Jet Aircraft Propulsion

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Lecture 36

Lecture Demonstration
of
Numerical Example
of
Off-design Performance
of
Aircraft Engines

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Problem -1

Design Point Data of a turbojet engine is given as :

Alt.=12 kM, M=2.0, T_a=216.7 K, P_a=19.40 kPa
Compr Pressure Ratio, π_c = 10;
Engine Max temperature, T_{03} =1800 K,
Heating value of the Fuel, Q = 42,800 kJ/kg
Intake design pressure recovery factor, $\pi_{I\text{-}loss}$= 0.95
Comb. chamber pressure recovery factor, π_{cc}=0.94
Nozzle pressure recovery factor, $\pi_{N\text{-}loss}$ = 0.96
Nozzle exit face pressure ratio, P_a/P_{ex} = 0.5
Polytropic efficiency of compr. Stages, $\eta_{c\text{-}poly}$ = 0.9
Polytropic efficiency of turbine stages, $\eta_{T\text{-}poly}$ = 0.9
Combustion efficiency, η_{cc}=0.98,
Mechanical efficiency of the shaft, η_{mech} =0.99

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Results obtained from Design Point analysis

Compr. Temp. ratio= $\pi_c^{\gamma_{air}-1/\gamma_{air}.\eta_{c-poly}}$ = 2.0771, η_c = 0.8641

Turbine Temp ratio = $\pi_T^{(\gamma_{air}-1)\eta_{T-poly}/\gamma_{air}}$ = 0.8155, η_T = 0.901

and Turbine Pressure Ratio, π_T = 0.375

Specific Thrust = 806.9 N/Kg/s; mass flow, $\dot{m}$ = 50 kg/s

Thrust, F=40.35 kN; s.f.c=44.21 mg/N-s=1.59 kg/N-hr

Fuel-air ratio, *f/a* = 0.03567

Thermal Efficiency, η_{th} = 41.9 %

Propulsive Efficiency, η_P = 74.4%

Overall Efficiency, η_O = 31.2%

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The defined engine is to be analyzed at off-design operating condition is defined as :

Altitude : 9 km ; M. No. $M_a = 1.5$, $T_a = 229.8$ K

$P_a = 30.8$ kPa

Turbine entry Temp. = 1670 K and

Exit face pressure ratio, $P_a / P_5 = 0.955$

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Solution: Off-design performance (Design values: red)

Gas constant at the operating condition:

For air : $R_{air} = [(\gamma_{air}-1)/\gamma_{air}]c_{p-air} = (04/1.4).1.004$
$= 0.2869$ kJ/kg.K

For gas : $R_{gas} = [(\gamma_{gas}-1)/\gamma_{gas}]c_{p-gas} = (03/1.3).1.239$
$= 0.2859$ kJ/kg.K

The sonic speed at 9 km, $a_{atm} = \sqrt{\gamma_{air}.R_{air}.T_a}$
$= 303.8$ m/s (295 m/s)

Flight velocity, $V_a = a_{atm}.M_a = 303.8 \times 1.5 = 455.7$ m/s
(590 m/s)

Inlet temp. rise, $\tau_I = T_{01}/T_a = 1 + \frac{\gamma_{air}-1}{2}.M_a^2 = 1.45$ (1.8)

Inlet pr. Rise, $\pi_I = (T_{01}/T_a)^{\gamma_{air}/(\gamma_{air}-1)} = 1.45^{3.5} = 3.671$ (7.825)

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(Design values: red)

Intake delivery total temp.=229.8x1.45=333 K (390K)

Now, **off-design analysis of intake** an emperical formula may be introduced for efficiency here :

$\eta_I = 1- 0.075(M_a-1)^{1.35} = 1-0.075\ (0.5)^{1.5} = 0.9706$ (0.925)

Intake off-design pr. recovery factor $\pi_{I\text{-loss}}=\eta_I.\pi_{I\text{-design}}$ = 0.922 (0.8788)

Max/Min Enthalpy ratio in the engine, $\tau_H = \dfrac{c_{p-gas}T_{03}}{c_{p-air}T_a}$

$$= \frac{(1.233 \times 1670)}{(1.004 \times 229.8)} = 8.97 \quad (10.25)$$

(Design values: red)

Off-design compression ratio is normally available from the compressor map. However, in absence of a compressor map it may be obtained by obtaining an estimate of the operating off-design temperature ratio

$$\tau_{oc} = \left(\frac{T_{02}}{T_{01}}\right)_{off-design} = 1 + \left[\left(\frac{T_{02}}{T_{01}}\right)_{design} - 1\right] \frac{\left(T_{03}/T_{02}\right)_{off-design}}{\left(T_{03}/T_{02}\right)_{design}}$$

$$= 1 + (2.0771 - 1) = \frac{1670/333}{1800/390}$$

$$= 2.170$$

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(Design values: red)

Compr. Pr Ratio, $\pi_{oc} = \left[1 + \eta_c . \left\{ \left({T_{02}}/{T_{01}} \right) - 1 \right\} \right]^{\left(\gamma / \gamma - 1 \right)_{air}}$

$= 1 + 0.864\ (2.17 - 1)]^{3.5} = 11.53$ (10)

Fuel-air ratio can be found from heat release in the combustion chamber for effecting $(T_{03} - T_{02})$

$$f = \frac{\tau_H - \tau_I . \tau_{oc}}{\dfrac{Q.\eta_{cc}}{c_{p-air} . T_{01}} - \tau_H} = \frac{8.968 - 1.45 \times 2.17}{\dfrac{42,800 \times 0.98}{1.004 \times 229.8} - 8.968}$$

= 0.0337 (0.03567)

(Design values: red)

The pressure ratio across the exit nozzle may be found from (assuming the nozzle is still choked)

$$\frac{P_{05}}{P_5} = \frac{P_a}{P_5}\,\pi_I.\pi_{I\text{-loss}}.\pi_{oc}.\pi_{cc}.\pi_{oT}.\pi_{N\text{-loss}}$$

$= 0.955 \times 3.671 \times 0.922 \times 11.53 \times 0.94 \times 0.375 \times 0.96$

[*assuming turbine and CC have same effective performance and the nozzle is still choked*]

$= 12.6$ (11.62)

The answer confirms that the nozzle pressure ratio is still high enough to be choked

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The Jet exhaust Mach no. can be calculated as :

$$M_5 = \sqrt{\frac{2}{\gamma_{gas}-1}\left[\left(\frac{P_{05}}{P_5}\right)^{\frac{\gamma_{gas}-1}{\gamma_{gas}}}-1\right]}$$

$$= \sqrt{\frac{2}{0.3}\left[(12.60)^{\frac{0.3}{1.3}}-1\right]} = 2.3 \quad (2.25)$$

From off-design engine temp ratio and turbine temp ratio (same as design) we can find at the exit

$$\frac{T_5}{T_a} = \frac{\tau_H . \tau_T}{\left(P_{05}/P_5\right)^{\gamma_{gas}/\gamma_{gas}-1}} \frac{c_{p-air}}{c_{p-gas}} = \frac{8.968 \; x \; 0.8155}{12.6^{03/1.3}} . \frac{1.004}{1.239} = 3.3$$

(3.85)

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Thus the exit static temp based sonic speed and exit jet velocity would be:

$$a_5 = \sqrt{\gamma . R . T_5} = \sqrt{1.3 \times 285.9 \times 3.3 \times 229.8} = 531 \text{ m/s}$$

$$V_5 = 1221 \text{ m/s}$$

Specific Thrust, $F/\dot{m} = (1+f)V_5 - V_a + (P_5 - P_a)A_5$

$$= 806.5 + (1-0.955)P_a . \dot{m}/(\rho_5 . V_5)$$

Mass flow

$$\dot{m} = \dot{m}_{des} \frac{P_a . \pi_I . \pi_{I\text{-loss}} . \pi_{oc}}{(P_a . \pi_I . \pi_{I\text{-loss}} . \pi_{oc})_{des}} . \sqrt{\frac{T_{03\text{-des}}}{T_{03}}}$$

$= 46.8$ kg/s (50 kg/s)

In absence of Compr. map

(Design values: red)

Complete Specific Thrust, $F/\dot{m}$ = 816 N/kg/s (806.9 N/kg/s)

Hence, Thrust, F = 816 x 46.8 = 38.2 kN (40.35 kN)

Thermal Efficiency $\eta_{th} = [(1+f)V_5^2 - V_a^2]/2.(Q.f)$ = 46.2 % (41.9%)

Propulsive Efficiency $\eta_p = F.V_a / ½.\dot{m}.[(1+f)V_5^2 - V_a^2]$ = 55.5 % (74.4%)

Overall Efficiency, $\eta_O = F.V_a/(Q.f.\dot{m})$ = 25.8% (31.2%)

Specific Fuel Consumption SFC = 41.3 mg/N-s (44.21)

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Turbine-Compr speed may be related through the normalized parameter, $N/\sqrt{T_{01}}$ to the design speed

$$N/N_{des} = \sqrt{\frac{T_{01}}{T_{01-des}} \frac{\pi_{oc}^{\frac{\gamma_{air}}{\gamma_{air}-1}} - 1}{\pi_{oc-des}^{\frac{\gamma_{air}}{\gamma_{air}-1}} - 1}} = 0.928$$

Similarly exit nozzle area may be related to the design nozzle area :

$$A_5 / A_{5\text{-}des} = (\dot{m} / \dot{m}_{des}) \left[(\rho_5 . V_5)_{des} / (\rho_5 . V_5) \right]$$
$$= 1.05$$

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Exercise Problem

The same engine is to be analyzed at an off-design operating condition defined as :

Altitude : 6 km ; M. No. M_a=1.1, T_a=249.2 K

P_a=47.18 kPa

Turbine entry Temp. = 1450 K and

Exit face pressure ratio, P_a/P_5 = 0.85

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Next Chapter

Ramjets
Pulsejets
Scramjets

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