbines