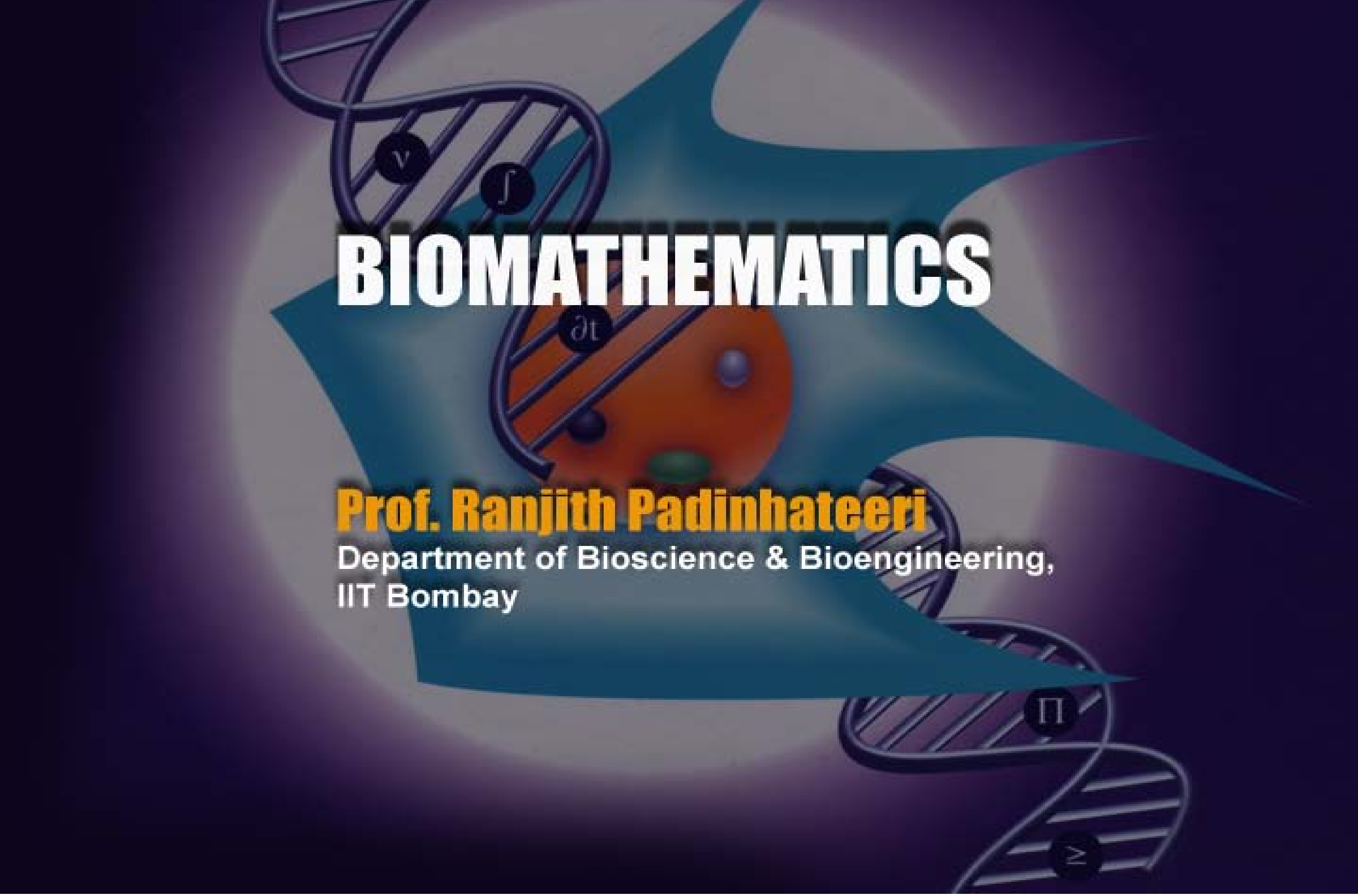
BIOMATHEMATICS
Prof. Ranjith Padinhateeri
Department of Bioscience & Bioengineering,
IIT Bombay

Lecture 27

Fourier Series

Fourier series

A period function f(x) can be written as sums of sines and cosines

Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) sin(nx) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

Some properties of sin and cos

$$\int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = \pi\delta_{mn}$$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = \pi\delta_{mn}$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0$$

Example of a Fourier series

$$f(x) = x, \quad \text{for } -\pi < x < \pi$$

$$f(x + 2\pi) = f(x), \quad \text{for } -\infty < x < \infty$$

=> Saw-tooth wave

Fourier coefficients

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = 0$$

Fourier coefficients

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos(nx) dx = 0$$

Fourier coefficients

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin(nx) dx$$

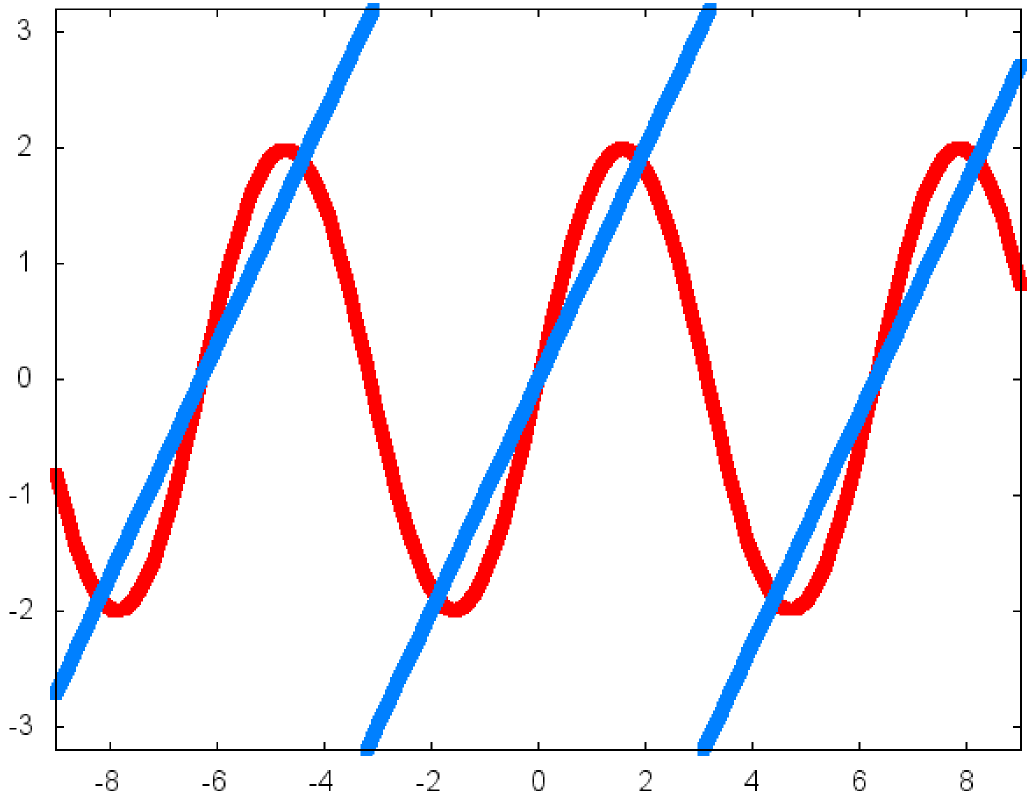
$$= -\frac{2}{n}\cos(n\pi) + \frac{2}{\pi n^2}\sin(n\pi)$$

$$= 2\frac{(-1)^{n+1}}{n}$$

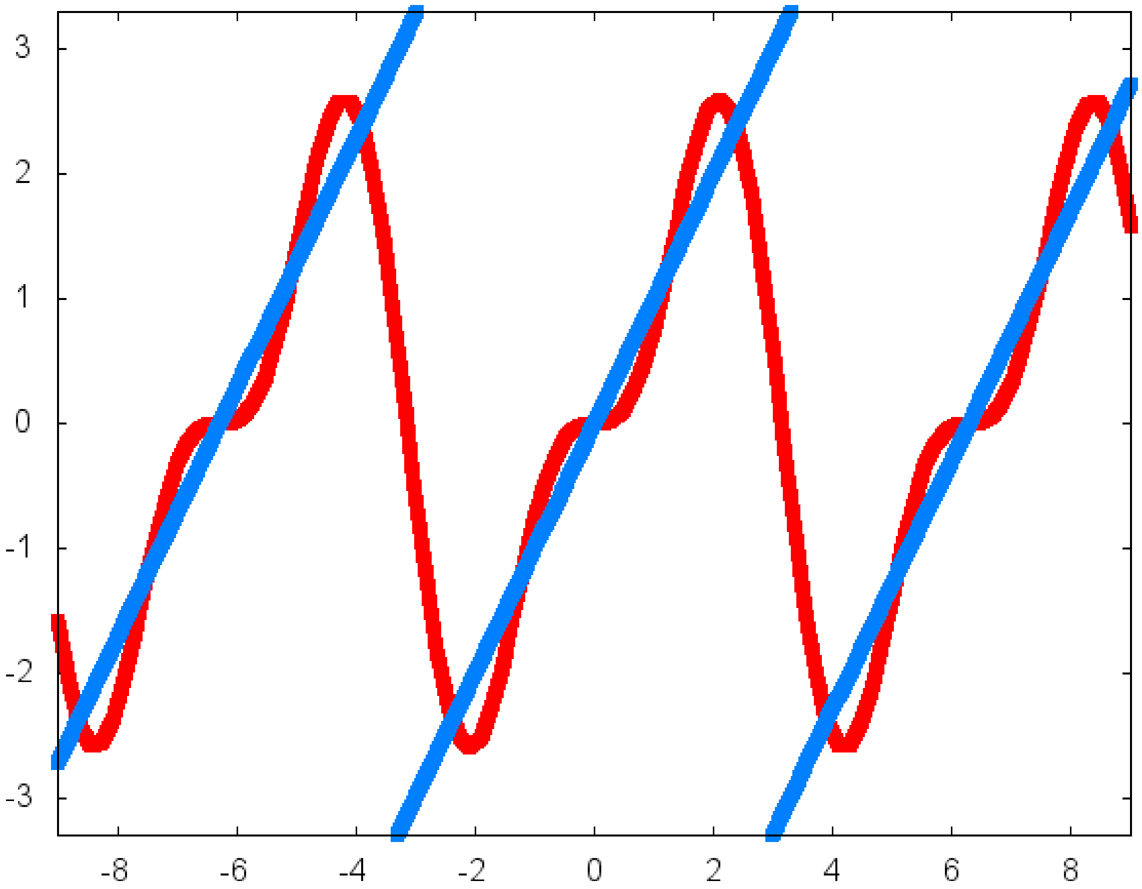
The series

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx)$$

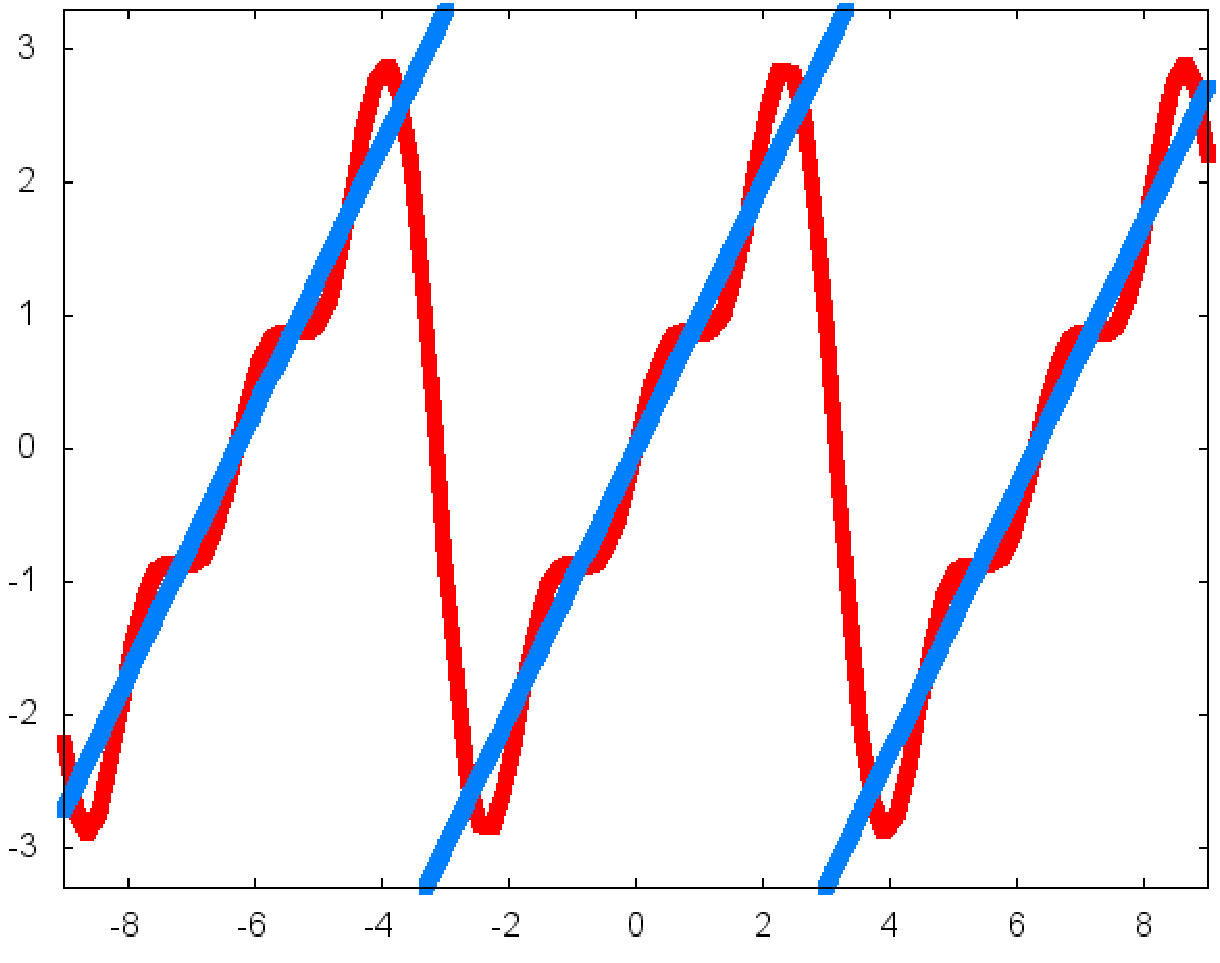
1 term



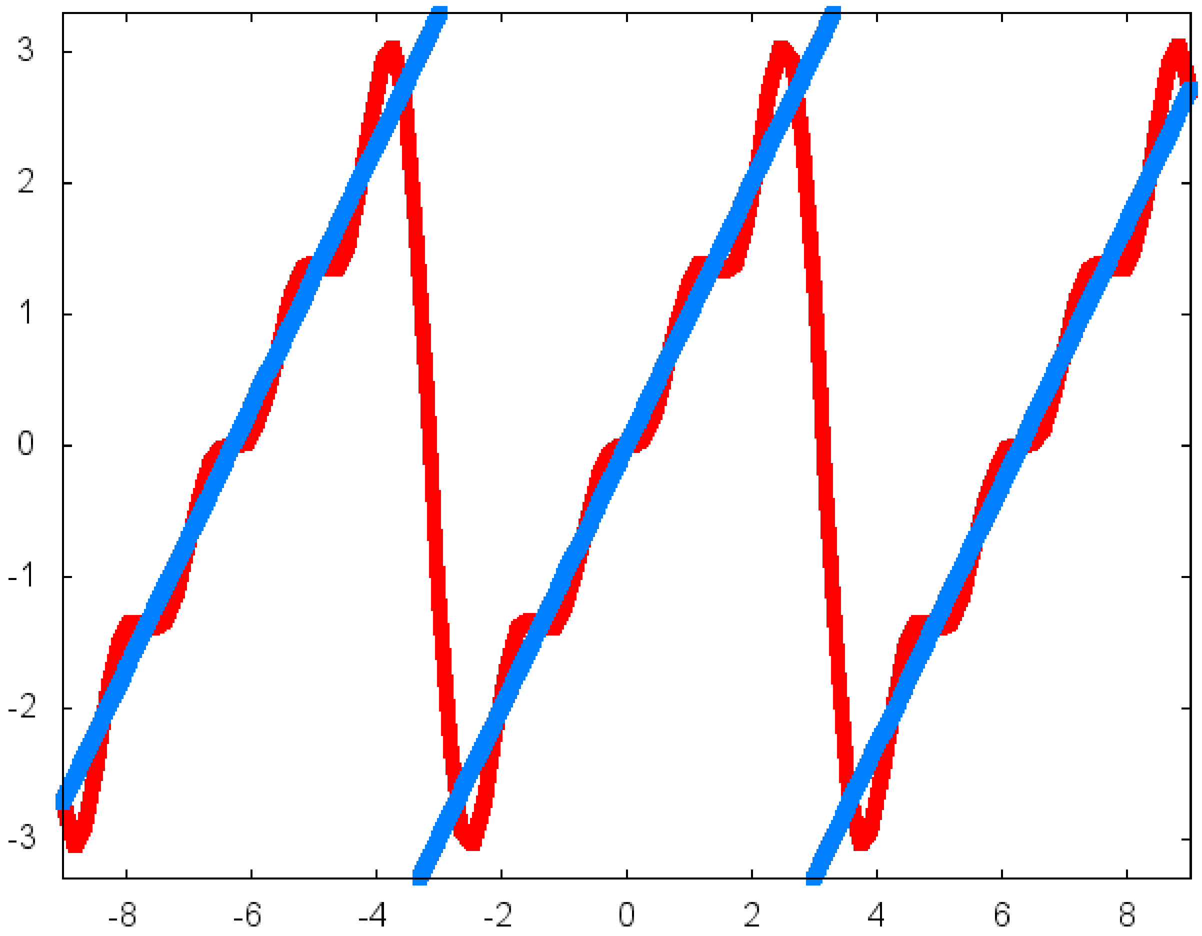
2 terms



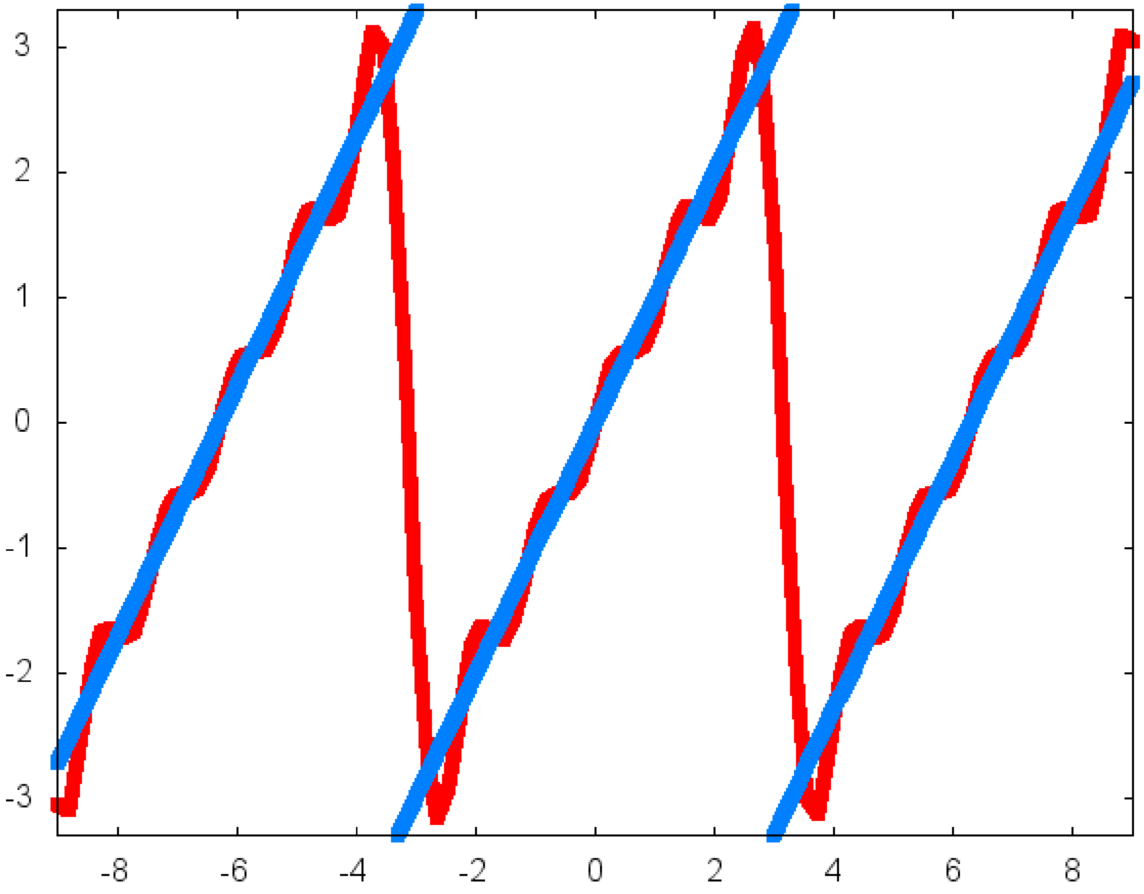
3 terms

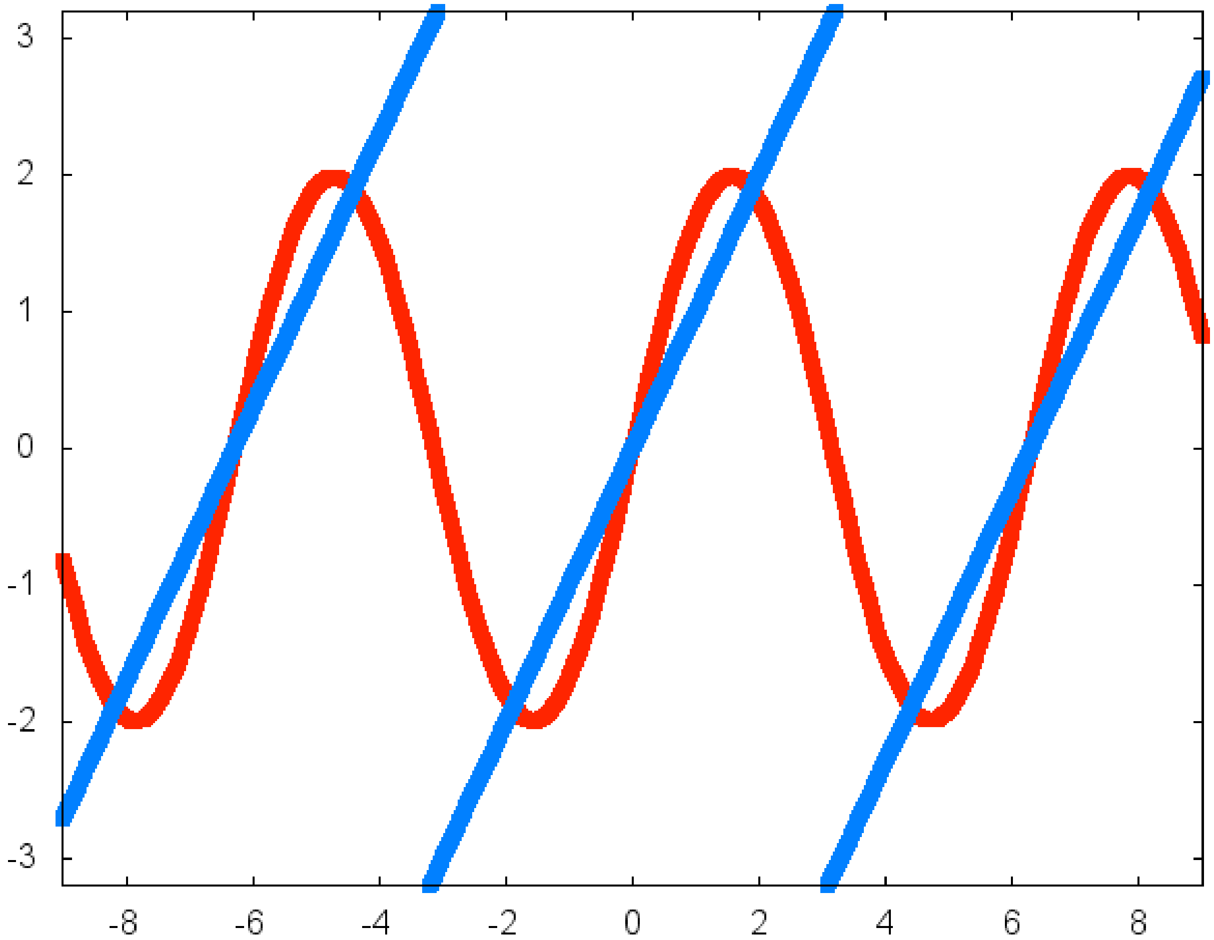


4 terms



5 terms




3
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0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8