TURBOMACHINERY AERODYNAMICS

Lect 40

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Fundamentals of CFD for use in Turbomachinery Analysis

- Physics of fluid mechanics are often captured in Partial Differential Equations (PDEs), mostly 2^{nd} order PDEs.

- Generally the governing equations are a set of coupled, non-linear PDEs valid within an arbitrary (or irregular) domain and are subject to various initial and boundary conditions.

- Purely analytical solutions of many fluid mechanic equations are limited due to imposition of various boundary conditions of typical fluid flow problems.

- Experimental data are often used for validation of CFD solutions. Together they are used for design purposes.

Linear and Non-linear PDEs

Linear :

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x} \quad \text{where, } a > 0$$

(1-d Wave Equation)

Non-Linear

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x}$$

(Inviscid Flow)

Laplace's Equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

where normally x and y are independent variables and ϕ is a dependant variable

Poisson's equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = f(x, y)$$

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$$A\frac{\partial^2\phi}{\partial x^2}+B\frac{\partial^2\phi}{\partial x.\partial y}+C\frac{\partial^2\phi}{\partial x^2}+D\frac{\partial^2\phi}{\partial y^2}$$

A,B,C,D,E,F,G are functions of x,y & ϕ

$$+E\frac{\partial\phi}{\partial y}+F\phi+G=0$$

Assume that $f = f(x,y)$ is a solution of the above differential equation.

This solution, typically is a surface in space, and the solutions produce space curves called characteristics.

2nd order derivatives along the characteristics are often indeterminate and may be discontinuous across the characteristics.

The 1st order derivatives are continuous.

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A simpler version of the 2nd order equation may be written as:

$$A\left(\frac{dy}{dx}\right)^2 - B\left(\frac{dy}{dx}\right) + C = 0$$

Solution of this yields the equations of the characteristics in the physical space :

$$\left(\frac{dy}{dx}\right) = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

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These characteristic curves can be real or imaginary depending on the values of $(B^2 - 4AC)$.

A 2nd order PDE is classified according to the sign of $(B^2 - 4AC)$:

(a) $(B^2 - 4AC) < 0$ – Elliptic - M<1.0 – Subsonic flow

(b) $(B^2 - 4AC) = 0$ -- Parabolic M =1.0 – Sonic flow

(c) $(B^2 - 4AC) > 0$ -- Hyperbolic M>1.0 –Supersonic flow

PDEs → **Algebraic equations : Finite Difference Equations**

↓

Various Finite Difference Techniques

- An Elliptic PDE has no real characteristics . A disturbance is propagated instantly in all directions within the region
- The domain solution of an elliptic PDE is a closed region. Providing the boundary condition uniquely yields the solution within the domain
- The solution domain for a parabolic PDE is open region.
- For Parabolic PDE one characteristic line exists
- A hyperbolic PDE has two characteristic lines
- A complete description of 2^{nd} order hyperbolic PDE requires two sets of initial conditions and two sets of boundary conditions

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Initial and Boundary conditions (*ICs and BCs*)

ICs : A dependant variable is prescribed at some initial condn

BCs : A dependent variable or its derivative must satisfy on the boundary of the domain of the PDE

1) **Dirichlet BC : Dependent Variable prescribed at boundary**
2) **Neumann BC: Normal gradient of the D.V. is specified**
3) **Robin BC : A linear combination of Dirichlet & Neumann**
4) **Mixed BC : Some part of the boundary has Dirichlet *BC* and some other part has Neumann *BC***

BCs → ***Body Surface*** | ***Far Field*** | ***Symmetry*** | ***In / Outflow***

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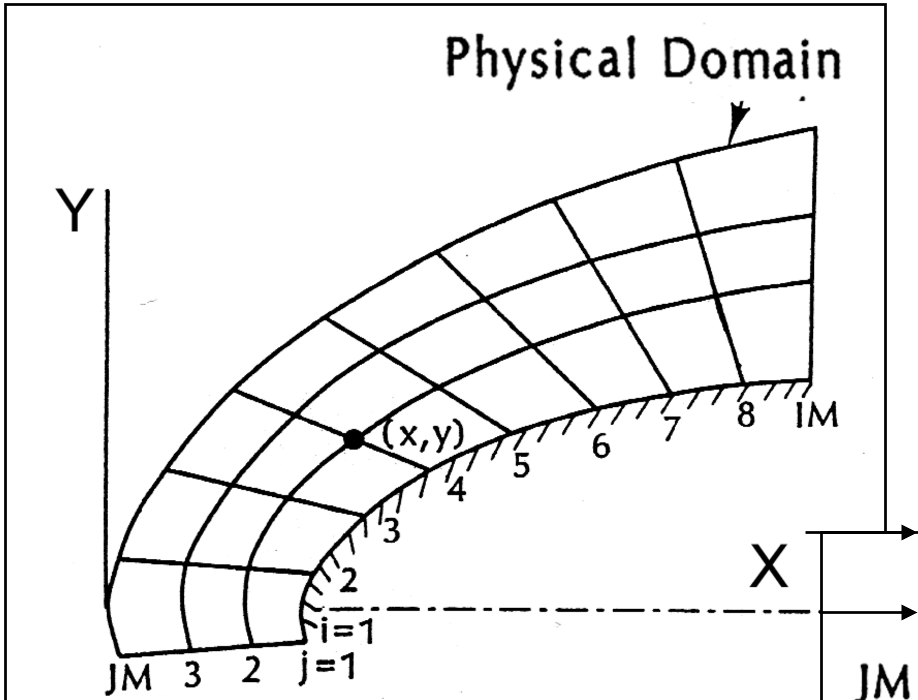
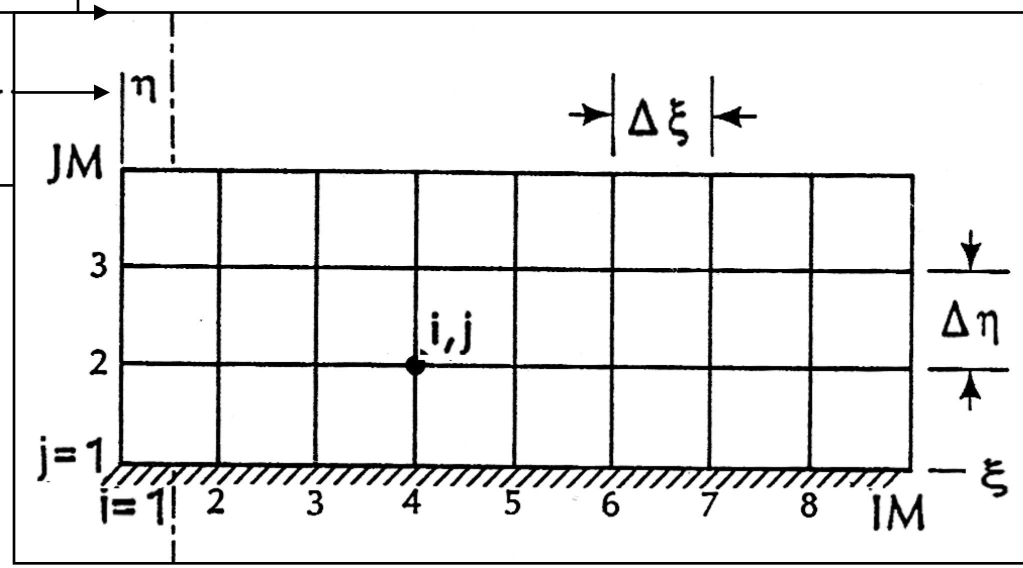
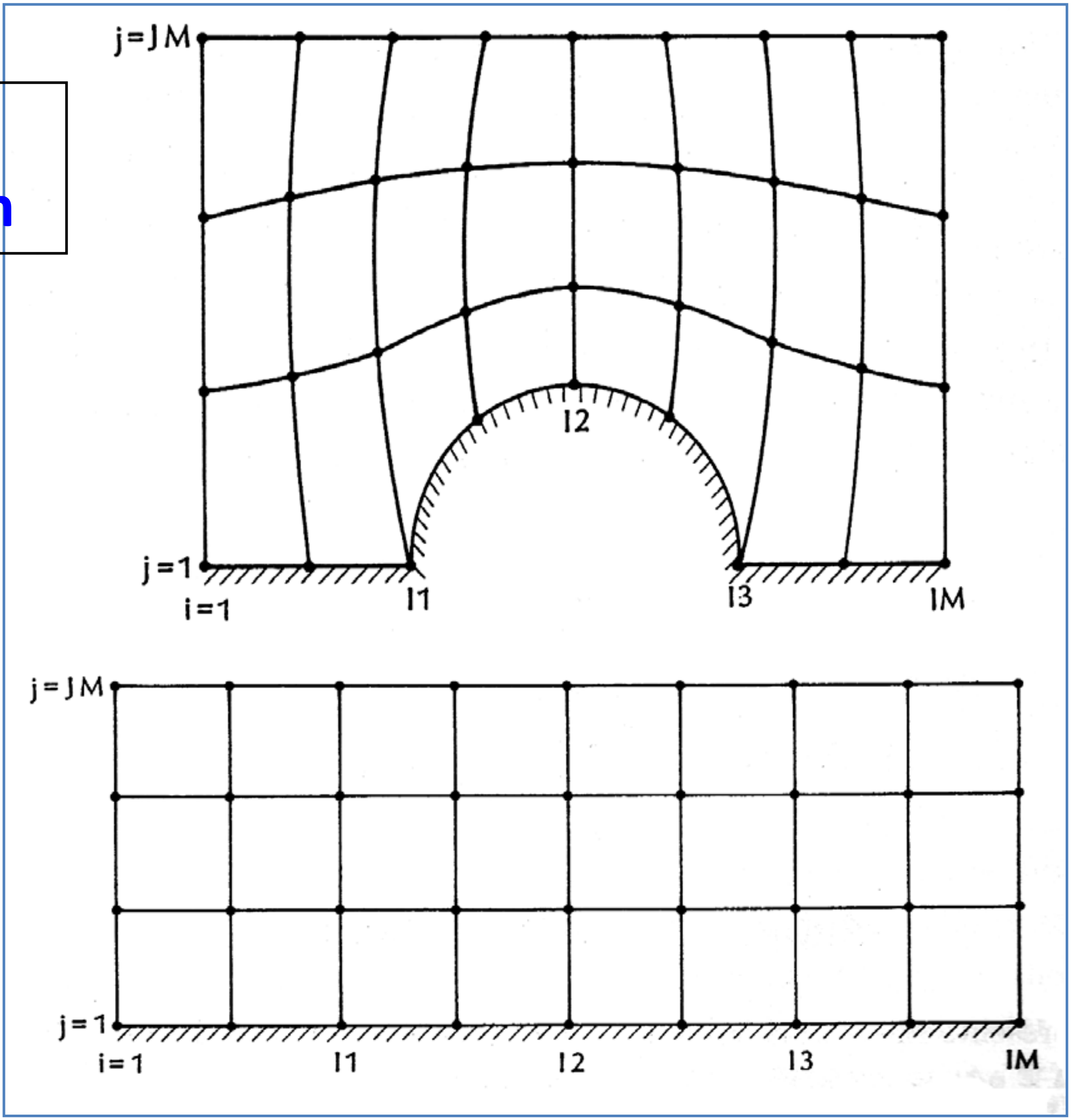


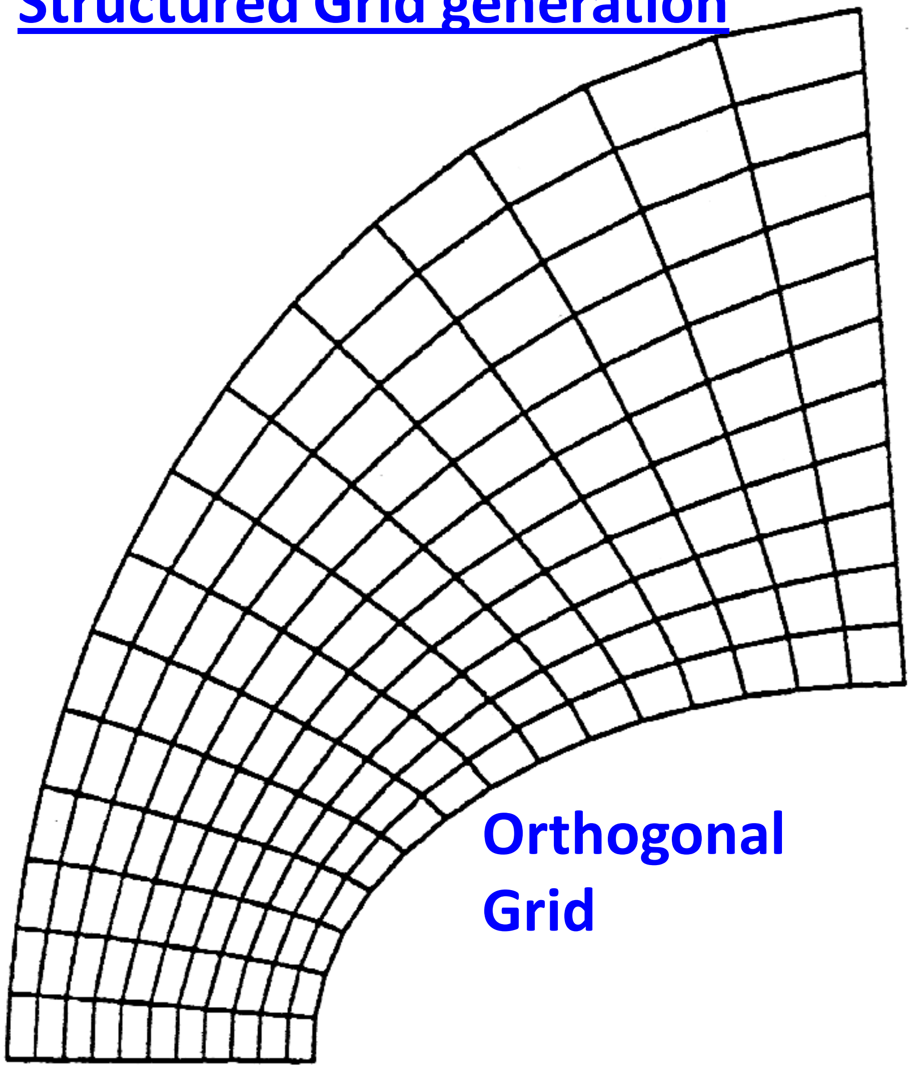
Computational space



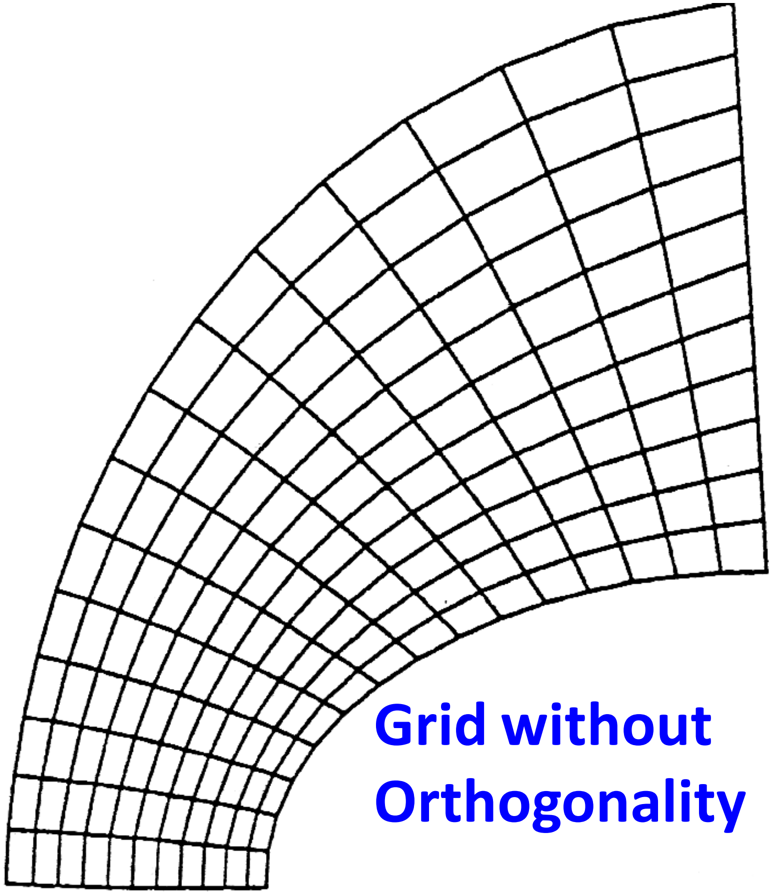
Domain Transformation



Structured Grid generation

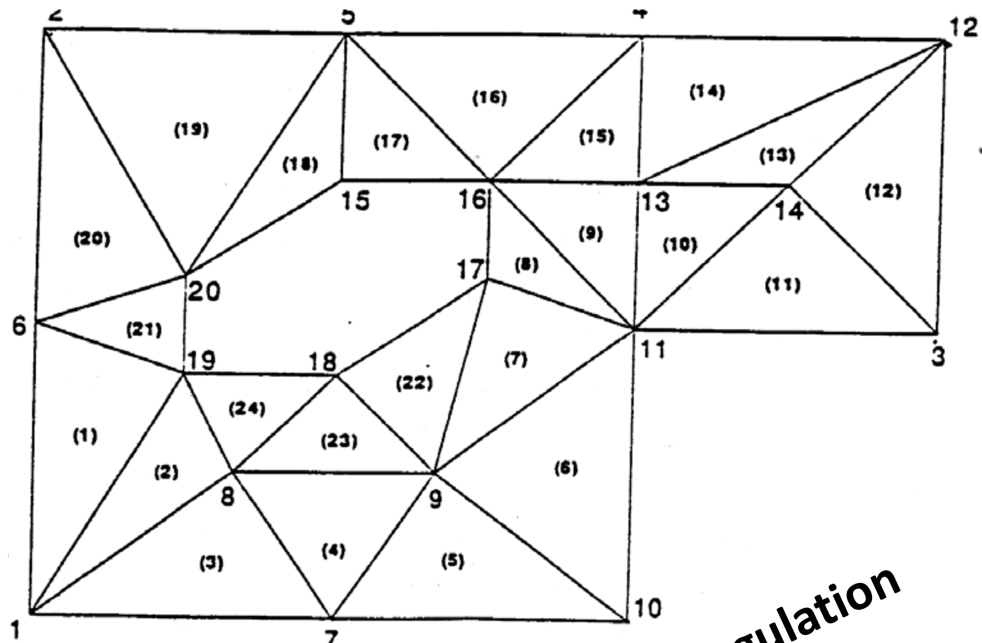


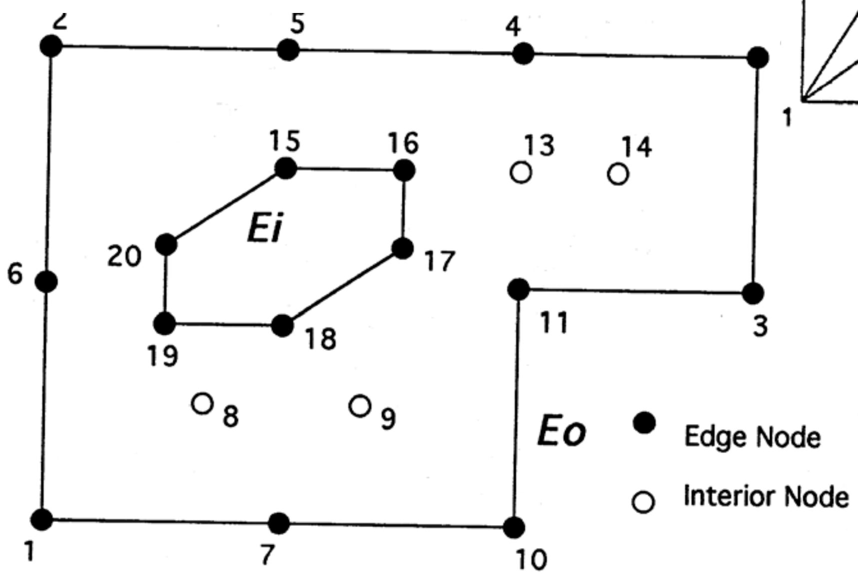
Orthogonal Grid



Grid without Orthogonality

Unstructured Grid generation





Domain Nodalization => Triangulation

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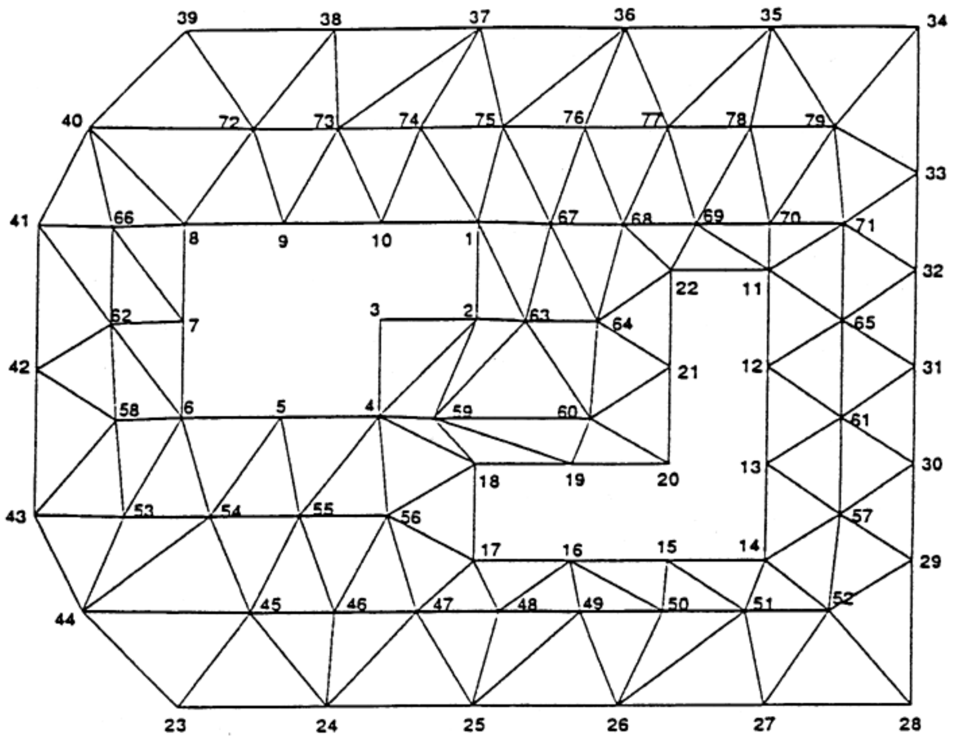
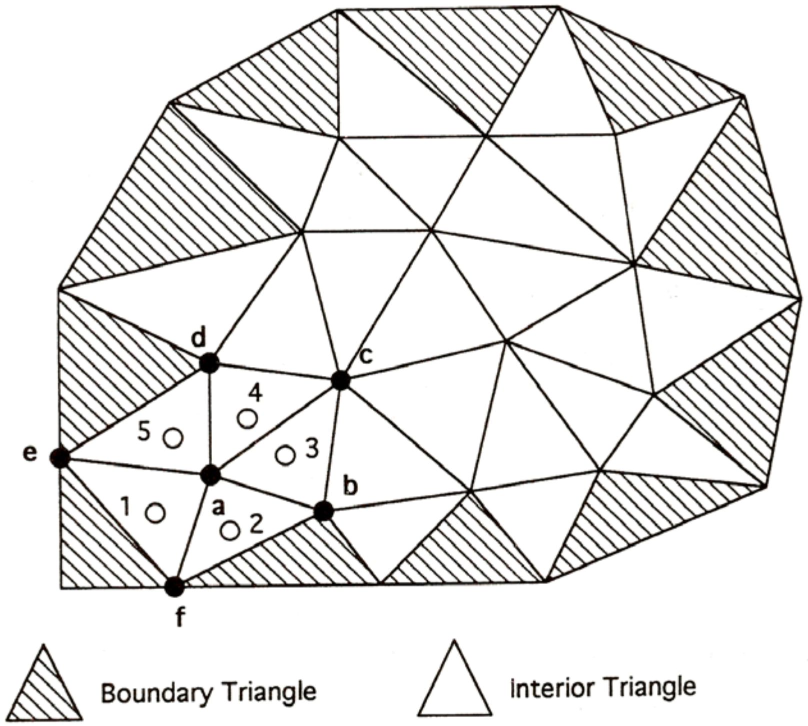


Unstructured Grid generation



CFD in Blade Design

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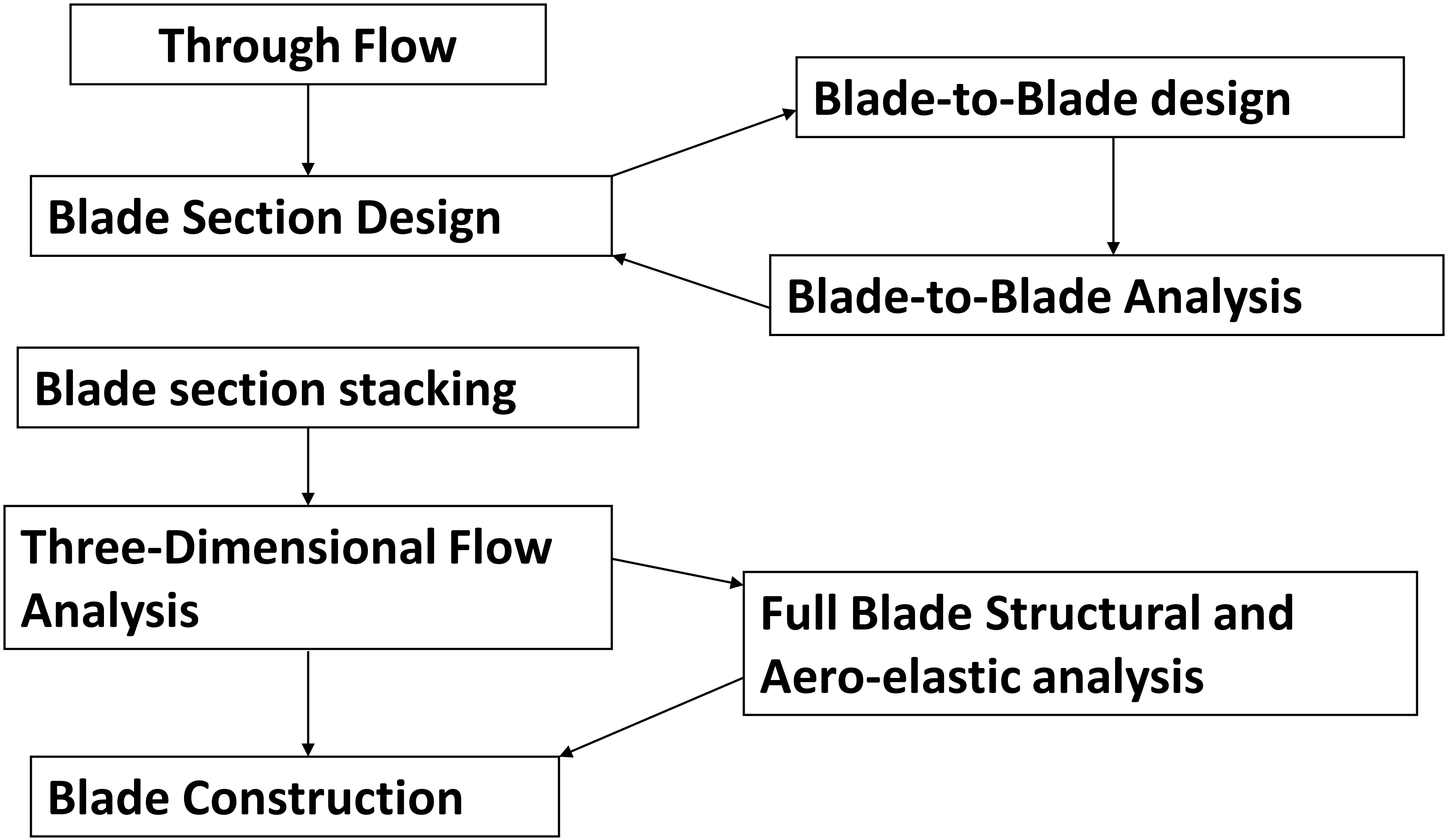
Blade design system



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Through Flow Program

Input : a) i) Annulus Information
ii) Blade row exit information
iii) Inlet profiles of Pr, Temp, a_1
iv) Inlet Mass flow
v) Rotational speeds of rotors
vi) Blade geometry, Loss distributions
vii) Passage averaged perturbation terms

Output : b) i) Blade row inlet and exit conditions
ii) Streamline definition and streamtube height

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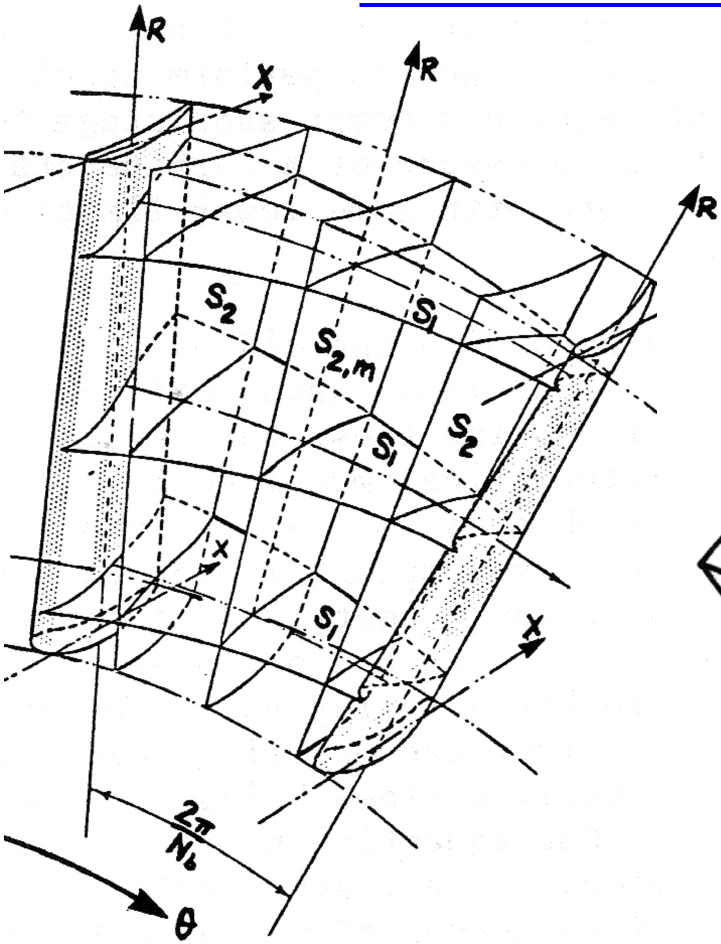
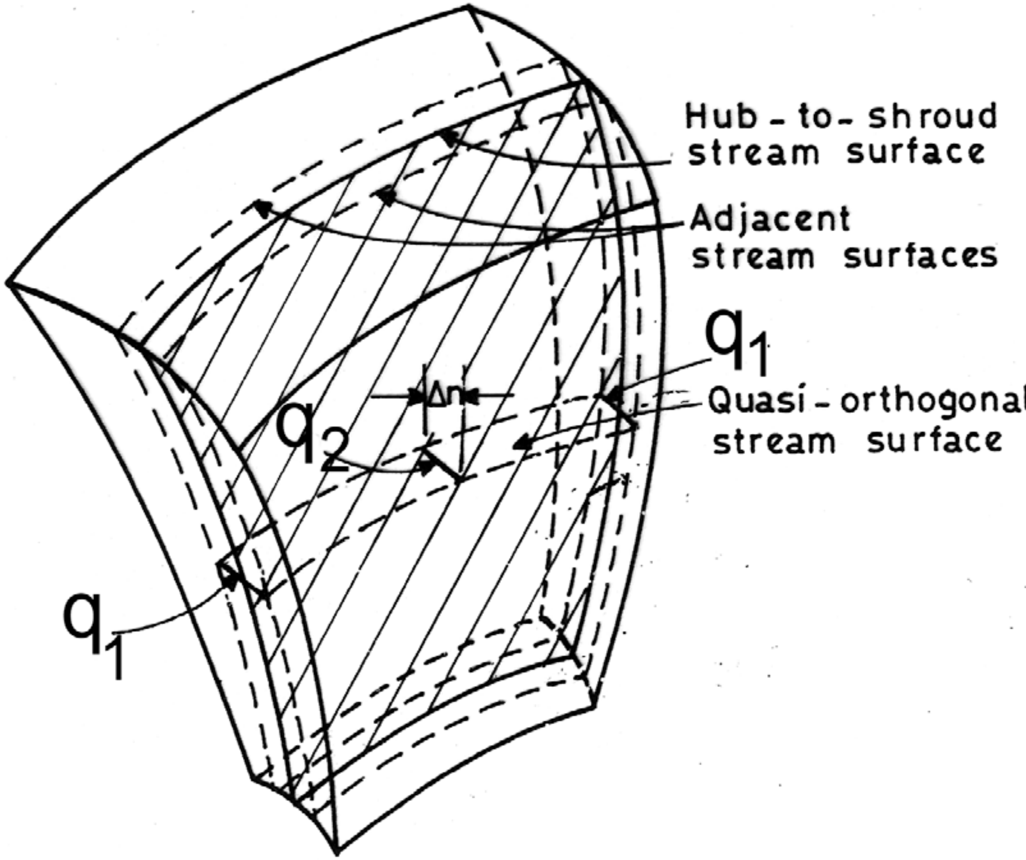
Blade-to-blade Flow Program





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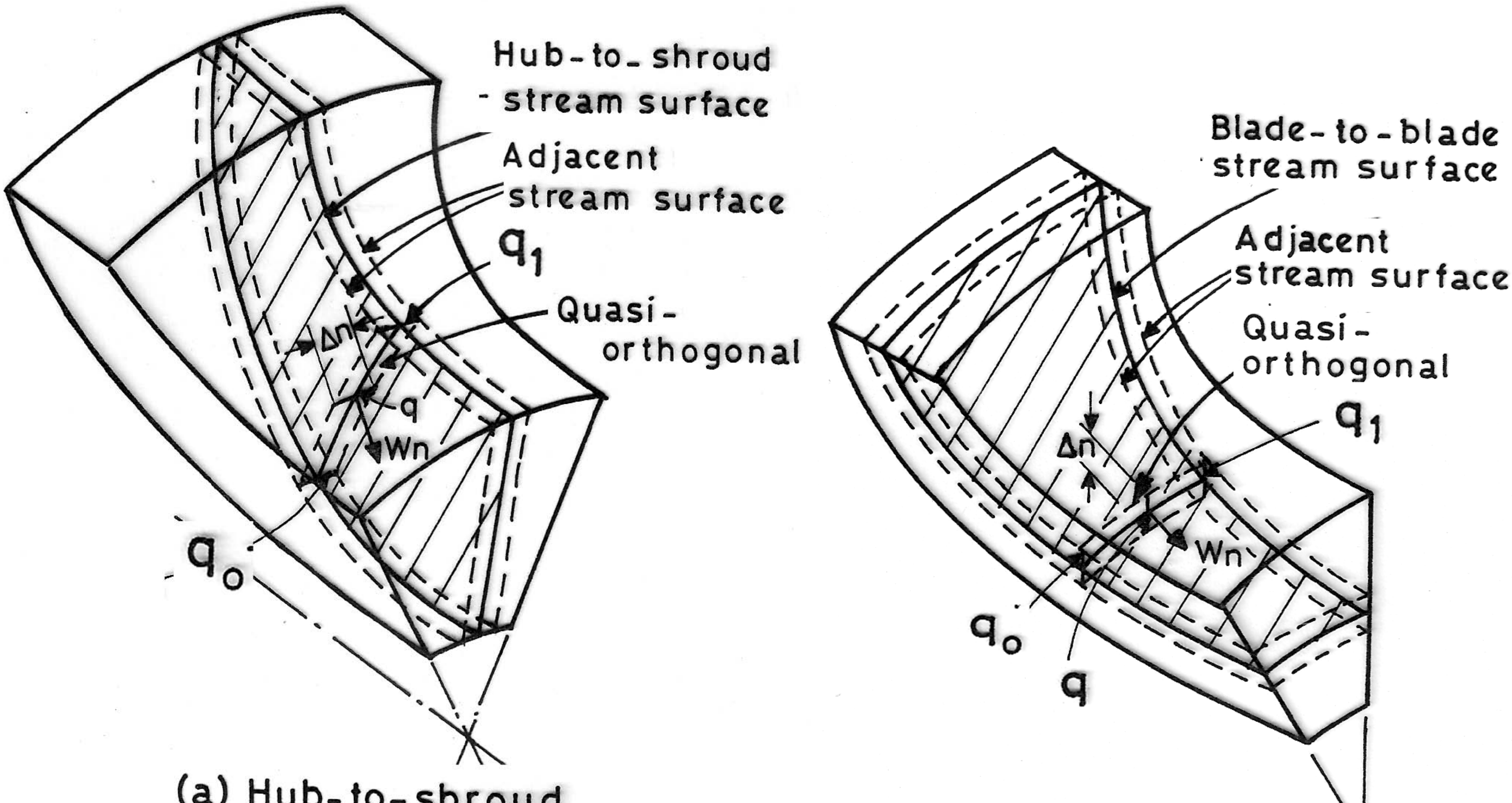
Blade-to-blade Flow Program



(a) Hub-to-shroud

(b) Blade-to-blade

Blade-to-Blade program

Input : Blade geometry
Inlet and Exit Velocity distribution
Streamline Definition

Output : Surface velocity distribution
Profile and loss distribution

Section Stacking Program

Input : Blade section geometry
Stacking points and stacking line
Axial and Tangential leans (sweep and Dihedral)

Output : Three-Dimensional blade geometry

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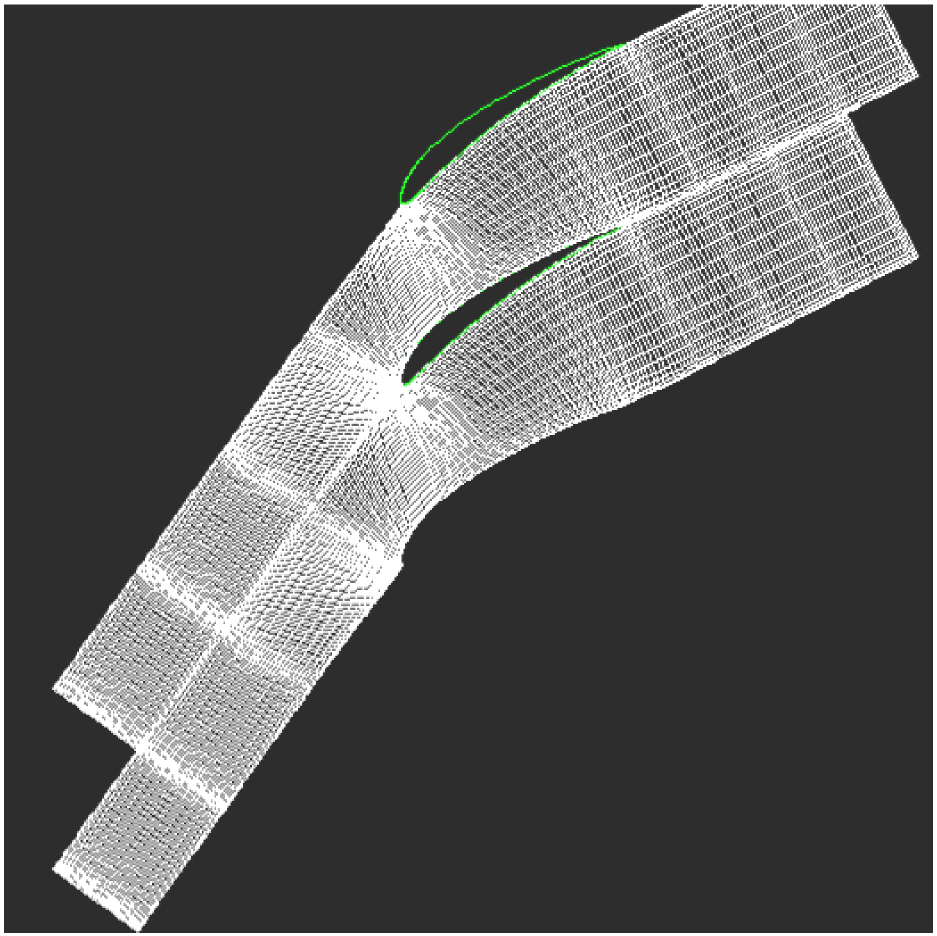
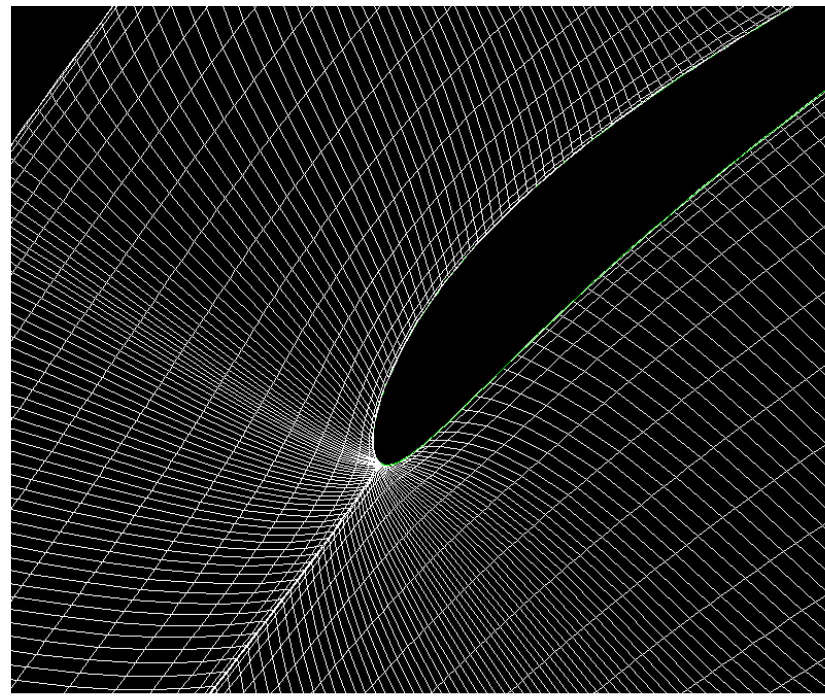
Blade-to-Blade program

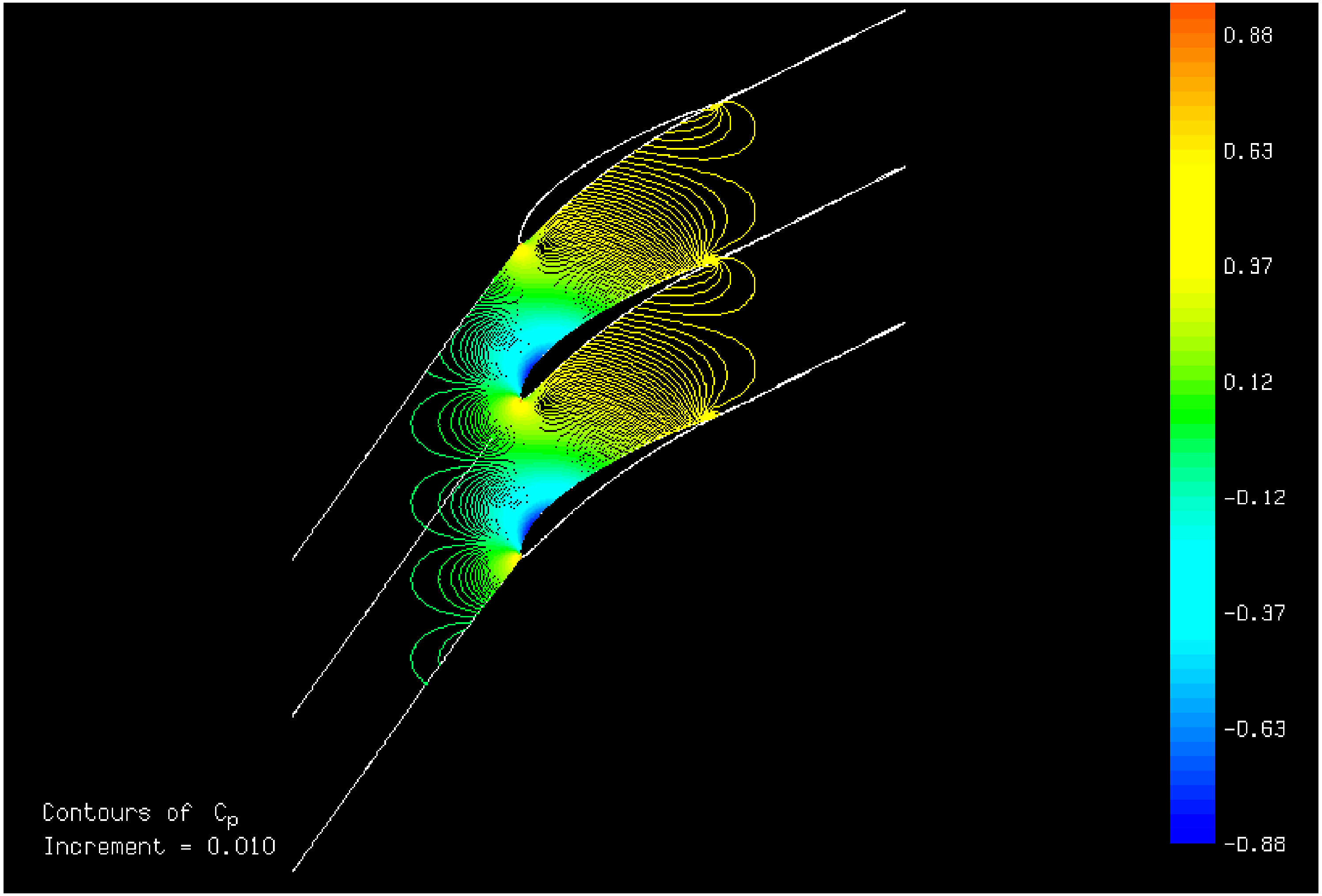
2D MISES code for Cascade Analysis





2D MISES code for Cascade Analysis

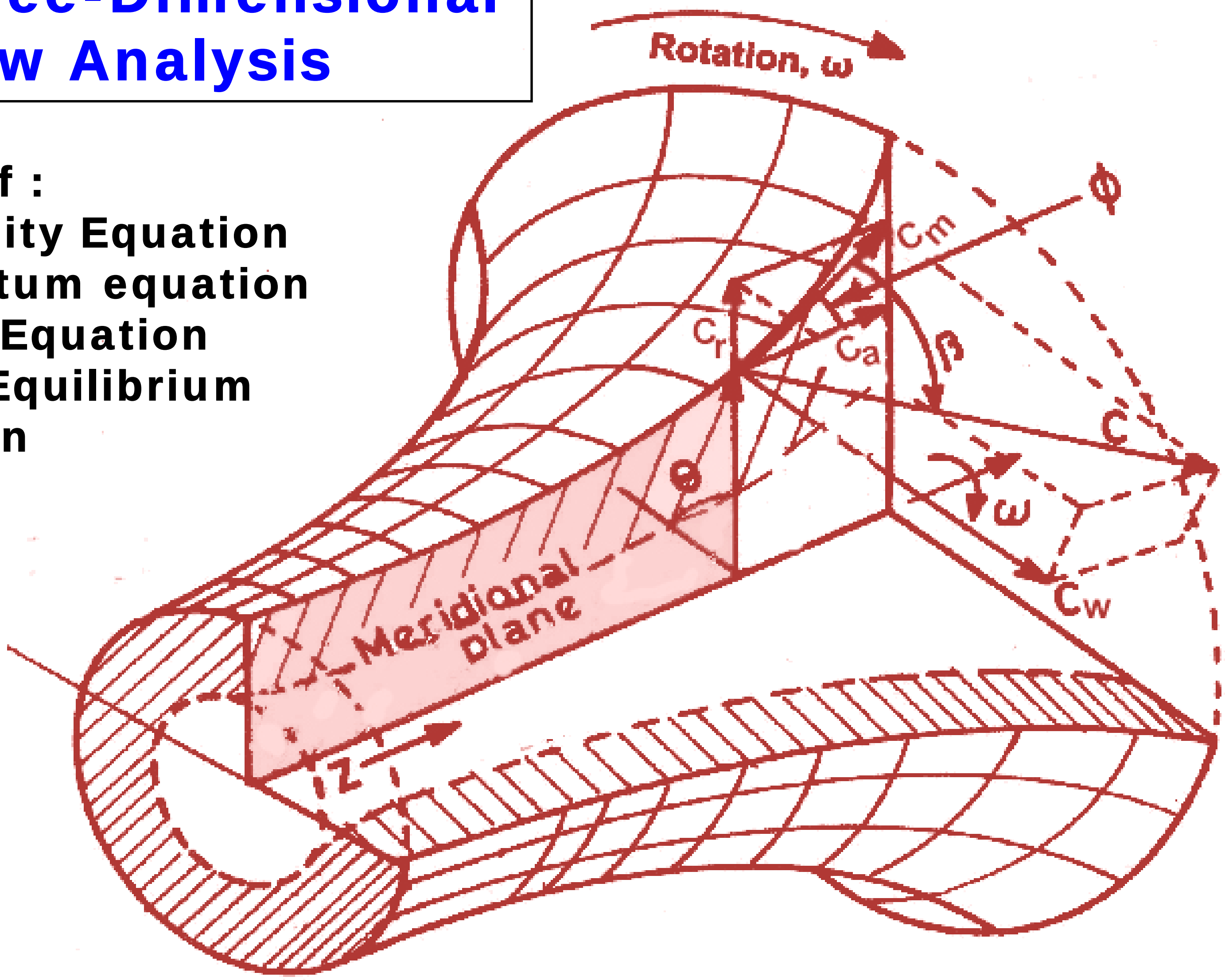
C_p contour



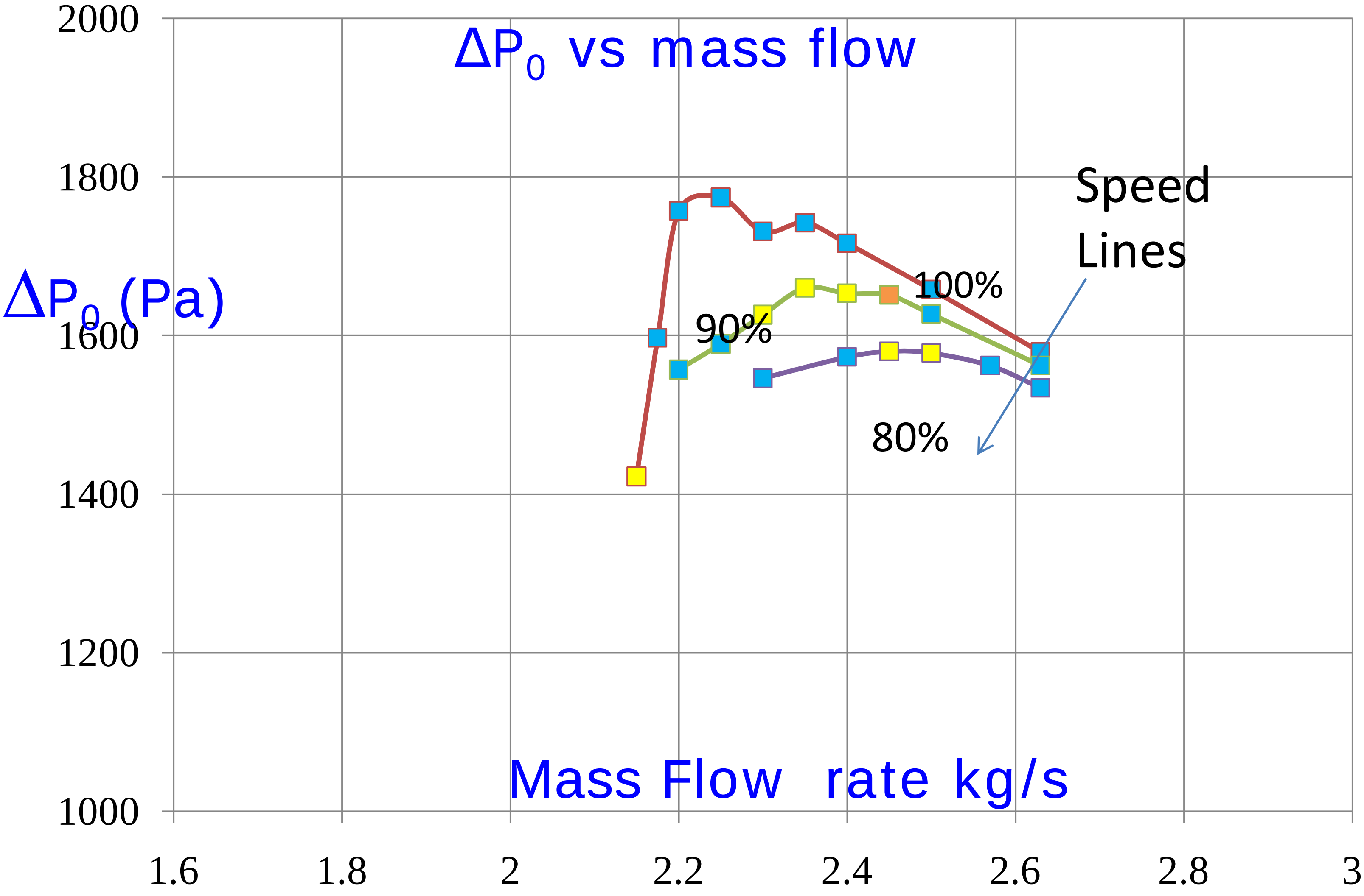
Three-Dimensional Flow Analysis

Solution of :

1) Continuity Equation
2) Momentum equation
3) Energy Equation
4) Radial Equilibrium Equation



Final Output : Compressor Rotor Characteristics



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