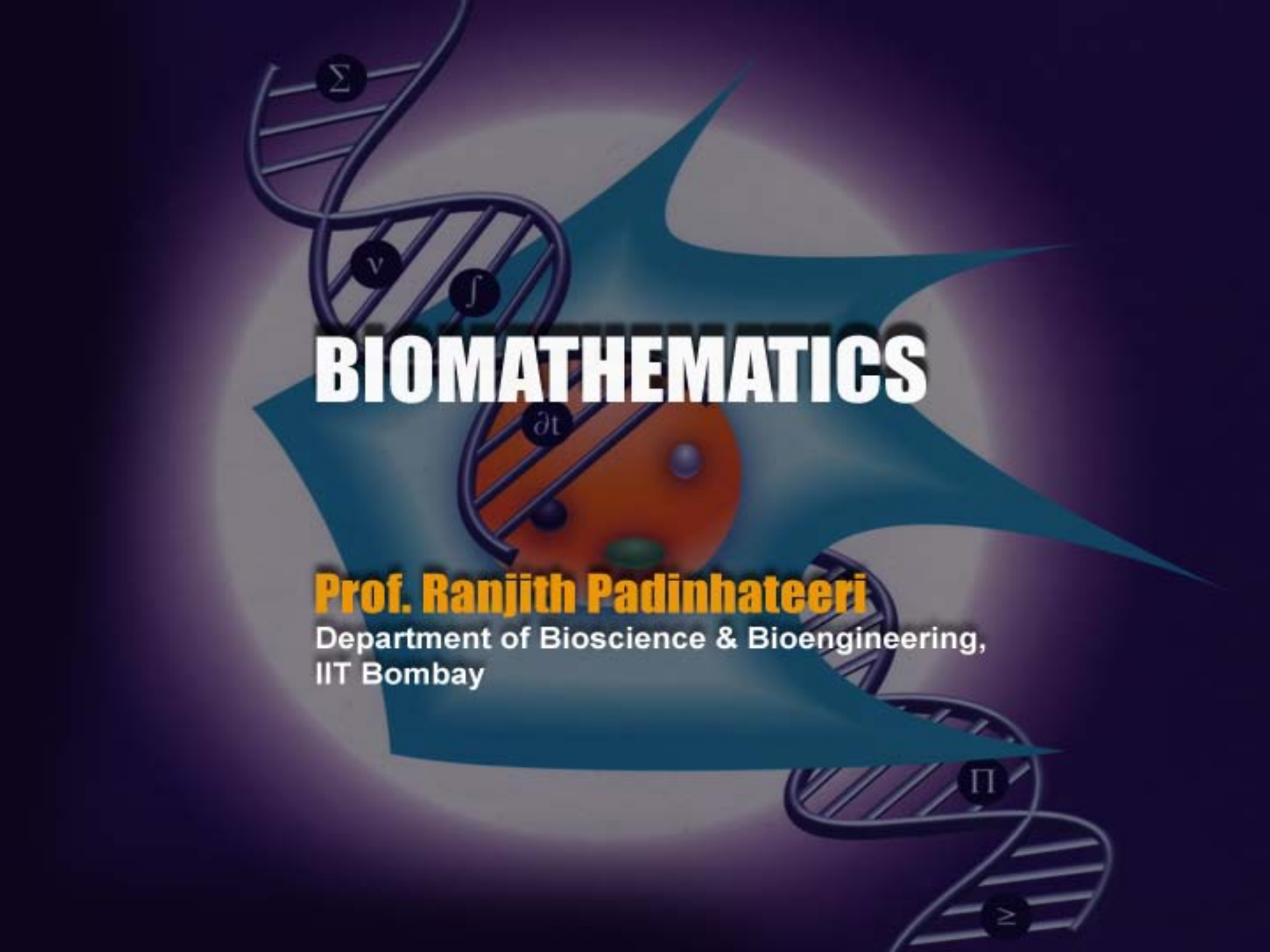


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# Graphs and Functions

# Experimental result → Graph

- Experimental results are typically presented as a graph -- not as a set of statements.
- A graph conveys much more information than a set of statements
- It is quantitative.

# Graph <--> Equation

- A graph, in principle, can be represented by a mathematical equation.
- Understanding that equation, we can learn more about the experimental data/biological system.

In this lecture, we will learn how simple equations can be plotted as graphs

# Function

- To plot a graph, we need X values and Y values.
- The relation between quantities that we plot in X axis and Y axis is called a "**Function**"

# Function

A function defines how the quantity in the Y axis is related to the quantity in the X axis

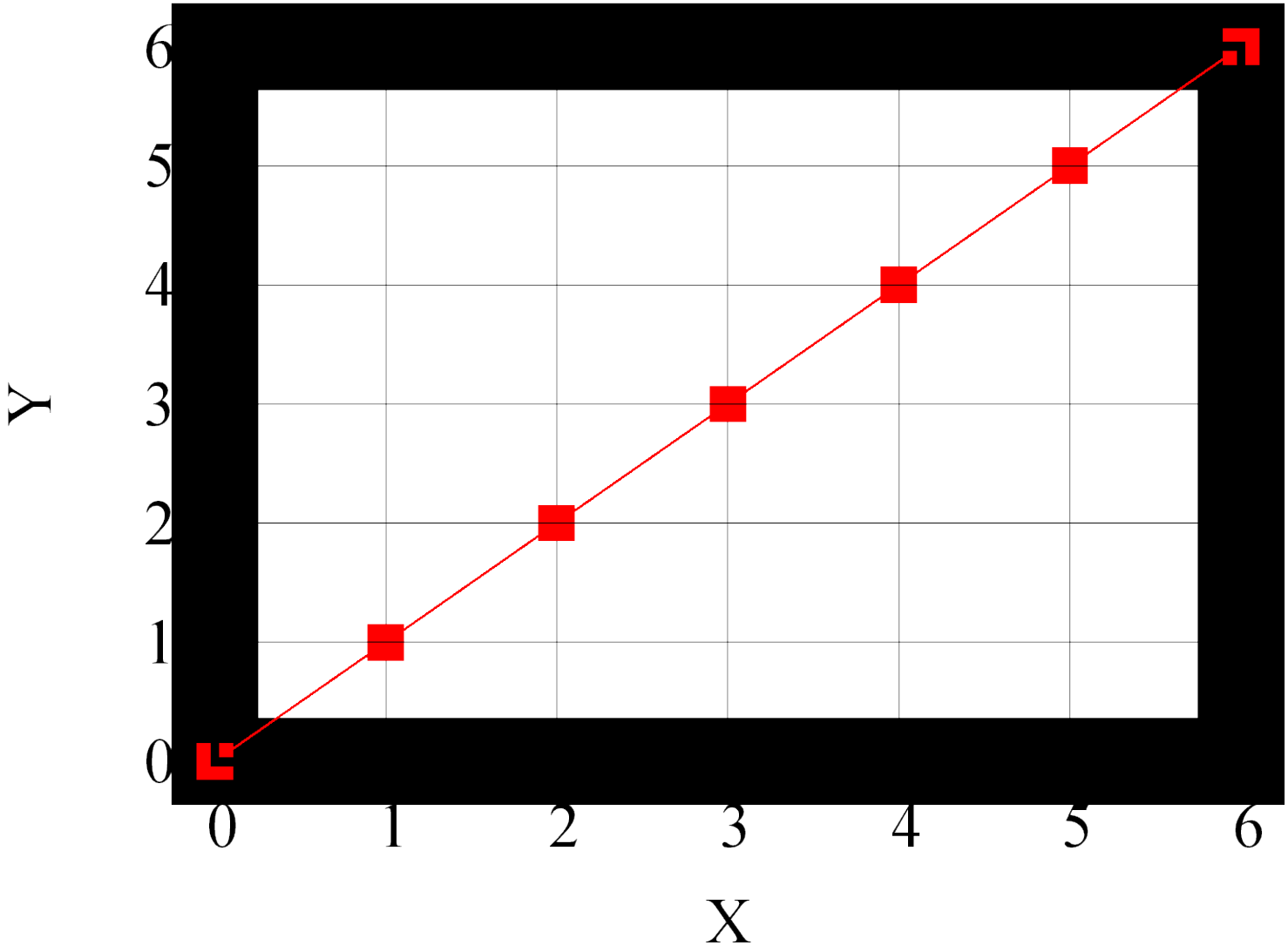
The simplest relation is

$$Y = X$$

# Y=X

| X | Y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |

# Y=X




| X | Y |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |

# Common notations

$Y=X$

$y(x)=x$

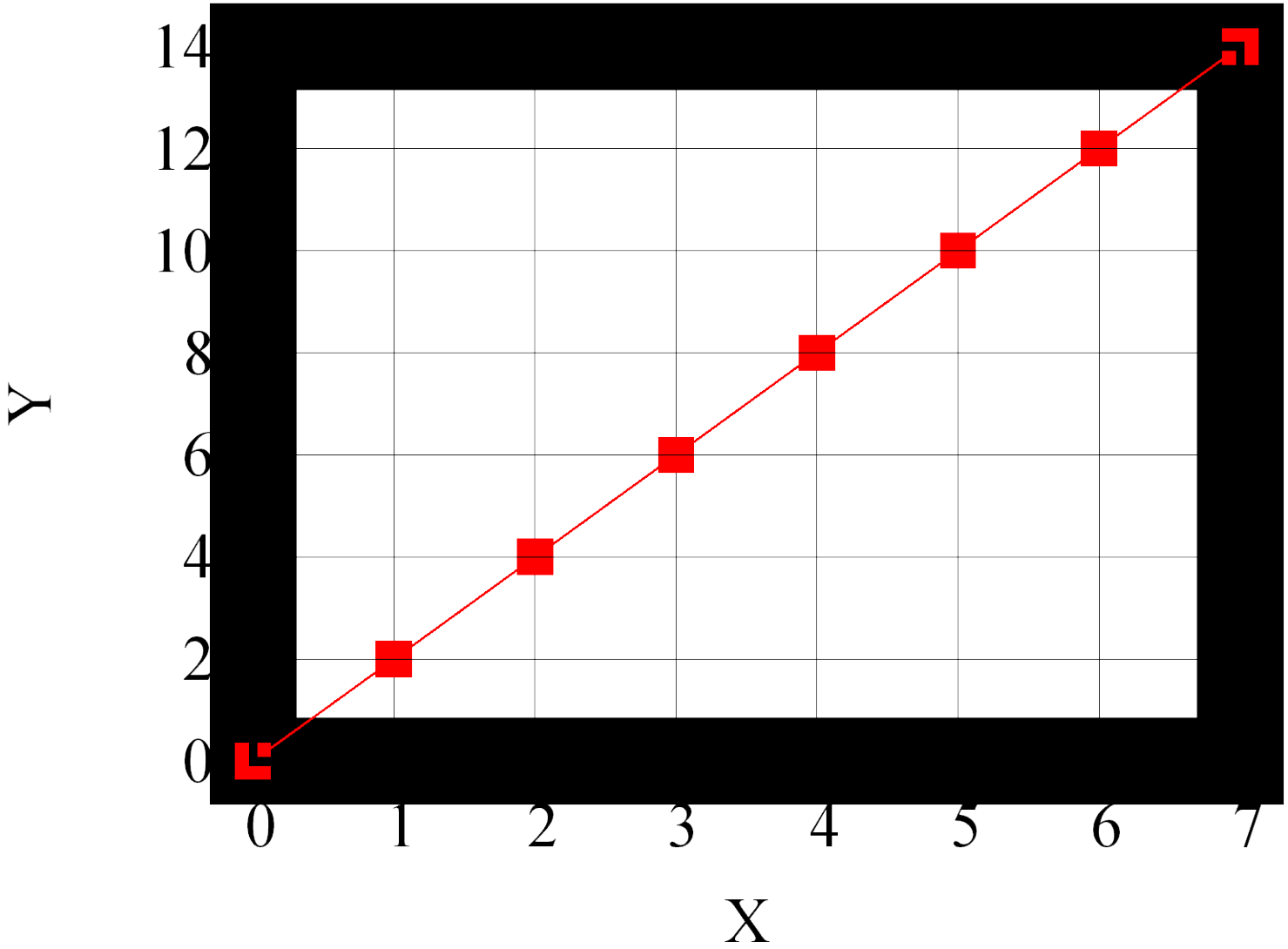
$f(x)= x$

All of the above mean the same thing.

# Y=2X

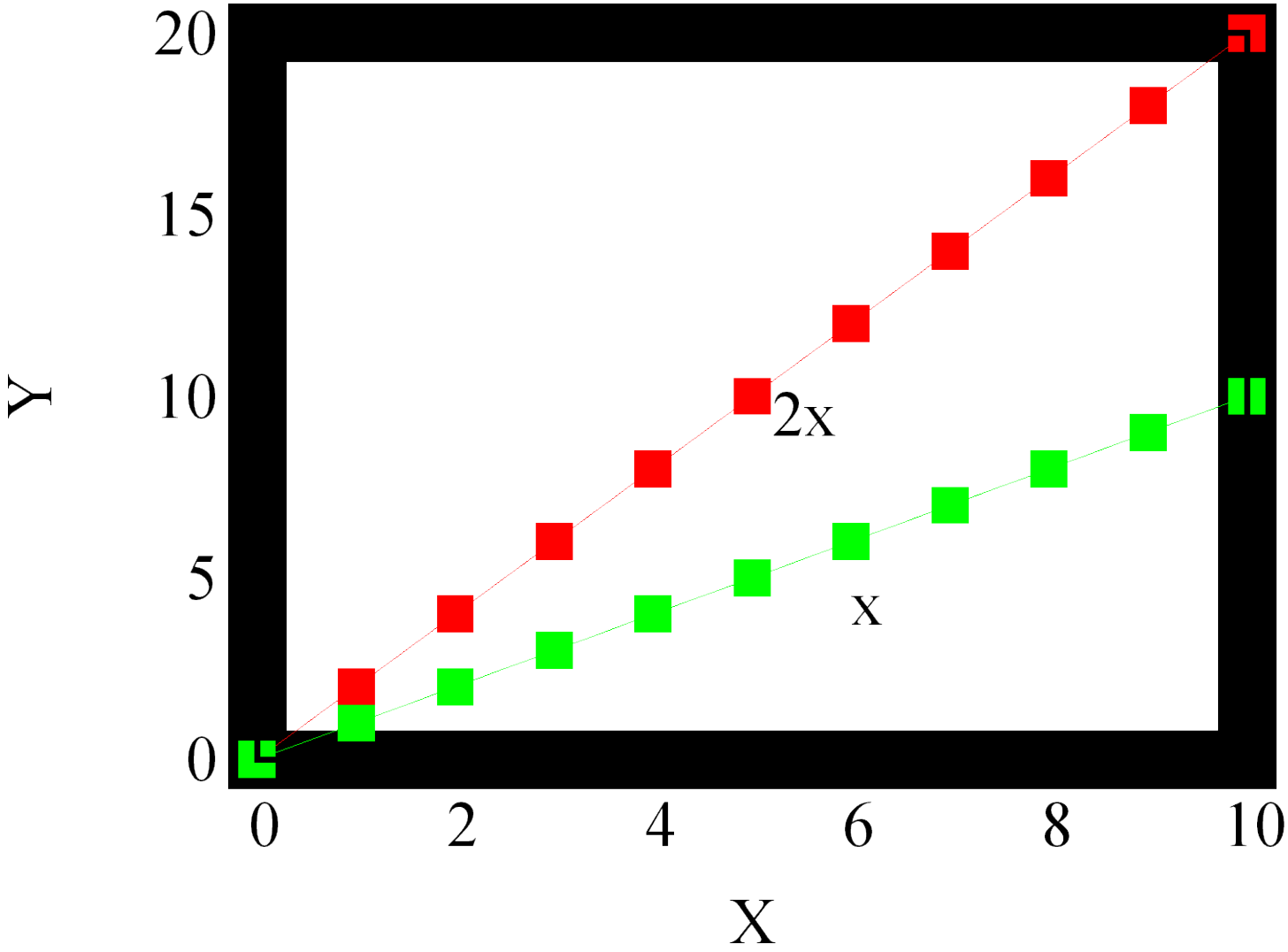
| X | Y |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |
| 5 | 10 |

# Y=2X




| X | Y |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |
| 5 | 10 |
| 6 | 12 |
| 7 | 14 |

# Compare: Y=X, and Y=2X

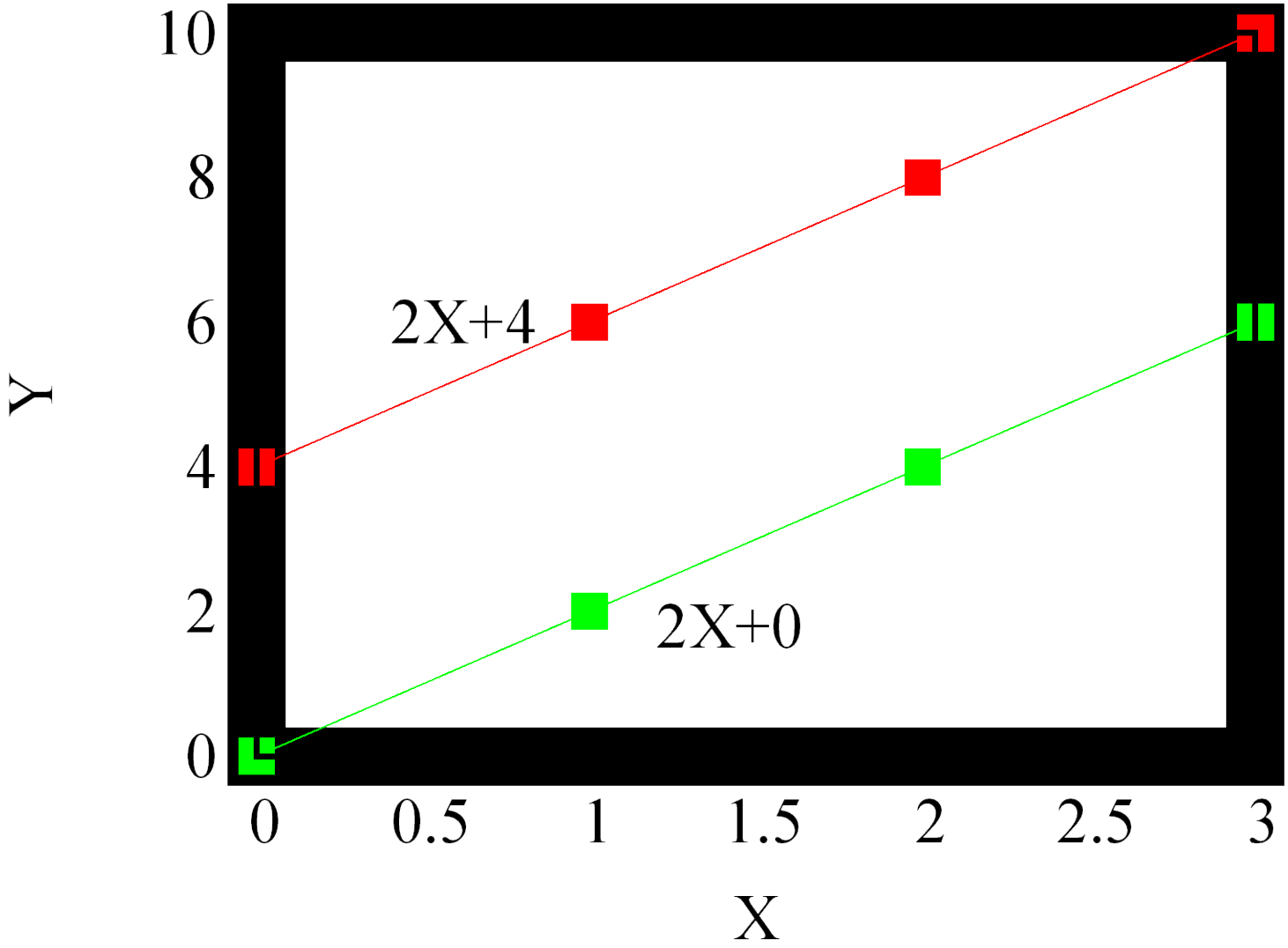


# In general : Y=mX

Y=mX is equation of a straight line passing through Y=0

m is a number

# Y=2X, and Y=2X+4



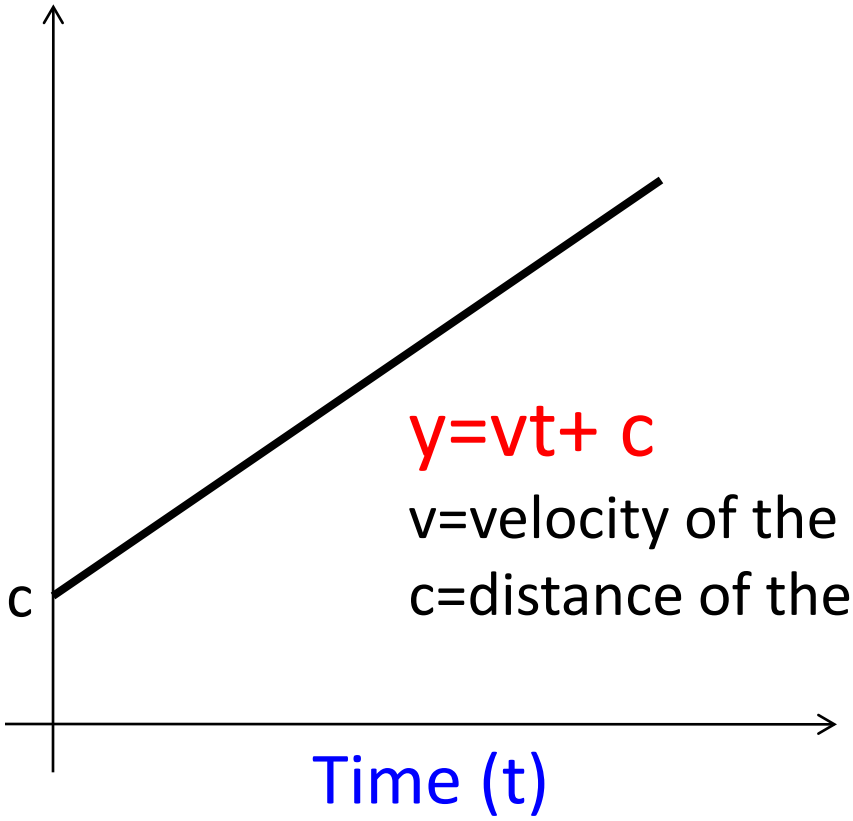
# Y=mX+C

Y=mX+C is equation of a straight line passing through Y=C

When X and Y are related using this equation, we say:

**Y is a linear function of X**

# Example: Molecular motor walking along microtubule




$y = vt + c$

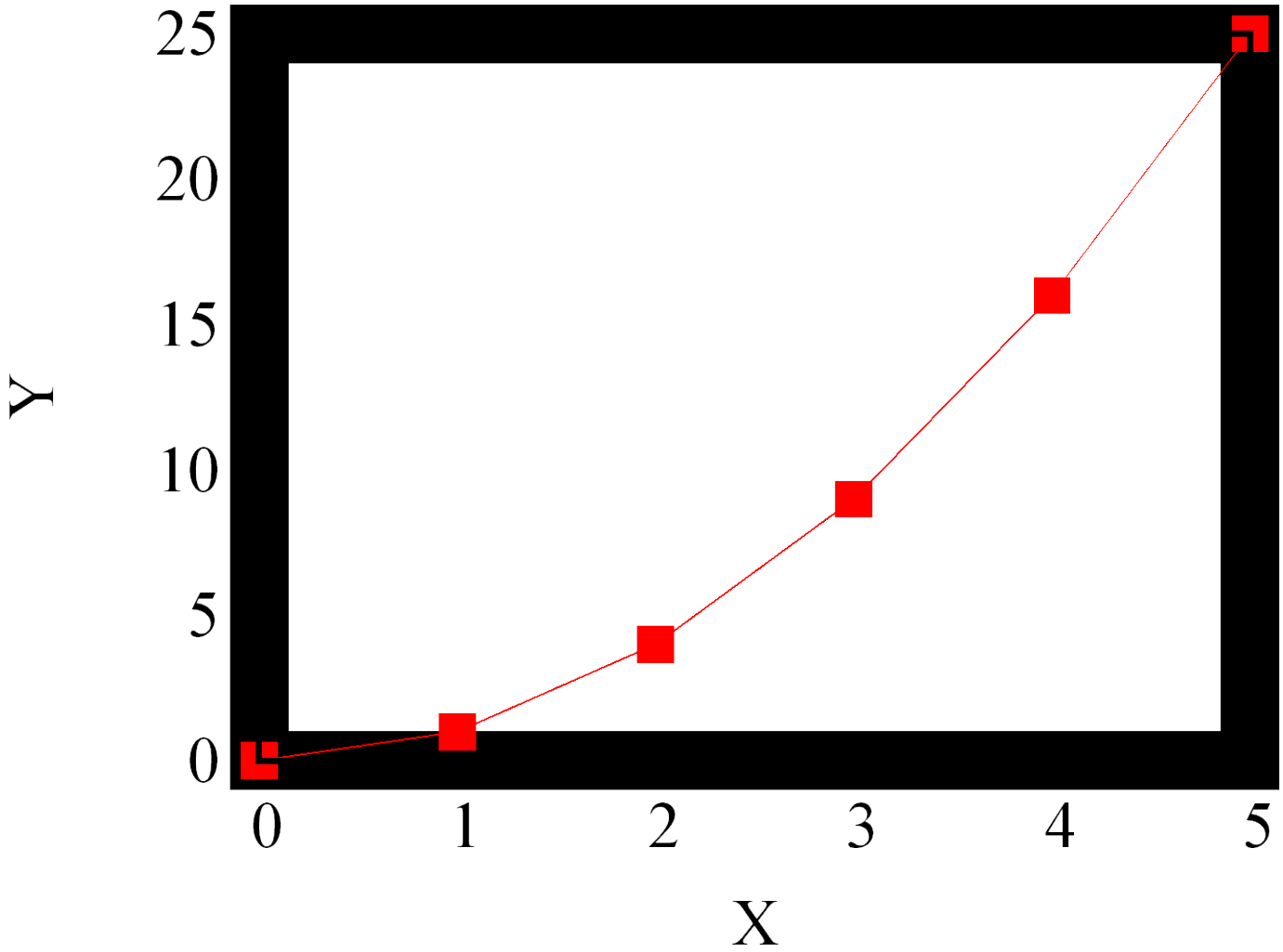
v=velocity of the motor

c=distance of the motor at time=0

# $Y=X^2$

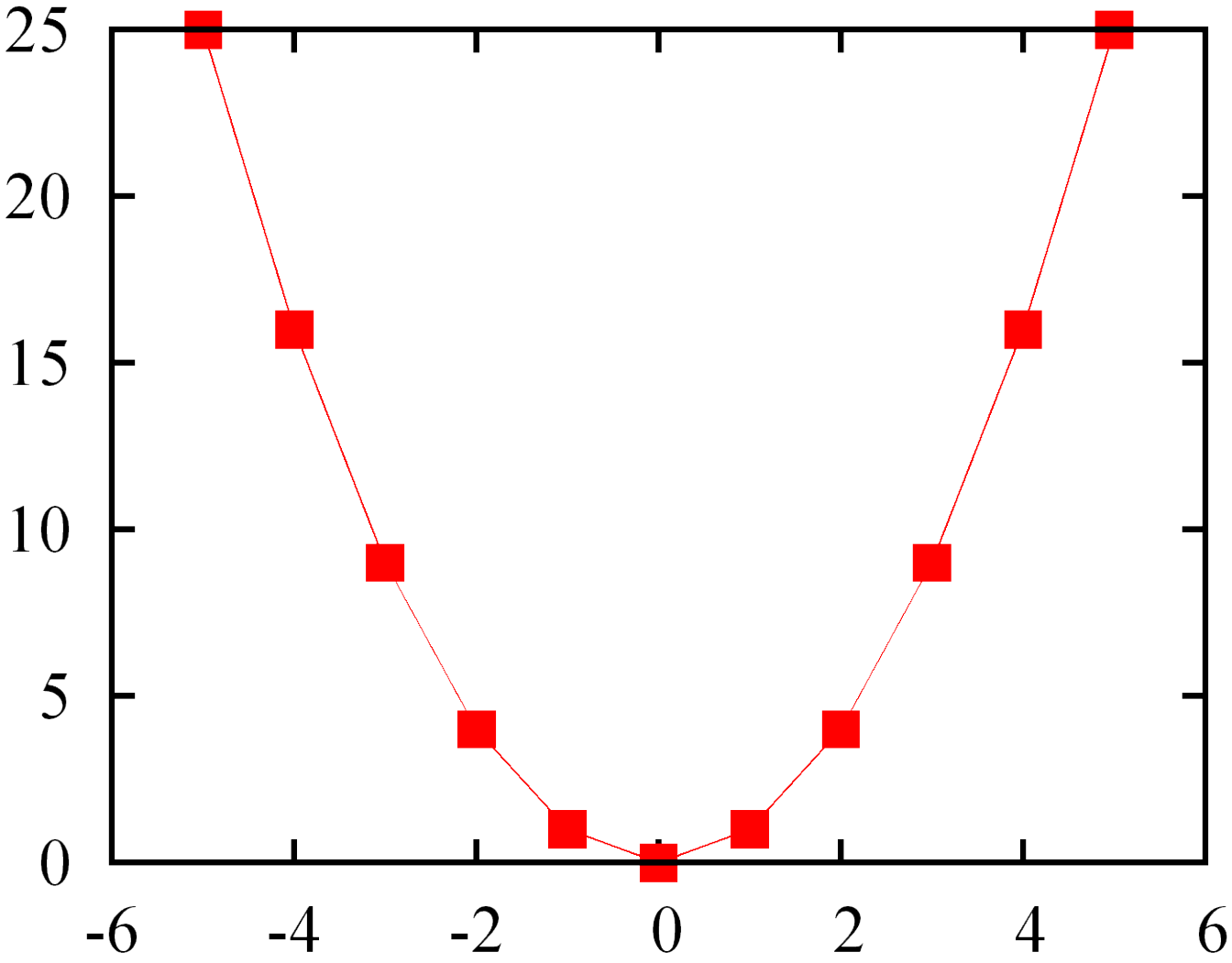
| X | $Y=X^2$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |

# $Y=X^2$

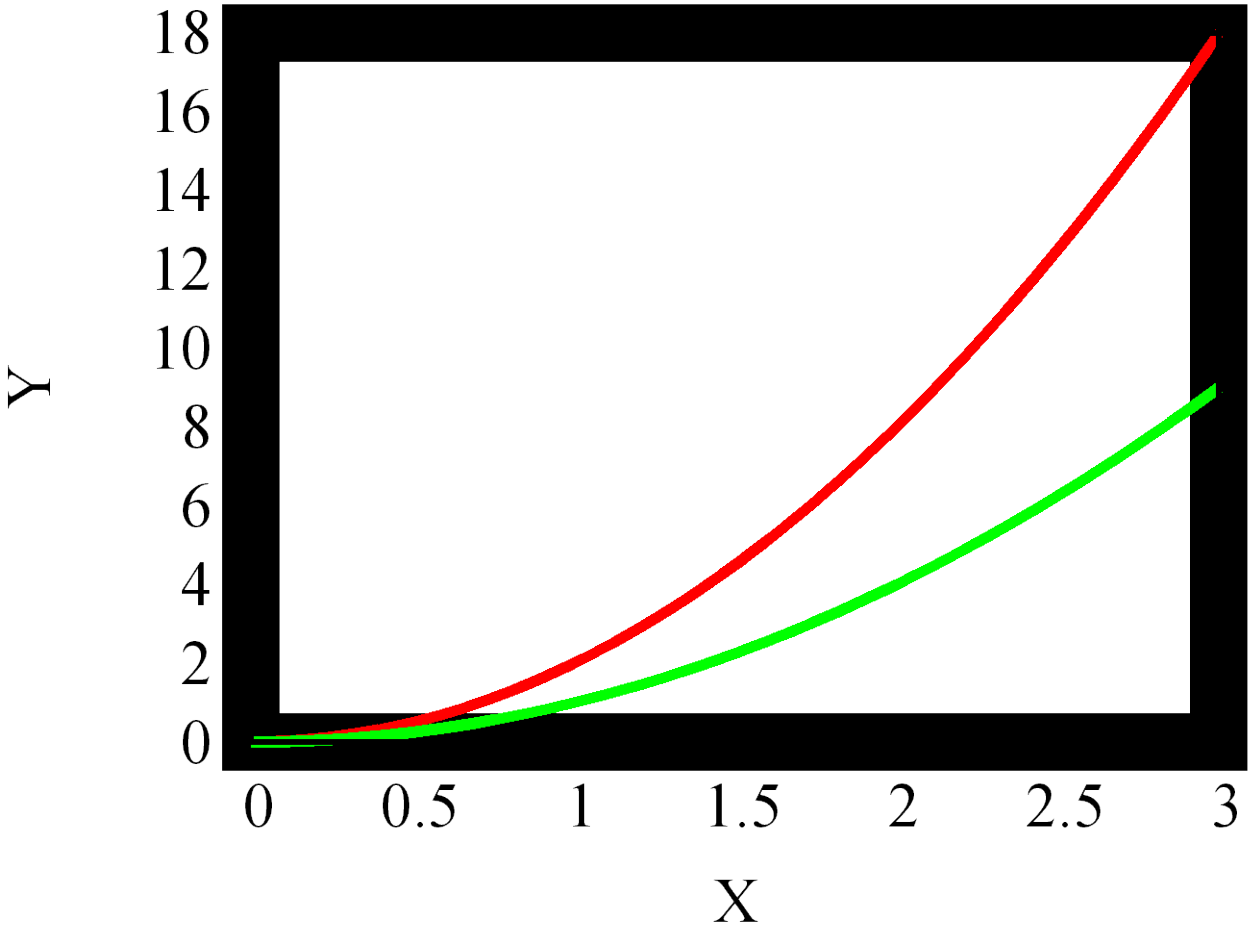



| X | $Y=X^2$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |

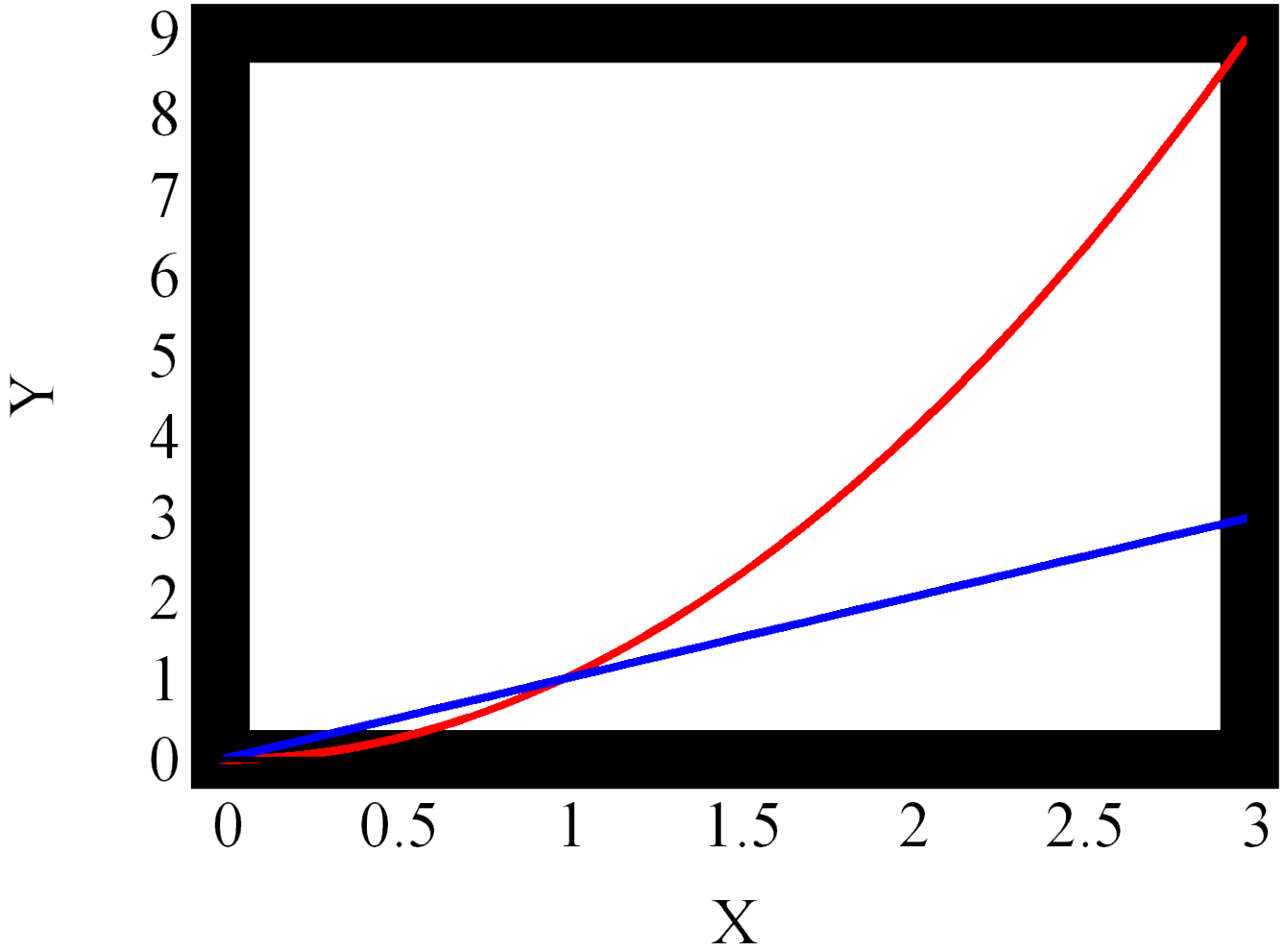
$Y=X^2$



# $Y = X^2$, and $Y = 2X^2$



# Y= X , and Y= $X^2$



# Common notations

$Y=X^2$

$y(x)=x^2$

$f(x)= x^2$

All of the above mean the same thing.

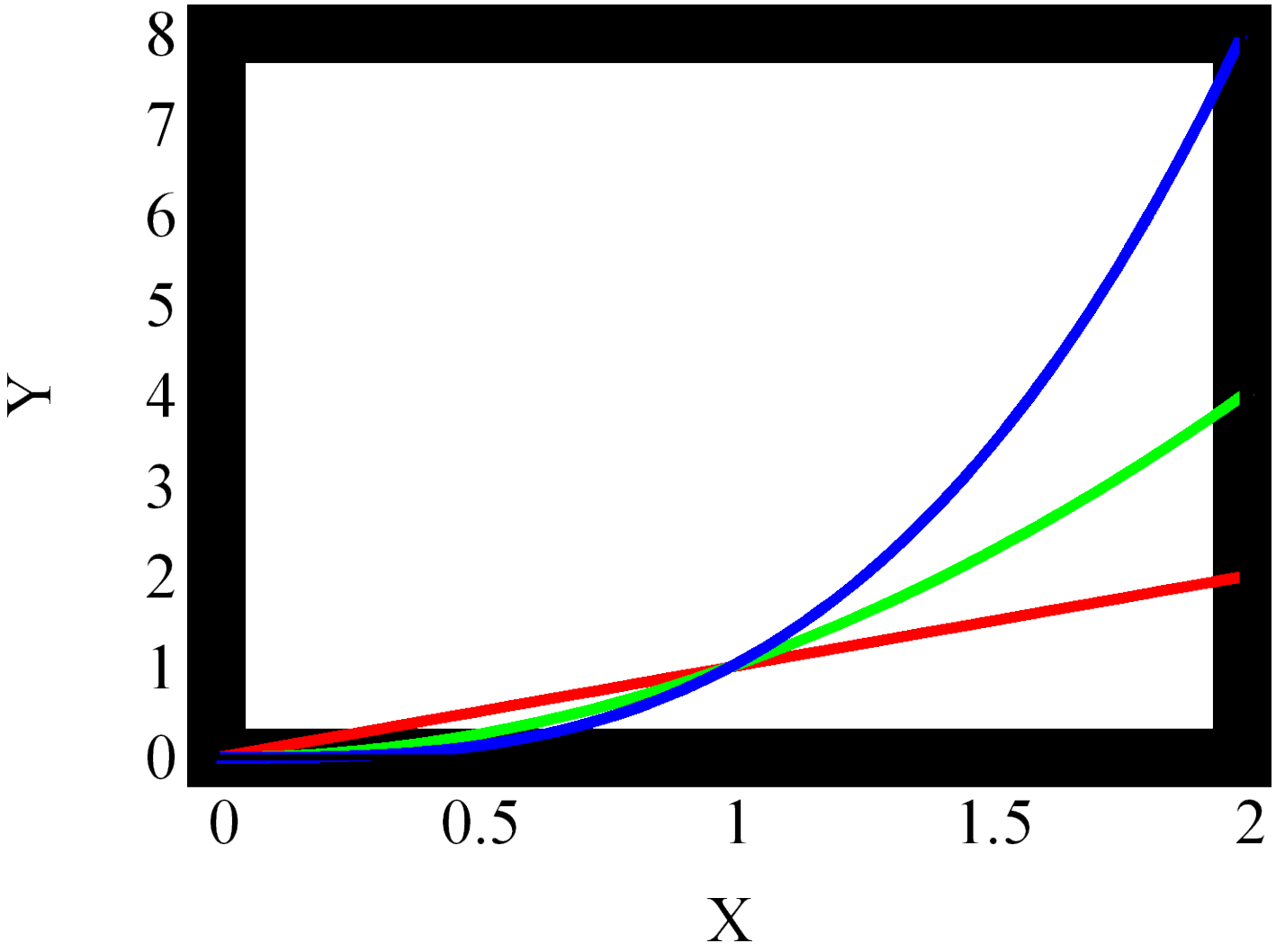
# Quadratic function : $kX^2$

Surface Area of an organism, having radius R = $4\pi R^2$

Basal metabolic rate of an animal is proportional to its surface area

Energy stored in spring-like molecule = $\frac{1}{2}kX^2$

# $Y = X$, $Y = X^2$, and $Y = X^3$



# Cubic function : $kX^3$

Volume of an organism, having radius R = $\frac{4}{3}\pi R^3$

# Combination of x, $x^2$, $x^3$ etc

Now that we understand x,$x^2$,$x^3$ etc, one can imagine many combination of these functions.

Most of the experimental curves you get can be obtained by appropriately adding and subtracting these functions

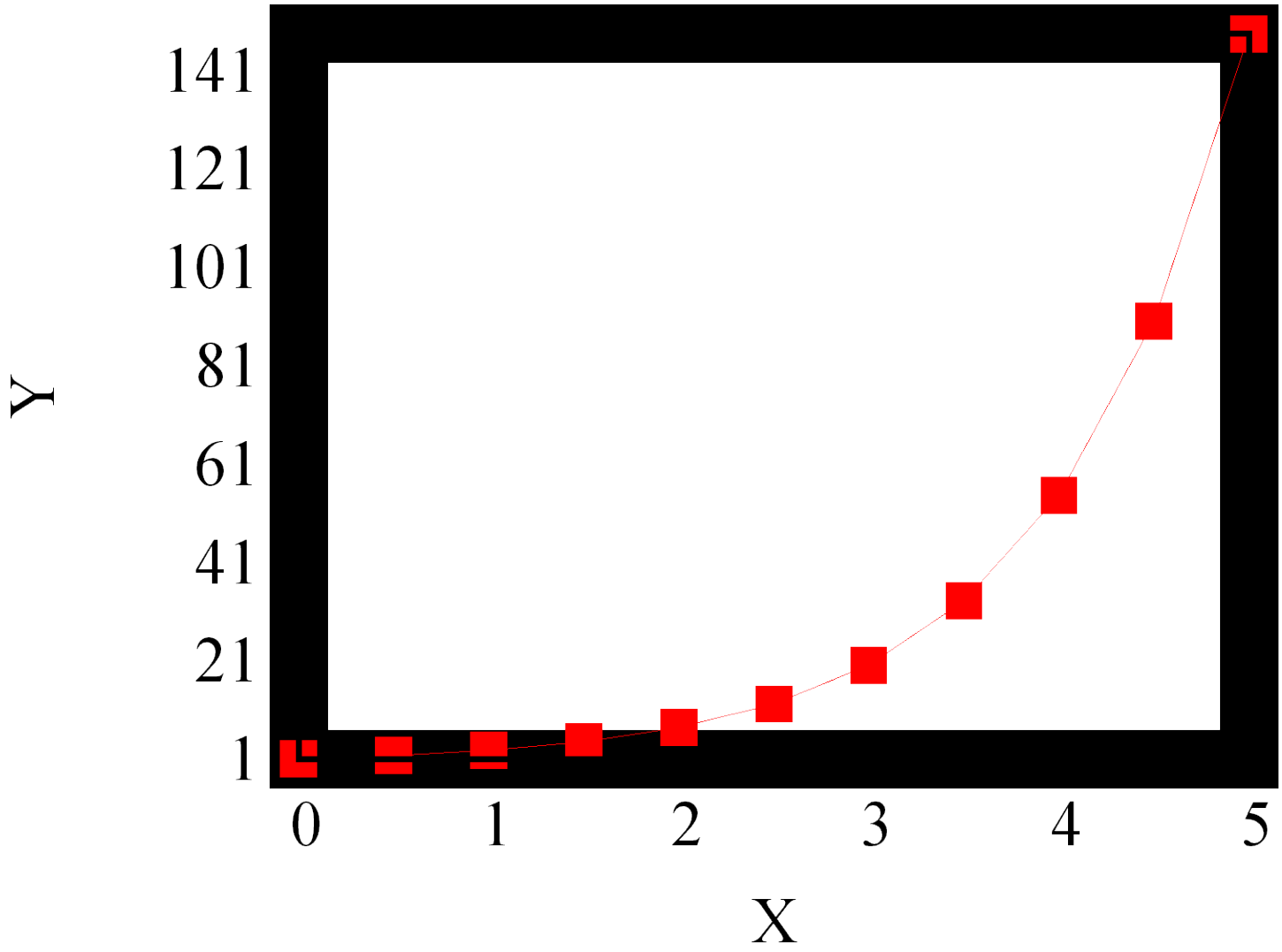
Also, many natural processes behave like a combination of these functions

# Exponential function : $e^x$

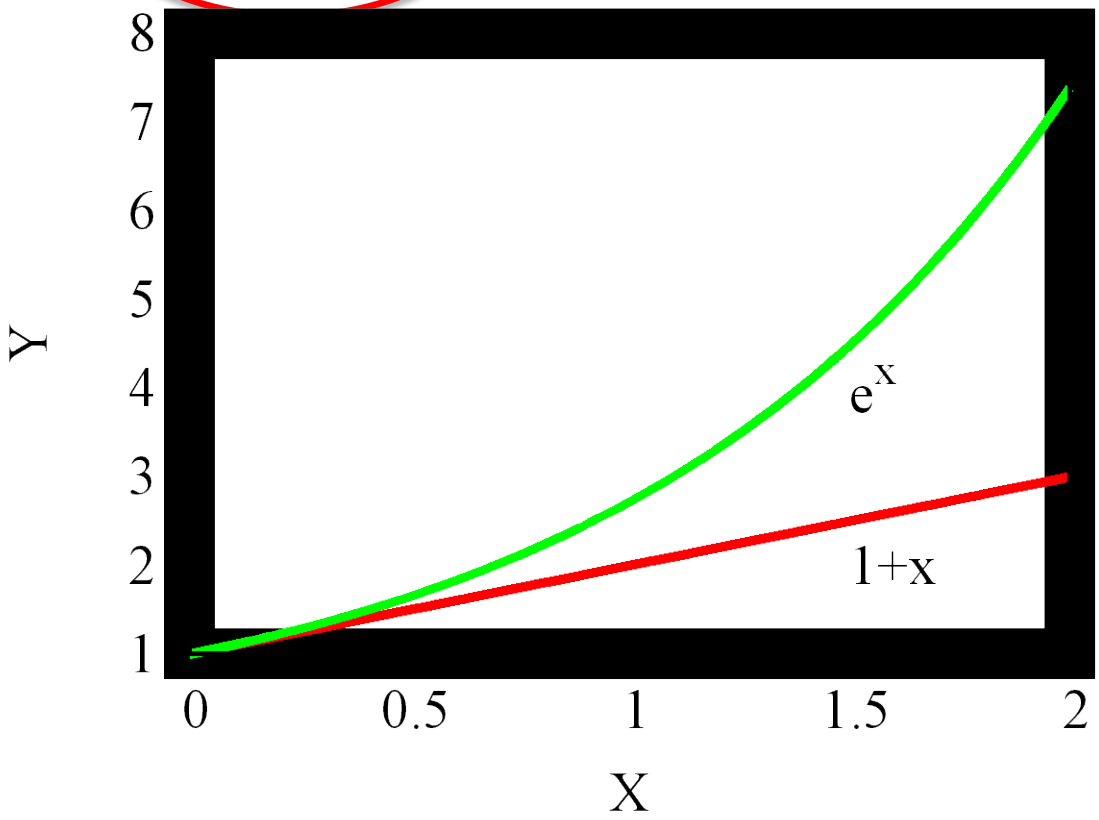
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$$

e is just a number, given by e=2.71828…

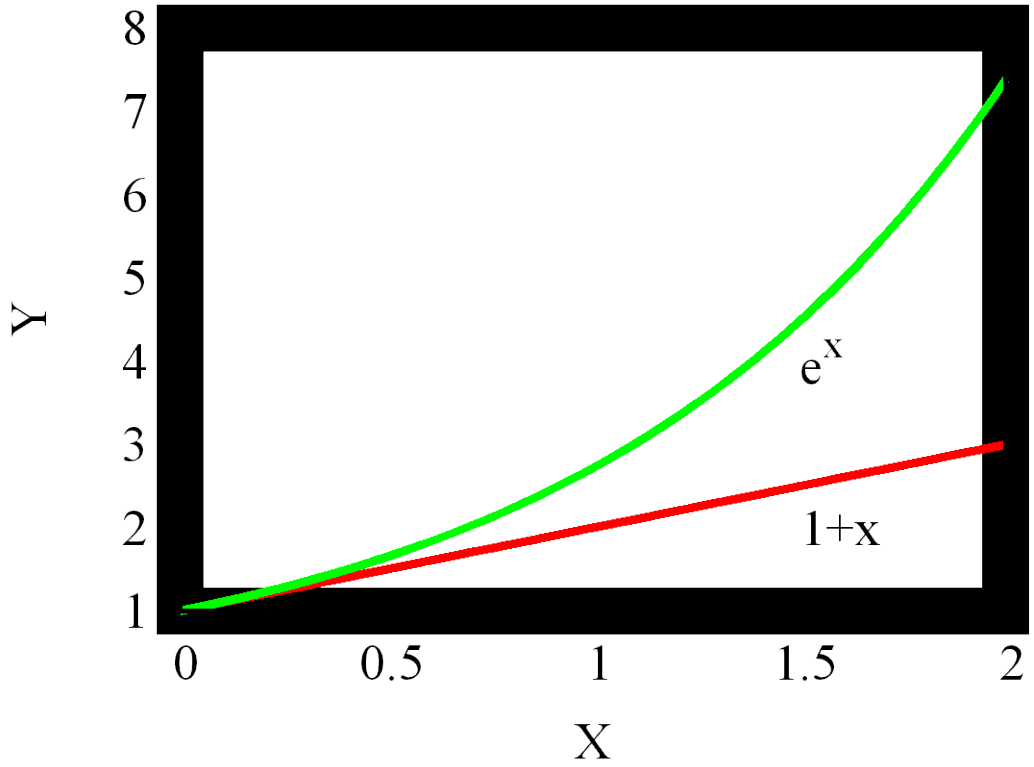
$Y = e^x$



$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$



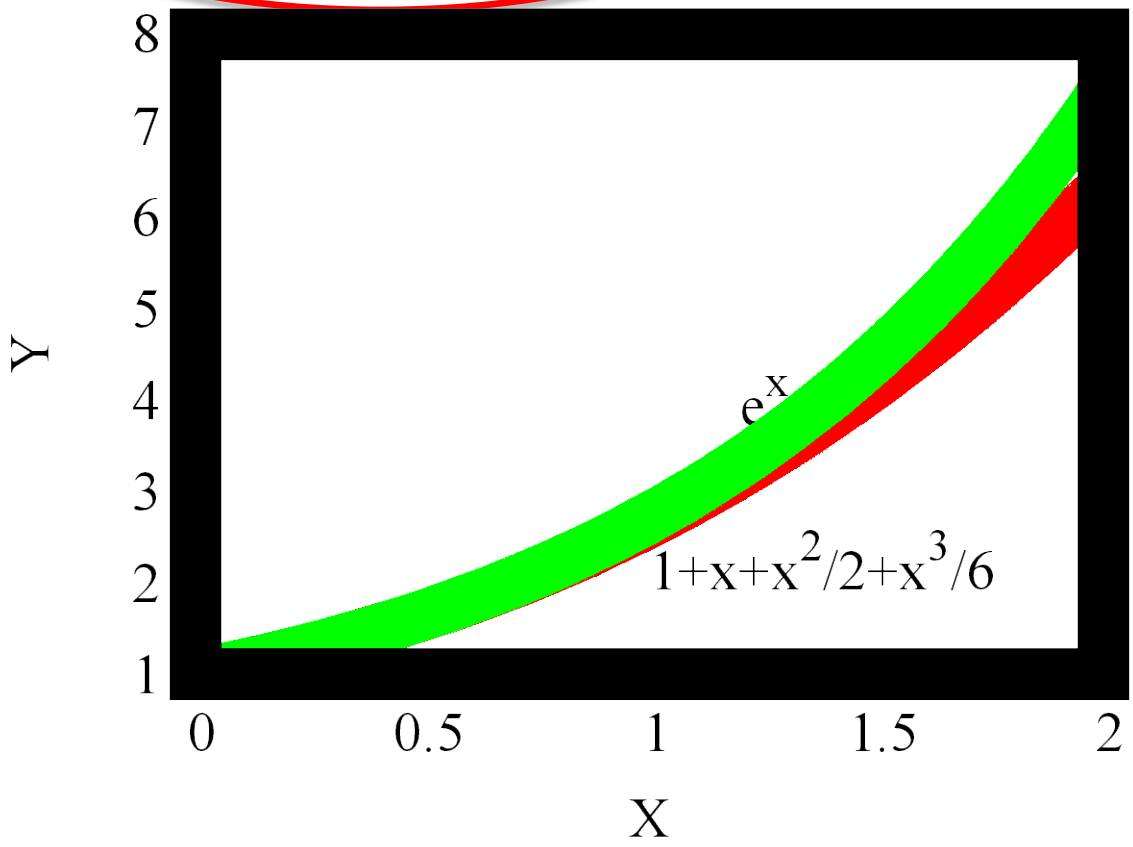
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$




When X is very close to zero :

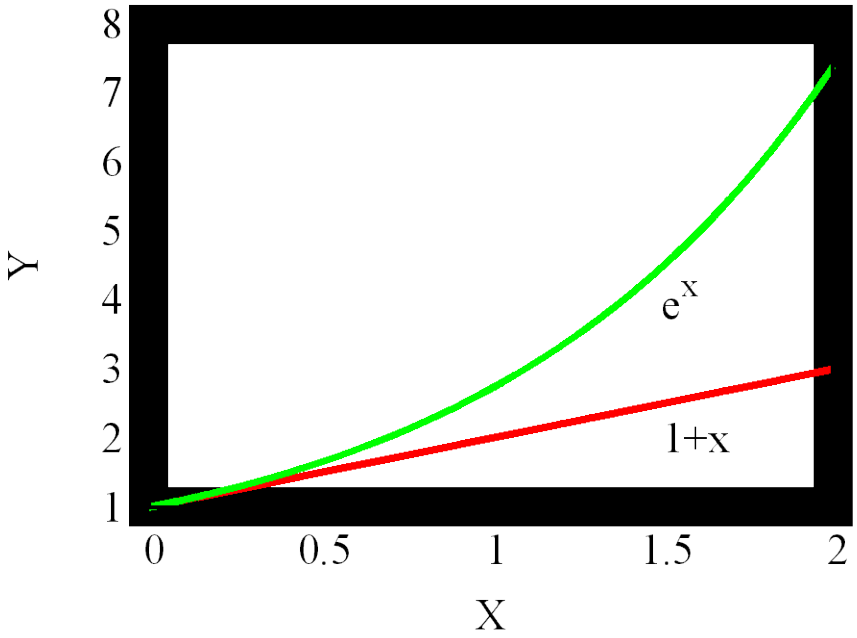
$$e^x \approx 1 + x$$

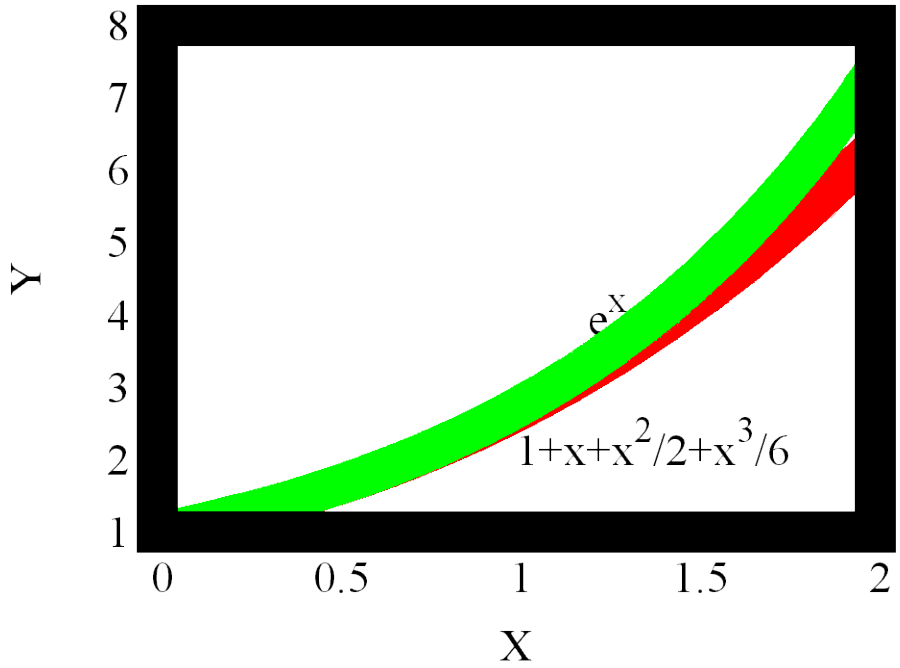
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$$



$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$$



$Y=e^x$

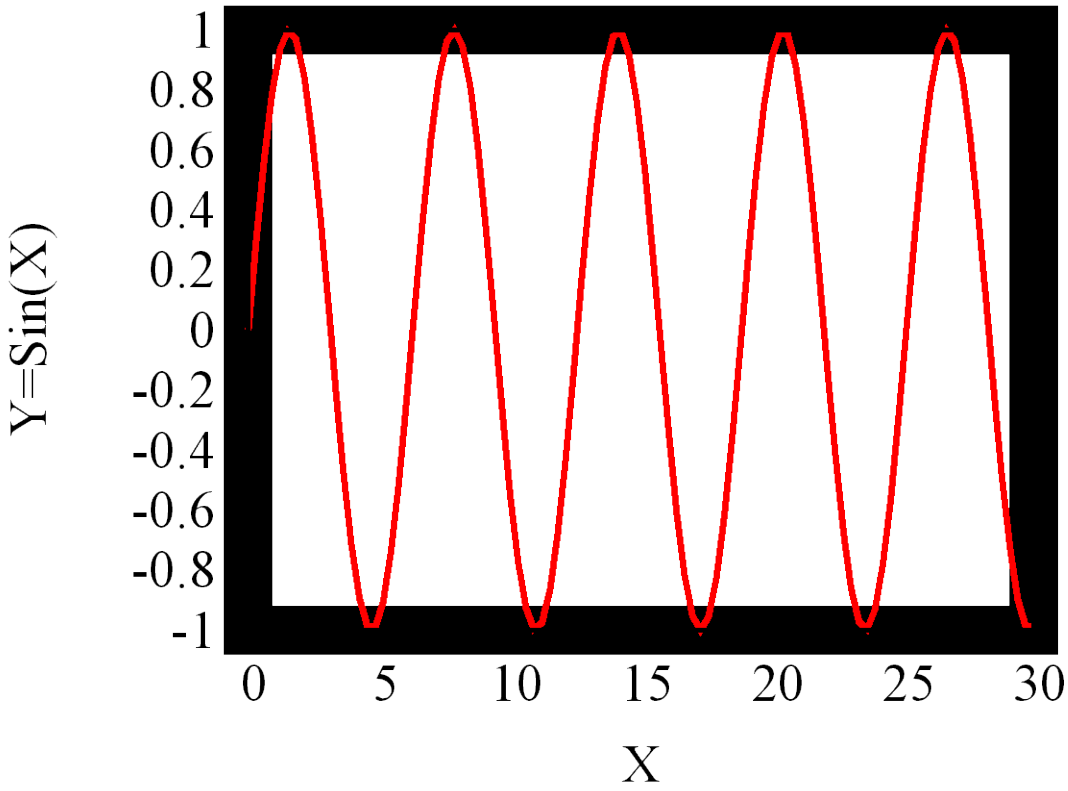
$y(x)=e^x$

$f(x)= e^x$

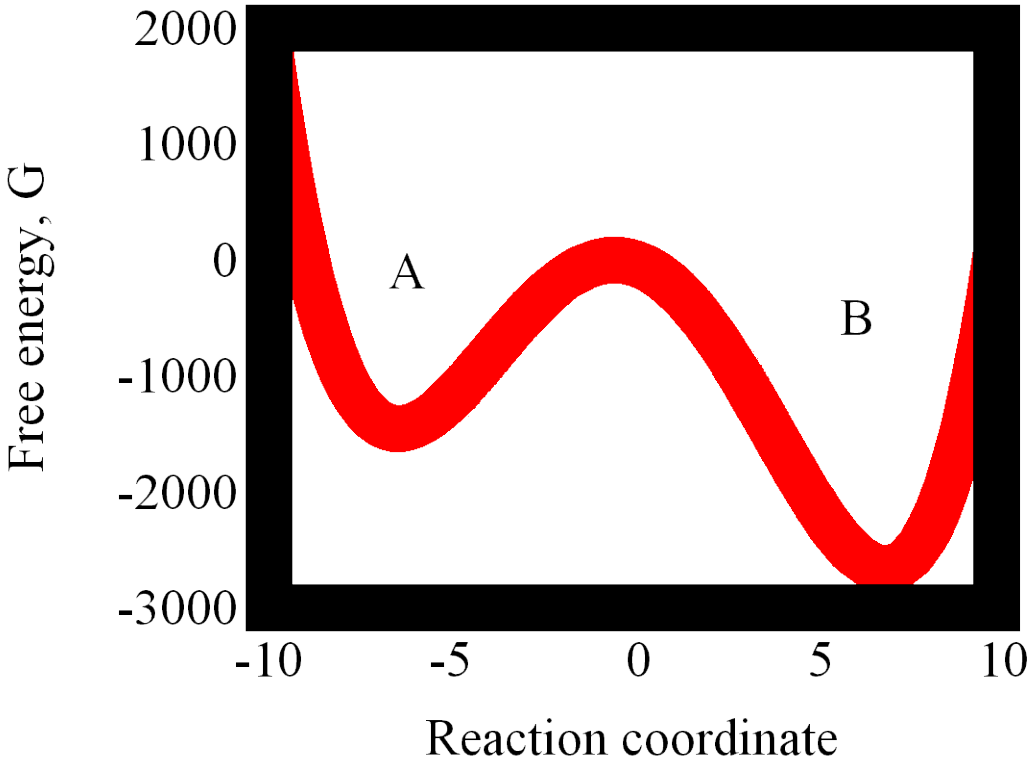
All of the above mean the same thing.

# Sin(x)

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots$$



# Free energy, G




$$Y = pX^4 - qX^2 - mX + c$$

p,q,m, and c are some numbers

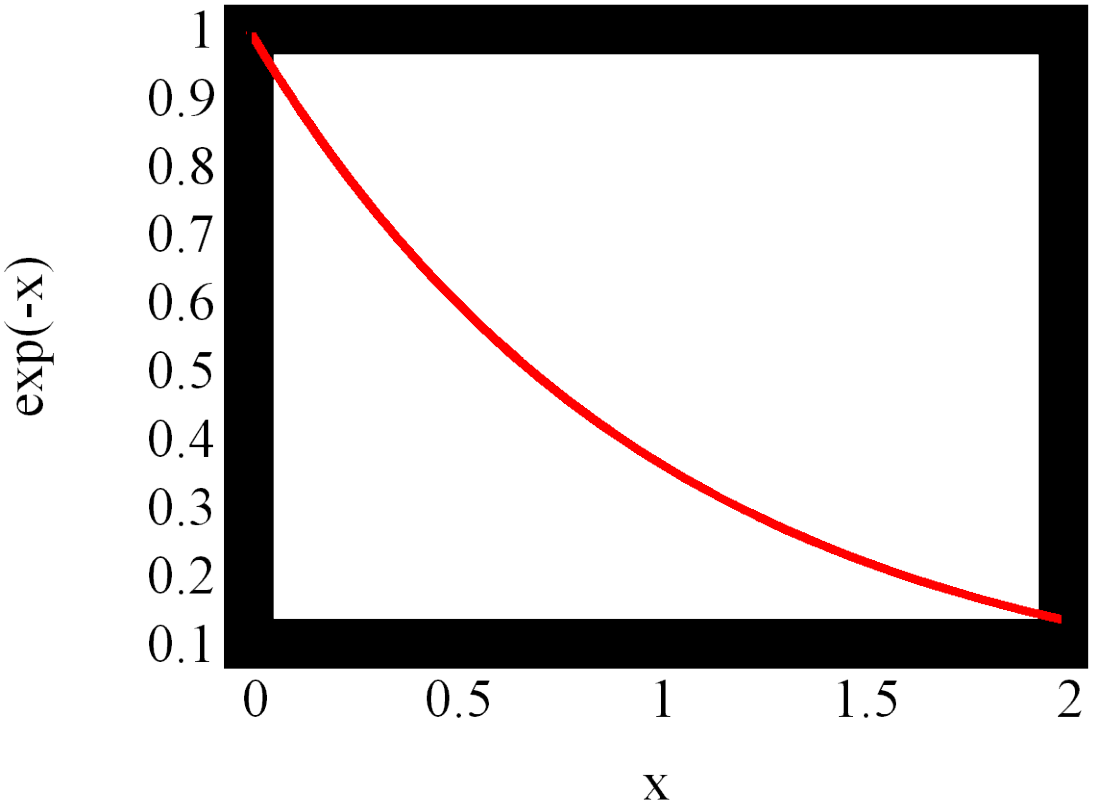
# $e^{-x}$

$$e^{-x} = 1 + (-x) + \frac{(-x)^2}{2} + \frac{(-x)^3}{6} + \ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

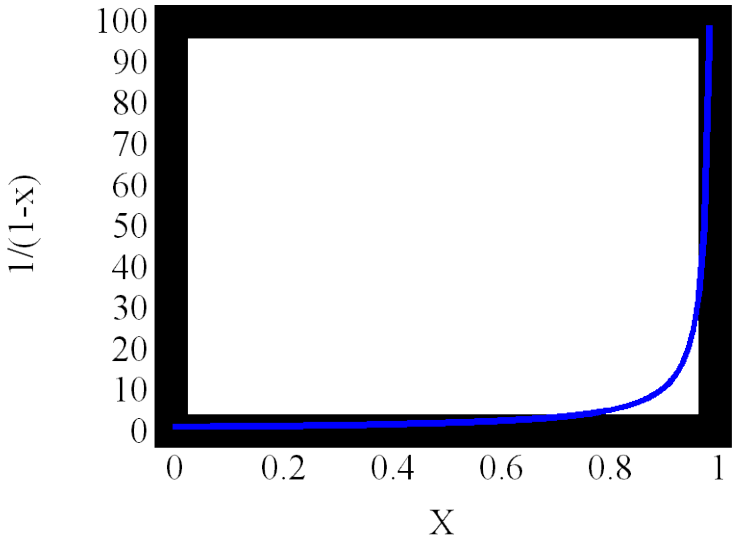
# $e^{-x}$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

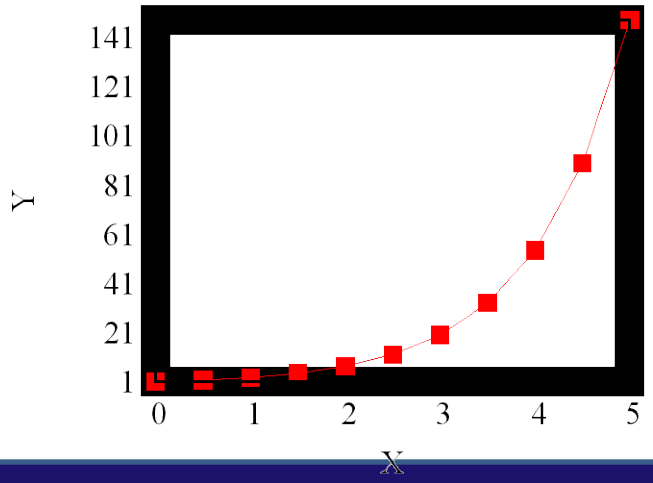


# 1/(1-x)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

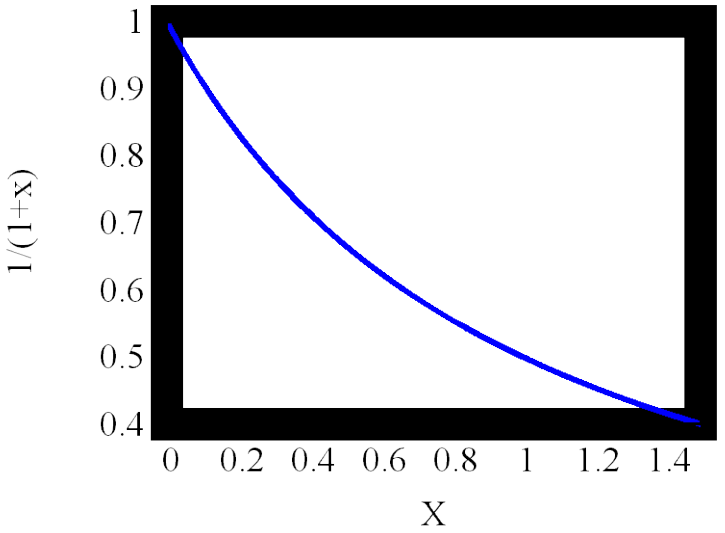



$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

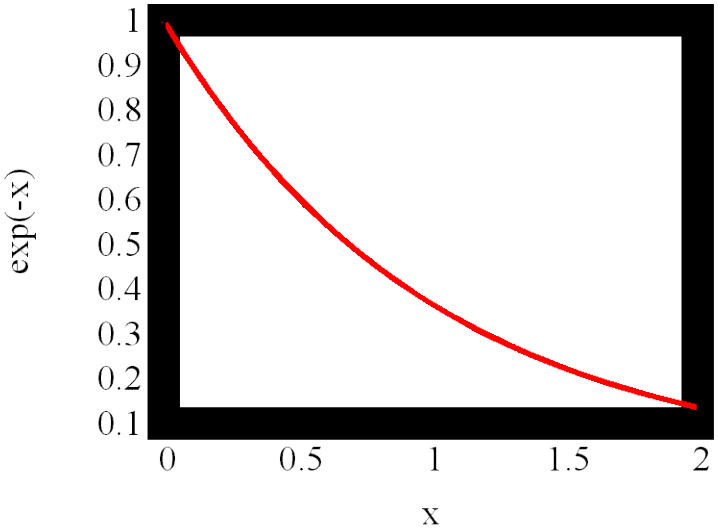


# 1/(1+x) and exp(-x)

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

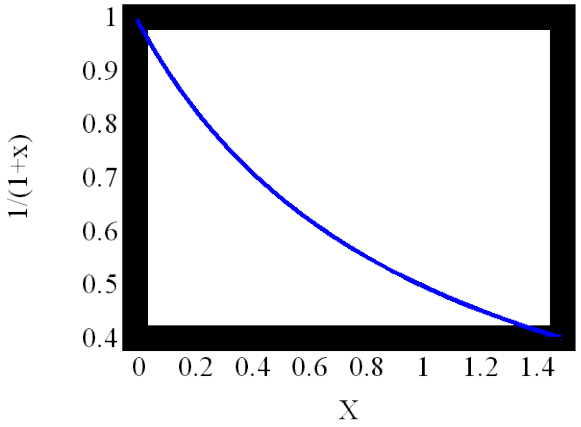



$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

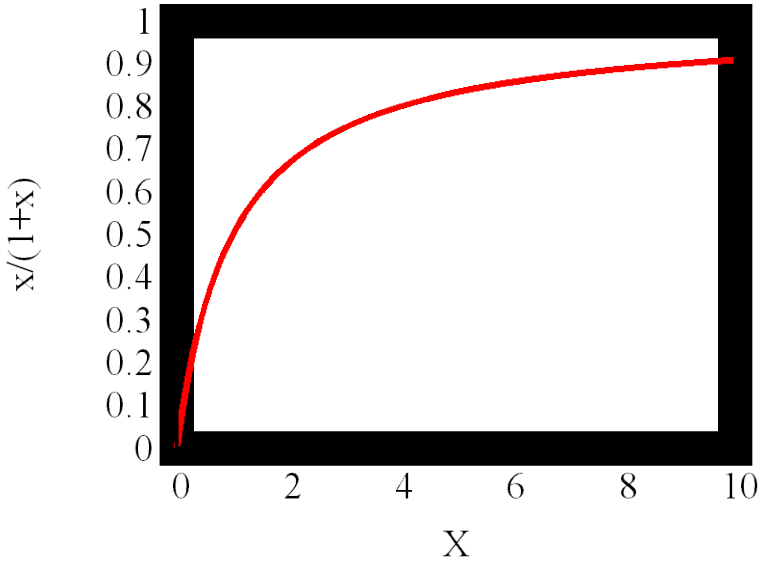


# 1/(1+x) and x/(1+x)

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

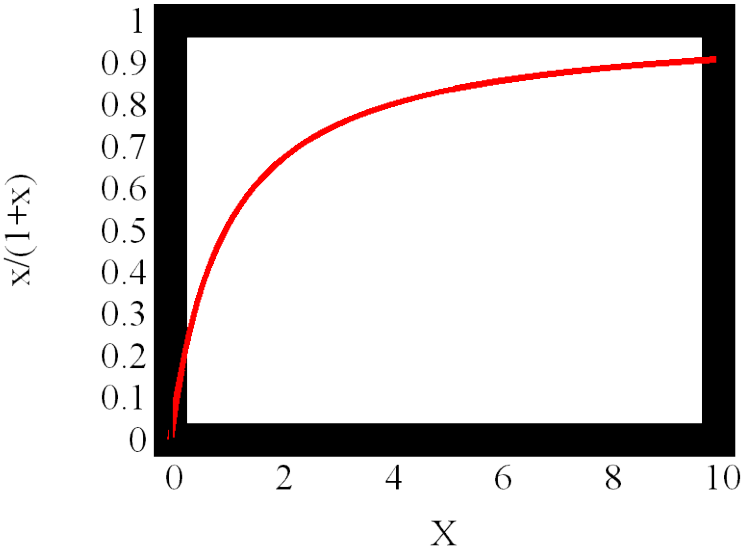



$$\frac{x}{1+x} = x - x^2 + x^3 - x^4 + \ldots$$



# x/(1+x)

$$\frac{x}{1+x} = x - x^2 + x^3 - x^4 + \ldots$$




This curve similar to many known curves in biology

(1) Part of a "growth curve"

(2) Enzyme kinetics curve

# Summary

- The relation between quantities that we plot in X and Y axis is called a "**Function**"

- Simple functions: $x$, $x^2$,$x^3$ etc

- Some known functions, like exp(x), can be represented as a combination of simple functions