Jet Aircraft Propulsion

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Lect-31

In this lecture...

- Tutorial on intakes and nozzles

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Problem 1

- Consider a turbofan engine operating at a Mach number of 0.9 at an altitude where the ambient temperature and pressure are -56.5°C and 22.632 kPa, respectively. The mass ingested into the engine is 235 kg/s through an inlet area of 3 m^2. If the diffuser efficiency is 0.9 and the Mach number at the fan entry is 0.45, calculate: (a) the capture area, (b) the static pressures at the inlet and fan face, (c) velocities at inlet and the fan face, (d) the diffuser pressure recovery.

Solution: Problem 1

- The ambient temperature, $T_a = 216.5$ K
- The flight speed is $u = M\sqrt{(\gamma RT)} = 271.6$ m/s
- The freestream density is

$$\rho = P_a / RT_a = 0.3479 \text{ kg/m}^3$$

Therefore, the capture area is

$$A_\infty = \dot{m} / \rho u = 2.486 \text{ m}^2$$

The capture area is smaller than the inlet area, typical of cruise operation.

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Solution: Problem 1

- From gas tables, for a Mach number of 0.9, the area ratio, A/A*=1.00886.
- Therefore, A*=2.486/1.00886=2.465 m^2
- Now, $A_1/A^*=3/2.465=1.217\ m^2$
- From the gas tables, the corresponding Mach number is $M_1=0.577$.
- We can also determine the temperature and pressure ratios for this Mach number from the gas tables.

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Solution: Problem 1

- $P_1/P_{01}=0.798$, $T_1/T_{01}=0.93757$
- Since, $T_{01}=T_{0a}$ and $P_{01}=P_{0a}$,
- $P_1=30.547$ kPa and $T_1=246.9$ K
- And, $u_1=M_1\sqrt{(\gamma RT_1)}=181.7$ m/s
- Since Mach number at the fan is $M_2=0.45$,
- For $M_2=0.45$, $P_2/P_{02}=0.87027$
- From the definition of diffuser efficiency,

$$\frac{P_{02}}{P_a}=\left(1+\eta_d\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

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Solution: Problem 1

$$\frac{P_{02}}{P_a} = \left(1 + \eta_d \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

Substituting, $P_{02} = 36.442$ kPa

Since, $P_2 = P_{02} \Big/ \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)} = 31.714$ kPa

and $T_2 = T_{02} \Big/ \left(1 + \frac{\gamma - 1}{2} M_2^2\right) = 253.1$ K

Therefore, $u_2 = M_2\sqrt{\gamma R T_2} = 143.5$ m/s

The pressure recovery is

$P_{02} / P_{0a} = 36.442 / 38.278 = 0.952$

Solution: Problem 1

This problem can also be solved in a different way :

From continuity equation :

$$\frac{\dot{m}}{A} = \rho u = \frac{P}{RT} M\sqrt{\gamma RT} = \frac{MP_0}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}}\sqrt{\frac{\gamma}{R}}\sqrt{\frac{(1+\frac{\gamma-1}{2}M^2)}{T_0}}$$

For station 1,

$$\frac{\dot{m}}{A_1 P_{01}}\sqrt{\frac{RT_{01}}{\gamma}} = \frac{M_1}{\left(1+\frac{\gamma-1}{2}M_1{}^2\right)^{(\gamma+1)/2(\gamma-1)}}$$

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Solution: Problem 1

In the above equation, M_1 is unknown. This can be solved iteratively.

T_1 can be determined by, $T_1 = \dfrac{T_{0a}}{1 + \dfrac{\gamma - 1}{2} M_1^2}$

$$P_1 = \frac{P_{0a}}{\left(1 + \dfrac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}}$$

To find P_{02}, $P_{02} / P_a = \left(1 + \eta_d \dfrac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$

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Solution: Problem 1

We can now determine the pressure recovery.

Now, $T_2 = \dfrac{T_{0a}}{1+\dfrac{\gamma-1}{2}M_2^2}$ and $P_2 = \dfrac{P_{02}}{\left(1+\dfrac{\gamma-1}{2}M_2^2\right)^{\gamma/(\gamma-1)}}$

Therefore, $u_2 = M_2\sqrt{\gamma R T_2}$

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Problem 2

- Consider the mixed compression two-dimensional supersonic intake as shown in the figure. The free stream Mach number is 3.0. The intake has a three shock system as shown. Determine the overall total pressure ratio and the overall static pressure ratio.

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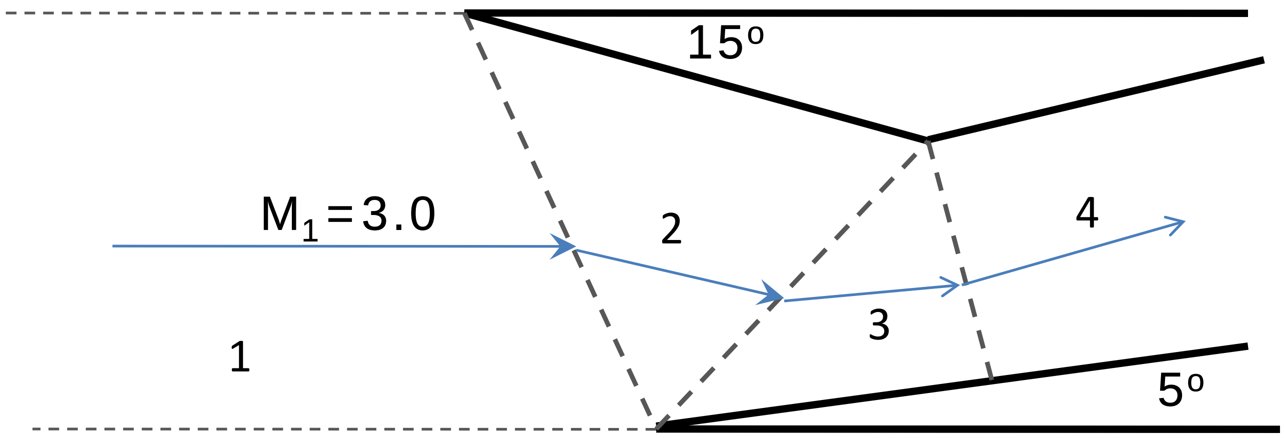
Problem 2



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Solution: Problem 2

- The first oblique shock has an upstream Mach number of 3.0 and $\delta_1 = 15^o$.
- From the shock tables, the shock angle is $\beta_1 = 32.25^o$.
- With $\beta_1 = 32.25^o$ and $\delta_1 = 15^o$
- $M_{1n} = M_1 \sin\beta_1 = 3.0 \sin 32.25 = 1.60$,
- From the normal shock tables, we can find M_{2n}.
- $M_2 = M_{2n}/\sin(\beta_1 - \delta_1) = 2.25$

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Solution: Problem 2

- From the normal shock tables, $P_{02}/P_{01}=0.8935$, $P_2/P_1=2.82$
- For region 2, the deflection angle, $\delta_2 = 15+5=20^o$
- For $M_2=2.25$ and $\delta_1 = 20^o$, $\beta_2 = 46.95^o$
- We find M_3 in the same way as we calculated M_2.
- $M_3=1.444$ and $P_{03}/P_{02}=0.878$, $P_3/P_2=2.992$

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Solution: Problem 2

- Similarly, $M_4=0.7219$ (from the normal shock tables)
- $P_{04}/P_{03}=0.9465$ and $P_4/P_3=2.333$
- The overall pressure ratios:
- $P_{04}/P_{01}=P_{04}/P_{03}xP_{03}/P_{02}xP_{02}/P_{01}$
  $=0.9465x0.878x0.8935=0.7865$
- $P4/P1=P_4/P_3xP_3/P_2xP_2/P_1$
  $=2.333x2.992x2.82=19.691$

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Problem 3

- Air enters a converging duct with varying flow area at T_1 =400 K, P_1=100 kPa, and M_1=0.3. Assuming steady isentropic flow, determine T_2, P_2, and M_2 at a location where the flow area has been reduced by 20 percent.

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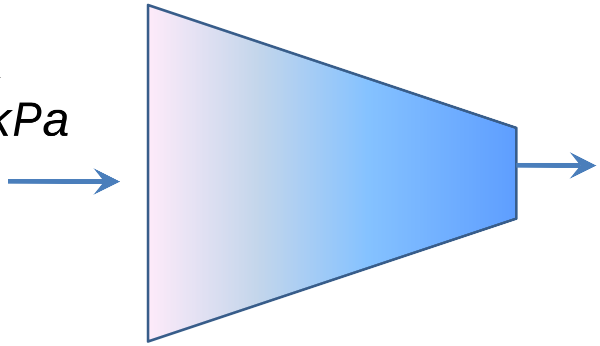
Problem 3

$T_1 = 400K$
$P_1 = 100\ kPa$
$M_1 = 0.3$
A_1



$T_2 = ?$
$P_2 = ?$
$M_2 = ?$
$A_2 = 0.8A_1$

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Solution: Problem 3

- From the isentropic tables, for a Mach number of 0.3,
- $A_1/A^*=2.0351$, $T_1/T_0=0.9823$, $P_1/P_0=0.9395$
- With a 20% area reduction, $A_2=0.8A_1$
- $A_2/A^*= A_2/A_1 \times A_1/A^* =0.8 \times 2.0351$
  $=1.6281$
- For this value of area ratio, from the isentropic tables, $T_2/T_0=0.9701$, $P_2/P_0=0.8993$ and therefore $M_2=0.391$

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Solution: Problem 3

$$\frac{T_2}{T_1} = \frac{T_2 / T_0}{T_1 / T_0} \rightarrow T_2 = T_1\left(\frac{T_2 / T_0}{T_1 / T_0}\right) = 400\left(\frac{0.9701}{0.9823}\right)$$

$$T_2 = 395 \text{ K}$$

$$\frac{P_2}{P_1} = \frac{P_2 / P_0}{P_1 / P_0} \rightarrow P_2 = P_1\left(\frac{P_2 / P_0}{P_1 / P_0}\right) = 100\left(\frac{0.8993}{0.9395}\right)$$

$$P_2 = 95.7 \text{ kPa}$$

The static temperature and temperature drops in flow through a converging nozzle. There is an increase in the Mach number.

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Problem 4

- Air enters a converging–diverging nozzle, shown in the Figure, at 1.0 MPa and 800 K with a negligible velocity. For an exit Mach number of M=2 and a throat area of 20 cm^2, determine (a) the throat conditions, (b) the exit plane conditions, including the exit area, and (c) the mass flow rate through the nozzle.

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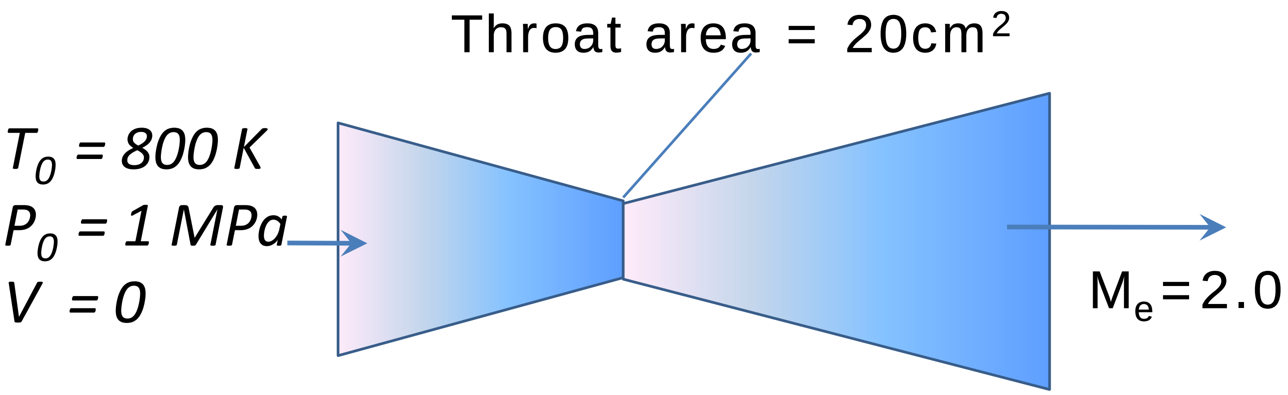
Problem 4



Solution: Problem 4

- The nozzle exit Mach number is given as 2.0. Therefore the throat Mach number must be 1.0.
- Since the inlet velocity is negligible, the stagnation pressure and stagnation temperature are the same as the inlet temperature and pressure, P_0=1.0 MPa and T_0=800 K.

$$\therefore \rho_0 = P_0 / RT_0 = 4.355 kg / m^3$$

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Solution: Problem 4

(a) At the throat, $M = 1$. From the isentropic tables,

$$\frac{P^*}{P_0} = 0.5283, \quad \frac{T^*}{T_0} = 0.8333, \quad \frac{\rho^*}{\rho_0} = 0.6339$$

$$P^* = 0.5283 P_0 = 0.5283 \text{ MPa}$$

$$T^* = 0.8333 T_0 = 666.6 \text{ K}$$

$$\rho^* = 0.6339 \rho_0 = 2.761 \text{ kg/m}^3$$

Therefore, $V^* = \sqrt{\gamma R T^*} = 517.5 \text{ m/s}$

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Solution: Problem 4

(b) At the nozzle exit, $M = 2$. From the isentropic tables,

$$\frac{P_e}{P_0} = 0.1278, \ \frac{T_e}{T_0} = 0.5556, \ \frac{\rho_e}{\rho_0} = 0.2300,$$

$$M^* = 1.6330, \ \frac{A_e}{A^*} = 1.6875$$

Therefore,

$$P_e = 0.1278 P_0 = 0.1278 \text{ MPa}$$

$$T_e = 0.5556 T_0 = 444.5 \text{K}$$

$$\rho_e = 0.2300 \rho_0 = 1.002 \text{ kg/m}^3$$

$$A_e = 1.6875 A^* = 33.75 \text{ cm}^2$$

Solution: Problem 4

The nozzle exit velocity can be determined from $V_e = M_e\sqrt{\gamma RT_e} = 2\sqrt{1.4 \times 287 \times 444.5}$

$= 845.2 \ m/s$

(c) The mass flow rate can be calculated based on the properties at the throat, since the flow is choked.

$\dot{m} = \rho * A * V* = 2.761 \, x \, 0.0002 \, x \, 517.5$

$= 2.86 \ kg/s$

This corresponds to the highest mass flow possible through the nozzle : choking mass flow rate.

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Exercise Problem # 1

- A turbofan engine ingests air at 500 kg/s through an inlet area of 3.0 m^2. If the ambient conditions are 288 K and 100 kPa, calculate the Mach number when the capture area will be equal to the inlet are.
- Ans: 0.405

Exercise Problem # 2

- An aircraft flies at a Mach number of 2.4 at an altitude where the ambient conditions are 70 kPa and 260 K. The aircraft has a two-dimensional intake with a wedge of half-angle 10°. If the axis of the intake and hence the wedge is tilted 25° with respect to the upstream airflow, determine the downstream Mach number, pressure, and temperature above the wedge.
- Ans: 3.105, 23.8 kPa, 191 K

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Exercise Problem # 3

- Air enters a nozzle at 0.2 MPa, 350 K, and a velocity of 150 m/s. Assuming isentropic flow, determine the pressure and temperature of air at a location where the air velocity equals the speed of sound. What is the ratio of the area at this location to the entrance area?
- Ans: 0.118 MPa, 301 K, 0.629

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Exercise Problem # 4

- Air enters a converging–diverging nozzle at 0.5 MPa with a negligible velocity. Assuming the flow to be isentropic, determine the back pressure that will result in an exit Mach number of 1.8.
- Ans: 0.087 MPa

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Exercise Problem # 5

- Air enters a converging–diverging nozzle of a supersonic wind tunnel at 1.5 MPa and 350 K with a low velocity. If a normal shock wave occurs at the exit plane of the nozzle at $M_e=2$, determine the pressure, temperature, Mach number, velocity, and stagnation pressure after the shock wave.
- Ans: 0.863 MPa, 328 K, 0.577, 210 m/s, 1.081 MPa

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