Jet Aircraft Propulsion

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Lecture 13

Thermodynamics of Turbines

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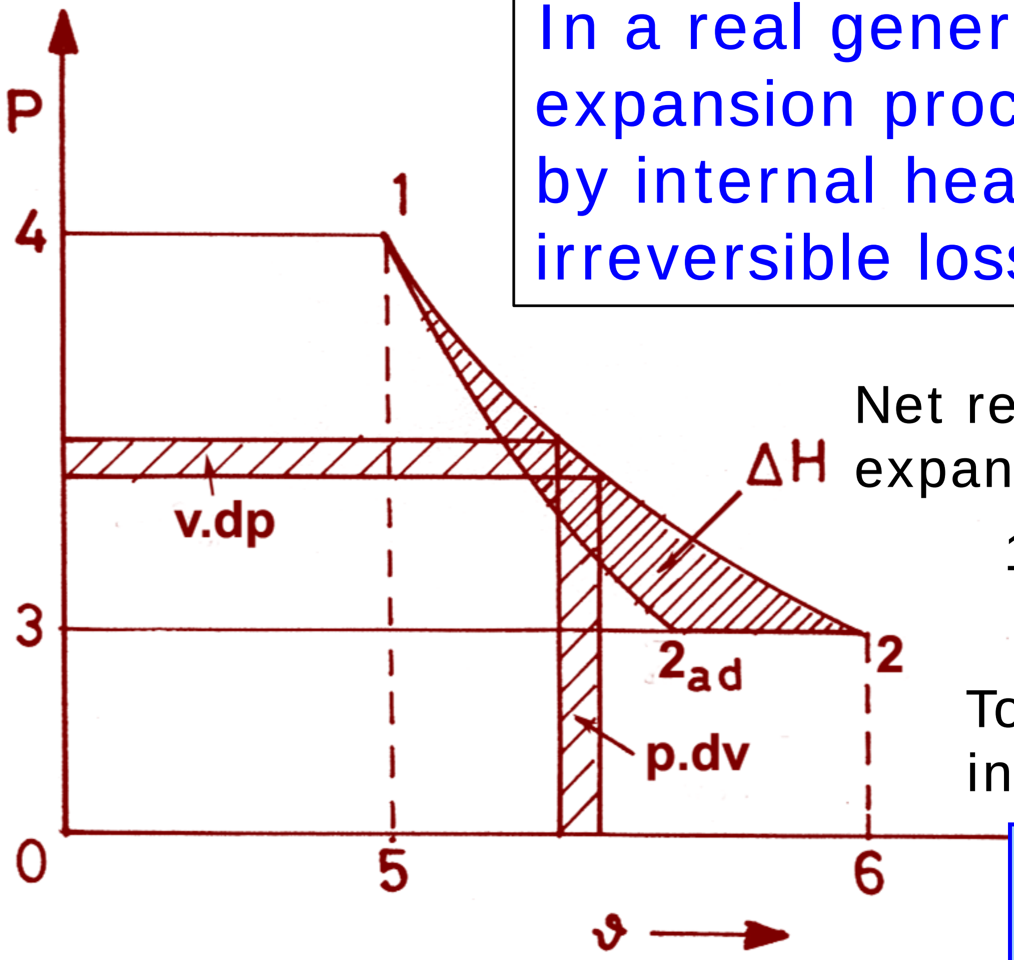
Thermodynamic Analysis of Turbine

In a real generalized situation the expansion process is accompanied by internal heat conversion of irreversible losses, L_R to Q_R



Net reversible polytropic expansion work = 1-4-3-2-1 = $\int_2^1 v.dp$

Total real expansion work in a fluid flow

$$= \int_2^1 v.dp + L_R + \frac{C_1^2 - C_2^2}{2}$$

Irreversible polytropic expansion work in the most general situation

$$H_{T_{poly}} = \frac{k_1}{k_1 - 1} p_1 . v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{k_2 - 1}{k_2}} \right]$$

Where, averaged polytropic indices are :

$$k_1 = \frac{\int_2^1 v.dp}{\int_2^1 p.dv} \qquad k_2 = \frac{\ln\left(p_1 / p_2 \right)}{\ln\left(v_1 / v_2 \right)}$$

For all practical purposes, $k_1 = k_2 = k$.

In aircraft turbines generally $\gamma = 1.33$ or 1.29, which tend to go up in the rear stages, as temperature drops

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The relation between *k* and γ for polytropic expansion is given by

$$k = \gamma . \frac{1 + \dfrac{(\gamma - 1)}{\gamma} . \dfrac{\partial Q_R \pm \partial Q_q}{R.dT}}{1 + (\gamma - 1) . \dfrac{\partial Q_R \pm \partial Q_q}{R.dT}}$$

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The difference between the reversible polytropic expansion work and isentropic expansion work is termed as 'reheat factor'.

Additional turbine work that need to be done,

$$\Delta H_T = H_{T_{poly}} - H_{T_{isen}} = \frac{k}{k-1} R\left(T_1 - T_2\right) - \frac{\gamma}{\gamma - 1} R(T_1 - T_{2ad})$$

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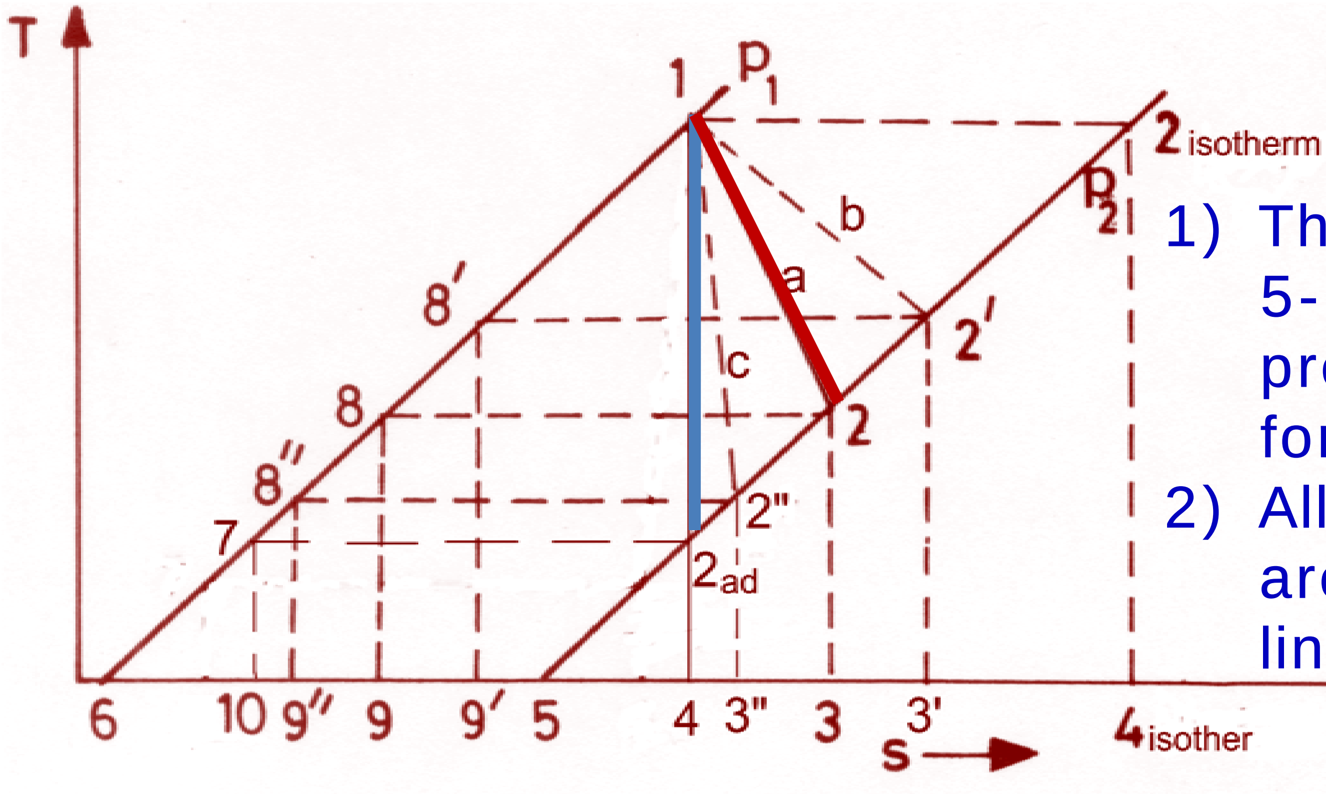

1) The lines 6-1 & 5-2 are the pressure lines for p_1 , p_2
2) All processes are assumed linear

1-2 – Polytropic process ; 1-2_{ad}- Isentropic process
1-2_{isoth}– Isothermal Process; 1-2'– Re-Heated Turbine
1-2" – Cooled Turbine

(a) Process 1-2, .

considering all the triangles and rectangles under the lines 5-2 (p_1) and 6-1 (p_2) to be comparable and equitable, $\partial Q_q = 0,\ \partial Q_R > 0,\ L_R = Q_R = 1-2-3-4-1$

Expansion work,

H_T = area 1-4-6-1 – area 2-3-5-2 = area 1-4-9-8-1

Reversible polytropic expansion work, $H_{T_{poly}} = H_T + L_R = 1-2-3-9-8-1$

Again, $H_{T\text{-}poly}$ = 1-2-2_{ad}-4-10-7-1

= $H_{T\text{-}isen} + \Delta H$ = 1-4-10-7-1 + 1-2-2_{ad}-1

From above eqn.s, $H_T = H_{T_{isen}} + \Delta H - L_R = H_{T_{isen}} - L'_R$

= (1-4-10-7-1) – (2-3-4-2_{ad}-2)

which means a part of the losses is usefully employed back in expansion. This is termed the "Reheat factor"

(b) Process 1-2' -considering all the above assumptions,

Expansion losses = $L_R = Q_R = 1-2^{/}-3^{/}-4-1$

Heat added externally = $Q_q = 1-2-3-3'-2'-1$

Reversible polytropic

work = $1-2'-2_{ad}-4-10-7-1 = H_{T_{isen}} + \Delta H'$

where $\Delta H' = 1-2'-2-1$ is the reheat factor.

So compared to case **a** there is a gain in expansion work by $\Delta H' - \Delta H$. This happens if the fuel continues to burn inside the 1st stage of turbine stator.

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(c) Process 1-2”

Considering a realistic situation- $1-2'', \; \partial Q_q < 0, \; \partial Q_R > 0$

$$Q_R - Q_q = 1-2''-3''-4-1$$

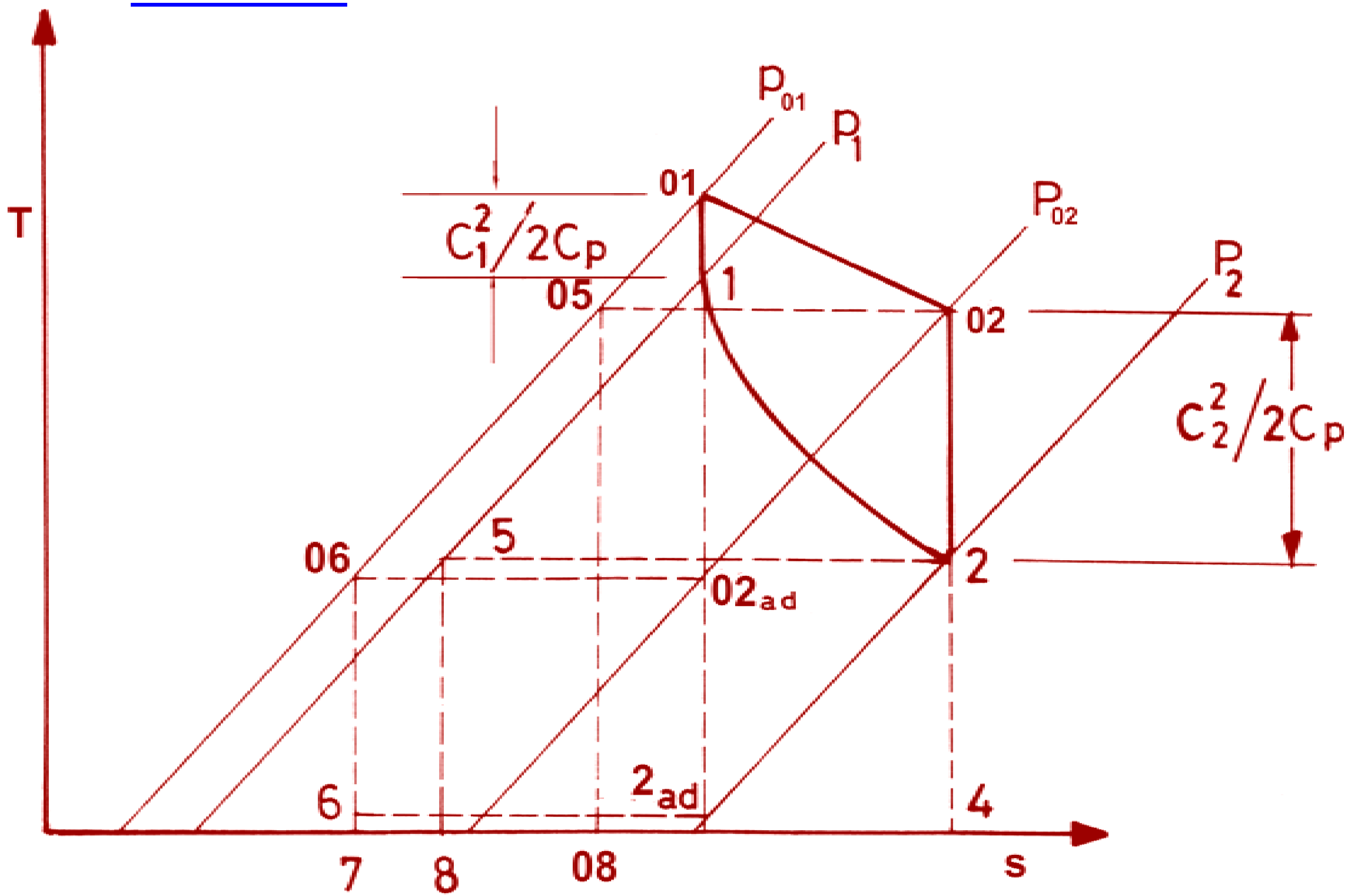
$$Q_q = 1-2-3-3''-2''-1$$

$$\Delta H'' = 1-2''-2_{ad}-1$$

-ve heat compared to 1-2 case (a)

So, there is a loss of expansion work compared to case ‘**a**’ by the amount $\Delta H - \Delta H''$

Thermodynamics of flowing fluid through turbine



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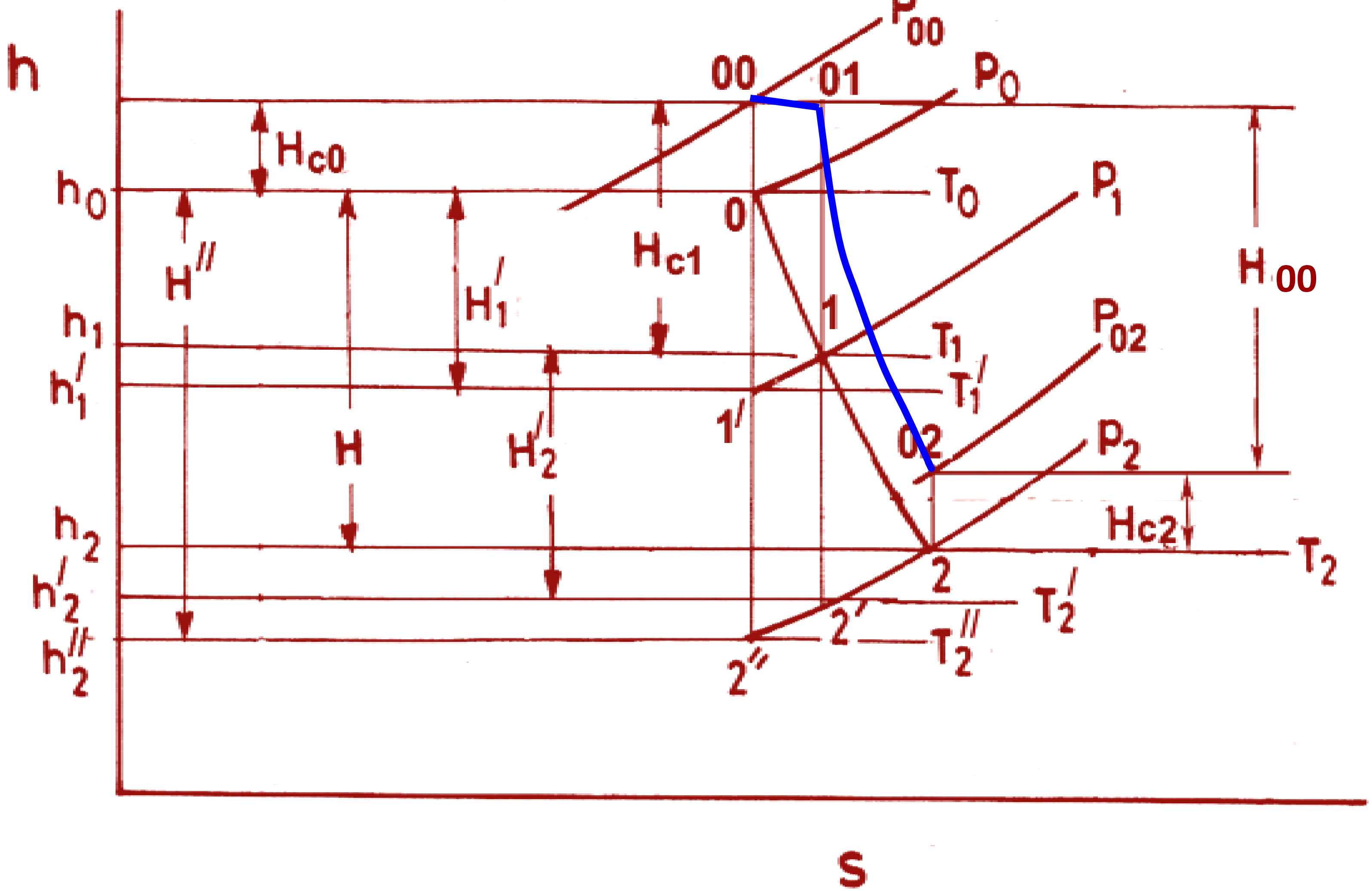
Thermodynamics of flowing fluid through turbine stage



L_R	Energy Loss parameter (thermodynamic), kJ/kg
h	Specific enthalpy, kJ/kg-K
H	Change or Exchange in specific Enthalpy, kJ/kg-K

<u>Subscripts</u>

1	Static parameter at entry to turbine
2	Static parameter at exit to turbine
01	Total parameter at entry to turbine
02	Total parameter at exit to turbine
00	Energy or Enthalpy change in a total head process
i, isen	Isentropic process based parameter
isother	Isothermal process based parameter
poly	Polytropic process based parameter
ad	Adiabatic process related parameter
C	Kinetic Energy of the total Enthalpy at a station

Component Efficiency Definitions

$$\eta_{T.blade} = \frac{H_{T.blade}}{H_{available}}$$

$$\eta_{T.internal} = \frac{\text{Work delivered to the rotor}}{\text{Work available ideally through same pressure ratio}} = \frac{H_t}{H_{available}}$$

$$\eta_{T.effective} = \frac{H_{te}}{H_{available}}$$

$$\eta_{T-poly} = \frac{H_T}{\int_{01}^{02} \frac{dp}{\rho}} \qquad \eta_{T-ad} = \frac{H_T}{\left(\int_{01}^{02} \frac{dp}{\rho}\right)_{available}}$$

Where

$\int_{01}^{02} \frac{dp}{\rho}$ Is the ideal total work

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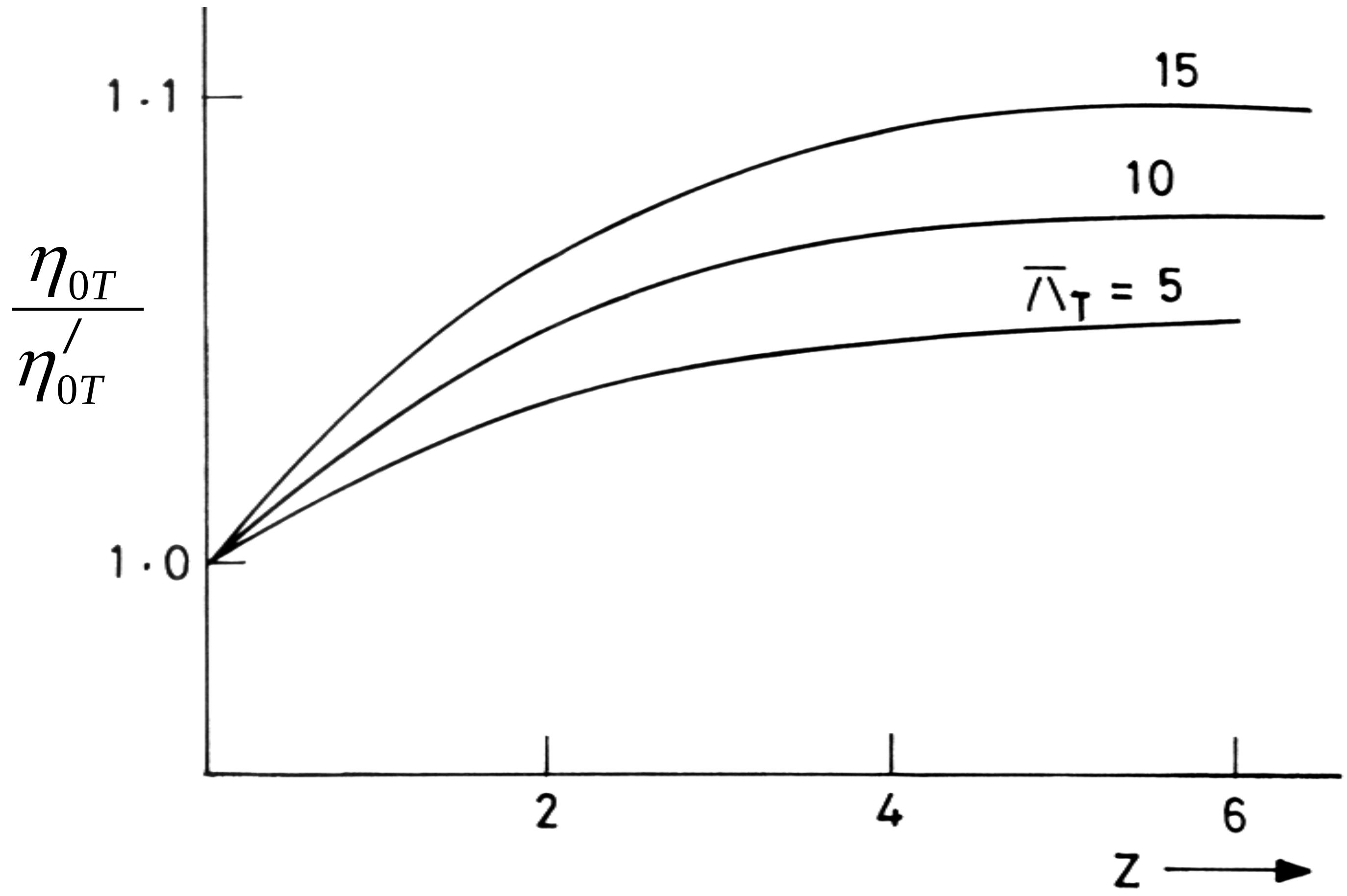
Assuming equal work in the stages and equal efficiency of each stage (η_{0T}), we can see the change in behavior of multistage turbine with increasing number of stages.



Polytropic Efficiency of Turbine

$$\eta^{*}_{T.poly} = \frac{H_T}{\int_{01}^{02} (v.dp)^{*} .k^{*}} = \frac{\frac{\gamma}{\gamma - 1} R\left(T_{01} - T_{02}\right)}{\frac{k}{k-1} R\left(T_{01} - T_{02}\right)} = \frac{\frac{\gamma}{\gamma - 1}}{\frac{k}{k-1}} = 1 - \frac{L_R^{*}}{\int_{01}^{02} (vdp)^{*} .k^{*}}$$

For modern turbines typically, $\gamma = 1.33,\ k^{*} = 1.29$

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For $\gamma_g = 1.33$, and $k_t^* = 1.29$

π_t^*	2.0	4.0	6.0	8.0	10.0
$\eta_{T_{ad}}^*$	0.91	0.915	0.918	0.922	0.925

Adiabatic Efficiency may be defined as,

$$\eta_{T_{ad}}^* = \frac{1 - \left(\frac{1}{\pi_t^*}\right)^{\frac{k_0^* - 1}{k_0^*}}}{1 - \left(\frac{1}{\pi_t^*}\right)^{\frac{\gamma_g - 1}{\gamma_g}}}$$

Where ***k_0*** is a total head based index, and **γ_g** is the gas sp. heat index

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Next Chapter

Compressor and Turbine Aerodynamics

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