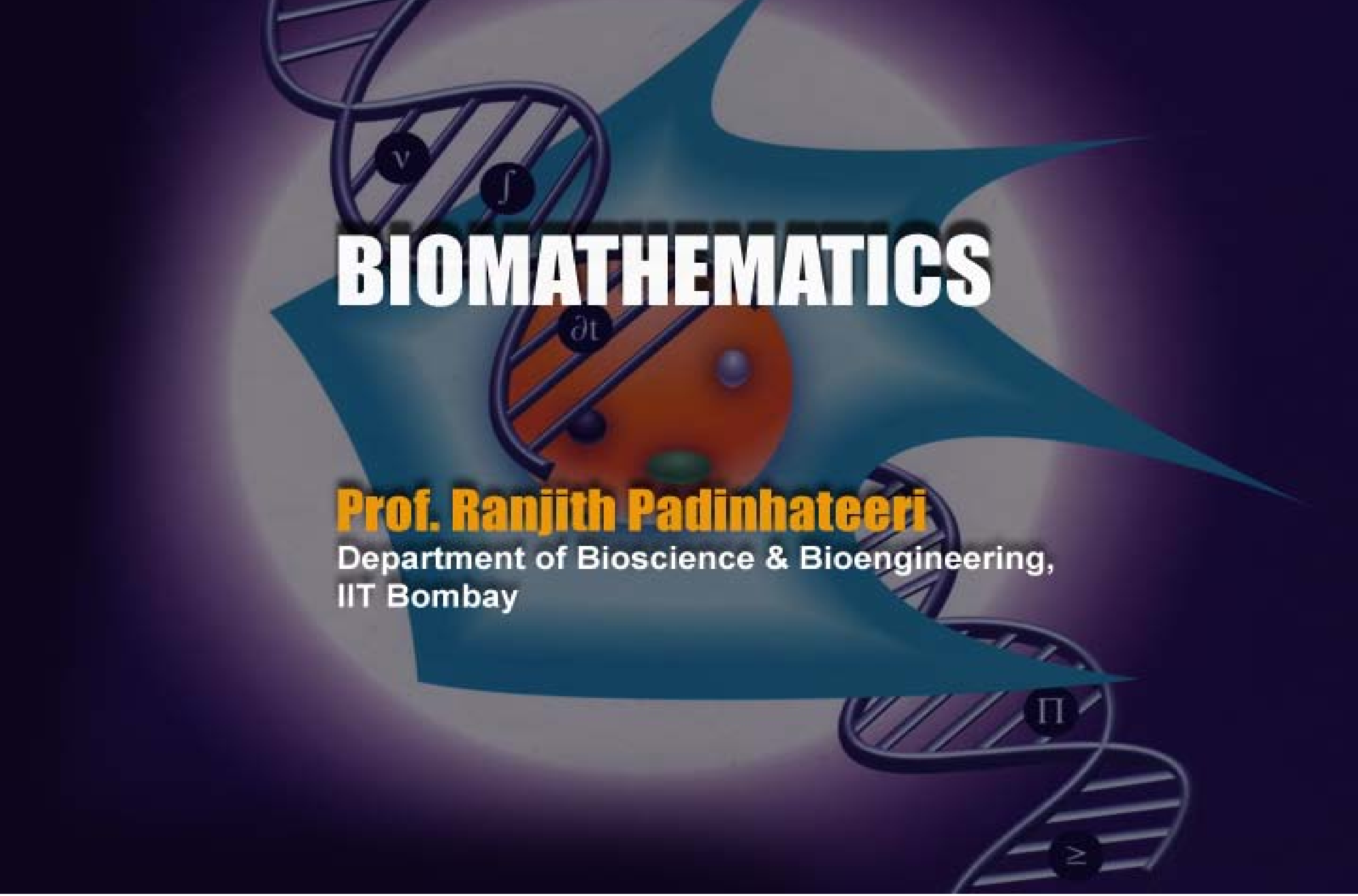
BIOMATHEMATICS
Prof. Ranjith Padinhateeri
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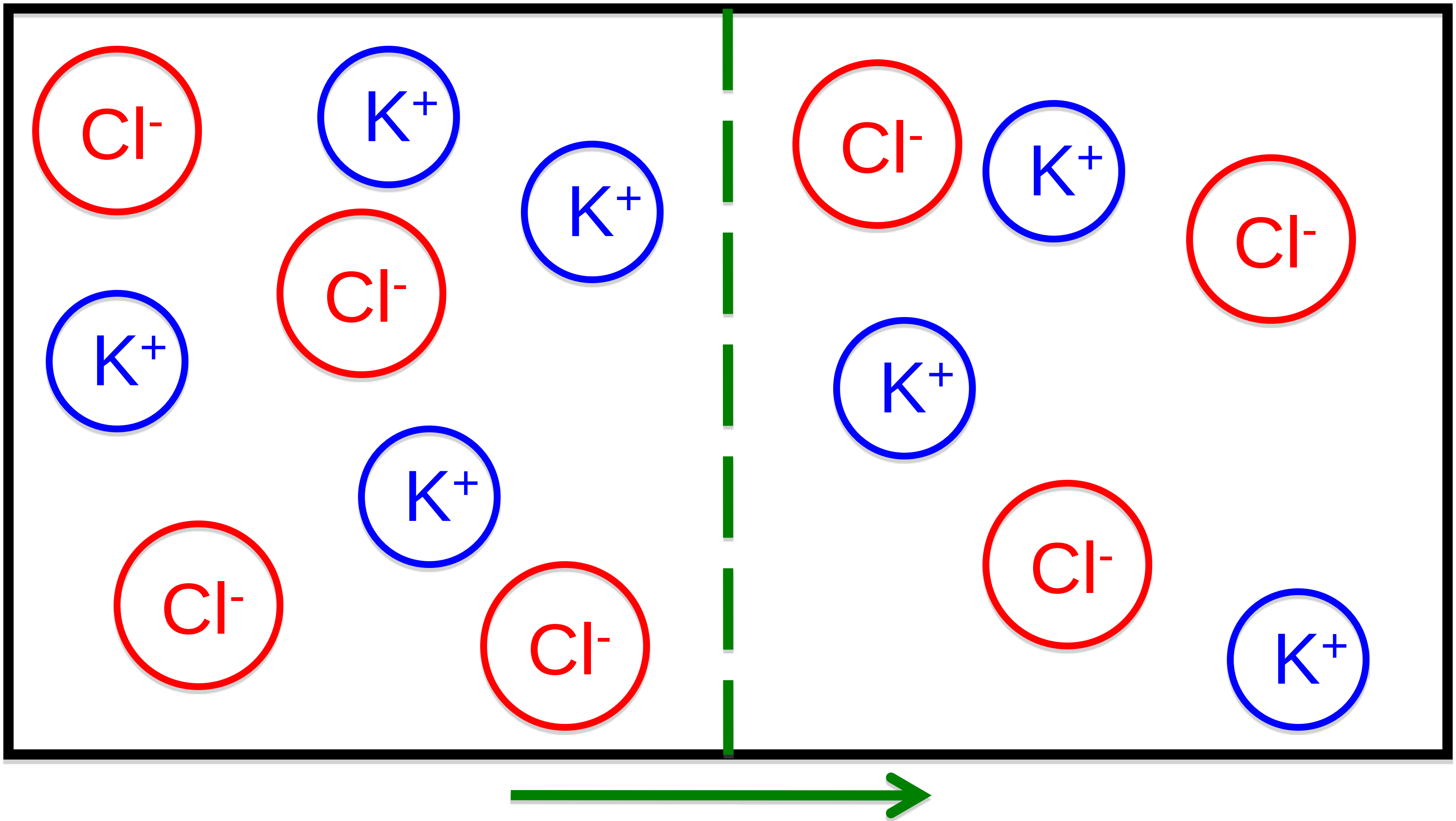
Lecture 17

Applications of calculus and vector algebra in biology

Nernst Equation

$C_1 > C_2$



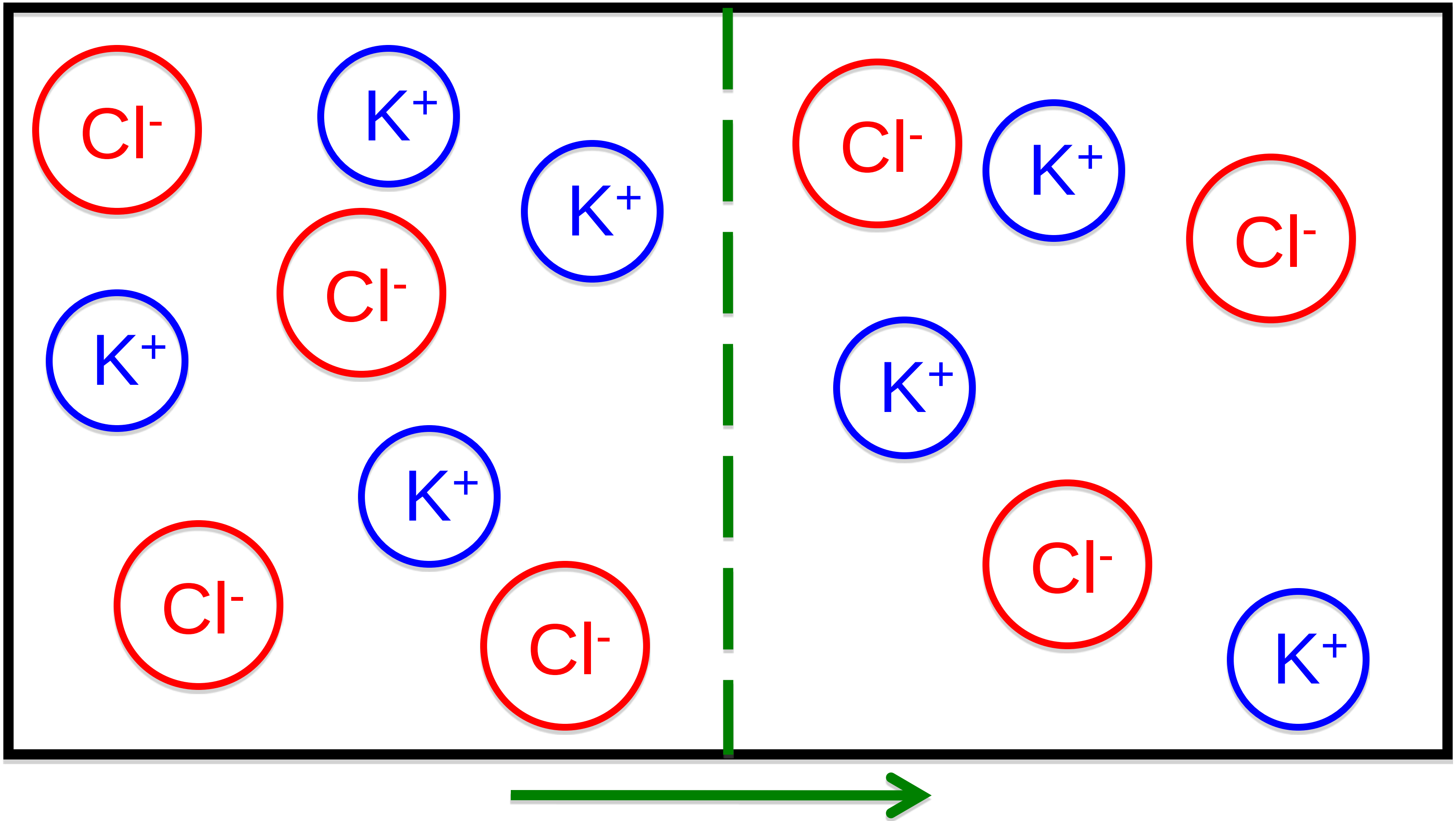
Diffusion (only K^+)

Diffusion Current

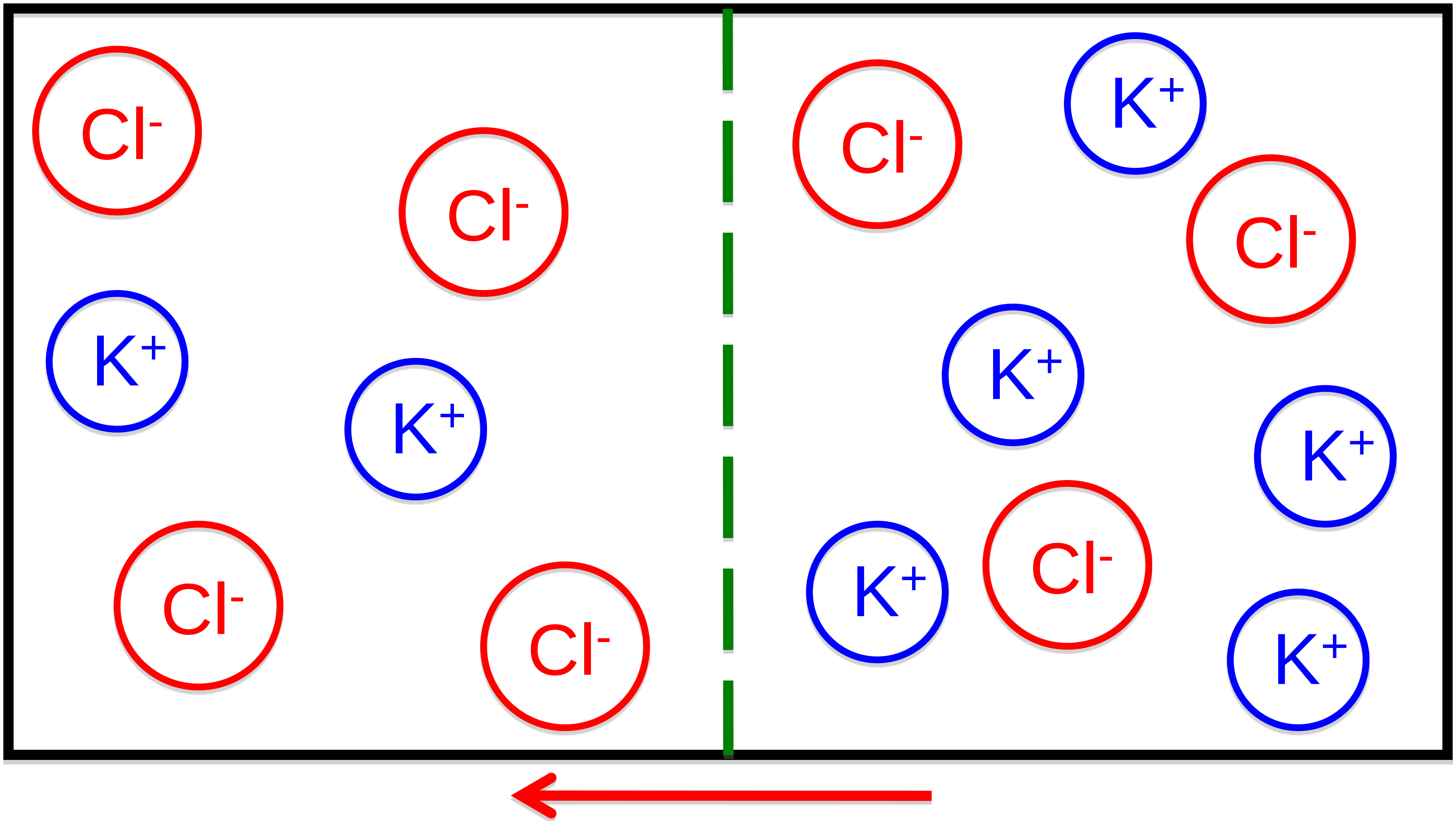
$$\vec{J}_D = -D\vec{\nabla}C$$

$$\vec{J}_D = -D\frac{\partial C}{\partial x}\hat{x}$$

$C_1 > C_2$



Diffusion (only K^+)



Electrostatic attraction on K^+

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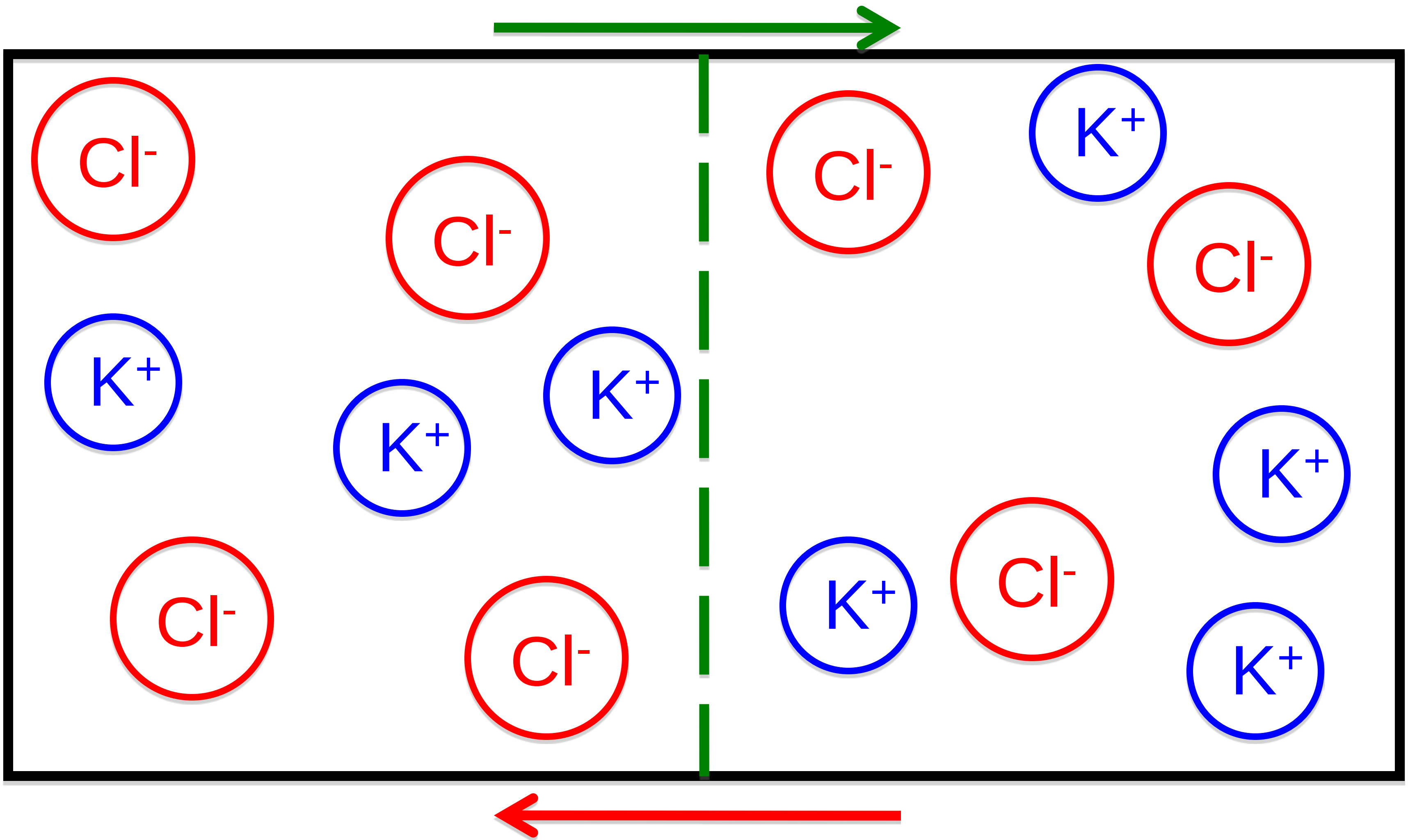
Current due to the electrostatic force

$$\vec{J}_E = C\vec{v}$$

$$\vec{f} = 6\pi\eta a\vec{v} = q\vec{E}; \quad \vec{v} = \frac{q\vec{E}}{6\pi\eta a}$$

$$\vec{J}_E = C\frac{q\vec{E}}{6\pi\eta a}$$

Diffusion of K^+



Electrostatic attraction on K^+

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When both the currents balance, we reach equilibrium.

Let C_1 and C_2 be the equilibrium concentrations of K^+ ions at either sides of the membrane

$$\vec{E} = -\vec{\nabla}\phi$$

ϕ: Electrostatic potential

$$\vec{J}_E = -\frac{qC\vec{\nabla}\phi}{6\pi\eta a}$$

$$\vec{J}_E = \frac{qC}{6\pi\eta a}\frac{\partial\phi}{\partial x}(-\hat{x})$$

At equilibrium: zero net flow

$$\vec{J}_D + \vec{J}_E = 0$$

$$\vec{J}_D = -\vec{J}_E$$

$$-D\frac{dC}{dx}\hat{x} = \frac{C}{6\pi\eta a}\frac{d\phi}{dx}\hat{x}$$

$$\int_{C_1}^{C_2}\frac{dC}{C} = \frac{-q}{6\pi\eta a D}\int_{\phi_1}^{\phi_2} d\phi$$

$$\ln\frac{C_2}{C_1} = \frac{-q}{6\pi\eta a D}(\phi_2 - \phi_1)$$

From Einstein's relation,

$$D = \frac{k_B T}{6\pi\eta a}$$

Nernst equation

$$\ln\frac{C_2}{C_1} = \frac{q}{k_B T}(\phi_1 - \phi_2)$$

$$\Delta\phi = \frac{k_B T}{q}\ln\frac{C_2}{C_1}$$

Potential difference across a membrane is related to concentrations of ions across the membrane

Summary

- Flow of ions due to concentration difference
- Flow of ions due to electrostatic attractions
- When these two flows balance, we get equilibrium
- At equilibrium there is a potential difference across the membrane

$$\Delta\phi = \frac{k_B T}{q} \ln \frac{C_2}{C_1}$$