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Lec. No. 11

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$$\int \underline{g'(x)}\,dx = g(x) + C$$

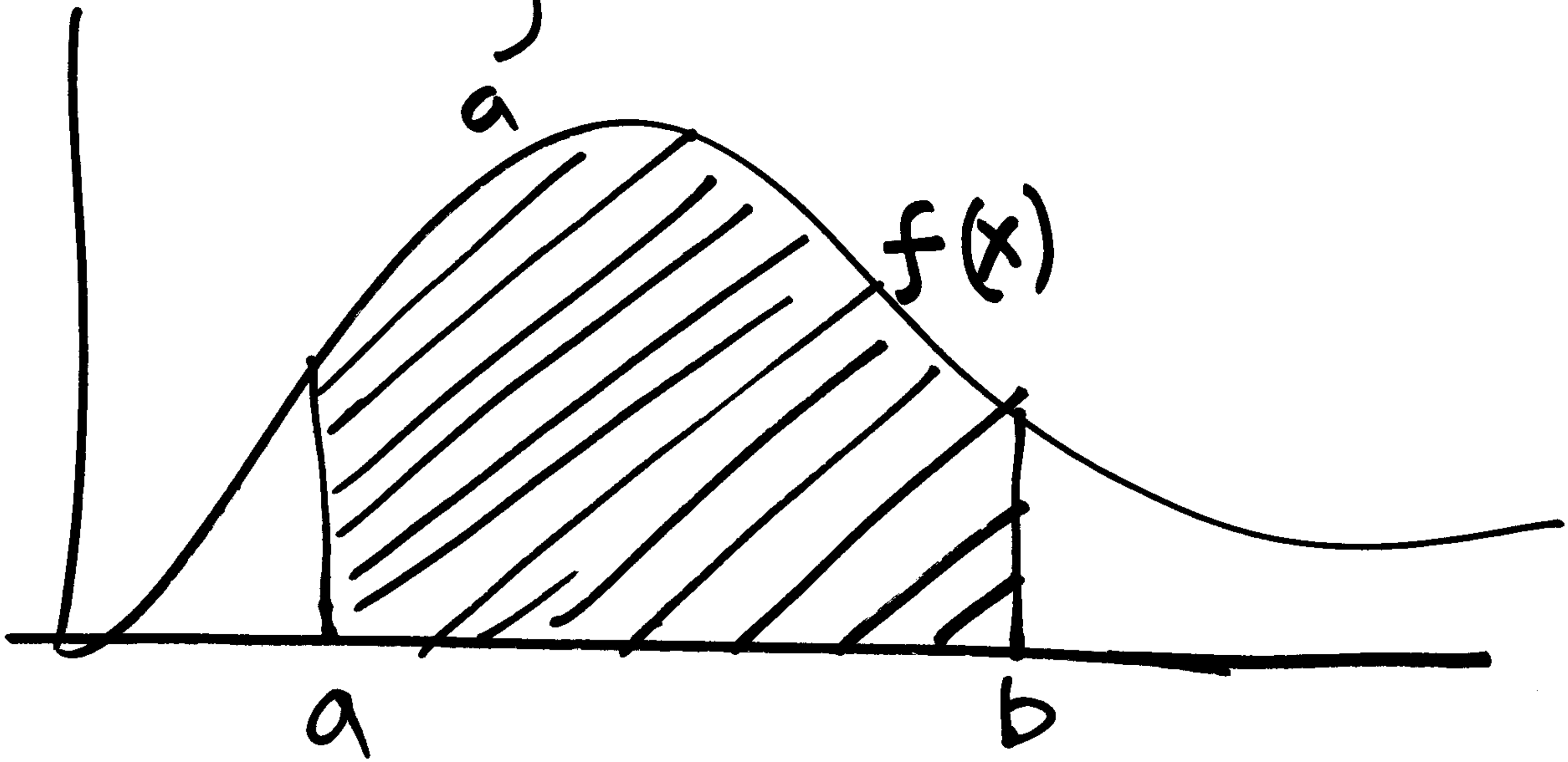
$$g'(x) = 2$$

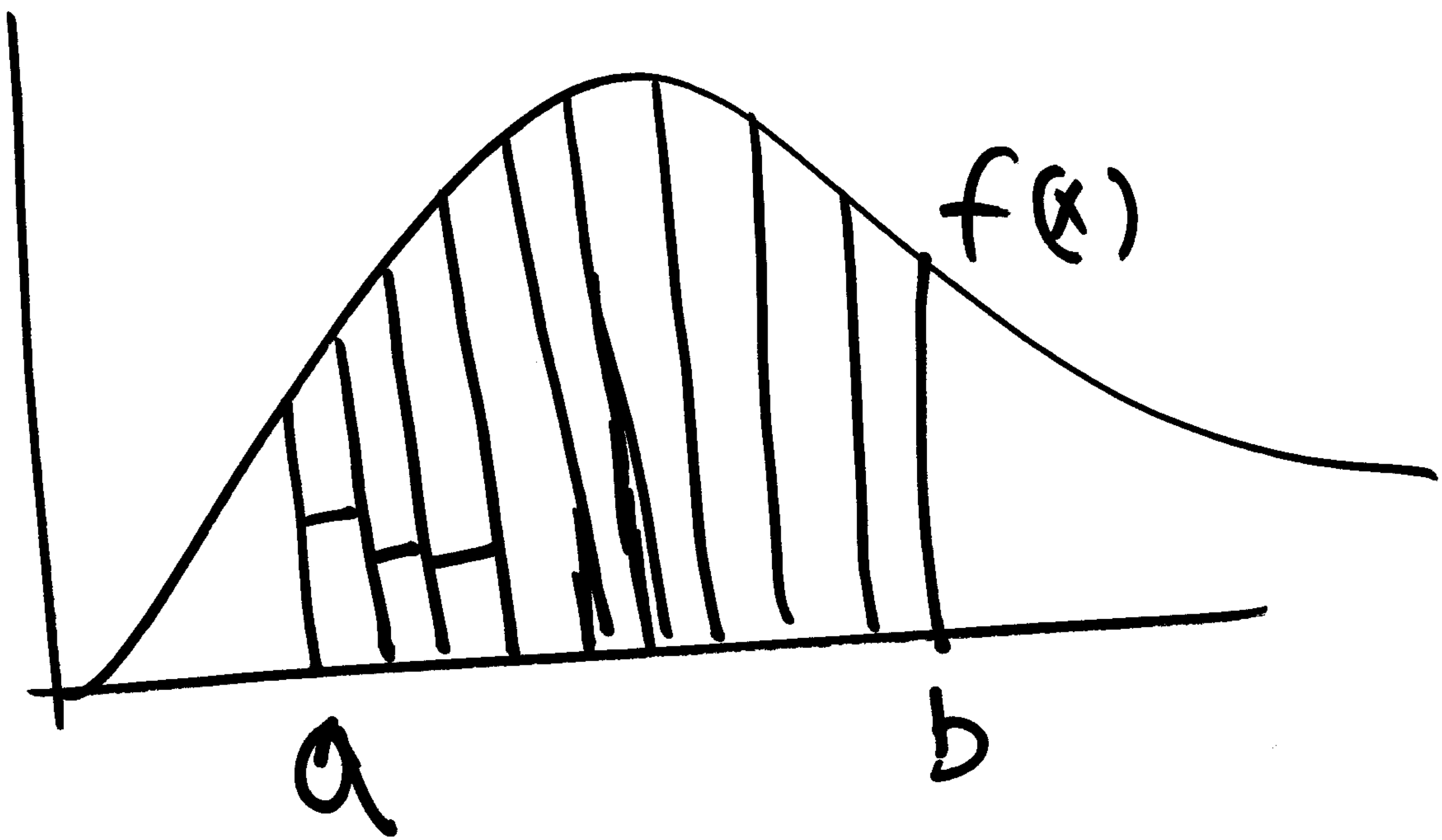
$$\int 2\,dx = 2x + C$$

$$\int_a^b \left(\frac{df}{dx}\right) dx$$

$$\int_a^b \underline{f'(x)} \, dx = f(b) - f(a)$$

$$\int_a^b f(x)\,dx = ?$$





$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} f(x_i)\,dx_i$$

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\,dx_i$$

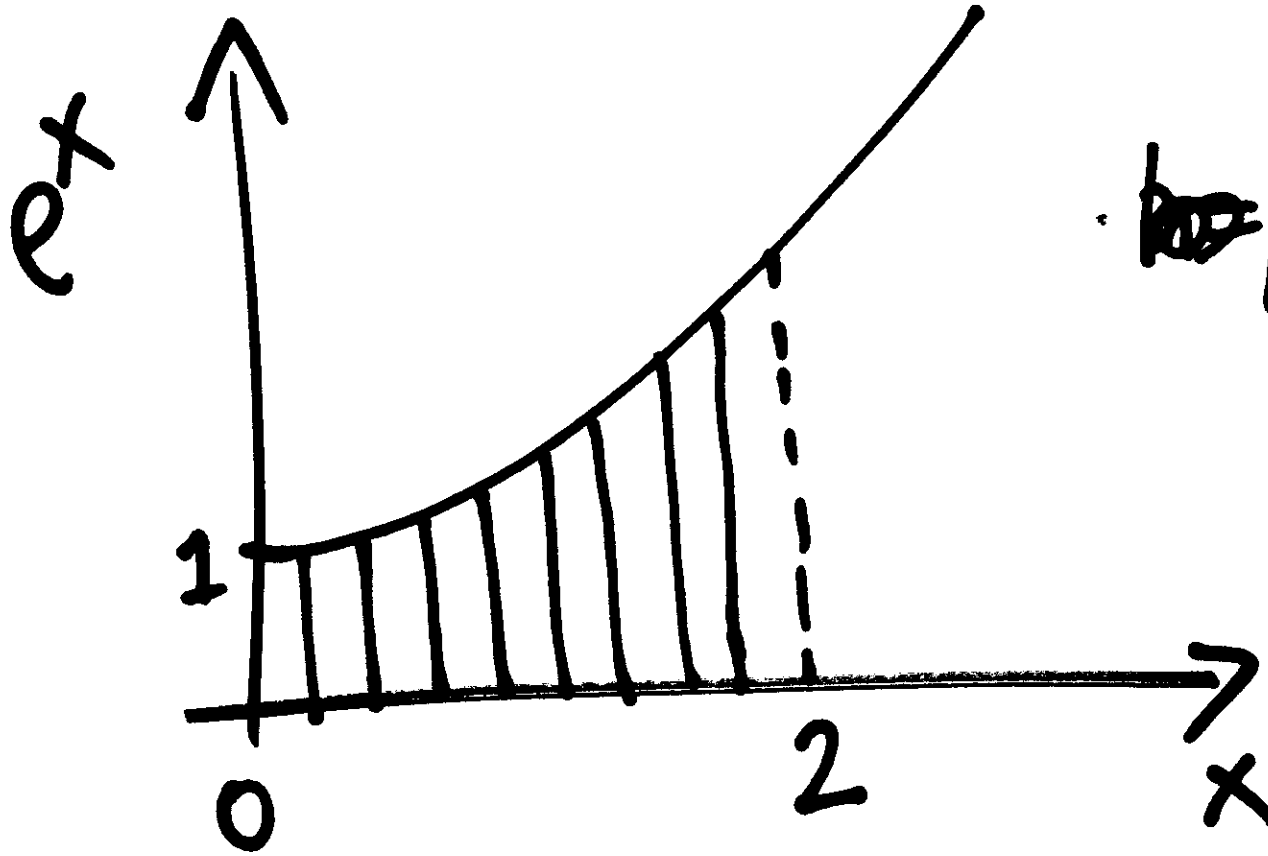
$$\frac{df}{dx} = \frac{f(x+dx) - f(x)}{dx}$$

$$f(x) = e^x$$

$$\int e^x dx = e^x$$

$$\int_0^2 e^x dx = e^x \Big|_0^2$$

$$= e^2 - e^0 = \underline{\underline{e^2 - 1}}$$



$$\boxed{\frac{2-0}{n} = d}$$

$$\int_0^2 e^x dx = e^0 \frac{2}{n} + e^{\frac{2}{n}} \frac{2}{n} + e^{\frac{4}{n}} \frac{2}{n} + \cdots$$

$$\frac{2}{n}\left[1 + r + r^2 + r^3 + \dots\right]$$

$$r = e^{2/n}$$

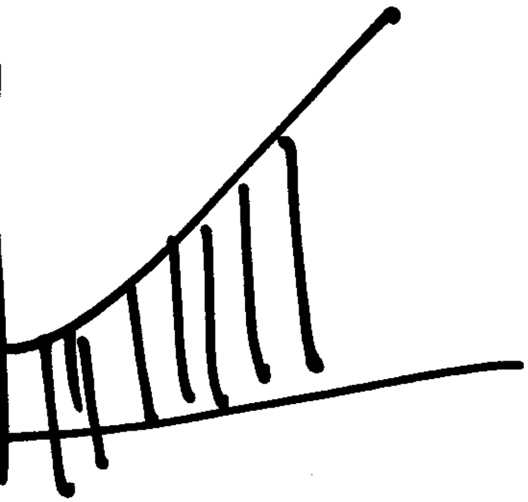
$$\frac{2}{n}\left[1 + e^{\frac{2}{n}} + \left(e^{\frac{2}{n}}\right)^2 + \cancel{e} \; \cancel{e} + \left(e^{2/n}\right)^3 + \dots\right]$$

$$S_n = \frac{1-r^n}{1-r}$$

$$= \frac{2}{n}\left[\frac{1-\left(e^{2/n}\right)^n}{1-e^{2/n}}\right]$$

$$S_n = \frac{2}{n} \frac{\left(e^{2/n}\right)^n - 1}{e^{2/n} - 1}$$

$$\boxed{S_n = \frac{2}{n} \frac{e^2 - 1}{e^{2/n} - 1}}$$

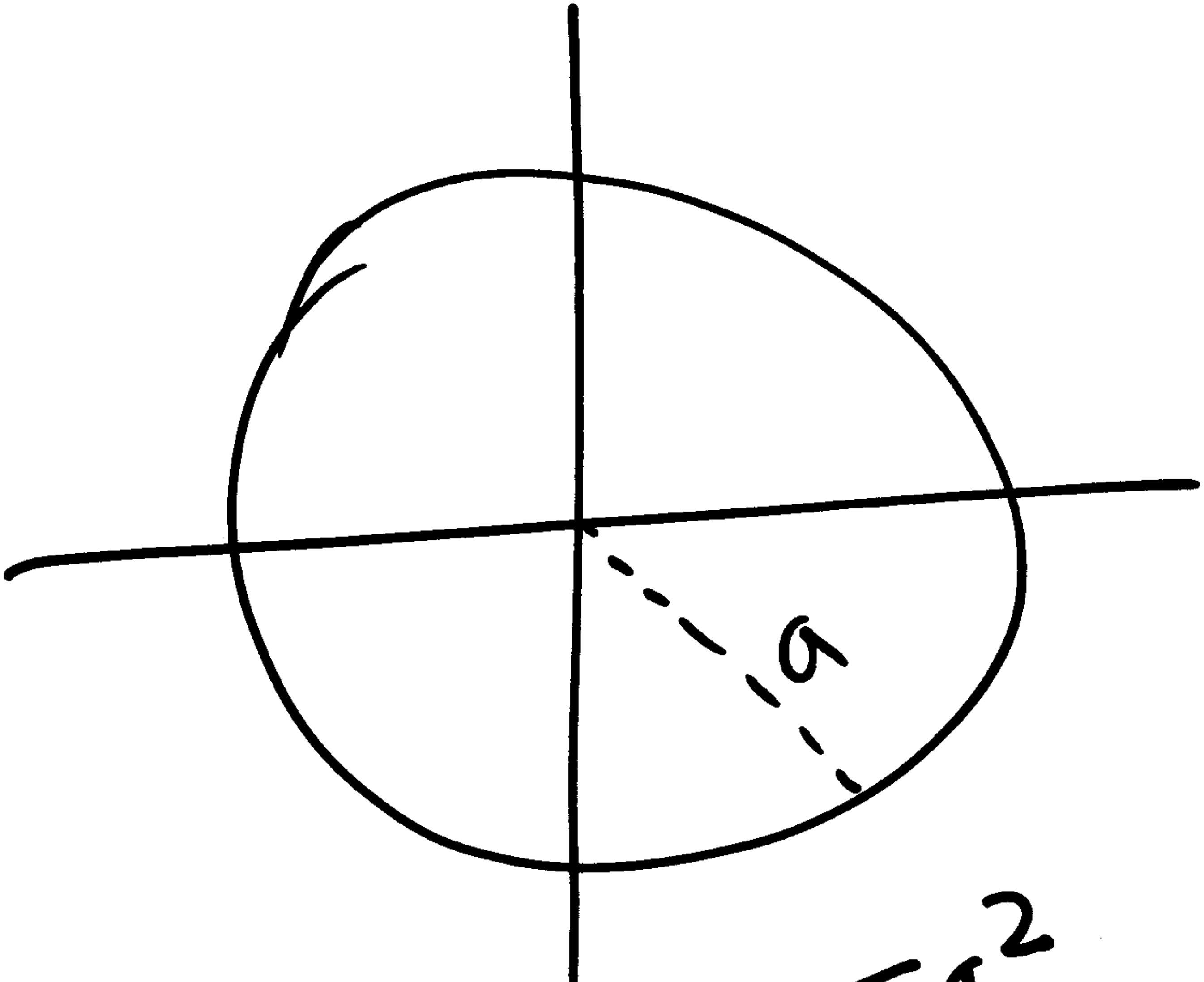


$$\lim_{n\to\infty} \frac{e^2-1}{\left(\frac{e^{2/n}-1}{2/n}\right)} = e^2-1$$

$$\int_0^2 e^x\,dx = e^x\Big|_0^2 = e^2-1$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int \sqrt{x} \, dx = \int x^{\frac{1}{2}} \, dx$$



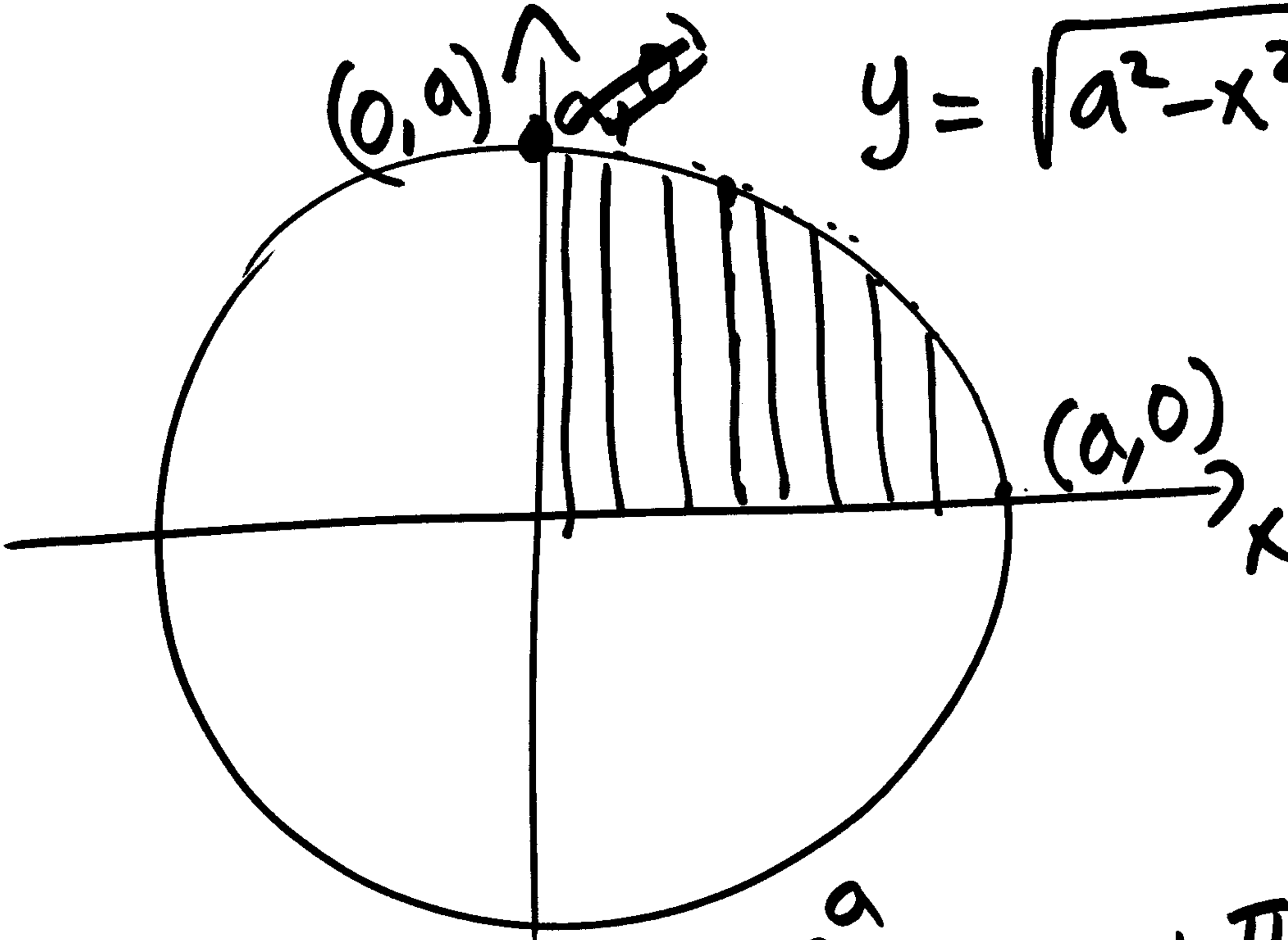
$\text{Area} = \pi a^2$

Equation of a circle

$$x^2 + y^2 = a^2$$

$$y^2 = a^2 - x^2$$

$$y(x) = \sqrt{a^2 - x^2}$$



$$y = \sqrt{a^2 - x^2}$$

$$\int_0^a y\,dx = \frac{1}{4}\pi a^2$$

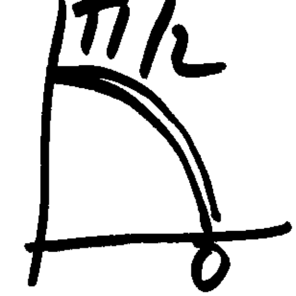
$$x^2+y^2=a^2$$

$$y=\sqrt{a^2-x^2}$$

$$\int_0^a y\,dx = \int_0^a \sqrt{a^2-x^2}\,dx$$

$$= \frac{\pi}{4}a^2$$

$$\int_0^a \sqrt{a^2 - x^2}\, dx$$



$x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$\frac{dx}{d\theta} = \cos\theta$

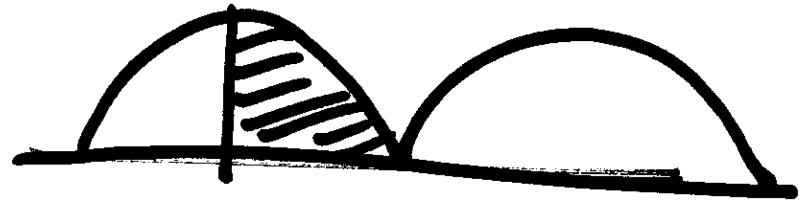
$$\int_0^{\pi/2} a\sqrt{a^2 - a^2\sin^2\theta}\, \cos\theta\, d\theta$$

$x = 0 \Rightarrow$

$\theta = 0$

$x = a$

$\theta = \pi/2$



$$a^2\int_0^{\pi/2}\cos^2\theta\,d\theta = a^2\frac{\pi}{4}$$

$$a\int_0^{\pi/2}\overset{\cos\theta}{\overline{\sqrt{a^2(1-\sin^2\theta)}}}\,\cancel{d\theta}\,\cos\theta\,d\theta$$

Integration by parts

$u(x), \ v(x)$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$uv\Big|_a^b = \int_a^b u\,dv + \int_a^b v\,du$$

$$\int_a^b \frac{d}{dx}(uv)\,dx = \int_a^b u\,\frac{dv}{dx}\,dx + \int_a^b v\,\frac{du}{dx}\,dx$$

$$\boxed{\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int_a^b \underbrace{x}_{u} \underbrace{\sin x\, dx}_{dv}$$

$v = -\cos x$

$dv = \sin x\, dx$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$= -x\cos x\Big|_a^b + \int_a^b \cos x\, dx$$

Derivative: Given data.

	x	$f(x)$	
x_1	0	23	$f(x_1)$
x_2	0.1	24	$f(x_2)$
x_3	0.2	25.3	$f(x_3)$
x_4	0.3	26.1	$f(x_4)$

$$\frac{df}{dx}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$\frac{df}{dx} = \frac{f(x+dx) - f(x)}{dx}$$

$$= \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

$$\frac{d^2f}{dx^2} = \frac{d}{dx}(m)$$

$$= \frac{m(x_{i+1}) - m(x_i)}{(x_{i+1} - x_i)}$$

$$\sum_{i=1}^{n} f(x_i)\, dx_i$$

$$\sum_{i=1}^{n} f(x_i)\,(x_{i+1} - x_1)$$

Integration

→ area under the curve

⇒ Definite integrals.

→ Data ⇒