Jet Aircraft Propulsion

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Lecture 23

Solved Examples and Tutorial problems

on

Axial Flow Turbines

and

Radial Flow Turbines

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1) A cooled axial flow turbine operates with following properties. T_{01} =1780 K, P_{01} = 1.4 Mpa, mass flow= 40 kg/s. The mean radius data are: M_1= 0.3, M_2= 1.15, U_2=400 m/s; T_{03} = 1550 K, $\alpha_1=\alpha_3=0^o$, r_m =0.4 m, C_{a2}/C_{a3} = 1 ; the loss coefficients are : $\bar{w}_{noz}$ = 0.04, $\bar{w}_{rotor}$ = 0.08; use : γ_{gas} = 1.3, R = 287 J/kg.K.

Compute :

1) the flow properties along the stage mean line

2) the Degree of reaction (DR) - R_x

3) total temperature ΔT_0 based stage loading Ψ

4) the isentropic efficiency

4) the flow areas at various axial stations, and at the hub and tip radii at: a) nozzle inlet, b) nozzle exit, c) rotor exit

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$T_{01} = 1780$ K, $P_{01} = 1.4$ MPa

$\alpha_1 = 0^o$, $M_1 = 0.3$

$\boxed{\overline{w}_{noz}} = 0.04$

$U_2 = 400$ m/s

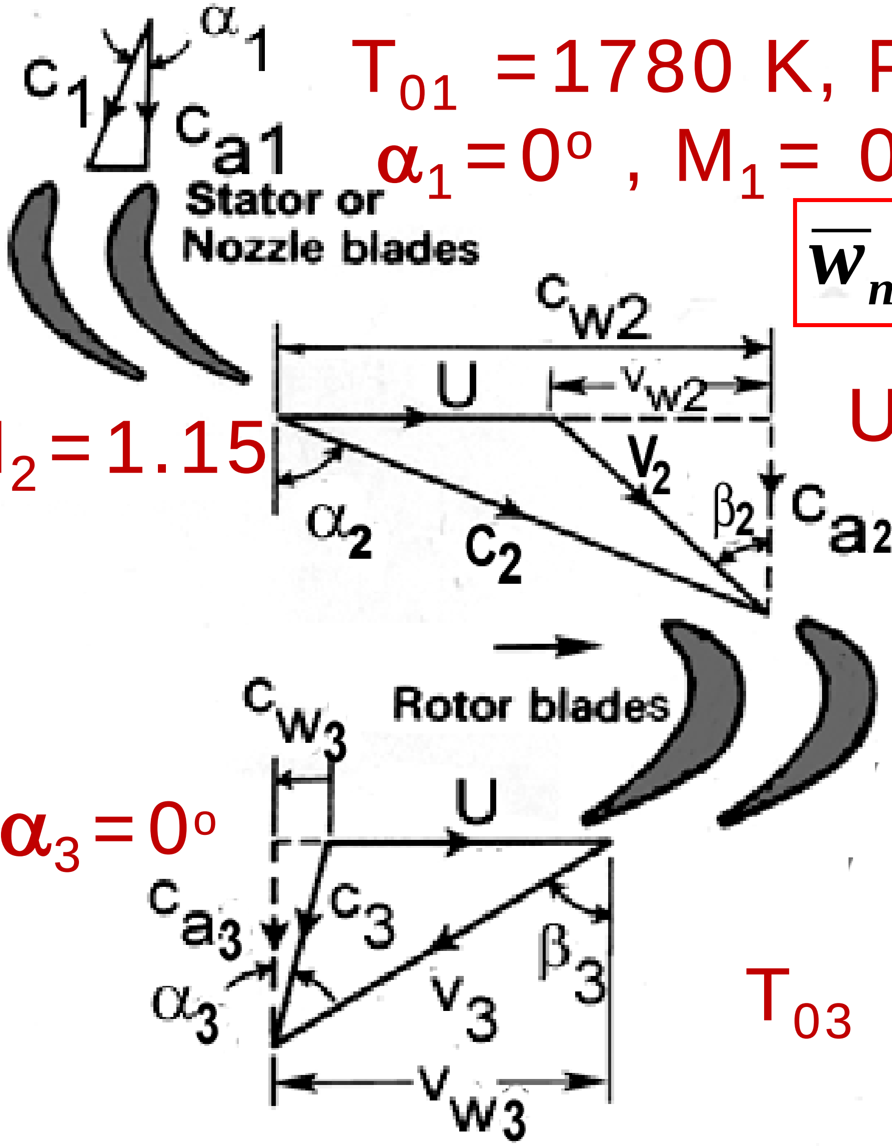
$M_2 = 1.15$

$C_{a2}/C_{a3} = 1$

$\boxed{\overline{w}_{rotor}} = 0.08$

$\alpha_3 = 0^o$

$T_{03} = 1550$ K



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$T_1 = T_{01} / [1+ (\gamma-1).M_1^2/2] = 1756.3\ K$; and

$P_1 = P_{01}.(T_1/T_{01})^{\gamma/(\gamma-1)} = 1321\ kPa$

Therefore,

$C_1 = M_1\sqrt{(\gamma R T_1)} = 0.3\sqrt{(1.3 \times 287 \times 1756.3)}$

$= 242.8\ m/s = C_{a1}$, and $C_{w1} = 0$

Area at stn.1, $A_1 = \dot{m}_{gas}/\rho_1.C_{a1} = 0.0628\ m^2$

Applying the same method as before,

$T_2 = T_{02} / [1+ (\gamma-1).M_2^2/2] = 1485.4\ K$, and

$C_2 = 856.1\ m/s$

If, as prescribed, 4% of the kinetic head is lost in the nozzle blades,

so, ideal $T_2' = 1503.55\ K$, and $C_2' = 877\ m/s$

and hence, from isentropic laws, one can find

P_{02} = 1370.2 kPa and P_2 = 625.5 kPa

Given that $\alpha_3 = 0$, $\alpha_2 = \sin^{-1}[\Psi.U/C_2]$,

as, from the definitions

$$\Psi = \Delta H_0/U^2 = (C_a/U).\tan\alpha_2$$

Using, $c_p = \gamma R/(\gamma - 1)$

we get $\Psi = \Delta H_0/U^2 = c_p.\Delta T_0/U^2 = 1.7878$;

Therefore, $\alpha_2 = 56.6^0$

$C_{a2} = C_2.\cos\alpha_2 = 470$ m/s; $C_{w2} = C_2.\sin\alpha_2 = 715$ m/s, and $V_{w2} = 715 - 400 = 315$ m/s

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Therefore, $\beta_2 = \tan^{-1}\{V_{w2}./C_{a2}\} = 33.80^0$; Now also it can be shown that $M_{2\text{-}rel} = 0.76$

Area at station 2,

$$A_2 = \dot{m}_{gas}/\ \rho_2.C_{a2} = (\dot{m}_{gas}.R.T_2)/(P_2.\ C_{a2}) = 0.58\ m^2$$

Axial velocity at stage exit,

$$C_{a3} = C_3.\cos\alpha_3 = \{(C_{a2}/C_{a3}).(\cos\alpha_2/\cos\alpha_3).C_2\}\cos\alpha_3 = 470\ m/s$$

Tangential velocity, $C_{w3} = 0$; and therefore $V_{w3} = 400$ m/s and $V_3 = \sqrt{(V_{w3}^2 + C_{a3}^2)} = 617$ m/s

Thus the exit flow angle is

$$\beta_3 = \tan^{-1}[V_{w3}/C_{a3}] = 40.3^0$$

Static temperature at station 3,

$T_3 = T_{03} - C_3^2/2c_p = 1461$ K; where $a_3 = 738.3$ m/s

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Degree of Reaction $R_x^{/} = 1 - \frac{C_{w1} + C_{w2}}{2.U} = 1 - \frac{\Psi}{2}$ = 0.106

Mach number at exit, $M_3 = 0.6375$, and relative exit mach number, $M_{3\text{-relative}} = V_3 / a_3 = 0.836$

Relative total temperature and pressure at exit, $T_{03\text{-rel}} = T_3 + V_3^2/2c_p = 1614$ K; and $P_{02\text{-rel}} = 897.3$ kPa from which we can obtain (by applying rotor loss coefficient), $P_{03\text{-rel}} = 872.7$ kPa

Using isentropic relation, $P_3 = 566$ kPa; & $P_{03} = 731$ kPa

The exit area at station-3, $A_3 = \dot{m}/\rho_3.C_{a3} = 0.063\ m^2$

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The final performance parameters are :

Temperature ratio, $\tau_T = T_{01}/T_{03} = 1780/1550 = 1.148$

Pressure ratio, $\pi_{0T} = P_{01}/P_{03} = 1400 / 731 = 1.915$

Efficiency, $\eta_{0T} = (1- \tau_T)/ [1- \pi_{0T}^{\gamma/(\gamma-1)}] = 92.9\%$

At each station, blade height $h_i = A_i/(2.\pi.r_m)$

Station	1	2	3
Area (m^2)	0.06285	0.05792	0.0629
Height (m)	0.025	0.023	0.025
Tip radius (m)	0.4125	0.4115	0.4125
Hub radius (m)	0.3875	0.3885	0.3875

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Tutorial problems on Axial Flow Turbines

1) An impulse turbine (R_x=0) operates with following pressures at various stations :
P_{01}=414 kPa, P_2=207 kPa, P_{02}=400 kPa,
P_3=200 kPa when operating with U_{mean} = 291 m/s at T_{01} = 1100 K and $\alpha_2 = 70^0$.
Assuming that $C_1 = C_3$ compute the total-to-total efficiency of the stage.

[Use c_p = 1148 kJ/kg.K, and γ = 1.333]

[91%]

2) Axial velocity through an axial flow turbine is held constant by design. The entry and the exit velocities are also axial by design. If the flow coefficient, $\Phi = 0.6$ and the gas leaves the nozzle with $\alpha_2 = 68.2^0$, all at mean diameter, compute :

i) Stage loading coefficient, Ψ
ii) Relative flow angles on rotor at mean diam, β_2, β_3
iii) The degree of reaction, R_x
iv) Total-to-total and total-to-static efficiencies, η_{0T}, η_{TS}

[i) 1.503, ii) 40^0, 59^0; iii) 0.25, iv) 90.5, 81.6%]

3) Following design data apply to an uncooled axial flow turbine: P_{01} = 400 kPa, T_{01} = 859 K, and at the mean radius, $\alpha_2 = 63.8^0$, $R_x = 0.5$, $\Phi = 0.6$, P_1 = 200 kPa , and η_{TS} = 85%. If the axial velocity is held constant through the stage

compute :

i) specific work done by the gas

ii) the blade speed

iii) stage exit static temperature

[131 kJ/kg; 301 m/s; 707.5 K]

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4) An axial flow turbine with only cooled rotor operates with following conditions at mean diameter : Mass flow , = 20 kg/s, T_{01} = 1000 K; P_{01} = 4 bar; C_a = 260 m/s (constant through the stage); $C_1 = C_3$; U_{mean} = 360 m/s; $\alpha_2 = 65^0$, $\alpha_3 = 10^0$, Nozzle loss coefficient, =0.05 .

Compute the following :

i) Relative flow angles on rotor at mean dia., β_2, β_3

ii) The degree of reaction, R_x

iii) Stage loading coefficient, Ψ

iv) Power output

v) Nozzle exit throat area, neglecting real flow effects

[37.2^0, 57.4^0, 0.29, 3.35; 4340 kW, 0.04 m^2]

5) An axial flow turbine with cooled nozzle and rotor blades operates with the following flow parameters at a reference diameter:

T_{01} = 1800 K, P_{01} = 1000 kPa, $C_{a3}/C_{a2} = 1$; $M_2 = 1.1$,
U_{mean} = 360 m/s ; $\alpha_2 = 45^0$; $\alpha_3 = 5^0$

Compute the following :

i) C_2 , C_{a2} , C_{w2} ; C_3 ; C_{a3} ; C_{w3}

ii) ΔT_0 and T_{01}/T_{03} for the stage

iii) π_{0T} and P_{03} for a polytropic efficiency of 89%

[830 m/s; 585 m/s, 585 m/s; 587 m/s, 585 m/s, 585 m/s; 184 K, 1.148; 1.792, 591 kpa]

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Radial Flow Turbine

A radial flow turbine with operates at following operating conditions :

$\dot{m}$=2 kg/s, P_{01} 400 kPa, T_{01}=1100 K, $P_{02}=0.99.P_{01}$; Nozzle exit angle, $\alpha_2=70^0$, Poly. efficiency, $\eta_{poly}=0.85$, Rotor maximum diameter=0.4 m , $V_{2r}=C_{a3}$, hub/tip radius ratio at rotor exit = 0.4, T_{03}=935 K;
[use γ =1.33; R =287 kJ/kg.K ; c_p =1.158 kJ/kg-K.]

Compute the following :

i) Rotor tip speed, rotational speed and rpm

ii) Mach number, velocities, rotor width at tip, & $T_{02\text{-}rel}$

iii) Stagnation pressure, Mach number and hub and tip radii at rotor exit

iv) At rotor exit V_3, $T_{03\text{-}rel}$, β_3, $M_{3\text{-}rel}$ at mean radius

v) Values of β_3, $M_{3\text{-}rel}$ at different radii at rotor exit

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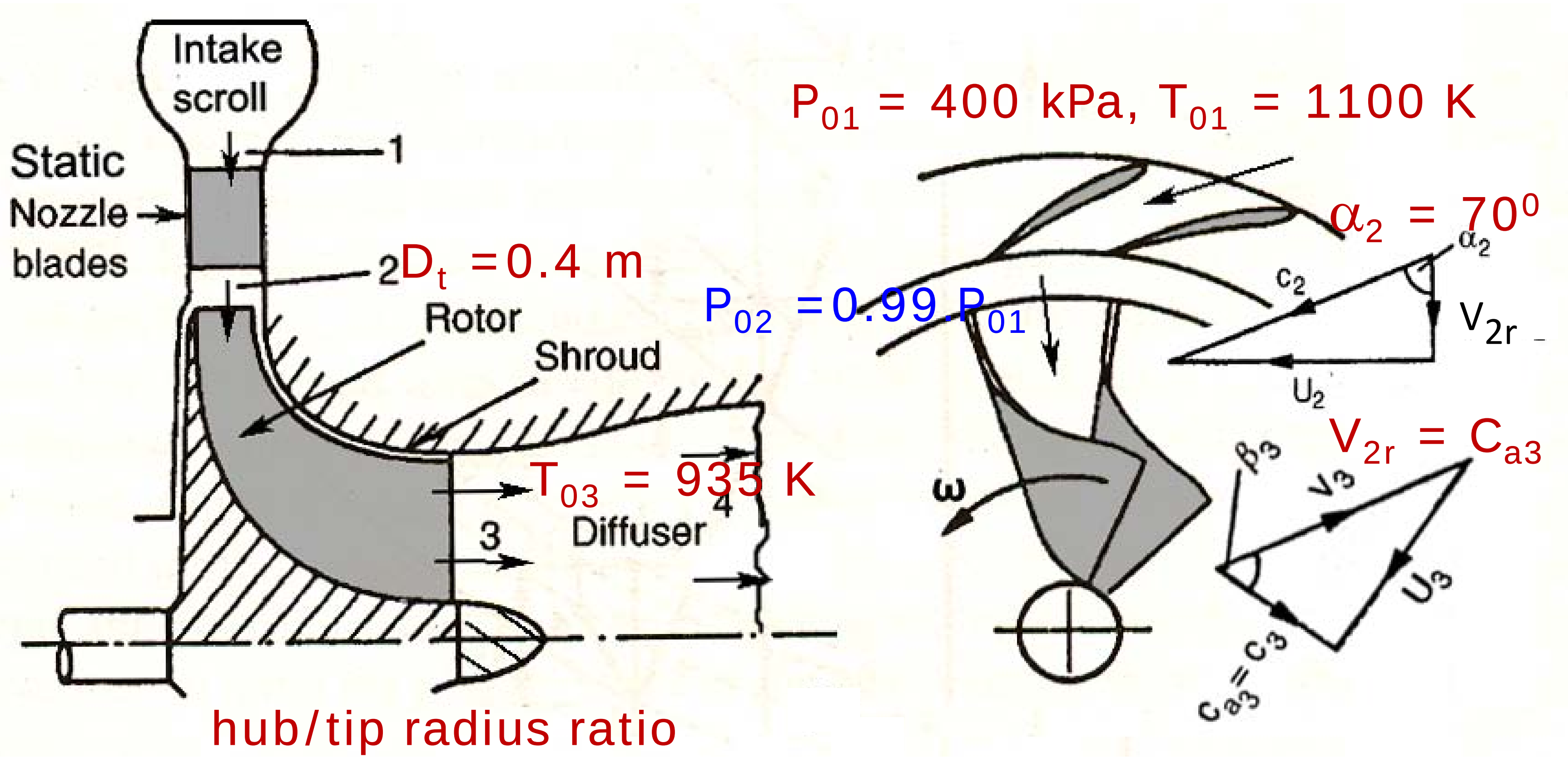

P_{01} = 400 kPa, T_{01} = 1100 K

$\alpha_2 = 70^0$

D_t = 0.4 m

$P_{02} = 0.99\ P_{01}$

V_{2r}

$V_{2r} = C_{a3}$

T_{03} = 935 K

hub/tip radius ratio
at rotor exit = 0.4

i) Rotor tip speed, U_2 =

$$\sqrt{H_{01}-H_{03}} = \sqrt{c_p.(T_{01}-T_{03})} = \sqrt{1158.(1100-935)} = 437 \text{ m/s}$$

rotational speed, $\omega = U_2 / r_2$ = 2185 rad/s,
whence, RPM, n = 20,870 rpm

ii) At rotor tip, $C_2 = U_2 / \sin\alpha_2$ = 437 /sin70^0 = 465 m/s,
and $V_{2r} = C_2.\cos \alpha_2$ = 159 m/s
the local speed of sound, $a_2 = (\gamma.R.T_2)^{1/2}$ where,
$T_2 = T_{02} - C_2^2 / 2.c_p$ = 707 K, so, a_2 = 620 m/s

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Hence, the nozzle exit Mach number is
$M_2 = 465/620 = 0.75$

Area at the rotor tip $A_2 = \dot{m} / \rho_2 . V_{2r}$,
where $\rho_2 = P_2 / R.T_2$ and $P_2 = P_{02} ./ (T_{02}/T_2)^{\gamma/(\gamma-1)}$

Thus A_2 is computed as = $0.0164\ m^2$

Whereupon, width of the rotor tip may be computed as, $b_2 = A_2/2.\pi.r_2 = 0.013\ m$

The relative total temperature,
$T_{02\text{-rel}} = T_{02} - C_2^2/2.c_p + V_2^2/2.c_p = 1017\ K$

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iii) The expansion ratio of the turbine at operating point, using the polytropic efficiency is given as :

$$\pi_{0T} = \frac{P_{01}}{P_{03}} = \left(\frac{T_{01}}{T_{03}}\right)^{\frac{\gamma}{(\gamma-1).\eta_{poly}}} = 2.1612,$$ which yields $P_{03} = 185$ kPa

Given that $V_{2r} = C_{a3} = 159$ m/s $= V_3$; we can therefore compute, $M_{3\text{-}rel} = V_3/(\gamma.R.T_3)^{1/2} = 0.267$, which is constant from the root to the tip at the rotor exit.

At station (rotor exit) $A_3 = \dot{m}/\rho_3.C_{a3}$ [Use isentropic relation as in (ii) to compute T_3 and P_3]

Therefore, $A_3 = 0.02363\ m^2$;

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iv) The rotor exit radii are:

r_{3tip} =0.0946 m; r_{3hub}=0.0378 m

Therefore at mean radius, r_{3mean} =0.06624 m , and

$C_{w3m} = U_{3m} = \omega.r_{3m}$ = 144.8 m/s

From velocity triangle at rotor exit, C_{3m} = 215 m/s, and hence, $T_{03m} = T_3 + C_{3m}^2/2.c_p$ = 944 K.

And, Mach number $M_{3m} = C_{3m} /(\gamma.R.T_3)^{1/2}$ = 0.362, and exit flow angle, $\beta_3 = 42.3^0$

iv) Radial variation of these two parameters β_3, $M_{3\text{-rel}}$ can now be found easily. Mach number and exit flow angle both go up with radius in a marginally non-linear manner.

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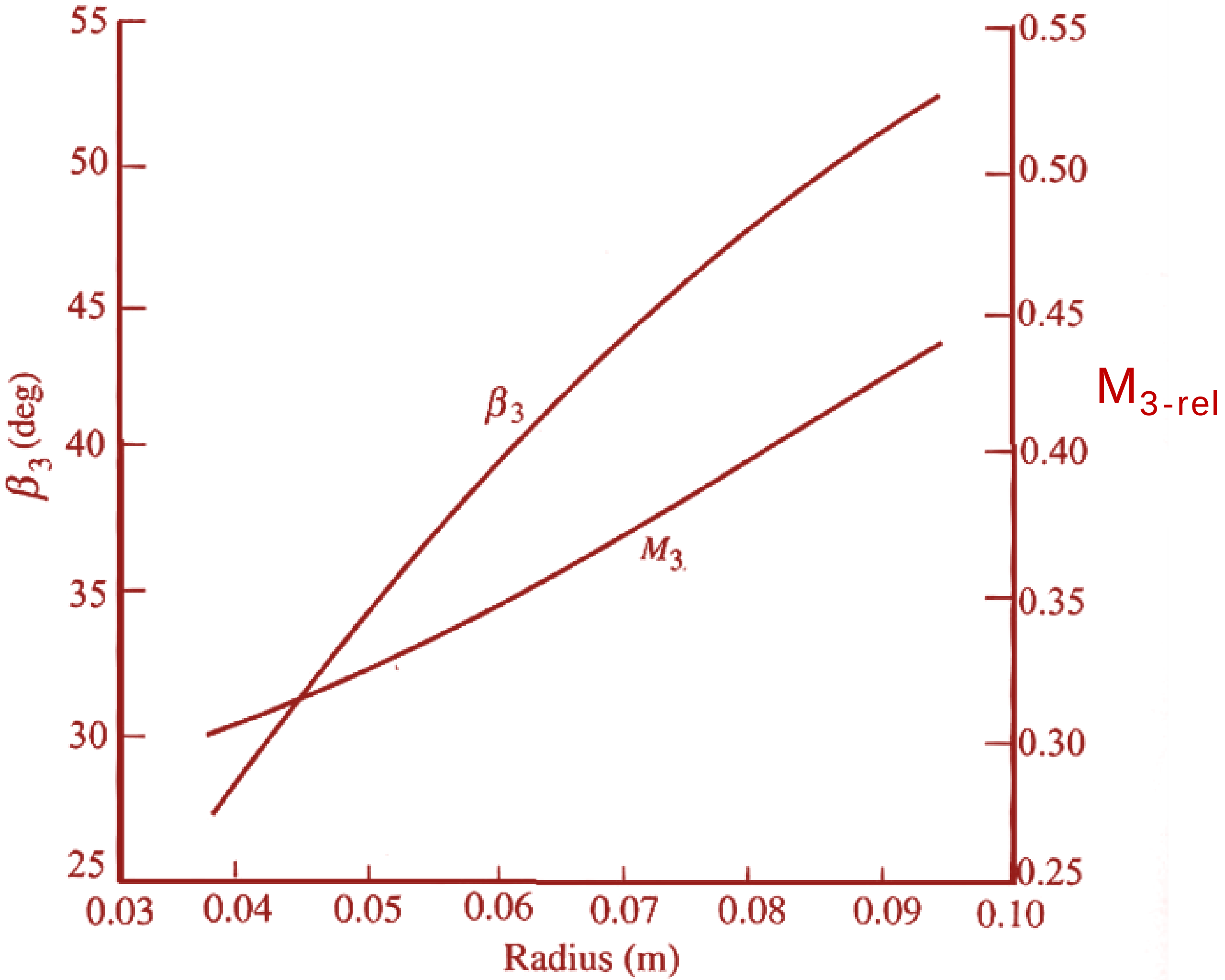

β_3 and M_3 vs radius

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Tutorial problem on Radial Turbine

The operating point data of a radial turbine are given as :

P_{01} = 699 kPa, T_{01} = 1145 K ; P_2 = 527.2 kPa, T_2 = 1029 K. P_3=384.7 kPa, T_3=914.5 K, and T_{03} =924.7 K. The impeller exit area mean diameter to the impeller tip diameter is chosen as 0.49 and the design rpm is 24,000 rpm. Assuming the relative velocity at the rotor inlet is radial and the absolute flow at the rotor exit is axial, compute :

i) the total-to-static efficiency of the radial turbine
ii) the impeller rotor tip diameter
iii) The loss coefficients in the nozzle and the rotor

[i) 90.5%; 0.27 m; iii) ξ_N = 0.05316; ξ_R=0.2]

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Next Class

Combustion Chambers

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