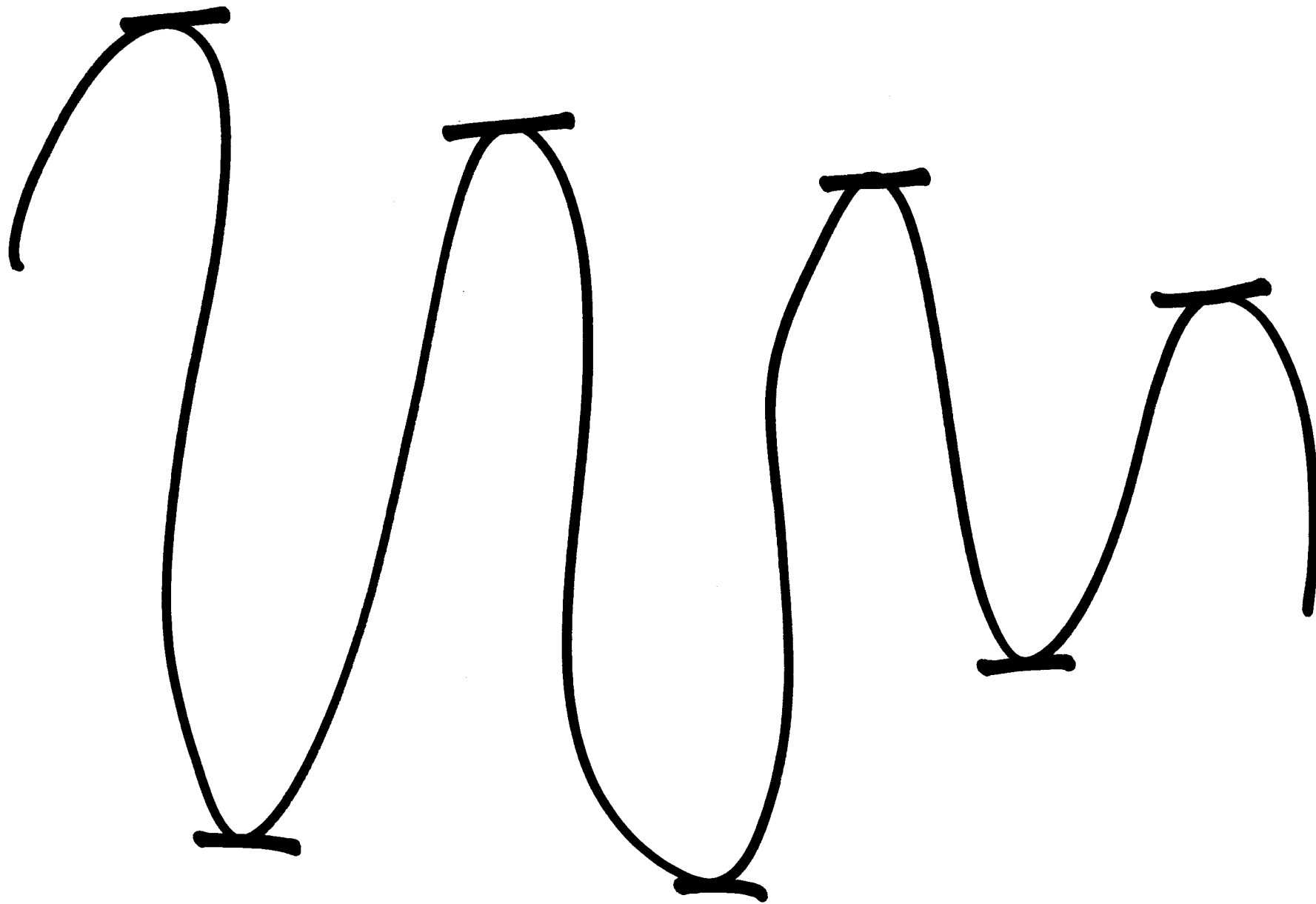
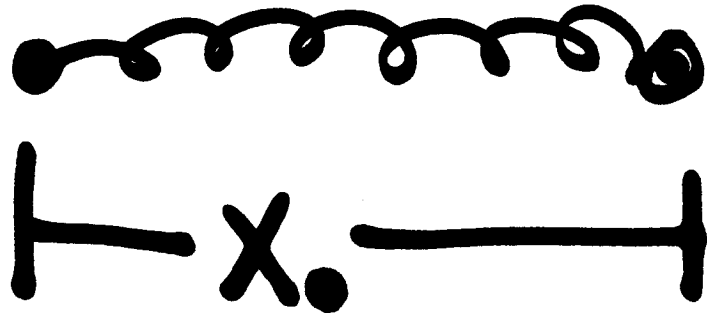


Prof. Ranjith P.
2008-2009

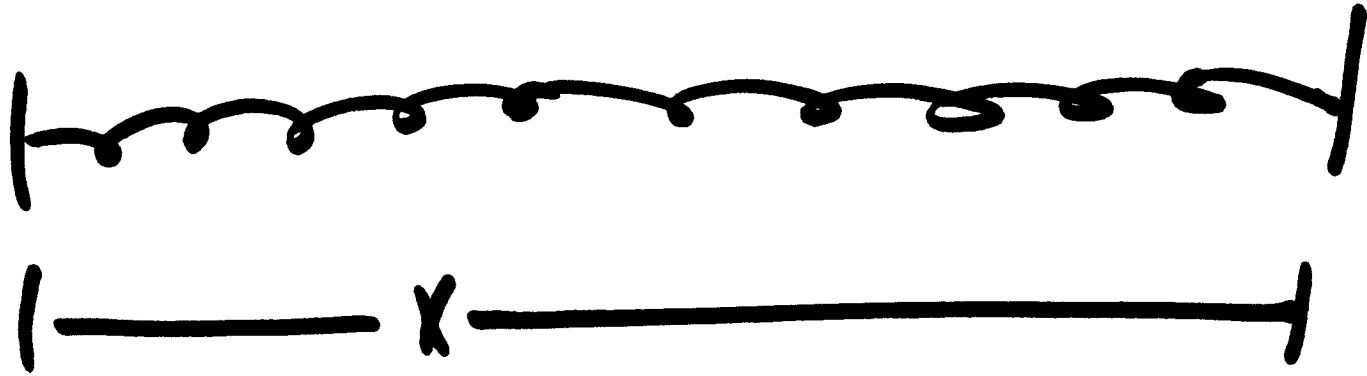
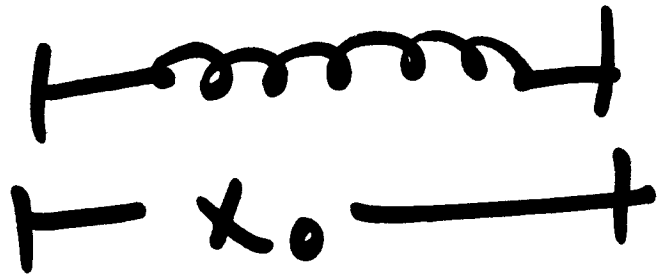




$$f \propto (x - x_0)$$

$$E = \frac{1}{2} k (x - x_0)^2$$

2



3

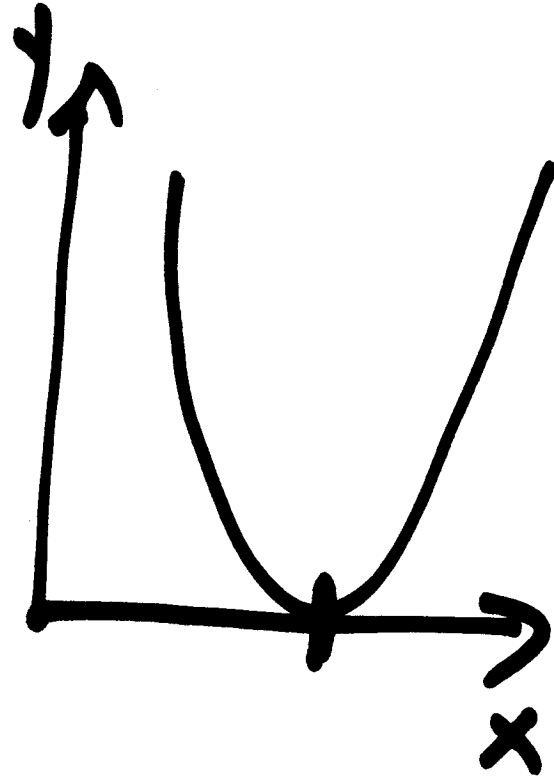
$$E = \frac{1}{2} K (x - x_0)^2$$

*

$$E = \frac{k}{2} (x - x_0)^2$$

$$\frac{4}{2} (x - 3)^2$$

$$E = 2 (x - 3)^2$$



5*

$$E = 2 \underline{(x-3)^2}$$

$$\frac{dE}{dx} = \frac{d}{dx} \left[\frac{2(x-3)^2}{4} \right]$$

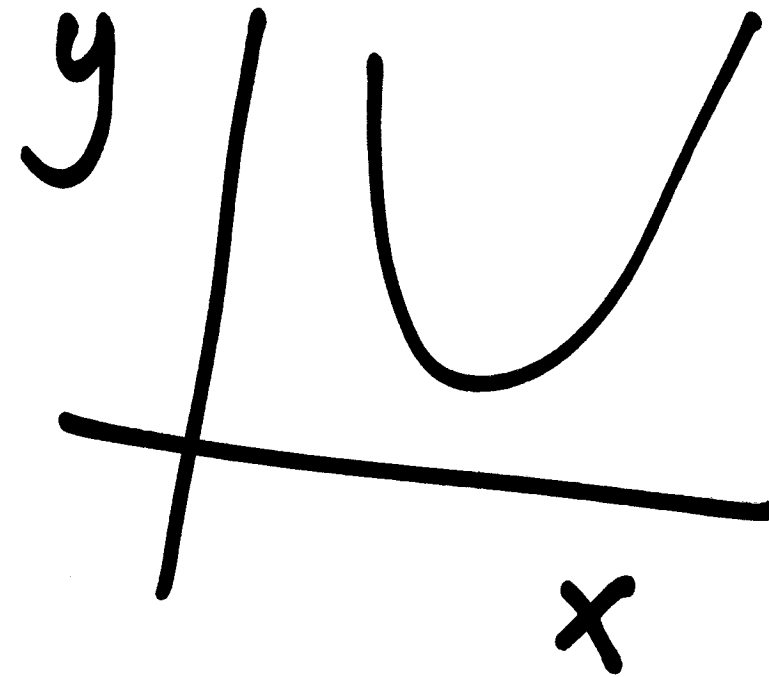
$$E = 2y^2$$

$$\text{where, } y = x-3$$

$$F = 2(u)^2, \quad u = x - 3$$

$$E[u(x)]$$

$$y(u(x))$$



6

x

$$E = 2u^2$$

where, $u = x - 3$

$$\frac{dE}{dx} = \frac{dE}{du} \cdot \frac{du}{dx}$$

$4u \cdot 1$

4

$$E = 2u^2, \quad u = x - 3$$

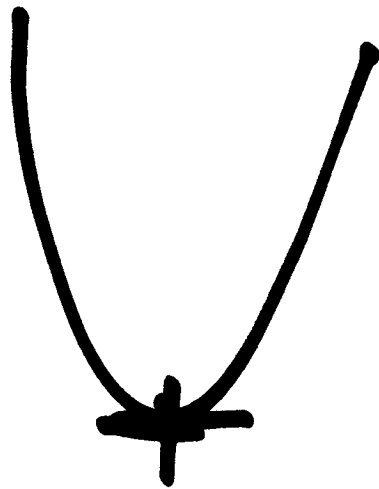
$$\frac{dE}{du} \cdot \frac{du}{dx}$$

$$\begin{aligned} \frac{dE}{du} &= \frac{d}{du} [2u^2] = 2 \frac{d}{du} u^2 \\ &= 2 \cdot 2u = \underline{\underline{4u}} \end{aligned}$$

$$\frac{du}{dx} = \frac{d}{dx} (x - 3) = \frac{dx}{dx} - 0 = \underline{\underline{1}}$$

a

$$\begin{aligned}\frac{dE}{dx} &= 4U \\ &= 4(x-3)\end{aligned}$$



$$\frac{dE}{dx} = 0$$

$$4(x-3) = 0$$

$$\Rightarrow x = 3$$

10

$$E = \frac{4}{2}(x-3)^2$$

$$\frac{dE}{dx} = 4(x-3)$$

$$\frac{d}{dx} \left(\frac{dE}{dx} \right) = \frac{d}{dx} [4(x-3)]$$

$$= 4 \frac{d}{dx} [x-3]$$

"

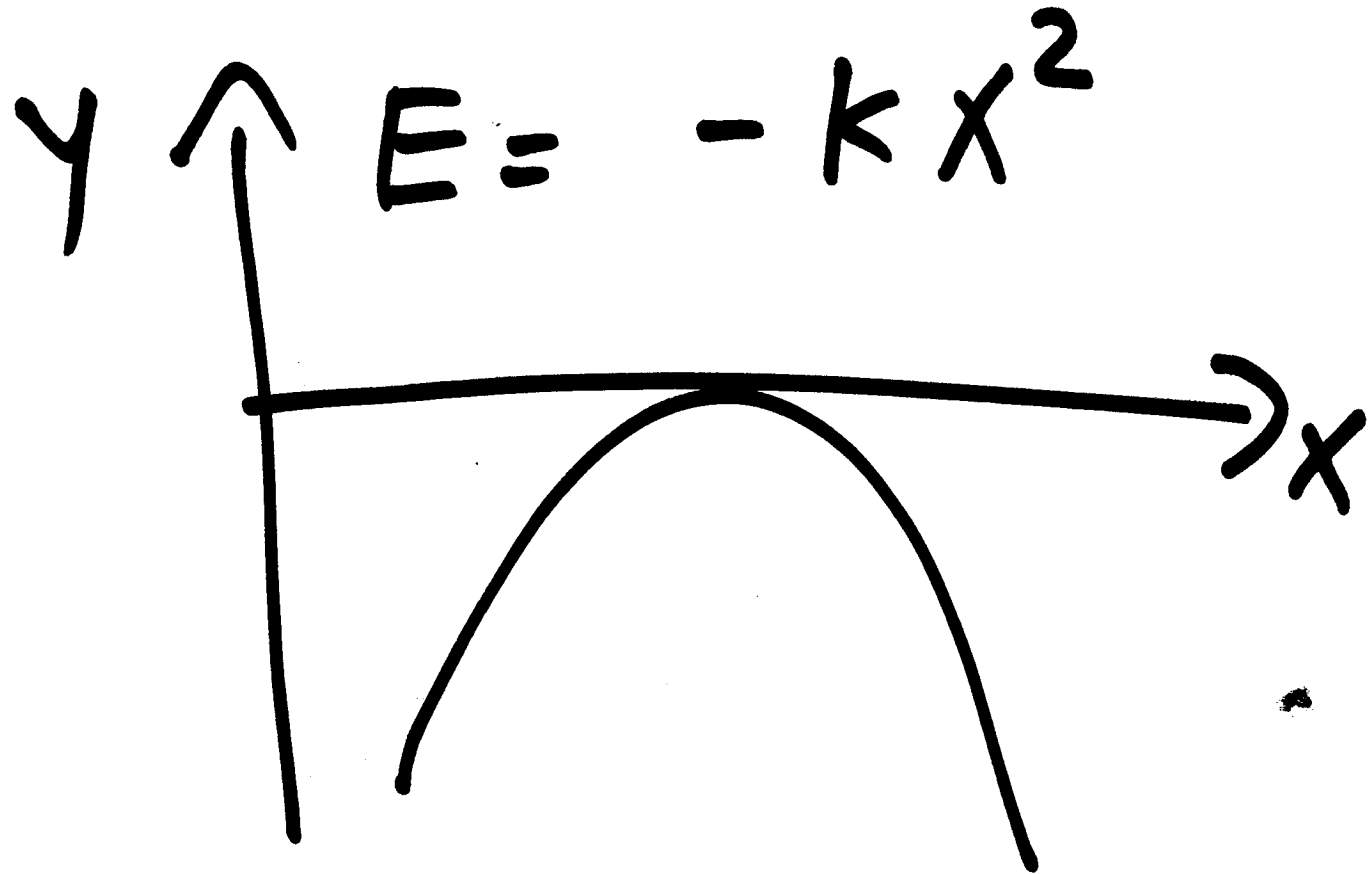
$$\frac{d}{dx} [x-3]$$

$$= \frac{d}{dx} x - \frac{d}{dx} 3$$

$$= 1 - 0 = \underline{\underline{1}}$$

$$\frac{d^2 E}{dx^2} = \frac{d}{dx} \left(\frac{dE}{dx} \right) = \frac{d}{dx} [4(x-3)] = \underline{\underline{4}}$$

12

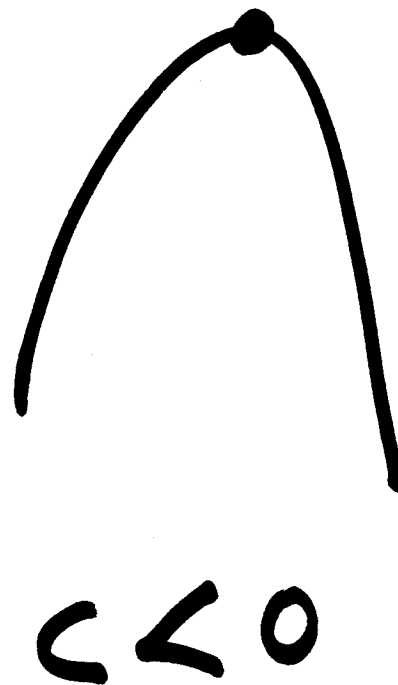
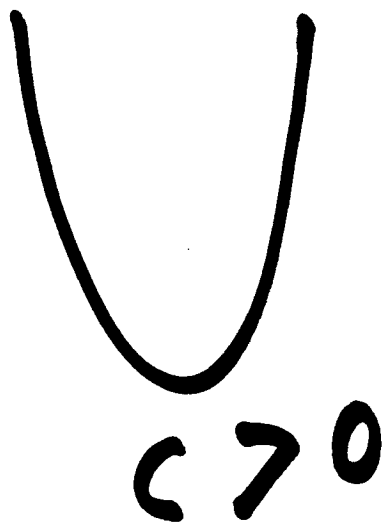


$$\frac{dE}{dx} = -2kx ; \quad \frac{d^2E}{dx^2} = \frac{d}{dx} (-2kx)$$

13

$$\frac{d}{dx} [-2kx]$$

$$= -2k$$



19

$$y = 2x^4 - 150x^2$$

$$\frac{dy}{dx} = \frac{d}{dx} [2x^4 - 150x^2]$$

$$= \frac{d}{dx} (2x^4) - \frac{d}{dx} (150x^2)$$

$$= (2 \cdot 4x^3) - 150 \cdot 2x$$

$$= 8x^3 - 300x$$

15

$$\frac{dy}{dx} = 8x^3 - 300x$$

$$8x^3 - 300x = 0$$

$$x [8x^2 - 300] = 0$$

① $x = 0$

② $8x^2 - 300 = 0 \Rightarrow 8x^2 = 300$

$$\Rightarrow x^2 = \frac{300}{8} = 37.5 \Rightarrow x = \pm \sqrt{37.5}$$

16

$$\textcircled{1} \quad \underline{x=0}$$

$$\textcircled{2} \quad + \sqrt{37.5} = \underline{+6.12}$$

$$\textcircled{3} \quad - \sqrt{37.5} = \underline{-6.12}$$

$$\frac{dy}{dx} = 8x^3 - 300x$$

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} [8x^3 - 300x]$$

$$= 8 \cdot 3x^2 - 300$$

$$\frac{d^2y}{dx^2} = 24x^2 - 300$$

17

18

$$\frac{d^2y}{dx^2} = 24x^2 - 300$$

$$x = 0$$

$$x = +6.12$$

$$x = -6.12$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=0} = -300$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=6.12} = 24(6.12)^2 - 300 > 0$$

summary

① Find derivative; Equate to 0

$$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \text{Extrema}$$

② Find second derivative; Evaluate at extrema

Last but one

Last

If $\frac{d^2y}{dx^2} > 0 \Rightarrow$ Minimum

If $\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum