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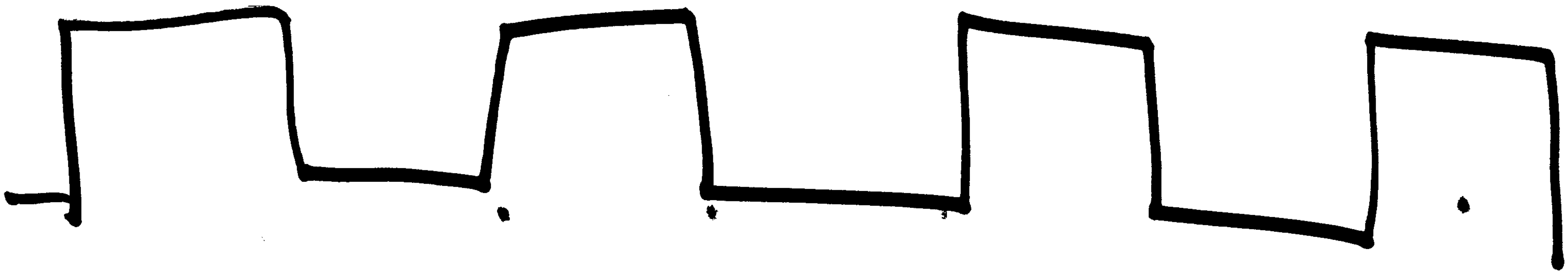
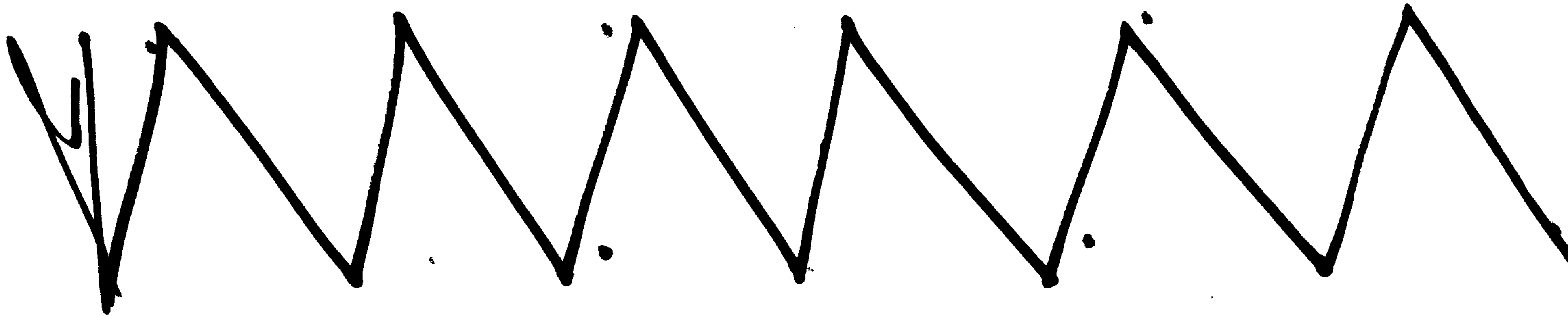
Lec. No. 27
Date: 7/11/11



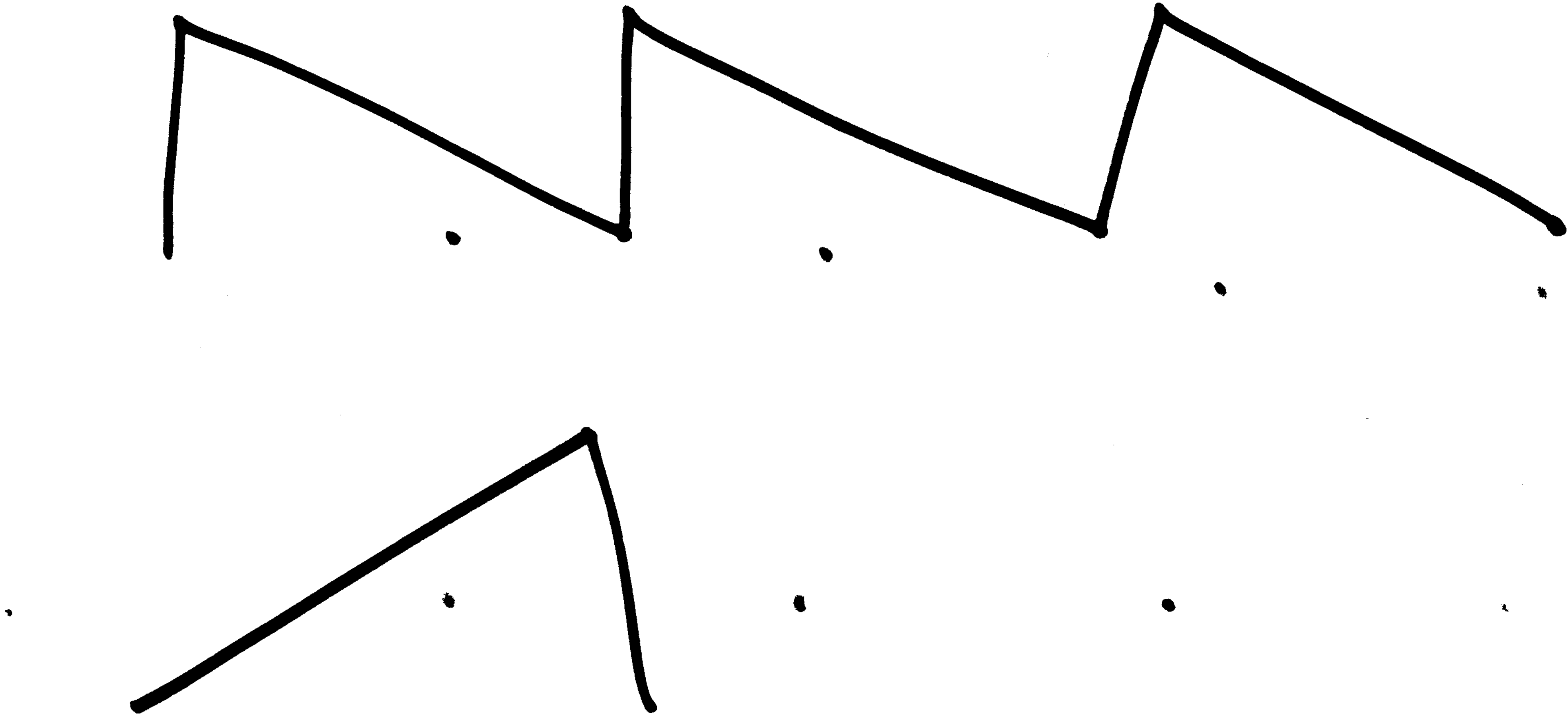
square - wave

$f(x) =$ combination of Sin & Cos



$f(x)$

saw-tooth wave



$f(x) =$

$$f(x) = ?$$

$$f(x) = a_1 \cos(k_1 x) + a_2 \cos(k_2 x)$$
$$+ a_3 \cos(k_3 x) + \cdots\cdots$$
$$b_1 \sin(k_1 x) + b_2 \sin(k_2 x)$$
$$+ \cdots\cdots$$
$$c$$

$$\underline{\vec{A}} = \underline{a}\hat{i} + \underline{b}\hat{j} + \underline{c}\underline{\hat{k}}$$

$$a = \vec{A}\cdot\hat{i}$$

$$\vec{A}\cdot\hat{i} = (a\hat{i} + b\hat{j} + c\hat{k})\cdot\hat{i}$$

$$= a$$

$$\vec{A}\cdot\hat{j} = b \quad , \quad \vec{A}\cdot\hat{k} = c$$

$$\hat{i}\cdot\hat{i} = 1$$

$$\hat{i}\cdot\hat{j} = 0$$

$$\vec{A} = \alpha \hat{i} + \beta \hat{j}$$

$$\alpha = \vec{A} \cdot \hat{i}$$

$$\beta = \vec{A} \cdot \hat{j}$$

$$f(x) = \sum_{n=1}^{\infty} \underline{\underline{a_n}} \, \underline{\cos(nx)} + \sum_{n=1}^{\infty} b_n \sin(nx) + c$$

$$a_n = \int f(x) \cos(nx) \, dx$$

$$b_n = \int f(x) \sin(nx) \, dx$$

$$f(x) = \sum_{n=0}^{\infty} a_n \cos n(x) + \sum_{n=0}^{\infty} b_n \sin nx$$

$$\int_{-\pi}^{\pi} f(x) \operatorname{Con}(nx) = \int \sum_{n} a_n \underline{\cos(nx) \cos(mx)}$$

$$\delta_{mn} = 0 \quad \text{if} \quad m \neq n$$

$$\delta_{mn} = 1 \quad \text{if} \quad m = n$$

$$f(x) = \sum_n a_n \cos(nx) + \sum_n b_n \cdot \sin nx$$

$$\int f(x) \cos m(x) = \int \sum_n \underline{a_n \cos n(x) \cos mx} \, dx$$

$$\int f(x) \cos mx = a_m$$

$$f(x) = \sum_{n} a_n \cos(nx) + \sum_{n} b_n \sin(nx)$$

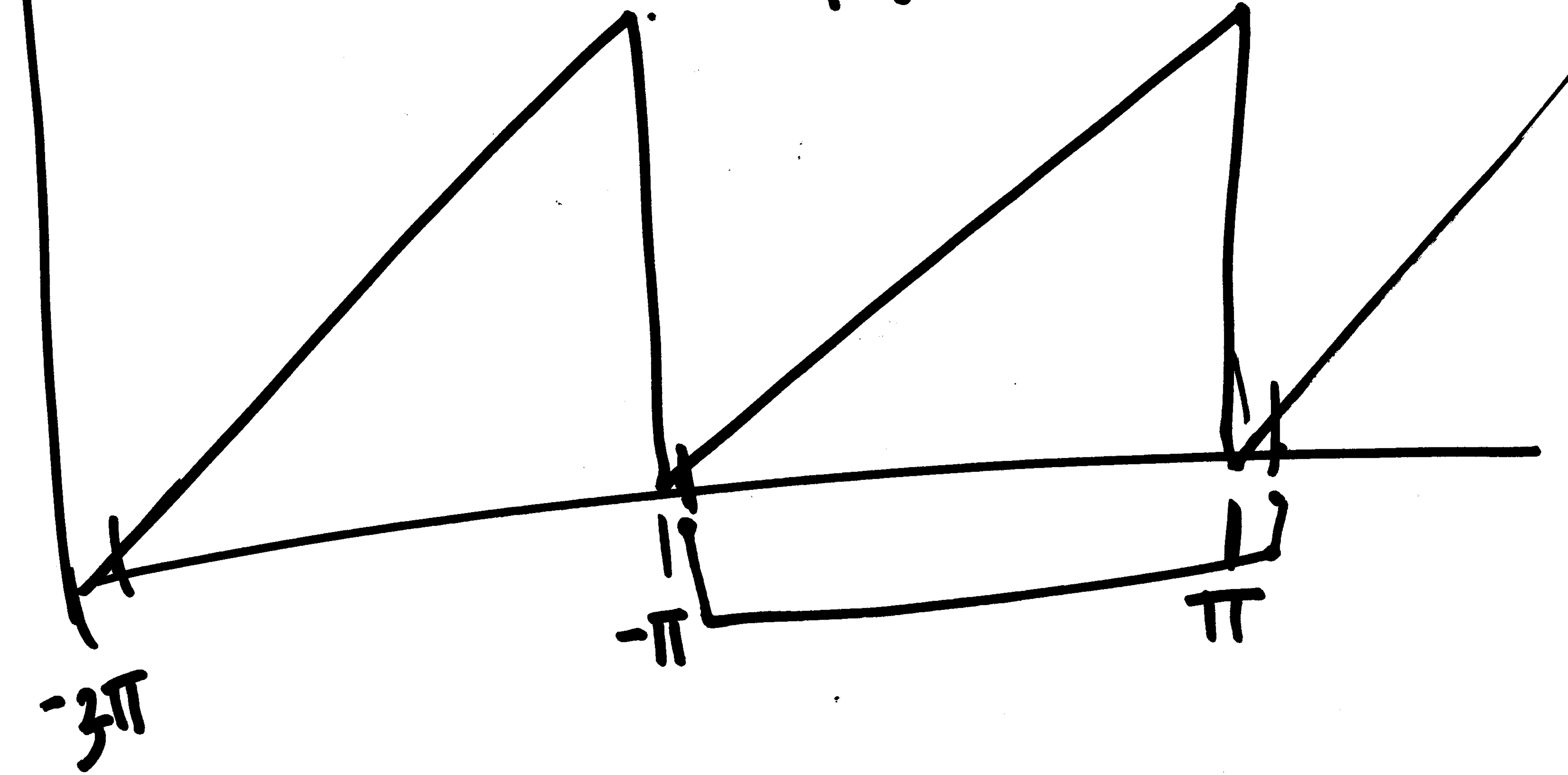
$$i \cdot i = 1$$

$$i \cdot j = 0$$

$$\boxed{\begin{matrix} i \cdot i = 1 \\ i \cdot j = 0 \end{matrix}}$$

$$f(x+2\pi) = f(x)$$

$$f(x) = x$$



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

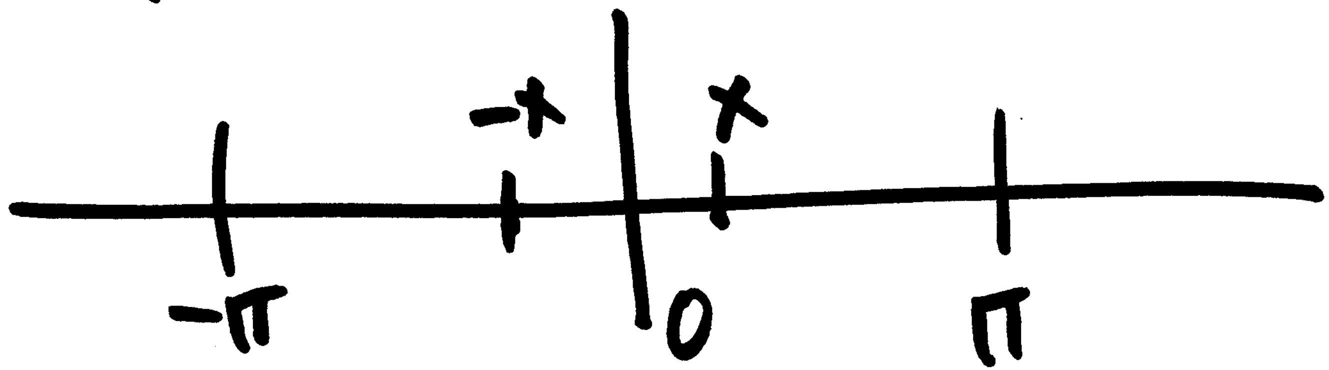
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx$$

$$= \frac{1}{\pi}\left(\frac{x^2}{2} \Bigg|_{-\pi}^{\pi} \right) = \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^2}{2} \right]$$

$$= 0$$

$$a_n = \int_{-\pi}^{\pi} x \cos(nx)\,dx$$

$$= \int_{-\pi}^{0} \underline{x \cos(nx)\,dx} + \int_{0}^{\pi} \underline{x \cos(nx)}\,dx$$



$$x\cos(x) = -x\cos(-x)$$

$$= 0$$

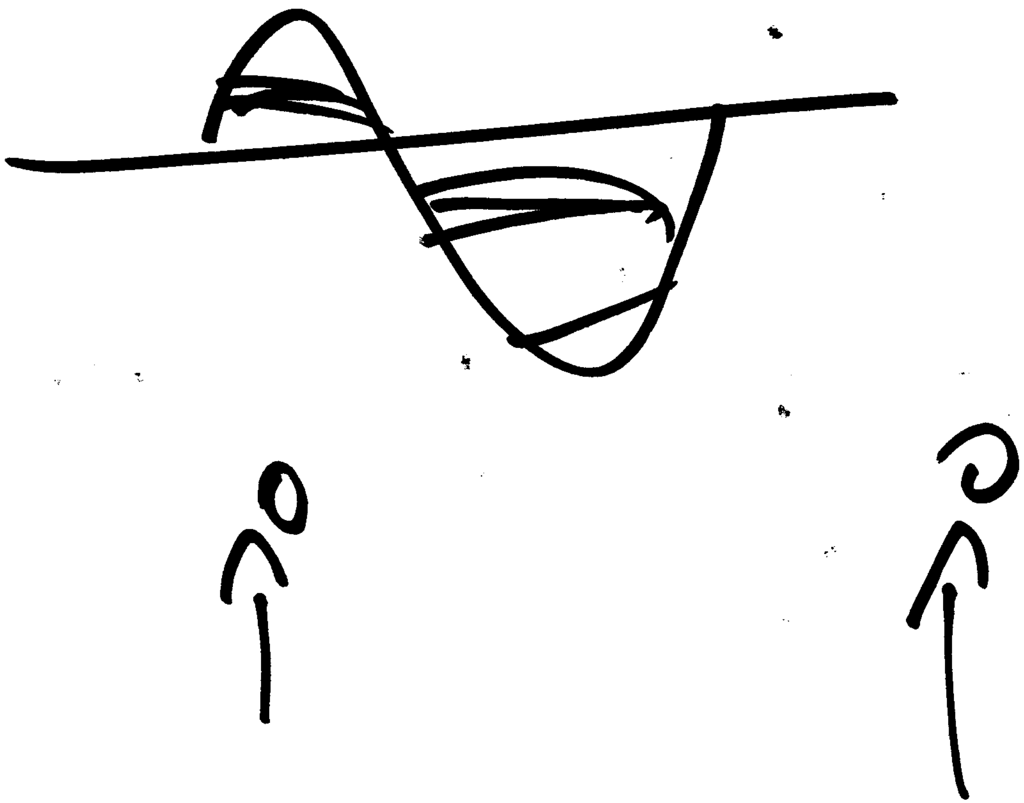
$$\cancel{-1\cos(-n) +}$$

$$-1\cos(-n) + 1\cos(n) = 0$$

$$\int_{-\pi}^{\pi} \underset{u}{x} \underbrace{\cos(nx)\,dx}_{dv}$$

$$\int_{-\pi}^{\pi} u\,dv = uv\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} v\,du$$

$$x\,\frac{\sin nx}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\sin nx}{n} = 0$$



$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{u}{\underline{x}}\ \underbrace{\sin(nx).dx}_{dv}$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v \cdot du$$

$dv = \sin nx \, dx$

$v = -\frac{\cos nx}{n}$

$$= -x\frac{\cos nx}{n}\Big|_{-\pi}^{\pi} + \underbrace{\int_{-\pi}^{\pi} \frac{\cos nx}{n}}_{0}$$

$$-\pi\frac{\cos n\pi}{n}-\pi\frac{\cos(-n\pi)}{n}$$

$$= -\frac{2}{n}\cos n\pi + (\ \ )$$

$$\boxed{-\frac{2}{n}\cos(n\pi) = \frac{2(-1)^{n+1}}{n}}$$

$n=1$

$-2\cos\pi$ $\cancel{=2}$ | 2

$=\underline{2}$

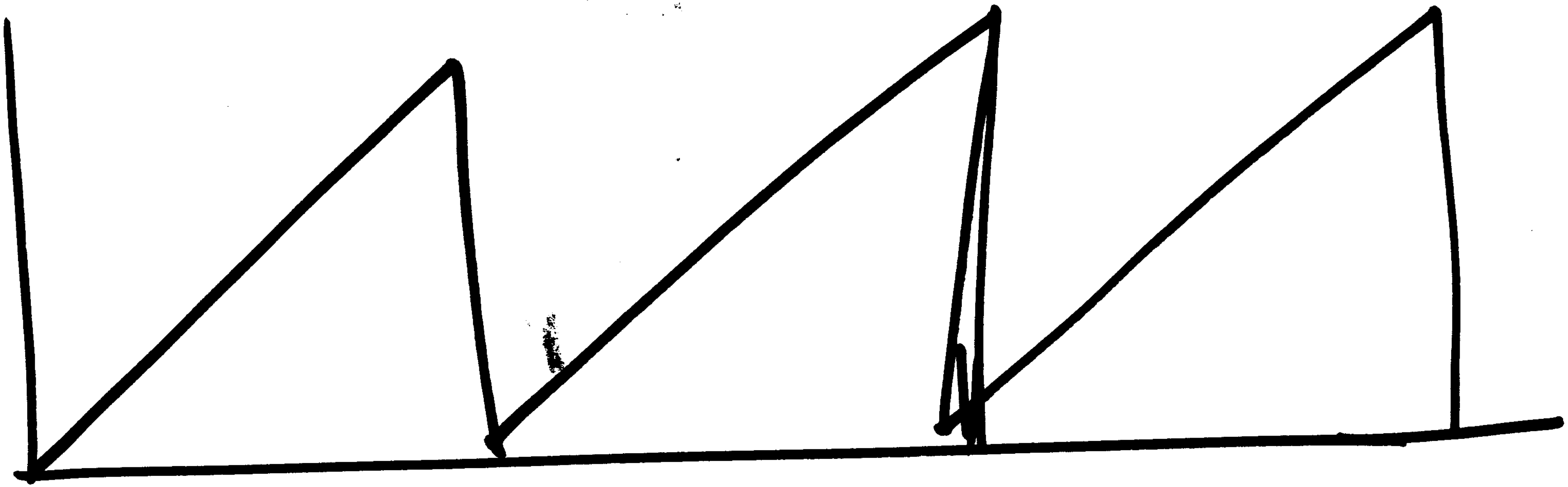
$\frac{2}{2}(-1)^{3} = \underline{-1}$

$n=2$: $-\frac{2}{2} = \underline{-1}$

$$f(x) = \cancel{\sum_{n=0}^{\infty} a_n \cos(nx)}$$

$$+ \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$



1st term

$$2 \sin(x)$$

2nd term $(n=2)$

$$2\left(\frac{-1}{2}\right)\sin(2x)$$

<u>3rd term</u> (n=3)

$$2 \frac{1}{4} \sin(3x) = \frac{1}{2} \sin(3x)$$

2 terms

$$f(x) = 2\sin x + (-1)\cdot \sin 2x$$

3terms

$$f(x) = 2\sin x - \sin 2x + \frac{1}{2}\sin 3x$$