Lecture – 21

# Pole Placement Control Design

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# Pole Placement Control Design

## <u>Assumptions:</u>

- The system is completely state controllable.
- The state variables are measurable and are available for feedback.
- Control input is unconstrained.

# Pole Placement Control Design

## Objective:

The closed loop poles should lie at $\mu_1, \ldots, \mu_n$, which are their 'desired locations'.

## Difference from classical approach:

Not only the "dominant poles", but "all poles" are forced to lie at specific desired locations.

**Necessary and sufficient condition:**

The system is completely state controllable.

# Closed Loop System Dynamics

$$\dot{X} = AX + BU$$

The control vector $U$ is designed in the following state feedback form

$$U = -KX$$

This leads to the following closed loop system

$$\dot{X} = (A - BK)X = A_{CL}X$$

where $A_{CL} \triangleq (A - BK)$

# Philosophy of Pole Placement Control Design

The gain matrix $K$ is designed in such a way that

$$\left| sI - (A - BK) \right| = (s - \mu_1)(s - \mu_2) \quad \cdots \quad (s - \mu_n)$$

where $\mu_1, \cdots, \mu_n$ are the desired pole locations.

# Pole Placement Design Steps: Method 1 (low order systems, $n \le 3$)

- Check controllability
- Define $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$
- Substitute this gain in the desired characteristic polynomial equation

$$\left| sI - A + BK \right| = (s - \mu_1) \cdots (s - \mu_n)$$

- Solve for $k_1, k_2, k_3$ by equating the like powers on both sides

# Pole Placement Control Design: Method – 2

$$\dot{X} = AX + Bu$$

$$u = -KX, \qquad K = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix}$$

**Let the system be in first companion (controllable canonical) form**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \ddots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & -a_{n-3} & \cdots & -a_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

After applying the control, the closed loop system dynamics is given by

$$\dot{X} = \left(A - BK\right)X = A_{CL}X$$

$$A_{CL} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \\ k_1 & k_2 & k_3 & \cdots & k_n \end{bmatrix}$$

$$A_{CL} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & 1 \\ \left(-a_n - k_1\right) & \left(-a_{n-1} - k_2\right) & \cdots & \cdots & \cdots & \left(-a_1 - k_n\right) \end{bmatrix} \cdots\cdots\cdots(1)$$

# Pole Placement Control Design: Method – 2

If $\mu_1, \cdots, \mu_n$ are the desired poles. Then the desired characteristic polynomial is given by,

$$(s-\mu_1)\cdots(s-\mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_n$$

This characteristic polynomial, will lead to the closed loop system matrix as

$$A_{CL} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & & & \\ & & & & \\ \vdots & & \ddots & & 1 \\ 0 & 0 & & \cdots & \\ -\alpha_n & -\alpha_{n-1} & -\alpha_{n-2} & \cdots & -\alpha_1 \end{bmatrix} \cdots\cdots(2)$$

State space form

# Pole Placement Control Design: Method – 2

**Comparing Equation (1) and (2), we arrive at:**

$$\begin{bmatrix} a_n + k_1 = \alpha_n \\ a_{n-1} + k_2 = \alpha_{n-1} \\ \vdots \\ a_1 + k_n = \alpha_1 \end{bmatrix} \Rightarrow \begin{bmatrix} k_1 = (\alpha_n - a_n) \\ k_2 = (\alpha_{n-1} - a_{n-1}) \\ \vdots \\ k_n = (\alpha_1 - a_1) \end{bmatrix}$$

$$K = (\alpha - a) \qquad (\text{Row vector form})$$

# What if the system is not given in the first companion form?

Define a transformation $X = T\hat{X}$

$$\dot{\hat{X}} = T^{-1}\dot{X}$$

$$\dot{\hat{X}} = T^{-1}\left(AX + Bu\right)$$

$$\dot{\hat{X}} = \left(T^{-1}AT\right)\hat{X} + \left(T^{-1}B\right)u$$

Design a $T$ such that $T^{-1}AT$ will be in first companion form.

Select $T = MW$

where $M \triangleq \left[B \quad AB \cdots\cdots\cdots A^{n-1}B\right]$ is the controllability matrix

# Pole Placement Control Design: Method – 2

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & & \iddots & \iddots & 0 \\ & \iddots & \iddots & \cdots & \vdots \\ a_1 & 1 & \cdots & \cdots & \vdots \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

Next, design a controller for the transformed system (using the technique for systems in first companion form).

$$u = -\hat{K}\hat{X} = -(\hat{K}T^{-1})X = -KX$$

***Note:* Because of its role in control design as well as the use of $M$ (Controllability Matrix) in the process, the '<u>first companion form</u>' is also known as 'Controllable Canonical form'.**

# Pole Placement Design Steps: Method 2: Bass-Gura Approach

- Check the controllability condition
- Form the characteristic polynomial for *A*

  $$|sI - A| = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n$$

  find $a_i$'s
- Find the Transformation matrix $T$
- Write the desired characteristic polynomial

  $$(s - \mu_1)\cdots(s - \mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_n$$

  and determine the $\alpha_i$'s
- The required state feedback gain matrix is

  $$K = [(\alpha_n - a_n) \quad (\alpha_{n-1} - a_{n-1}) \quad \cdots \quad (\alpha_1 - a_1)]\, T^{-1}$$

# Pole Placement Design Steps: Method 3 (Ackermann's formula)

Define $\tilde{A} = A - BK$

desired characteristic equation is

$$\left| sI - (A - BK) \right| = (s - \mu_1) \cdots (s - \mu_n)$$

$$| sI - \tilde{A} | = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_{n-1} s + \alpha_n = 0$$

Caley-Hamilton theorem states that every matrix $A$ satisfies its own characteristic equation

$$\phi(\tilde{A}) = \tilde{A}^n + \alpha_1 \tilde{A}^{n-1} + \alpha_2 \tilde{A}^{n-2} + \cdots + \alpha_{n-1} \tilde{A} + \alpha_n = 0$$

For the case $n = 3$ consider the following identities.

$$\mathbf{I} = \mathbf{I}$$

$$\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{BK}$$

$$\tilde{\mathbf{A}}^2 = (\mathbf{A} - \mathbf{BK})^2 = \mathbf{A}^2 - \mathbf{ABK} - \mathbf{BK}\tilde{\mathbf{A}}$$

$$\tilde{\mathbf{A}}^3 = (\mathbf{A} - \mathbf{BK})^3 = \mathbf{A}^3 - \mathbf{A}^2\mathbf{BK} - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2$$

# Pole Placement Design Steps: Method 3: (Ackermann's formula)

Multiplying the identities in order by $\alpha_3$, $\alpha_2$, $\alpha_1$ respectively and adding we get

$$\alpha_3 \mathbf{I} + \alpha_2 \tilde{\mathbf{A}} + \alpha_1 \tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3$$

$$= \alpha_3 \mathbf{I} + \alpha_2 (\mathbf{A} - \mathbf{BK}) + \alpha_1 (\mathbf{A}^2 - \mathbf{ABK} - \mathbf{BK}\tilde{\mathbf{A}}) + \mathbf{A}^3 - \mathbf{A}^2\mathbf{BK} - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2$$

$$= \alpha_3 \mathbf{I} + \alpha_2 \mathbf{A} + \alpha_1 \mathbf{A}^2 + \mathbf{A}^3 - \alpha_2 \mathbf{BK} - \alpha_1 \mathbf{ABK} - \alpha_1 \mathbf{BK}\tilde{\mathbf{A}} - \mathbf{A}^2\mathbf{BK} - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2 \quad \text{..........(1)}$$

From Caley-Hamilton Theorem for $\tilde{\mathbf{A}}$

$$\alpha_3 \mathbf{I} + \alpha_2 \tilde{\mathbf{A}} + \alpha_1 \tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3 = \phi(\tilde{\mathbf{A}}) = \mathbf{0}$$

And also we have for A

$$\alpha_3 \mathbf{I} + \alpha_2 \mathbf{A} + \alpha_1 \mathbf{A}^2 + \mathbf{A}^3 = \phi(\mathbf{A}) \neq \mathbf{0}$$

# Pole Placement Design Steps: Method 3 (Ackermann's formula)

Substituting $\varphi(\tilde{\mathbf{A}})$ and $\varphi(\mathbf{A})$ in equation (1) we get

$$\phi(\tilde{\mathbf{A}}) = \phi(\mathbf{A}) - \alpha_2\mathbf{BK} - \alpha_1\mathbf{BK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2 - \alpha_1\mathbf{ABK} - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{A}^2\mathbf{BK}$$

0

$$\phi(\mathbf{A}) = \mathbf{B}(\alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2) + \mathbf{AB}(\alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}}) + \mathbf{A}^2\mathbf{BK}$$

$$= [\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \mathbf{A}^2\mathbf{B}]\begin{bmatrix} \alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K} \end{bmatrix}$$

**Since system is completely controllable inverse of the controllability matrix exists we obtain**

$$[\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \mathbf{A}^2\mathbf{B}]^{-1}\phi(\mathbf{A}) = \begin{bmatrix} \alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K} \end{bmatrix} \quad \text{.........}\mathbf{(2)}$$

# Pole Placement Design Steps: Method 3 (Ackermann's formula)

Pre multiplying both sides of the equation **(2)** with **[0 0 1]**

$$[0 \quad 0 \quad 1][\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \mathbf{A}^2\mathbf{B}]^{-1}\phi(\mathbf{A}) = [0 \quad 0 \quad 1]\begin{bmatrix} \alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K} \end{bmatrix} = \mathbf{K}$$

- For an arbitrary positive integer *n* ( number of states) *Ackermann's formula* for the state feedback gain matrix *K* is given by

$$K = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots & 1 \end{bmatrix}\begin{bmatrix} B & AB & A^2B & \cdots\cdots\cdots & A^{n-1}B \end{bmatrix}^{-1}\phi(A)$$

$$\text{where} \quad \phi(A) = A^n + \alpha_1 A^{n-1} + \cdots\cdots\cdots + \alpha_{n-1}A + \alpha_n I$$

$\alpha_i$'s are the coefficients of the desired characteristic polynomial

# Choice of closed loop poles : Guidelines

- Do not choose the closed loop poles far away from the open loop poles, otherwise it will demand high control effort

- Do not choose the closed loop poles very negative, otherwise the system will be fast reacting (i.e. it will have a small time constant)
  - In frequency domain it leads to large bandwidth, and hence noise gets amplified

# Choice of closed loop poles : Guidelines

- Use “Butterworth polynomials”

$$\left(\frac{s}{w_o}\right) = (-1)^{\frac{n+1}{2n}} = \left(\underbrace{e^{(j(2k+1)\pi)}}_{-1}\right)^{\frac{n+1}{2n}} \qquad k = 0, 1, 2 - - -$$

$w_o$ = a constant (like " natural frequency")

$n$ = system order (number of closed loop poles)

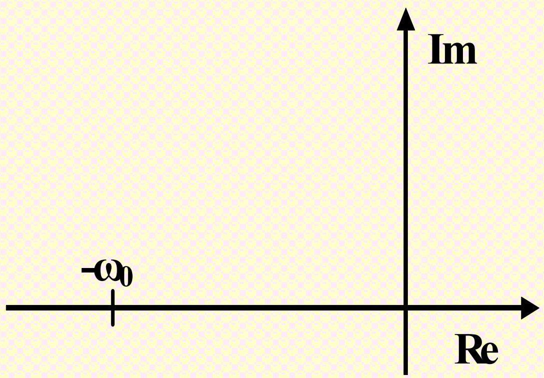
choose only stable poles.

Example: 1

let $n = 1$ only one pole

use $k = 1$

$s = \omega_0(\cos\pi + j\sin\pi) = -\omega_0$



# Choice of closed loop poles : Guidelines

Example 2:

Let n = 2 we know $(\cos\theta + j\sin\theta)^m = (\cos m\theta + j\sin m\theta)$

$$\frac{n+1}{2n} = \frac{3}{4}$$

$$s = \omega_0[\cos\left((2k+1)3\pi/4\right) + j\sin\left((2k+1)(3\pi/4)\right)]$$

$k = 0 \Rightarrow s_1 = \omega_0[\cos\left(3\pi/4\right) + j\sin\left((3\pi/4)\right)]$

stabilizing: accept.

$k = 1 \Rightarrow s_2 = \omega_0[\cos\left(9\pi/4\right) + j\sin\left((9\pi/4)\right)]$

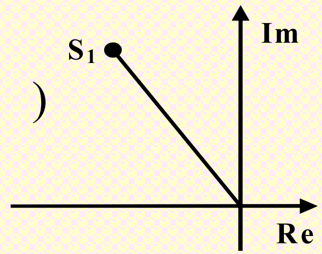
Destabilizing: reject.

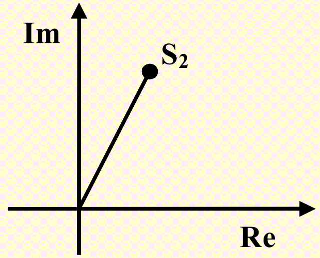
$k = 2 \Rightarrow s_3 = \omega_0[\cos\left(15\pi/4\right) + j\sin\left((15\pi/4)\right)]$

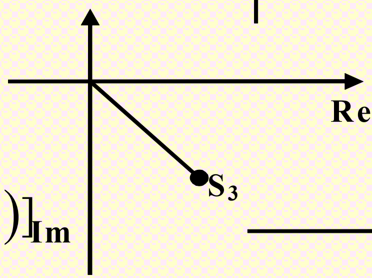
Destabilizing: reject.

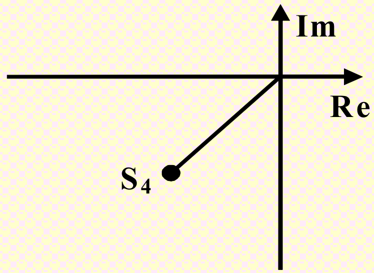
$k = 3 \Rightarrow s_4 = \omega_0[\cos\left(21\pi/4\right) + j\sin\left((21\pi/4)\right)]$

stabilizing: accept.



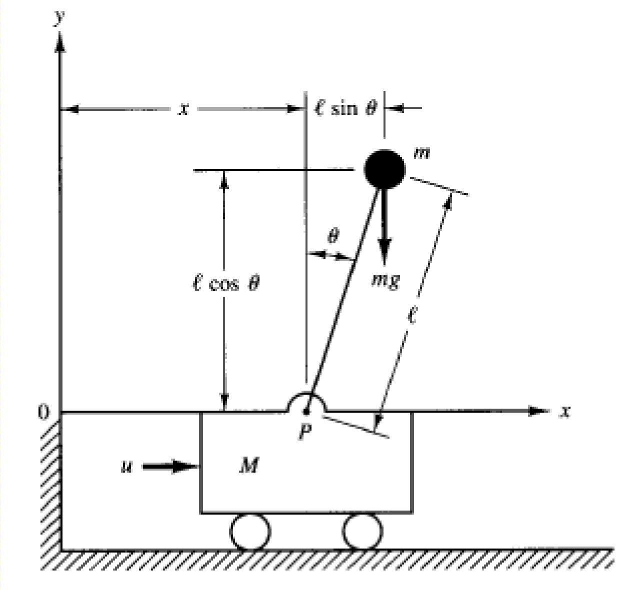



# Example: Inverted Pendulum

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$




$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

## Step 1: Check controllability

$$\mathbf{M} = [\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \mathbf{A}^2\mathbf{B} \;\vdots\; \mathbf{A}^3\mathbf{B}] = \begin{bmatrix} 0 & -1 & 0 & -20.601 \\ -1 & 0 & -20.601 & 0 \\ 0 & 0.5 & 0 & 0.4905 \\ 0.5 & 0 & 0.4905 & 0 \end{bmatrix}$$

$$\left| M \right| \neq 0$$

Hence, the system is controllable.

## Step 2: Form the characteristic equation and get $\boldsymbol{a_i}$'s

$$\left|s\mathbf{I}-\mathbf{A}\right| = \begin{bmatrix} s & -1 & 0 & 0 \\ -20.601 & s & 0 & 0 \\ 0 & 0 & s & -1 \\ 0.4905 & 0 & 0 & s \end{bmatrix}$$

$$= s^4 - 20.601s^2$$

$$= s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0$$

$$a_1 = 0, \qquad a_2 = -20.601, \qquad a_3 = 0, \qquad a_4 = 0$$

## Step 3: Find Transformation $\boldsymbol{T = MW}$ and its inverse

$$\mathbf{W} = \begin{bmatrix} a_3 & a_2 & a_1 & 1 \\ a_2 & a_1 & 1 & 0 \\ a_1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -20.601 & 0 & 1 \\ -20.601 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{T} = \mathbf{MW} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -9.81 & 0 & 0.5 & 0 \\ 0 & -9.81 & 0 & 0.5 \end{bmatrix}$$

$$\mathbf{T}^{-1} = \begin{bmatrix} -\frac{0.5}{9.81} & 0 & -\frac{1}{9.81} & 0 \\ 0 & -\frac{0.5}{9.81} & 0 & -\frac{1}{9.81} \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

## Step 4: Find $\alpha_i$'s *from desired poles* $\mu_1, \mu_2, \mu_3, \mu_4$

$$\mu_1 = -2 + j2\sqrt{3}, \qquad \mu_2 = -2 - j2\sqrt{3}, \qquad \mu_3 = -10, \qquad \mu_4 = -10$$

$$\begin{aligned}(s-\mu_1)(s-\mu_2)(s-\mu_3)(s-\mu_4) &= (s+2-j2\sqrt{3})(s+2+j2\sqrt{3})(s+10)(s+10)\\ &= (s^2+4s+16)(s^2+20s+100)\\ &= s^4+24s^3+196s^2+720s+1600\\ &= s^4+\alpha_1 s^3+\alpha_2 s^2+\alpha_3 s+\alpha_4 = 0\end{aligned}$$

$$\alpha_1 = 24, \qquad \alpha_2 = 196, \qquad \alpha_3 = 720, \qquad \alpha_4 = 1600$$

## Step 5: Find State Feed back matrix *K* and input *u*

$$\mathbf{K} = [\alpha_4 - a_4 \;\vdots\; \alpha_3 - a_3 \;\vdots\; \alpha_2 - a_2 \;\vdots\; \alpha_1 - a_1]\,\mathbf{T}^{-1}$$

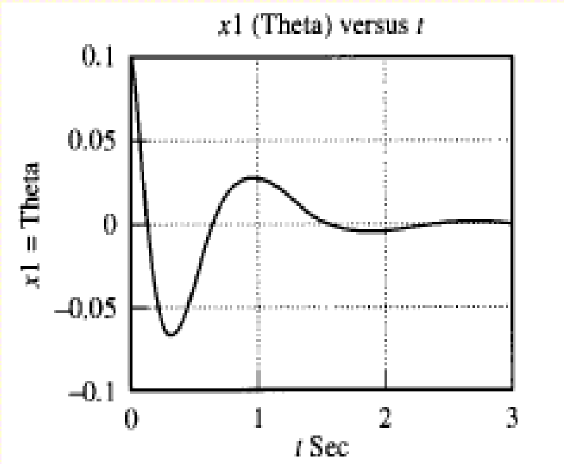
$$= [1600 - 0 \;\vdots\; 720 - 0 \;\vdots\; 196 + 20.601 \;\vdots\; 24 - 0]\,\mathbf{T}^{-1}$$

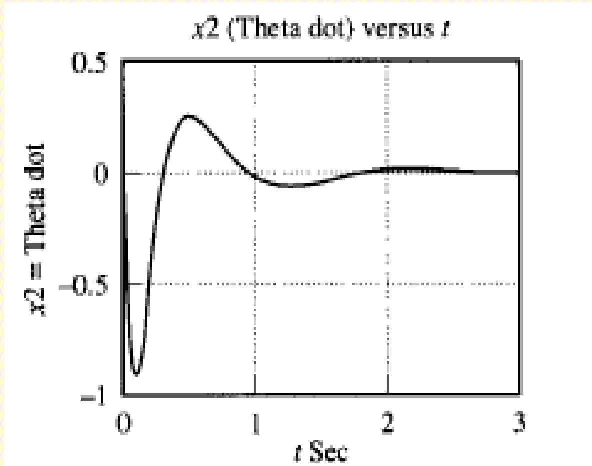
$$= [1600 \quad 720 \quad 216.601 \quad 24]\begin{bmatrix} -\frac{0.5}{9.81} & 0 & -\frac{1}{9.81} & 0 \\ 0 & -\frac{0.5}{9.81} & 0 & -\frac{1}{9.81} \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

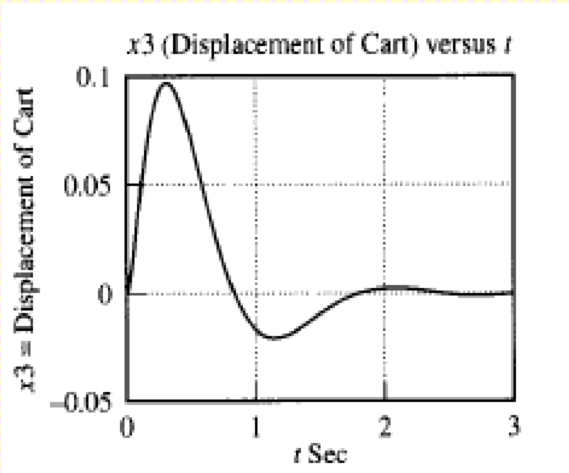
$$= [-298.1504 \quad -60.6972 \quad -163.0989 \quad -73.3945]$$

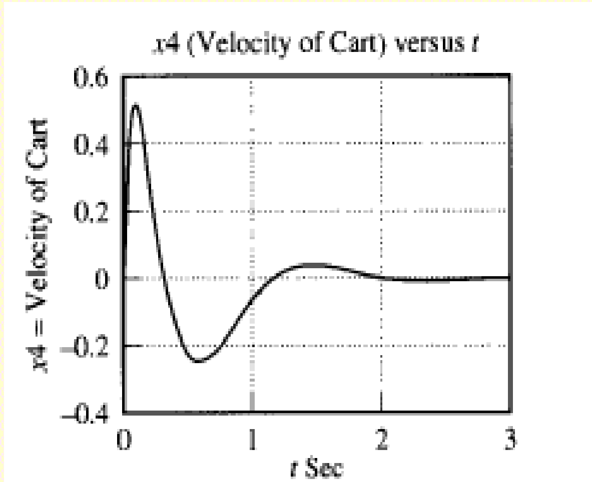
$$u = -\mathbf{Kx} = 298.1504x_1 + 60.6972x_2 + 163.0989x_3 + 73.3945x_4$$

# Time Simulation: Inverted Pendulum



# Choice of closed loop poles : Guidelines

- Do not choose the closed loop poles far away from the open loop poles. Otherwise, it will demand high control effort.

- Do not choose the closed loop poles very negative. Otherwise, the system will be fast reacting (i.e. it will have a small time constant)

  In frequency domain it leads to a large bandwidth, which in turn leads to amplification of noise!

# Multiple input systems

- The gain matrix is not unique even for fixed closed loop poles.
- Involved (but tractable) mathematics

$$\dot{X} = AX + BX$$

$$U = -KX$$

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix}$$

# Multiple input systems: Some tricks and ideas

- Eliminate the need for measuring some $\boldsymbol{x_j}$ by appropriately choosing the closed loop poles.

$$Example: \quad u = \mu_1 x_1 + (\mu_2 - \beta) x_2$$

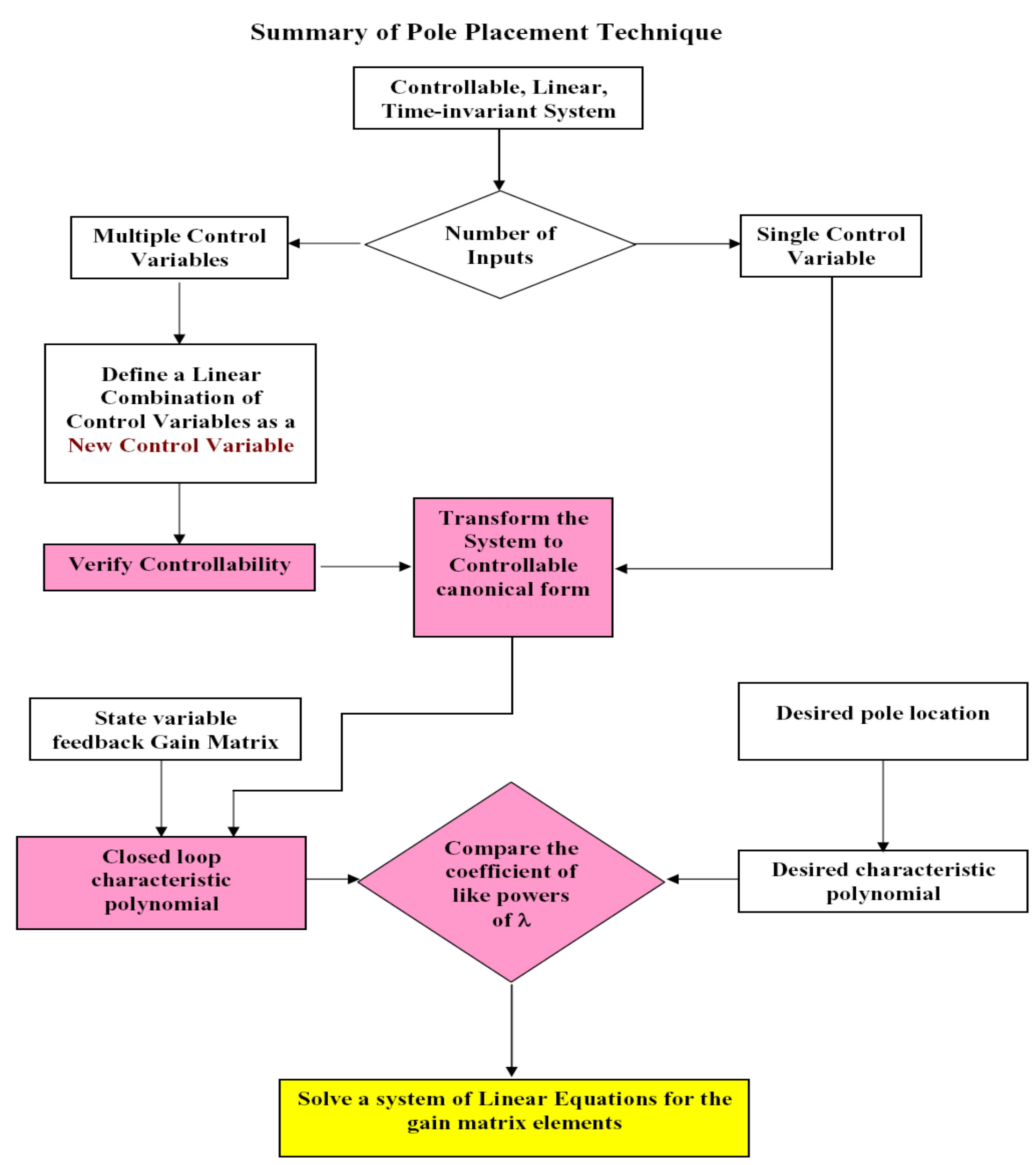
$$\text{select } \mu_2 = \beta \text{ provided } \beta < 0$$

- Relate the gains to proper physical quantities

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & g_{13} & 0 \\ 0 & g_{22} & 0 & g_{24} \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

- Shape eigenvectors: “Eigen structure assignment control”
- Introduce the idea of optimality: “optimal control”

## Control Allocation:



# References


- K. Ogata: *Modern Control Engineering*, 3rd Ed., Prentice Hall, 1999.
- B. Friedland: *Control System Design*, McGraw Hill, 1986.

# Thanks for the Attention....!