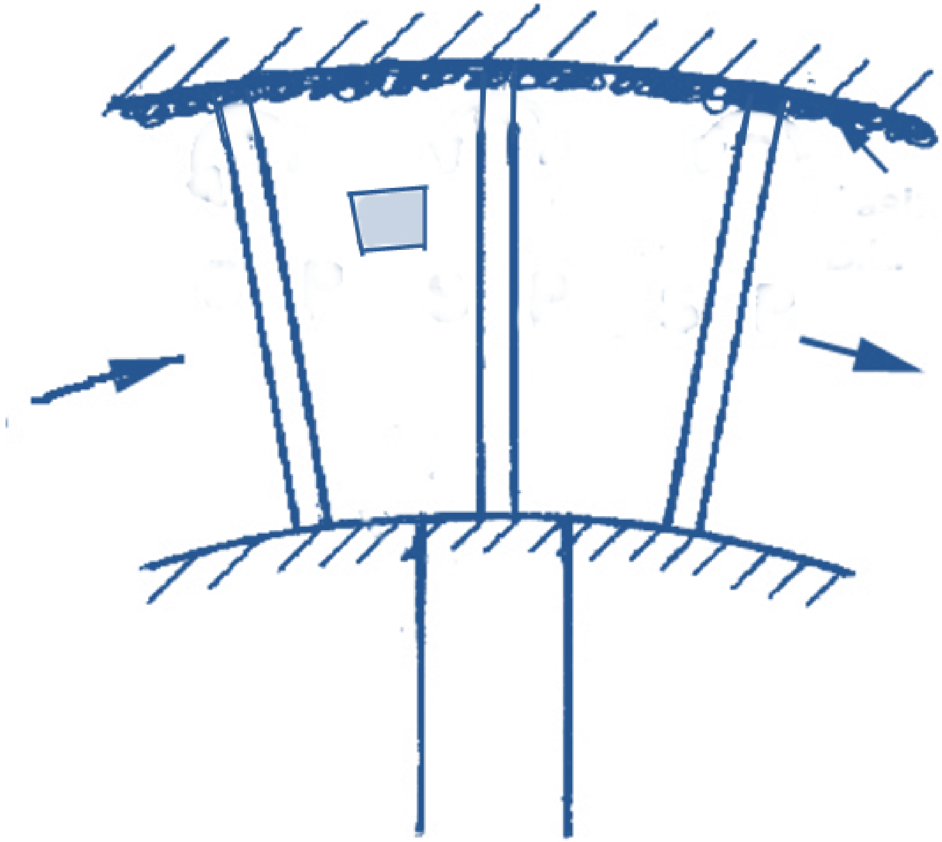
TURBOMACHINERY AERODYNAMICS

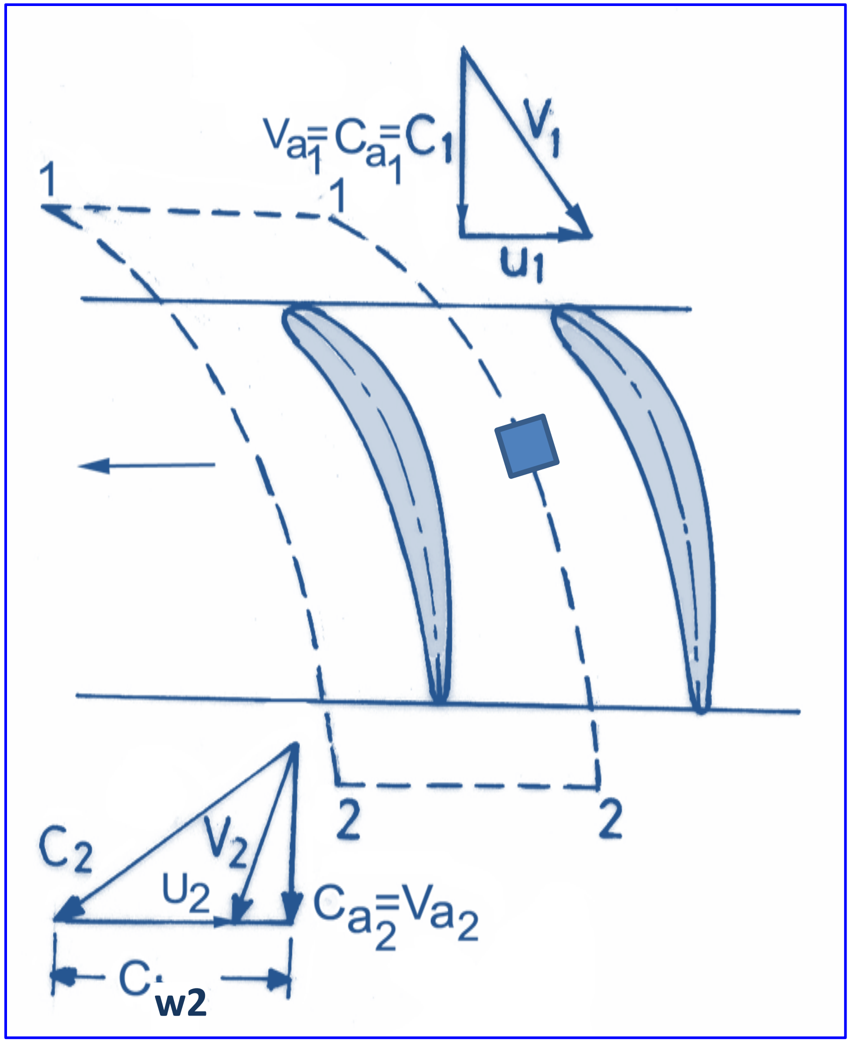
Lect - 7

Prof. Bhaskar Roy, Prof. A M Pradeep
Department of Aerospace Engineering,
IIT Bombay

Three Dimensional Flow Analysis in Axial Compressor



Let us assume that a small element inside the rotating blade passage represents the fluid flow inside the rotor, such that the analysis of the status of this element may wholly represent the status of the whole flow inside the rotor passage



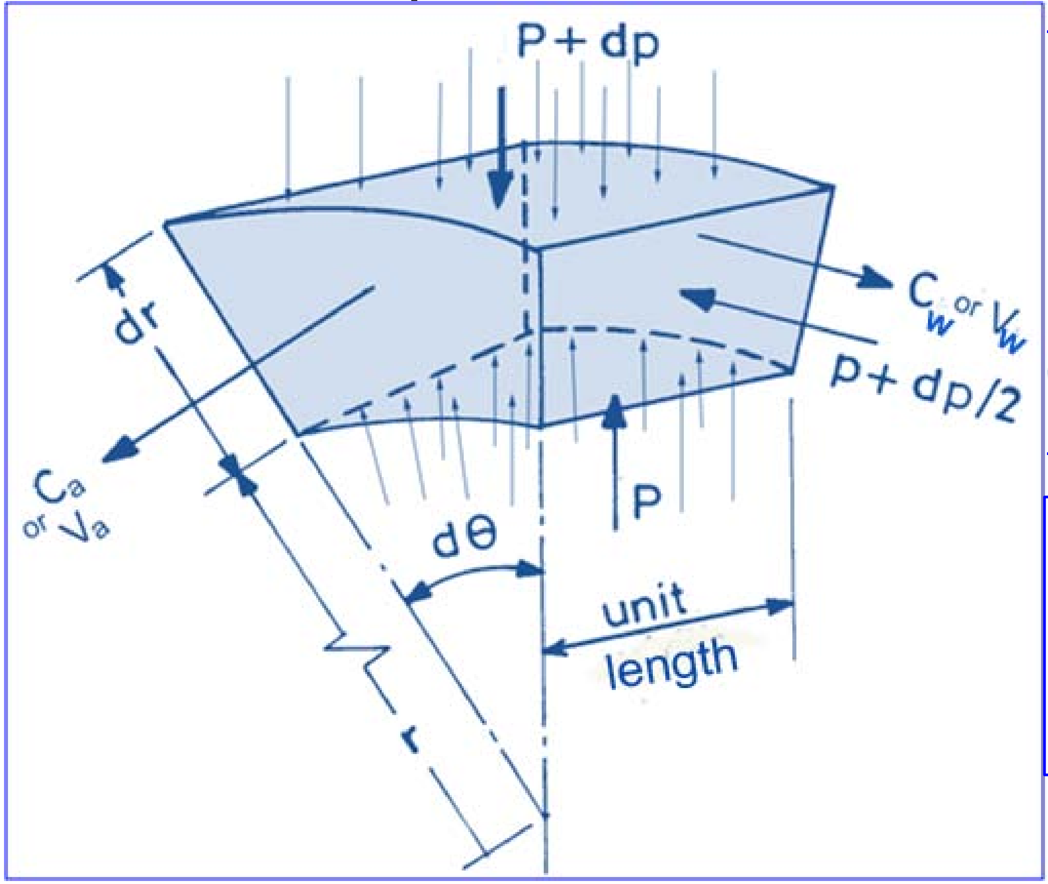
It may be recalled that this element is also executing a path through the curved diffusing passage between the rotor blades.

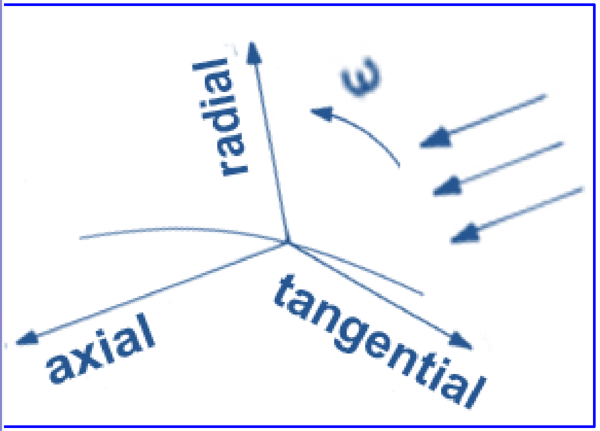
Simple three dimensional flow analysis : *Initial assumptions*

1) Radial movement of the flow is governed by the radial equilibrium of forces
2) Radial movements occur within the blade passage only and not outside it
3) Flow analysis involves balancing the radial force exerted by the blade rotation
4) Gravitational forces can be neglected

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Consider this *fluid element* of unit axial length subtended by an angle $d\theta$, of thickness dr, along which the pressure variation is from p to $p+dp$.





Subscript w refers to tangential / whirl component of the flow

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Resolving all the aerodynamic forces, acting on this element, in the radial direction,

we get,

$$(p+dp)(r+dr).d\theta.1 - p.r.1.d\theta - 2(p+dp/2).dr.(d\theta/2).1$$

$$= \rho.\ dr.\ r.\ C_w^2\ / r$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Neglecting the second order terms (products of small terms e.g. **dp.dr** etc) the equation reduces to

$$\boxed{\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}.C_w^2}$$

This is called the

Simple Radial Equilibrium Equation

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Invoke the laws of fluid and thermo-dynamics

1) $H = h + C^2/2 = c_pT + ½\,(C_a^{\,2} + C_w^{\,2})$

→ Energy Eqn

2) $c_p.T = \dfrac{\gamma}{\gamma - 1}\dfrac{p}{\rho}$ → From Equation of state

3) $\dfrac{p}{\rho^{\gamma}} = c$ → Isentropic Law

Where, H is total enthalpy, h is static enthalpy pressure p, density ρ, are the fluid properties and c_p and γ are the thermal properties of air at the operating condition substituting for c_p from Eqn(2) and then differentiating the eqn (1) w.r.t. r ,

we get

$$\frac{dH}{dr} = C_a \frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{\gamma}{\gamma - 1} \cdot \left[\frac{1}{\rho} \frac{dp}{dr} - \frac{p}{\rho^2} \frac{dp}{dr} \right]$$

Differentiating the eqn 3 (isentropic law) we get

$$\frac{d\rho}{dr} = \frac{\rho}{\gamma . p}\frac{dp}{dr}$$

Substituting this in the new energy equation we get

$$\frac{dH}{dr} = C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr} + \frac{1}{\rho}\frac{dp}{dr}$$

Now invoking the simple *radial equilibrium equation* developed earlier in the energy equation

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{1}{r}.C_w^2$$

We get

$$\frac{dH}{dr} = C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr} + \frac{{C_w}^2}{r}$$

- At entry to the compressor, except near the hub and the casing, enthalpy H (r) = constant.

- Using the condition of *uniform work distribution along the blade length (i.e. radially constant)* we can say

$$\frac{dH}{dr} = 0$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Thus, the energy equation would be written as,

$$\frac{dC_a}{dr} + C_w \frac{dC_w}{dr} + \frac{C_w^{\ 2}}{r} = 0$$

Now, if Ca = constant at all radii, then the first term is zero and the above equation reduces to

$$C_w \frac{dC_w}{dr} = -\frac{C_w^{\ 2}}{r}$$

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Therefore, the equation becomes

$$\frac{dC_W}{dr} = -\frac{dr}{r}$$

This yields, on integration

$C_w \cdot r =$ constant.

This condition is commonly known as the ***<u>Free Vortex</u>* <u>Law</u>**

- The term Free Vortex essentially denotes that the strength of the vortex (*or lift per unit length*) created by each airfoil section used from the root to the tip of the blade remains constant

$$\text{Lift}, L = \rho.V.\Gamma$$

where, ρ is the density,
V is the inlet velocity, and
Γ is the strength of circulation

- It, therefore, means that at the trailing edge of the blade the trailing vortex sheet has constant strength from the root to the tip of the blade

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay

Next Class ---

Free Vortex Design Law
and
Other Blade design laws

Prof. Bhaskar Roy, Prof. A M Pradeep, Department of Aerospace, IIT Bombay