



Jet Aircraft Propulsion

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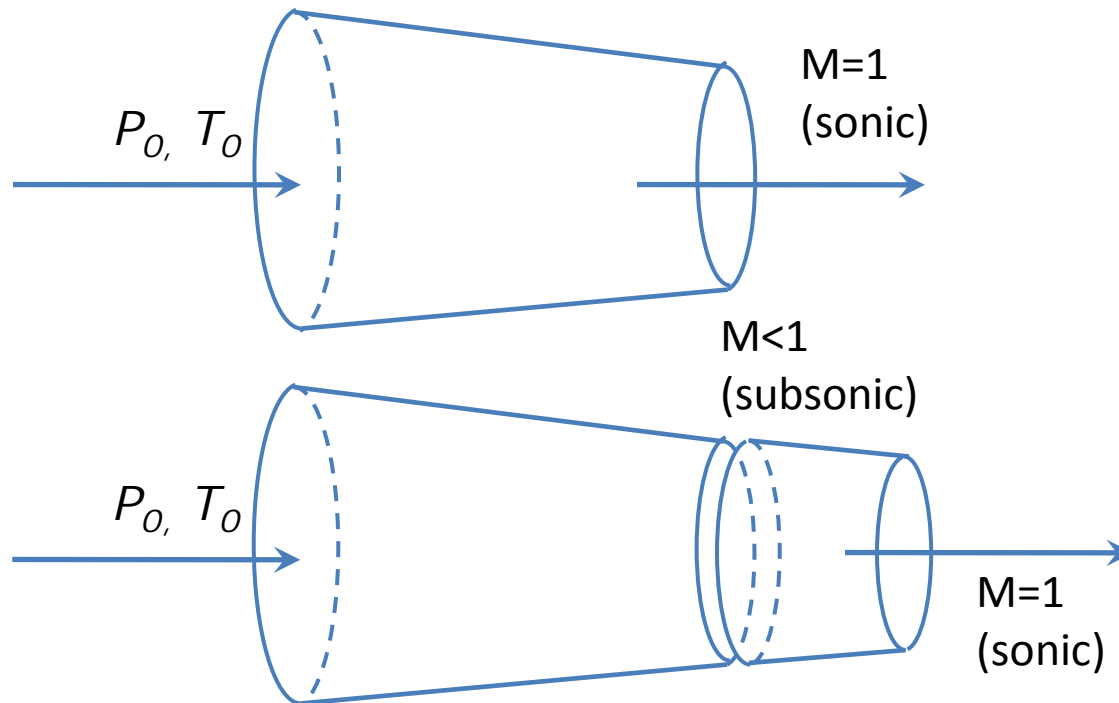
Department of Aerospace Engineering,
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Lect-30

In this lecture...

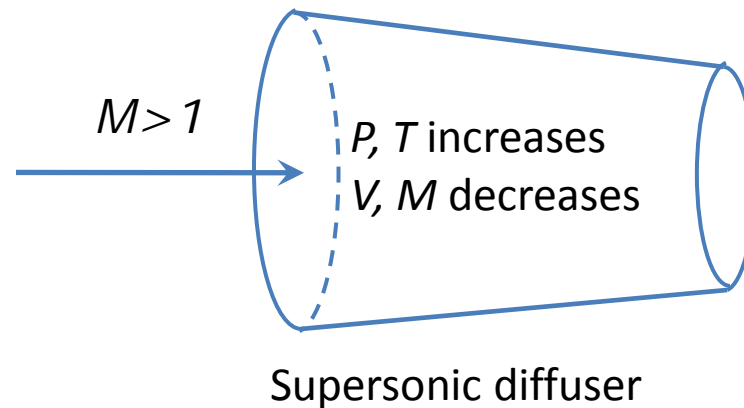
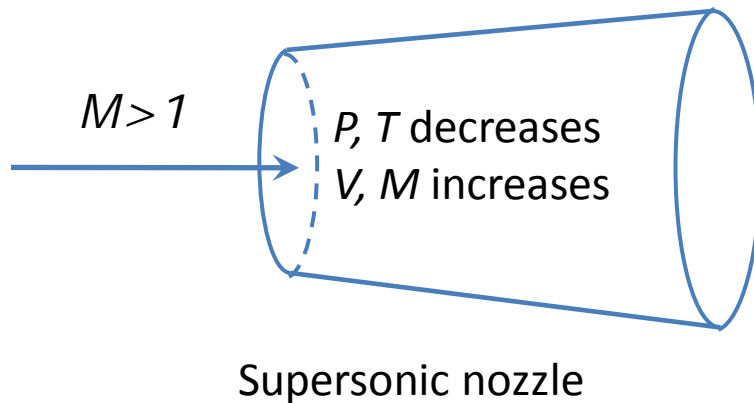
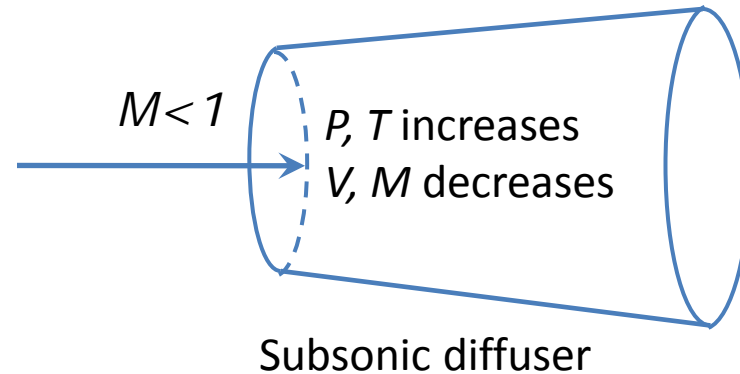
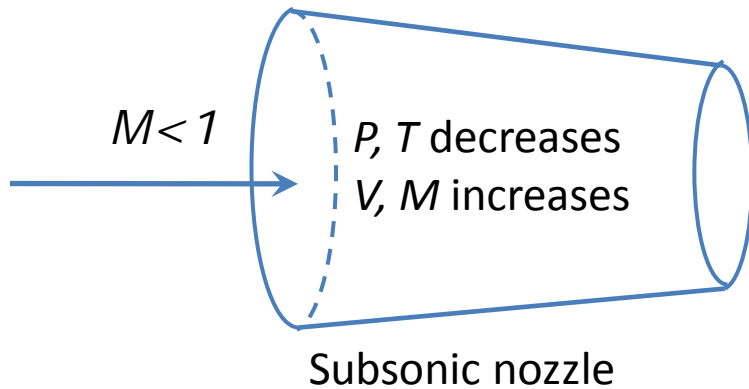
- Subsonic and supersonic nozzles
- Working of these nozzles
- Performance parameters for nozzles

Variation of fluid velocity with flow area



Sonic velocity will occur at the exit of the converging extension, instead of the exit of the original nozzle, and the mass flow rate through the nozzle will decrease because of the reduced exit area.

Variation of fluid velocity with flow area



Governing equations

Let us consider a calorically perfect gas flow through a nozzle.

The mass flow through the nozzle is

$$\begin{aligned} \dot{m} &= \rho u A = \left(\frac{P}{RT} \right) (M \sqrt{\gamma RT}) A = (MA) \left(\frac{P}{P_0} \right) P_0 \frac{\sqrt{\gamma}}{\sqrt{RT}} \sqrt{\frac{T}{T_0}} \\ &= \frac{\sqrt{\gamma} P_0}{\sqrt{T_0 R}} MA \frac{\left\{ 1 + ((\gamma - 1) / 2) M^2 \right\}^{1/2}}{\left\{ 1 + ((\gamma - 1) / 2) M^2 \right\}^{\gamma / (\gamma - 1)}} \end{aligned}$$

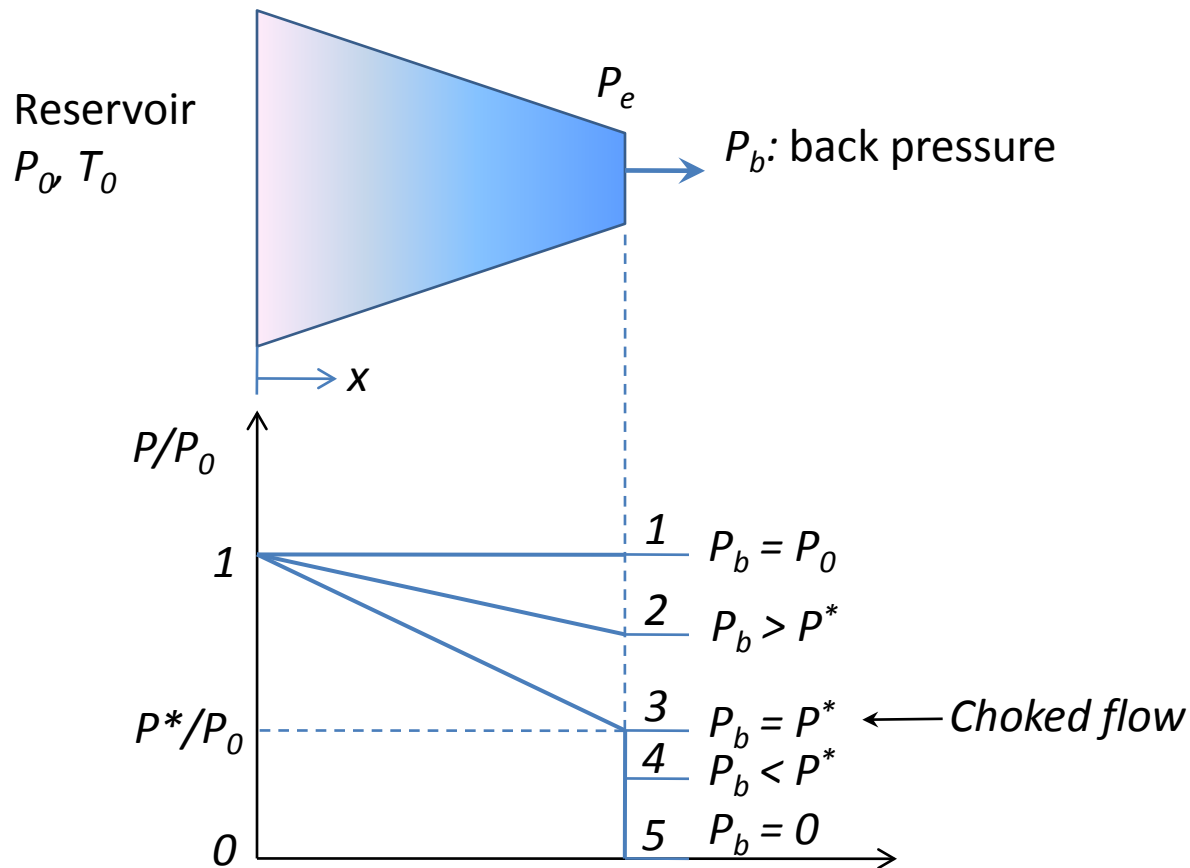
This on simplification reduces to

$$\dot{m} = \frac{A P_0}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} \frac{M}{\left\{ 1 + ((\gamma - 1) / 2) M^2 \right\}^{(\gamma + 1) / 2 (\gamma - 1)}}$$

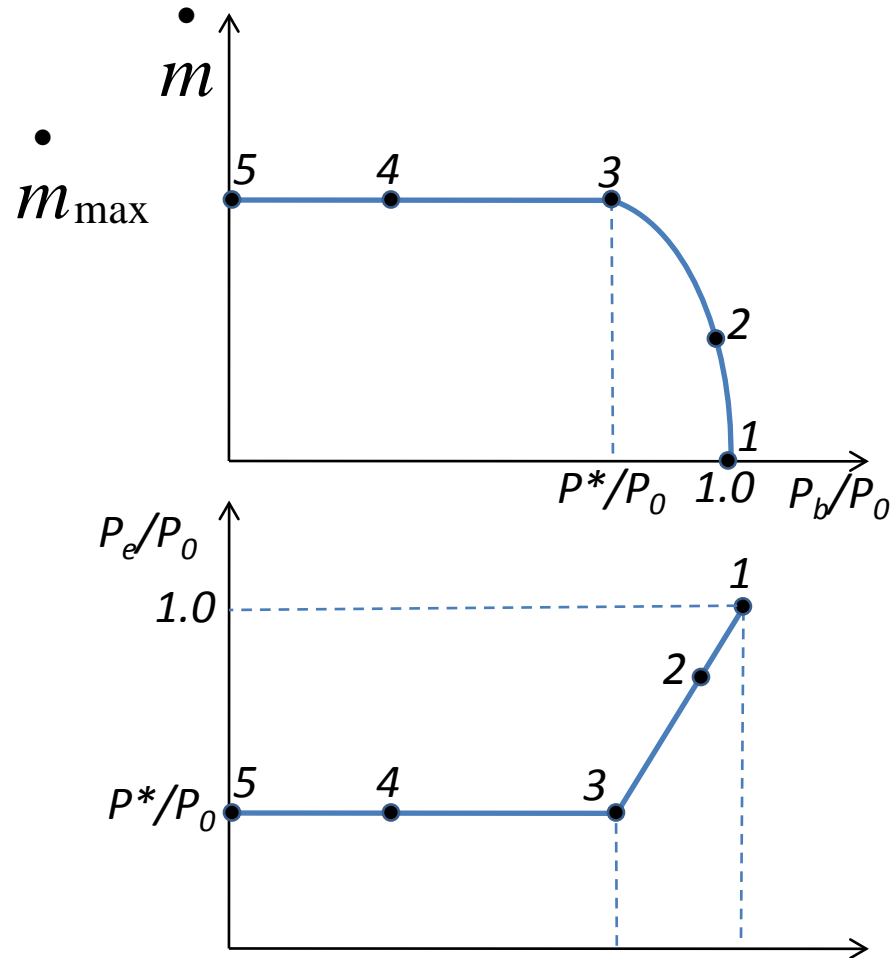
Isentropic flow through converging nozzles

- Converging nozzle in a subsonic flow will have decreasing area along the flow direction.
- We shall consider the effect of back pressure on the exit velocity, mass flow rate and pressure distribution along the nozzle.
- We assume flow enters the nozzle from a reservoir so that inlet velocity is zero.
- Stagnation temperature and pressure remains unchanged in the nozzle.

Isentropic flow through converging nozzles



Isentropic flow through converging nozzles



The effect of back pressure P_b on the mass flow rate and the exit pressure P_e .

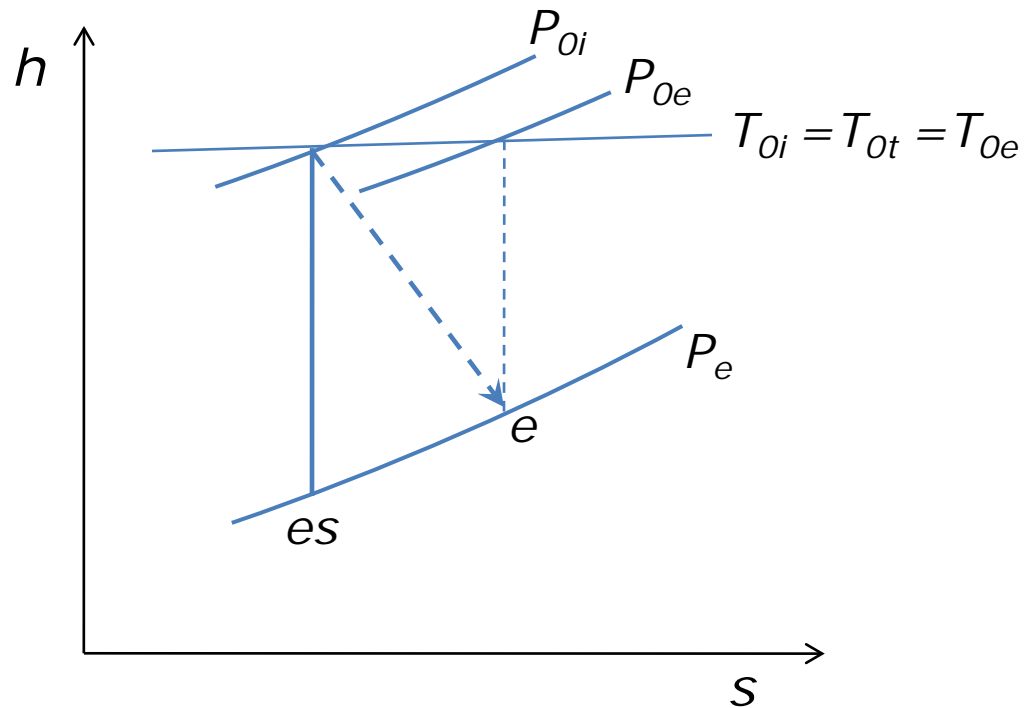
Isentropic flow through converging nozzles

- From the above figure,

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

- For all back pressures lower than the critical pressure, exit pressure = critical pressure, Mach number is unity and the mass flow rate is maximum (choked flow).
- A back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

Nozzle efficiency



Converging nozzles

The efficiency of a nozzle is defined as

$$\eta_n = \frac{h_{0i} - h_e}{h_{0i} - h_{es}}, \text{ where } h_{0i} \text{ is the stagnation enthalpy}$$

at the nozzle inlet, h_e is the actual static enthalpy at the nozzle exit, h_{es} is the isentropic static enthalpy at the nozzle exit.

In terms of the corresponding temperatures,

$$\eta_n = \frac{T_{0i} - T_e}{T_{0i} - T_{es}} = \frac{1 - T_e / T_{0e}}{1 - T_{es} / T_{0i}}$$

Converging nozzles

For choked flow, $M = 1$,

$$\eta_n = \frac{1 - (2 / (\gamma + 1))}{1 - (P_c / P_{oi})^{(\gamma-1) / \gamma}}$$

The pressure ratio is therefore,

$$\frac{P_{oi}}{P_c} = \frac{1}{(1 - (1 / \eta_n) ((\gamma - 1) / (\gamma + 1)))^{\gamma / (\gamma - 1)}}$$

$$\text{If } \frac{P_{oi}}{P_c} < \frac{P_{oi}}{P_a},$$

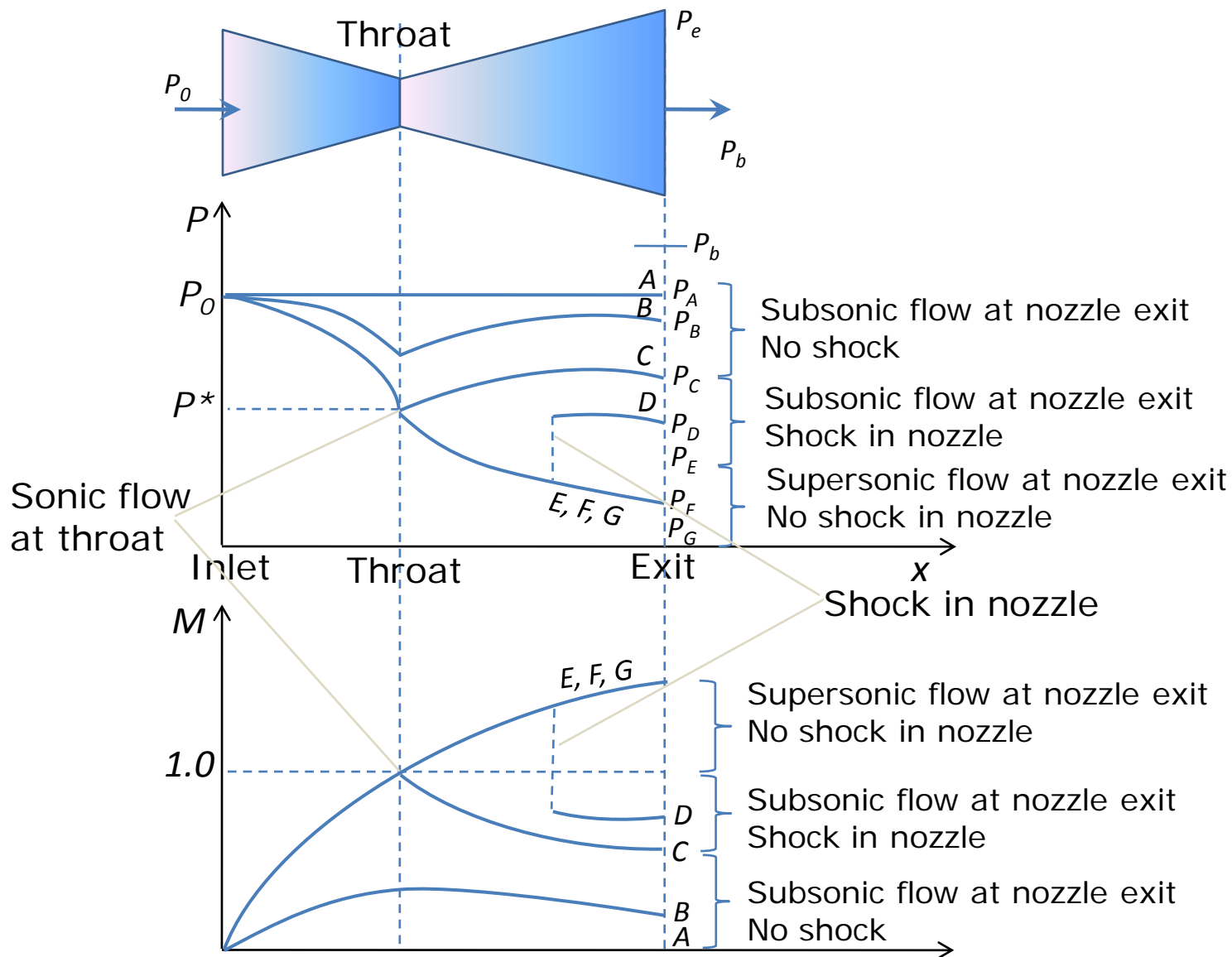
the nozzle is operating under choked condition.

Converging nozzles

- If a convergent nozzle is operating under choked condition, the exit Mach number is unity.
- The exit flow parameters are then defined by the critical parameters.
- To determine whether a nozzle is choked or not, we calculate the actual pressure ratio and then compare this with the critical pressure ratio.
- If the actual pressure ratio $>$ critical pressure ratio, the nozzle is said to be choked.

Isentropic flow through converging-diverging nozzles

- Maximum Mach number achievable in a converging nozzle is unity.
- For supersonic Mach numbers, a diverging section after the throat is required.
- However, a diverging section alone would not guarantee a supersonic flow.
- The Mach number at the exit of the converging-diverging nozzle depends upon the back pressure.



Converging-diverging nozzles

- The flow through nozzles is normally assumed to be adiabatic as the heat transfer per unit mass is much smaller than the difference in enthalpy between the inlet and outlet.
- The flow from the inlet to the throat can be assumed to be isentropic, but the flow from the throat to exit may not be due the possible presence of shocks.

Converging-diverging nozzles

The efficiency of a nozzle is defined as

$$\eta_n = \frac{h_{0i} - h_e}{h_{0i} - h_{es}} = \frac{T_{0i} - T_e}{T_{0i} - T_{es}} = \frac{1 - T_e / T_{0e}}{1 - T_{es} / T_{0i}}$$

$$= \frac{1 - (P_e / P_{0e})^{(\gamma-1)/\gamma}}{1 - (P_e / P_{0i})^{(\gamma-1)/\gamma}}$$

Therefore,
$$\left(\frac{P_e}{P_{0e}} \right) = \left[1 - \eta_n \left\{ 1 - \left(\frac{P_e}{P_{0i}} \right)^{(\gamma-1)/\gamma} \right\} \right]^{\gamma/(\gamma-1)}$$

Since,
$$\frac{P_{0i}}{P_{0e}} = \frac{P_e}{P_{0e}} \frac{P_{0i}}{P_e} \Rightarrow \frac{P_{0i}}{P_{0e}} = \frac{P_{0i}}{P_e} \left[1 - \eta_n \left\{ 1 - \left(\frac{P_e}{P_{0i}} \right)^{(\gamma-1)/\gamma} \right\} \right]^{\gamma/(\gamma-1)}$$

Converging-diverging nozzles

The exit velocity can be calculated from

$$\begin{aligned}
 u_e &= \sqrt{2(h_{oi} - h_e)} = \sqrt{2\eta_n(h_{oi} - h_{es})} \\
 &= \sqrt{2c_p\eta_n(T_{oi} - T_{es})} = \sqrt{2c_p\eta_n T_{oi} \left\{ 1 - \left(\frac{P_e}{P_{oi}} \right)^{(\gamma-1)/\gamma} \right\}} \\
 &= \sqrt{\frac{2\gamma R}{(\gamma-1)} \eta_n T_{oi} \left\{ 1 - \left(\frac{P_e}{P_{oi}} \right)^{(\gamma-1)/\gamma} \right\}}
 \end{aligned}$$

Converging-diverging nozzles

The exit Mach number is $M_e^2 = \frac{u_e^2}{a_e^2} = \frac{u_e^2}{\gamma RT_e}$

Since, $\frac{T_{0e}}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 = \frac{T_{0i}}{T_e}$

$$M_e^2 = \frac{2\eta_n}{\gamma - 1} \left\{ 1 + \frac{\gamma - 1}{2} M_i^2 \right\} \left\{ 1 - \left(\frac{P_i}{P_{0i}} \right)^{(\gamma-1)/\gamma} \right\}$$

$$= \frac{2}{\gamma - 1} \left[\frac{\eta_n \left\{ 1 - (P_e / P_{0i})^{(\gamma-1)/\gamma} \right\}}{1 - \eta_n \left\{ 1 - (P_e / P_{0i})^{(\gamma-1)/\gamma} \right\}} \right]$$

Converging-diverging nozzles

From the governing equation discussed earlier, and also assuming isentropic flow upto throat, the ratio between the throat area and the exit area is,

$$\frac{A_t}{A_e} = \frac{P_{0e}}{P_{ot}} \frac{M_e}{M_t} \left[\frac{1 + ((\gamma - 1) / 2)M_t^2}{1 + ((\gamma - 1) / 2)M_e^2} \right]^{(\gamma+1 / 2(\gamma-1))}$$

$$= \frac{P_{0e}}{P_{oi}} \frac{M_e}{M_t} \left[\frac{1 + ((\gamma - 1) / 2)M_t^2}{1 + ((\gamma - 1) / 2)M_e^2} \right]^{(\gamma+1 / 2(\gamma-1))}$$

If the throat is choked, $M_t = 1$,

$$\frac{A^*}{A_e} = \frac{P_{0e}M_e}{P_{oi}^*} \left[\frac{(\gamma + 1) / 2}{1 + ((\gamma - 1) / 2)M_e^2} \right]^{(\gamma+1 / 2(\gamma-1))}$$

Converging-diverging nozzles

The mass flow rate will therefore be,

$$\dot{m} = \frac{A^* P_{oi}^*}{\sqrt{T_{oi}^*}} \sqrt{\frac{\gamma}{R}} \frac{1}{((\gamma + 1) / 2)^{(\gamma+1) / 2(\gamma-1)}}$$

The mass flow rate is a function of the inlet stagnation pressure, temperature and throat area.

By design one would like to keep the area ratio A_i/A_e as close as possible to unity. This is to keep the external drag under control. However this may result in the nozzle exit pressure to be different from the ambient pressure : incomplete expansion.

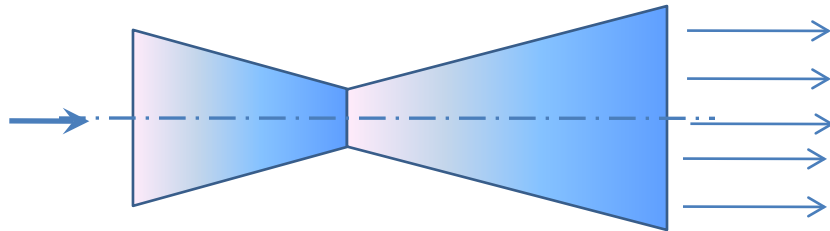
Converging-diverging nozzles

- Underexpanded nozzle:
 - $P_e > P_a$
 - The flow is capable of additional expansion.
 - Expansion waves originating from the lip of the nozzle.
- Overexpanded nozzle:
 - $P_e < P_a$
 - Shock waves originate from the nozzle lip.

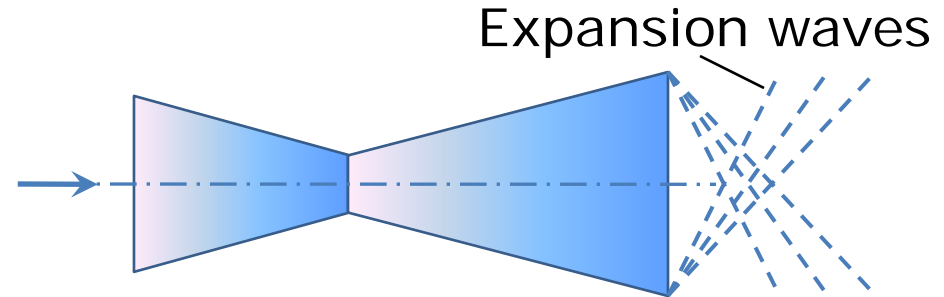
Converging-diverging nozzles

- Fully expanded nozzle:
 - $P_e = P_a$
 - No shock waves/expansion waves.
- If $P_e \ll P_a$
 - Shock waves will occur within the divergent section of the nozzle.

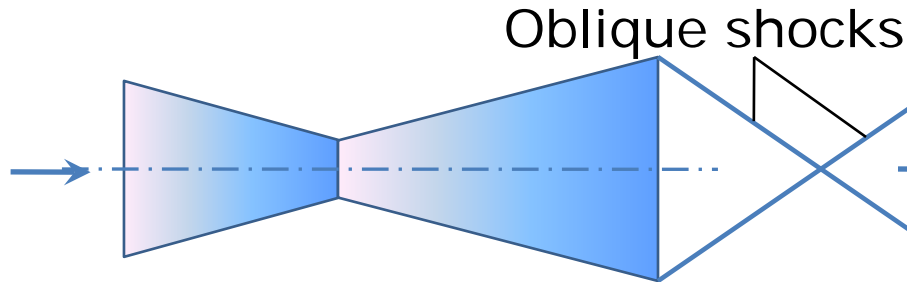
Converging-diverging nozzles



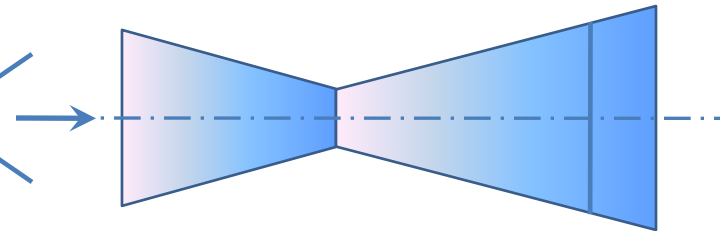
Fully expanded: $P_e = P_a$



Underexpanded: $P_e = P_a$



Overexpanded: $P_e = P_a$



$P_e \ll P_a$

In this lecture...

- Subsonic and supersonic nozzles
- Working of these nozzles
- Performance parameters for nozzles

In the next lecture...

- Tutorial on intakes and nozzles