BIOMATHEMATICS

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Graphs and Functions – part II

In this lecture, we will continue discussing some more simple functions and their graphs.

We will also see how some natural phenomena behave as if they are mathematical functions

Function

- To plot a graph, we need X values and Y values.
- The relation between quantities that we plot in X axis and Y axis is called a "**Function**"

We learned :

- Linear function : $Y= mX + C$
- Quadratic function : $Y = kX^2$
- Cubic function : $Y= kX^3$

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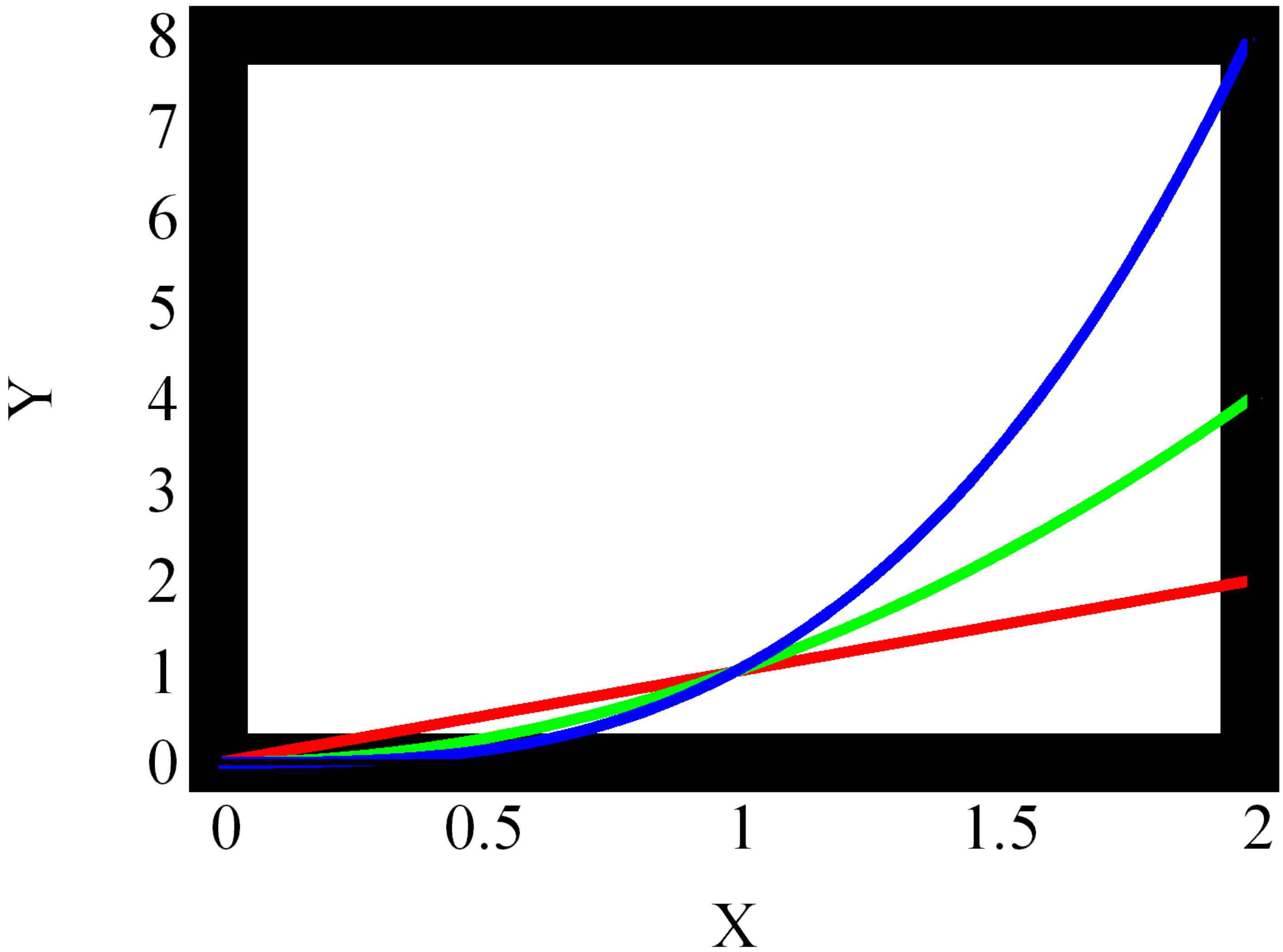
Y= X, Y= X^2, and Y= X^3



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Combination of x, x^2, x^3 etc

Combination of x, x^2, x^3 etc can give us new functions, like:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$$

Notation: $e^x = \exp(x)$

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$$e^{x} = 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \ldots$$

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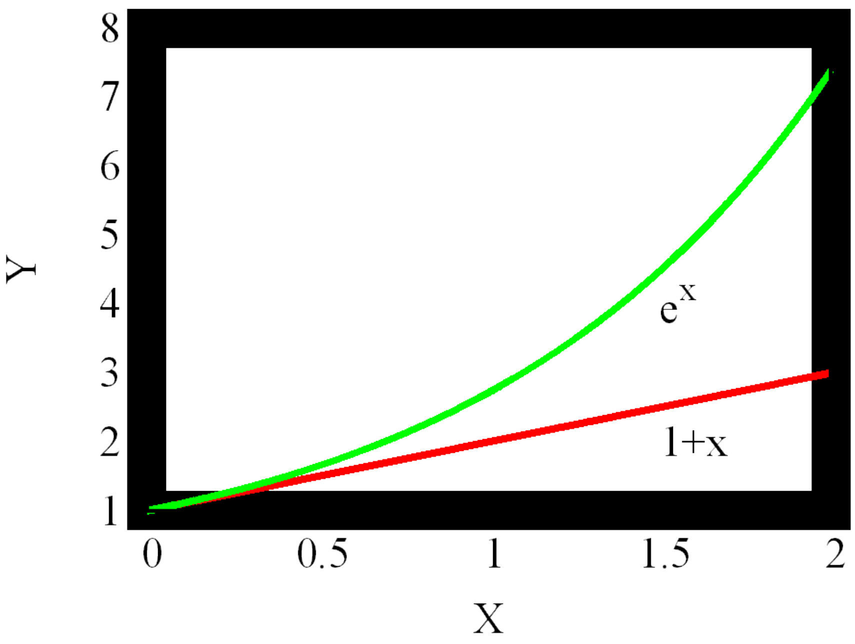

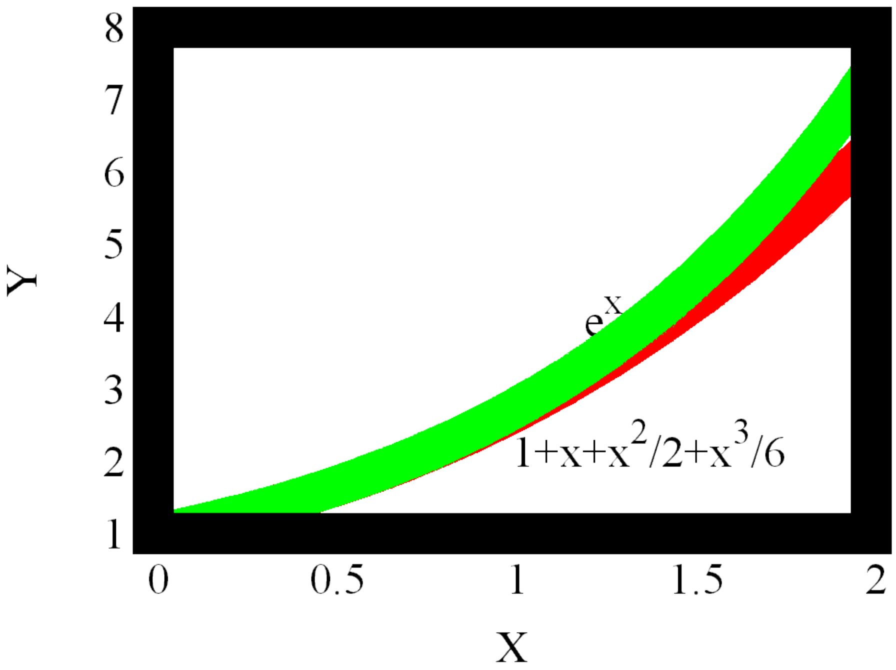

Example in biology: e^x

- Microbial growth : when rapidly multiplying
- Arrhenius equation $r = A\exp\left(\frac{-E_a}{RT}\right)$

How do we plot these graphs ?

- Use a scientific calculator, create a table, and mark it on a graph sheet

- Use a computer to plot it -- use a software
  - Gnuplot (http://www.gnuplot.info/)
  - Microsoft Excel

- We will have a short demonstration on how to use some of these software

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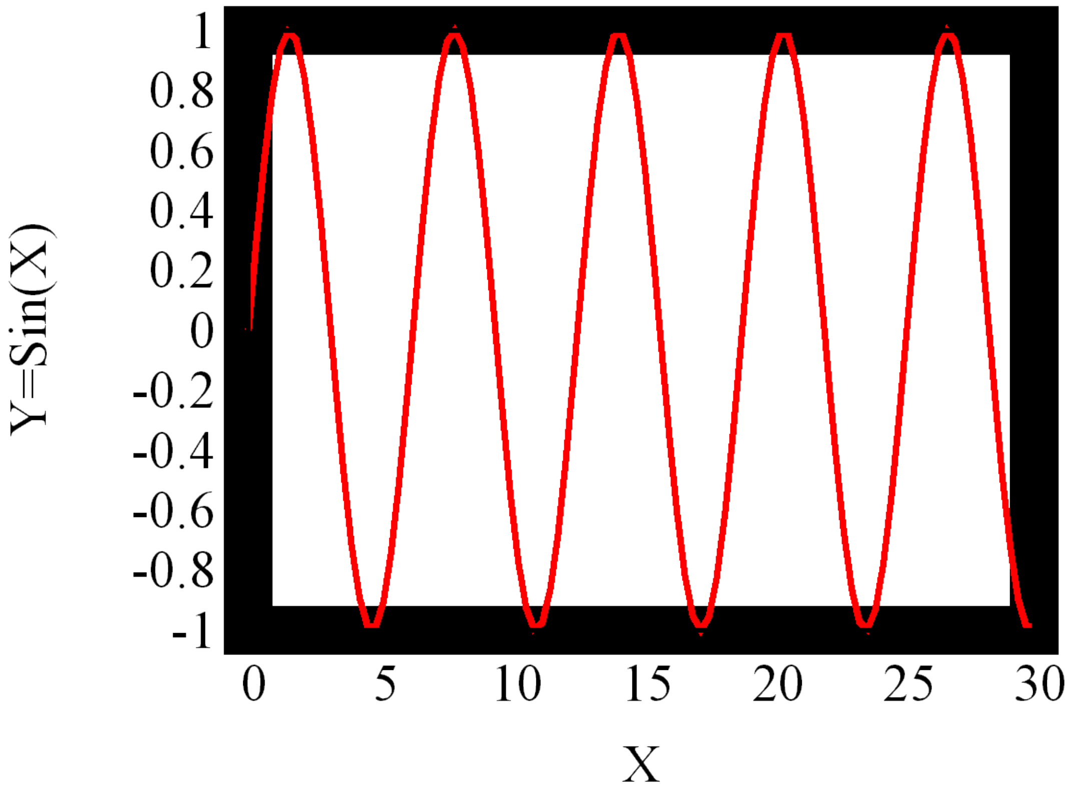
Sin(x)

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots$$



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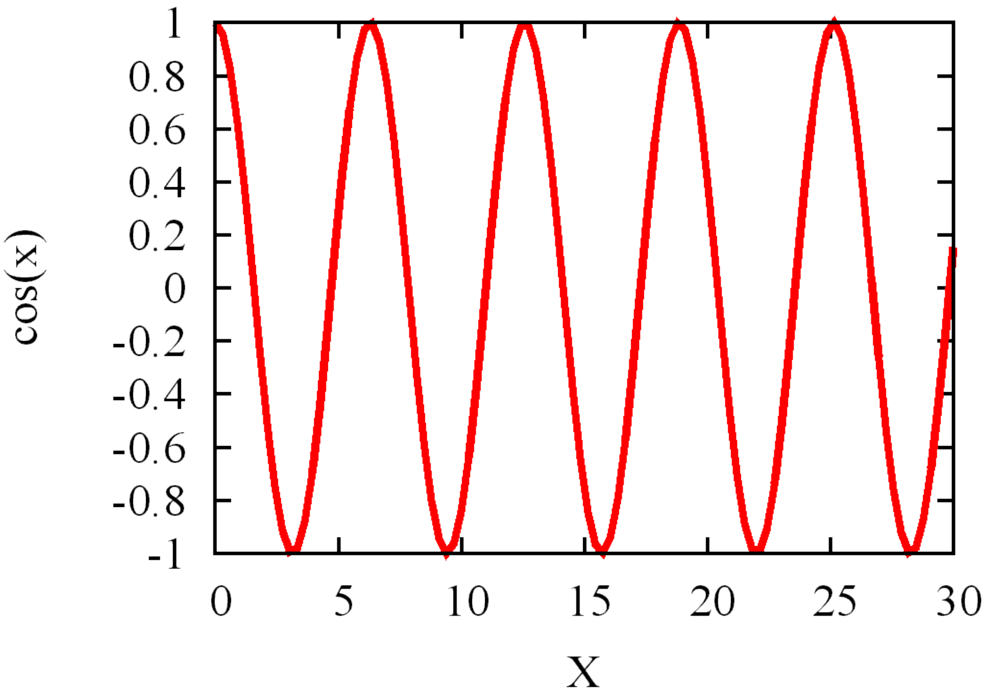
Cos(x)

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} \ldots$$



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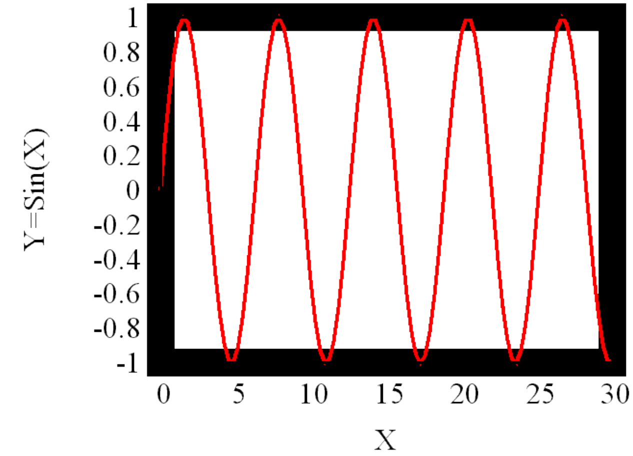
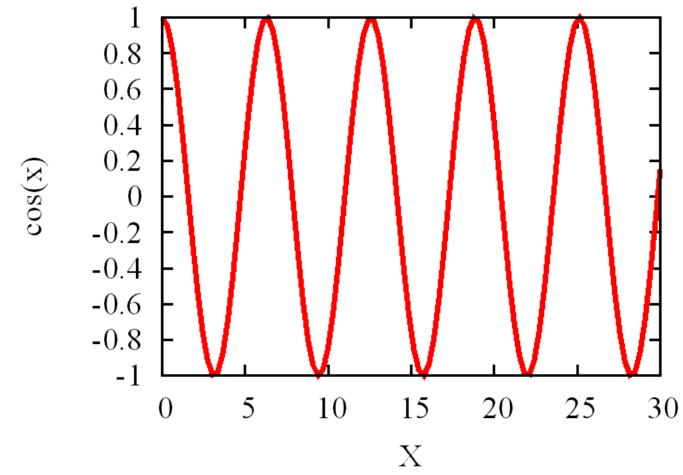
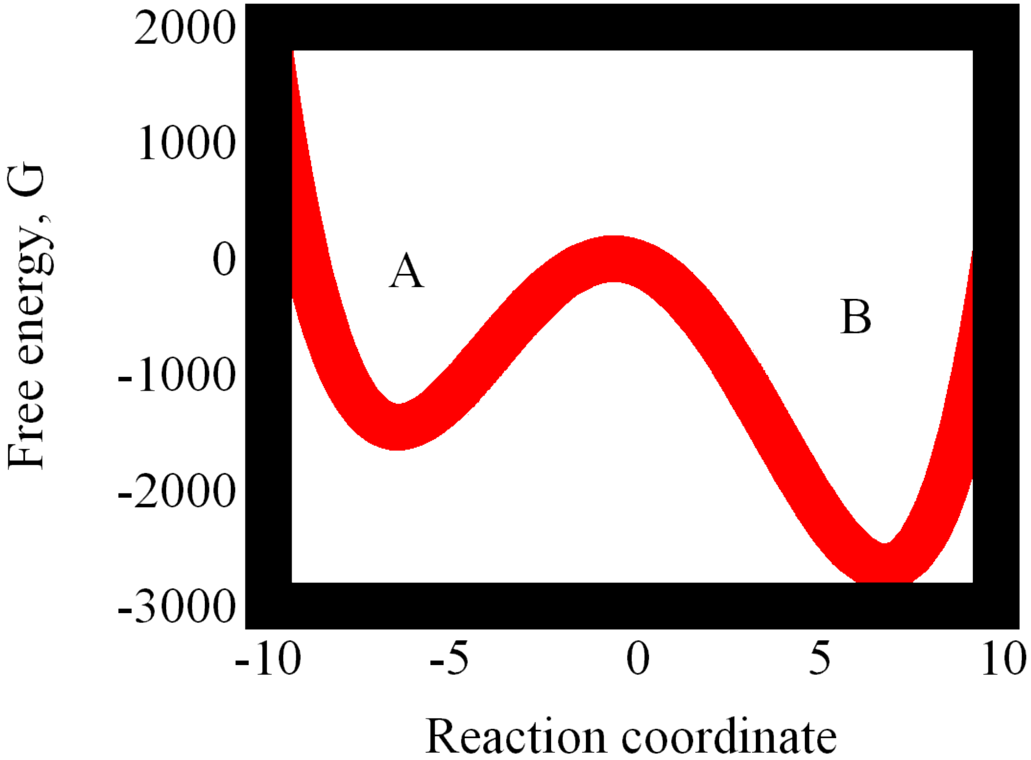
Oscillating or periodic functions

- Temperature over seasons
- Biological clock
  (eg. Insulin secretion)
- Cell cycle : Cyclin activity





Free energy, G



$$Y = pX^4 - qX^2 - mX + c$$

p,q,m, and c are some numbers

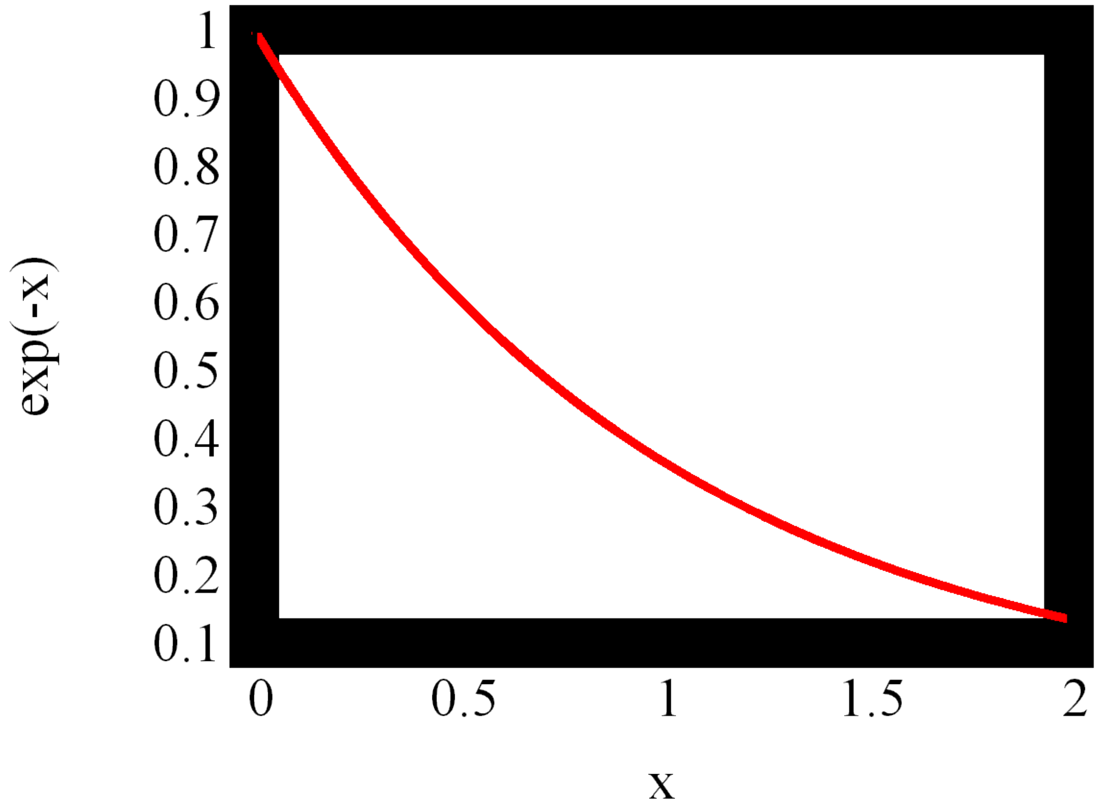
e^{-x}

$$e^{-x} = 1 + (-x) + \frac{(-x)^2}{2} + \frac{(-x)^3 \ \text{[redacted]}}{6} + \ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3 \ \text{[redacted]}}{6} + \ldots$$

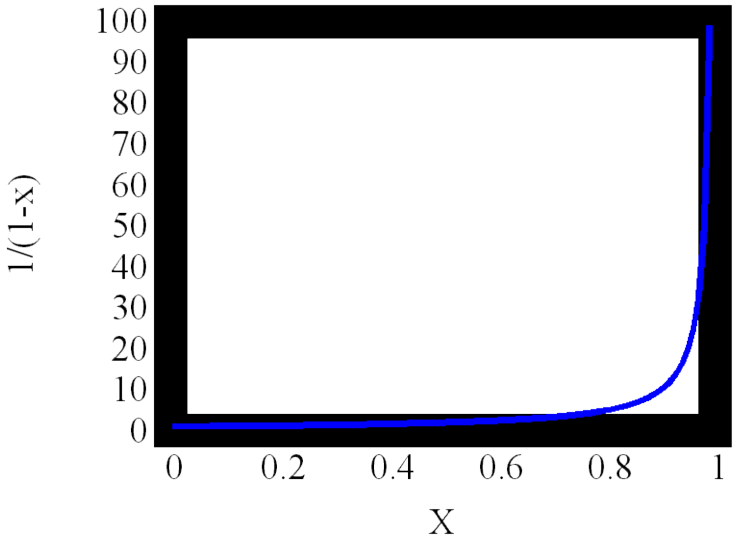
e^{-x}

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

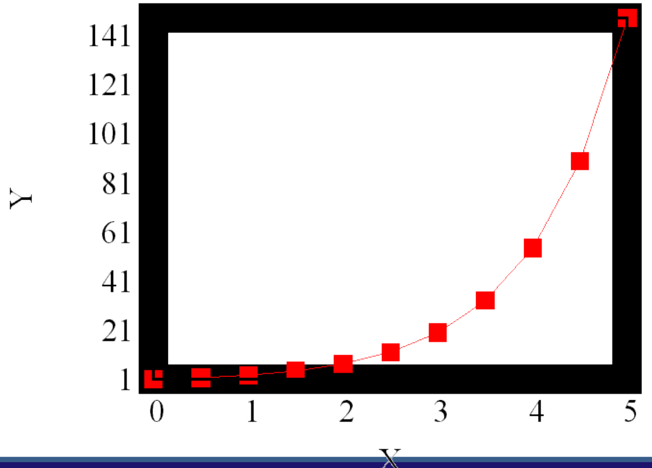


1/(1-x)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

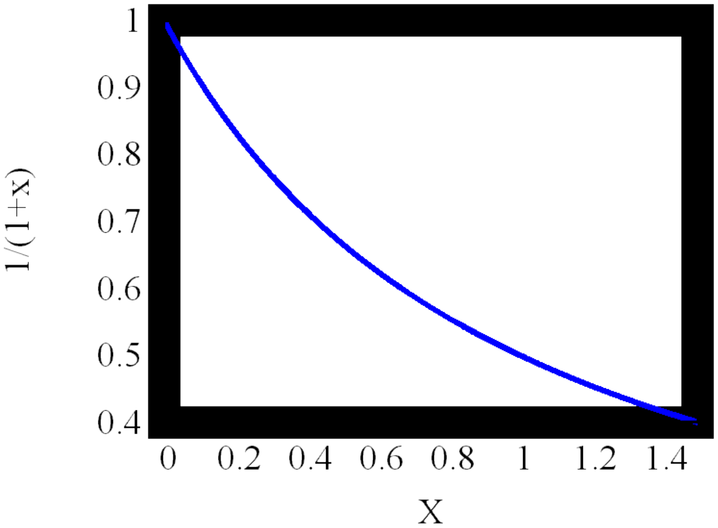


$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

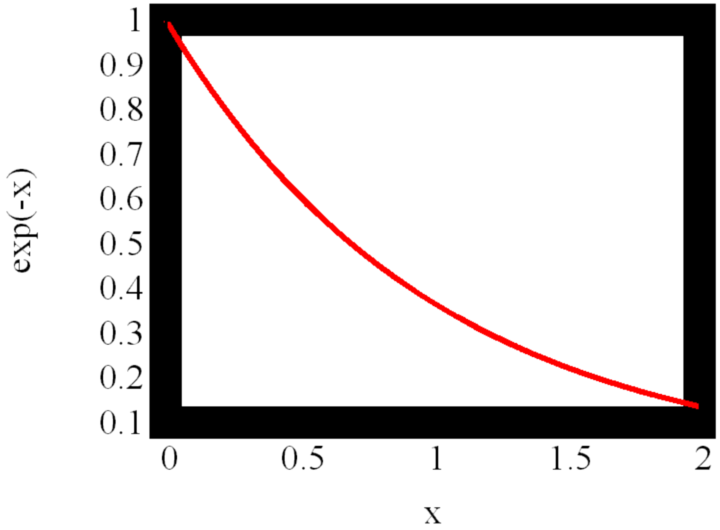


1/(1+x) and exp(-x)

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

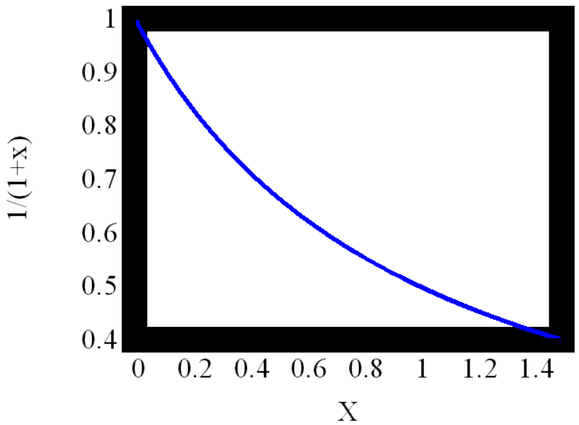


$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \ldots$$

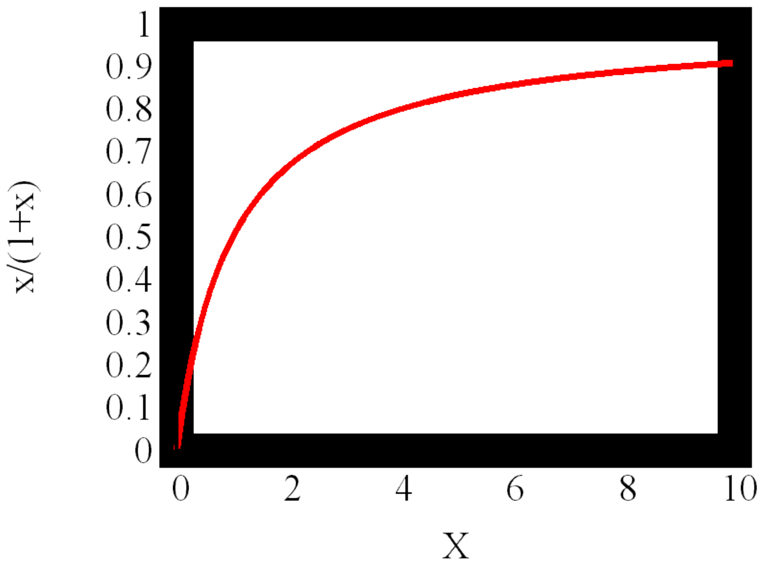


1/(1+x) and x/(1+x)

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$



$$\frac{x}{1+x} = x - x^2 + x^3 - x^4 + \ldots$$



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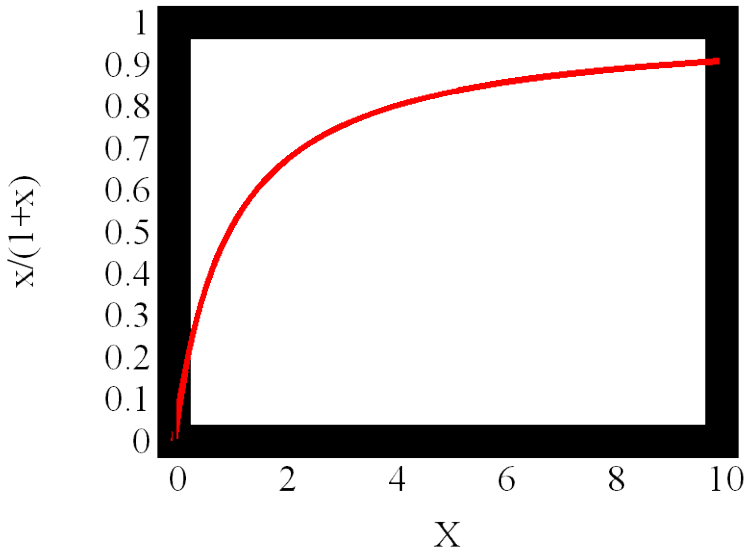
x/(1+x)

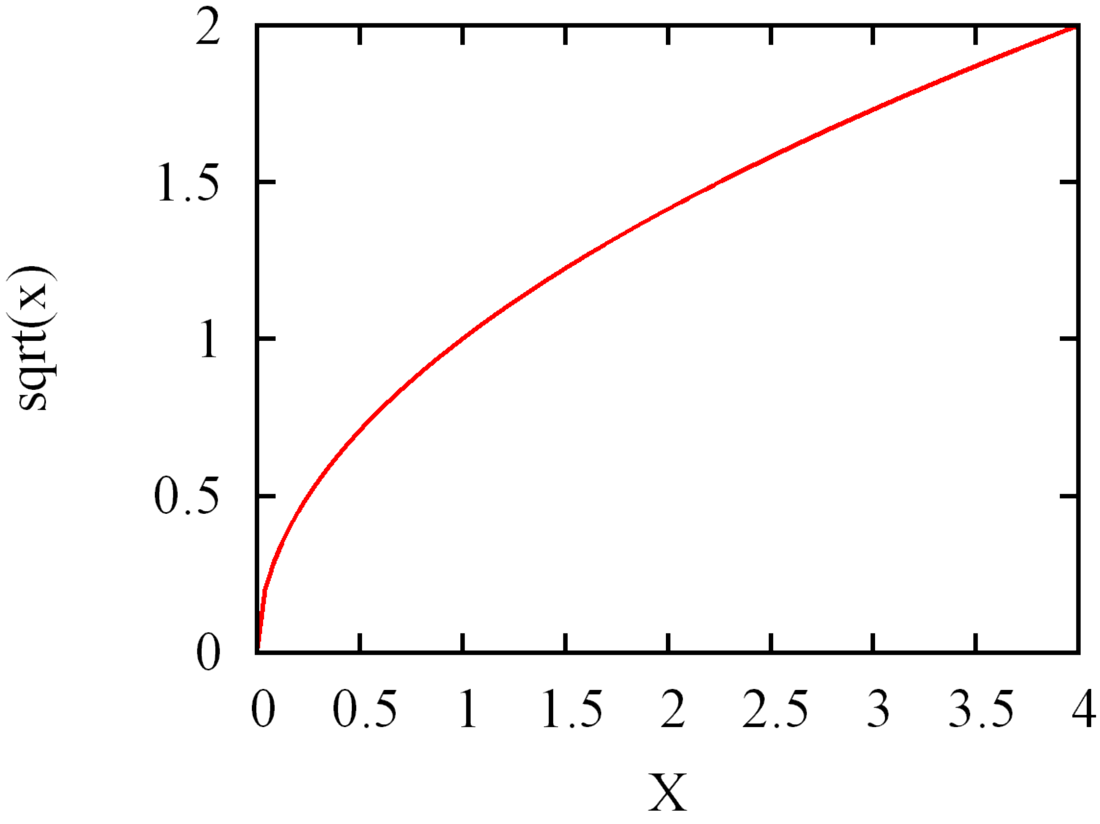
$$\frac{x}{1+x} = x - x^2 + x^3 - x^4 + \ldots$$



This curve similar to many known curves in biology

(1) Part of a "growth curve"
(2) Enzyme kinetics curve

$$Y = \sqrt{X}$$



$$\sqrt{X} = X^{\frac{1}{2}}$$

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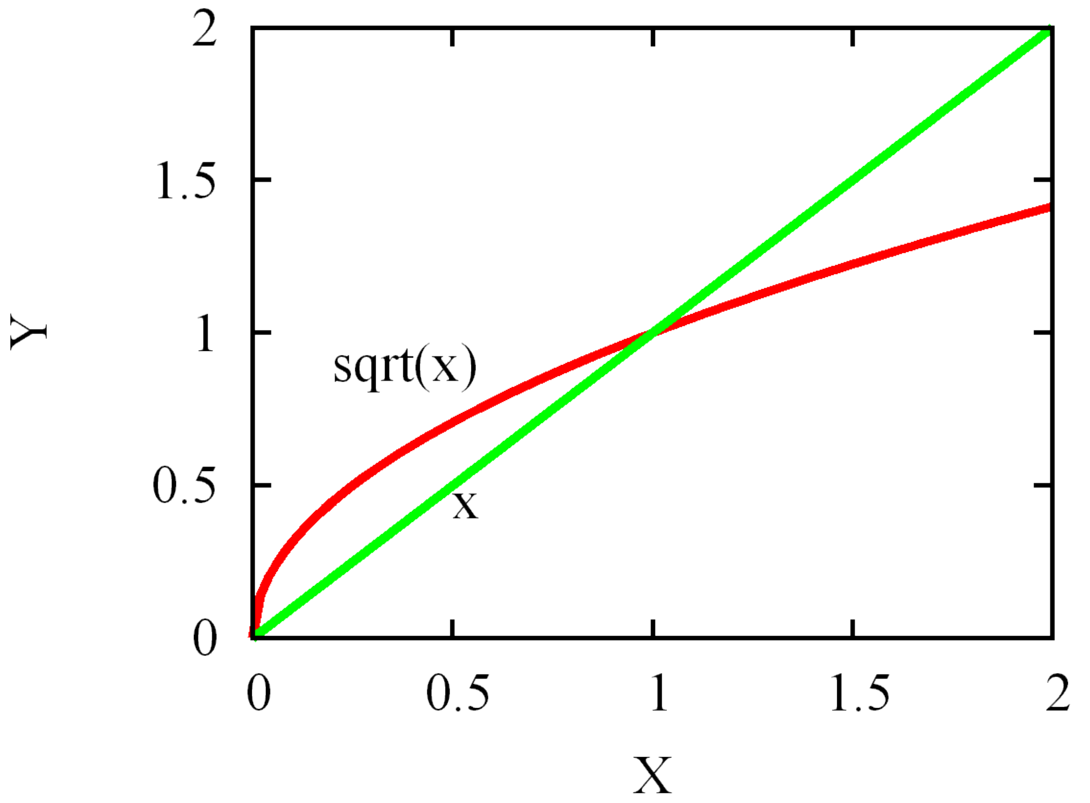
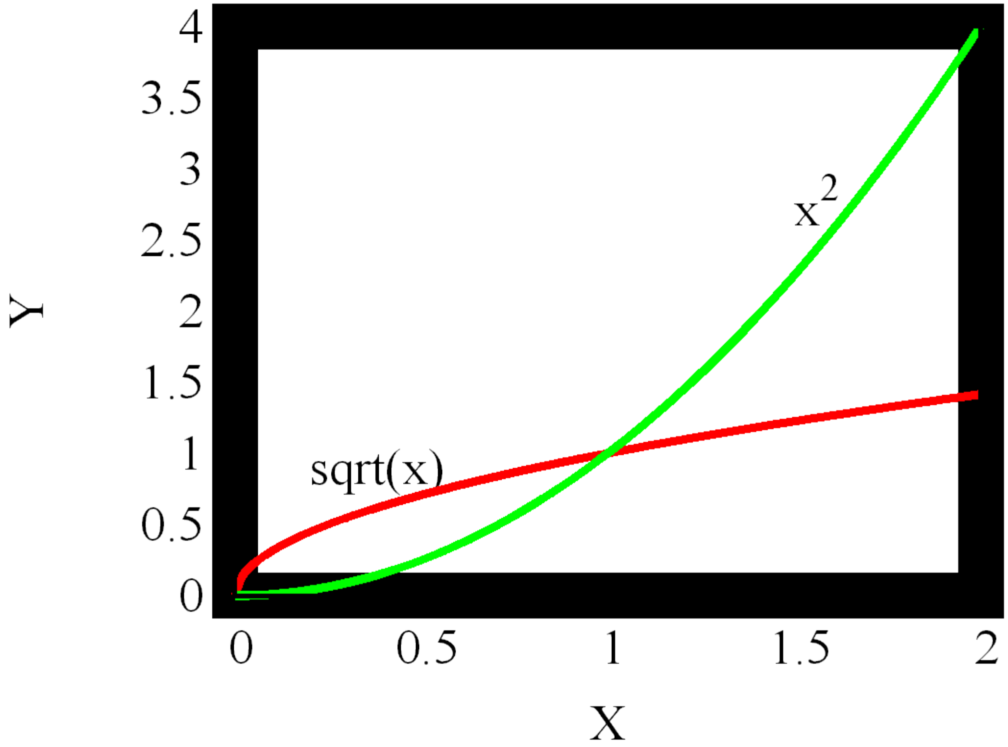
Compare: Y= X and Y= $X^{1/2}$



Compare: $Y = X^2$ and $Y = X^{1/2}$

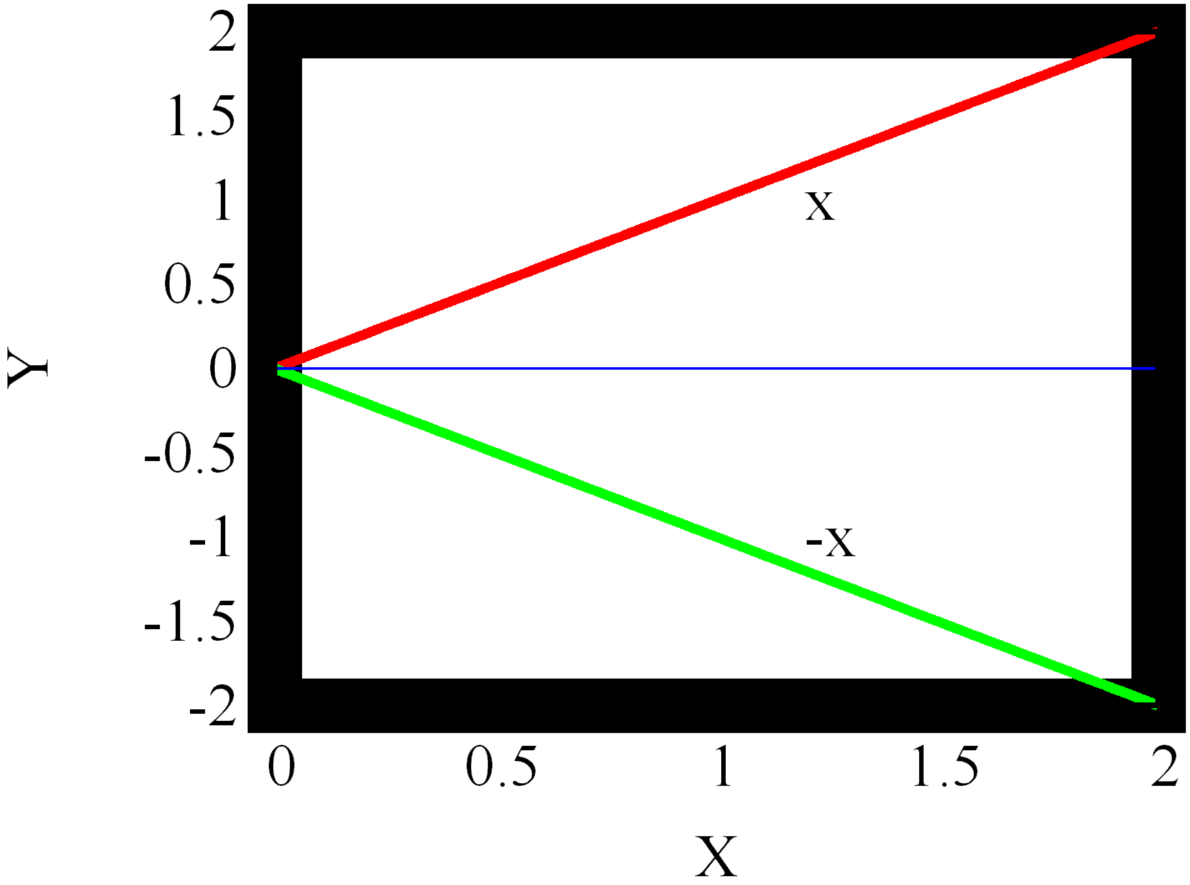


Functions that decrease with X

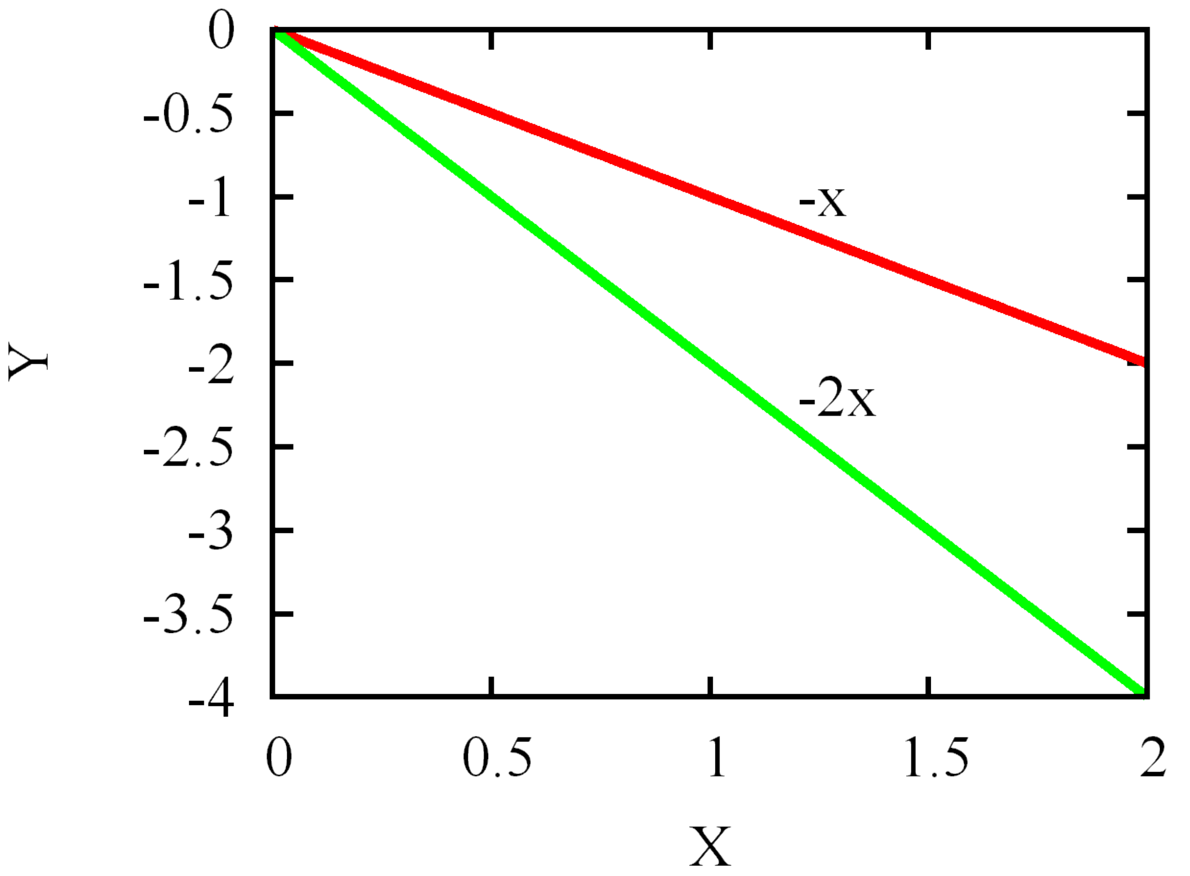
- Height of a falling object decreases with time
- Concentration of nutrient in bacterial culture
- Free energy vs Temperature

$$\Delta G = \Delta H - T\Delta S$$

Compare: Y=X and Y=-X



Compare: Y=-X and Y=-2X



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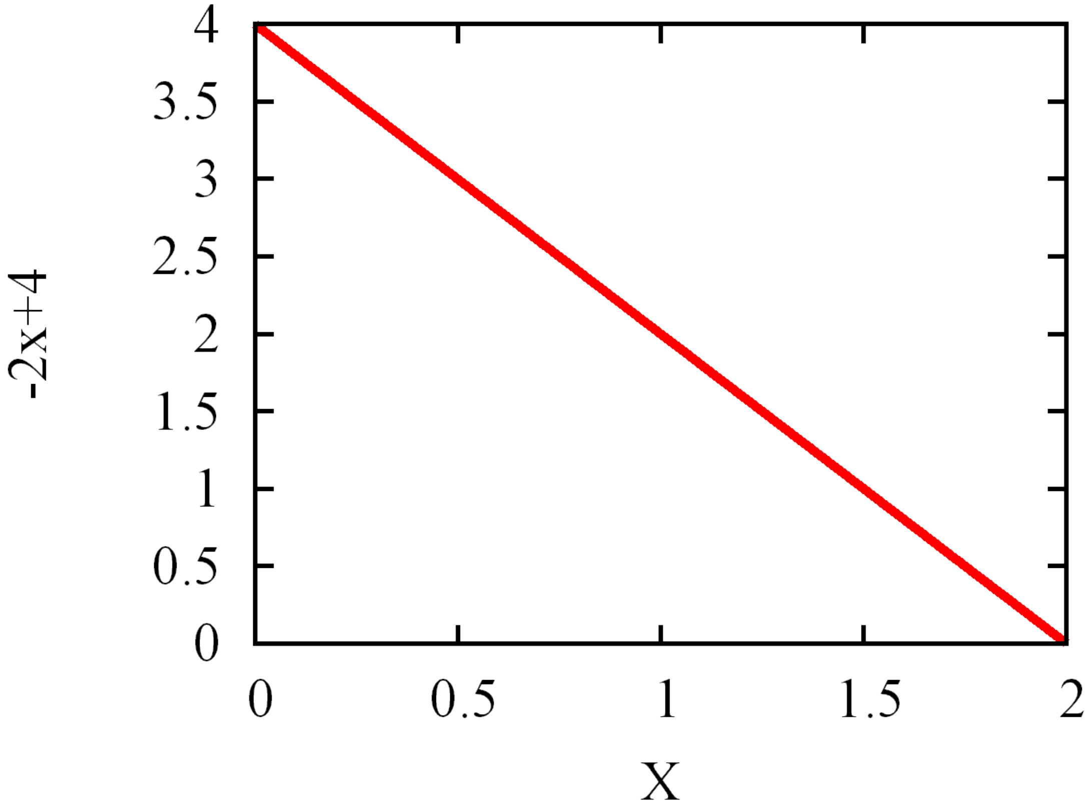
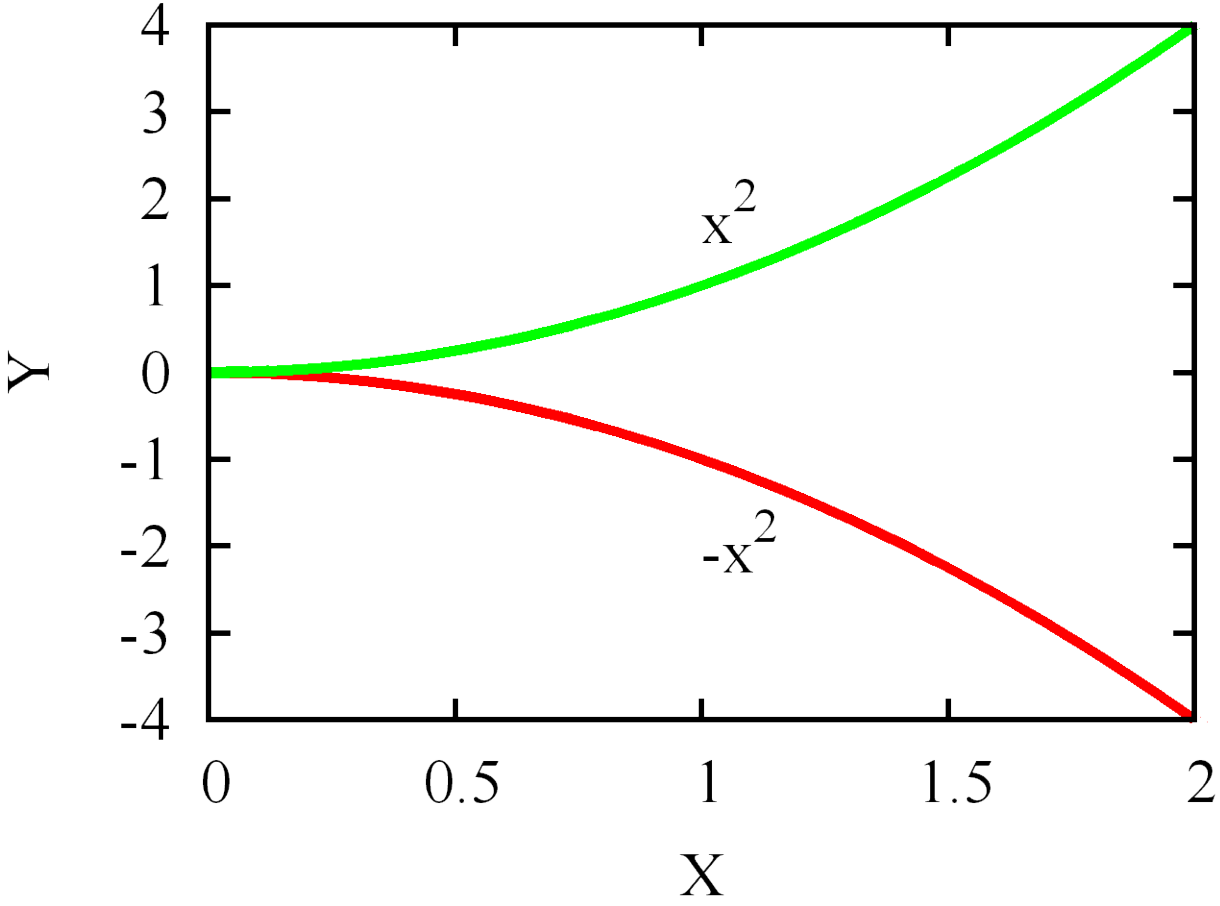
Y=4-2X



$$\Delta G = \Delta H - T\Delta S$$

Compare: $Y = X^2$ and $Y = -X^2$



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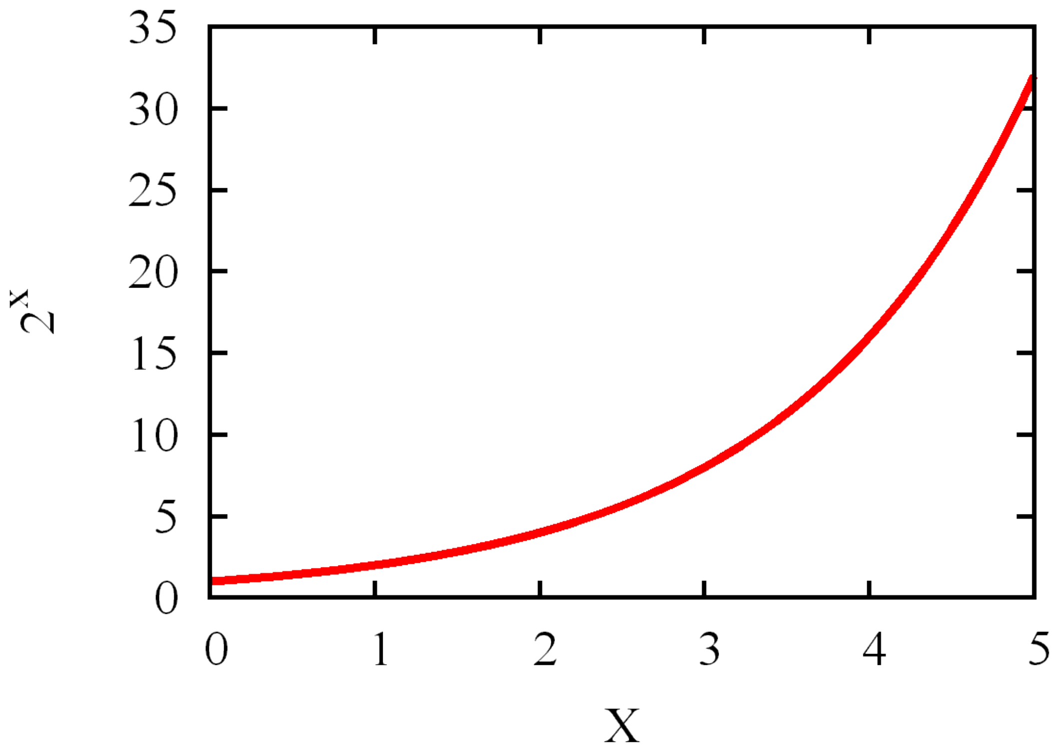
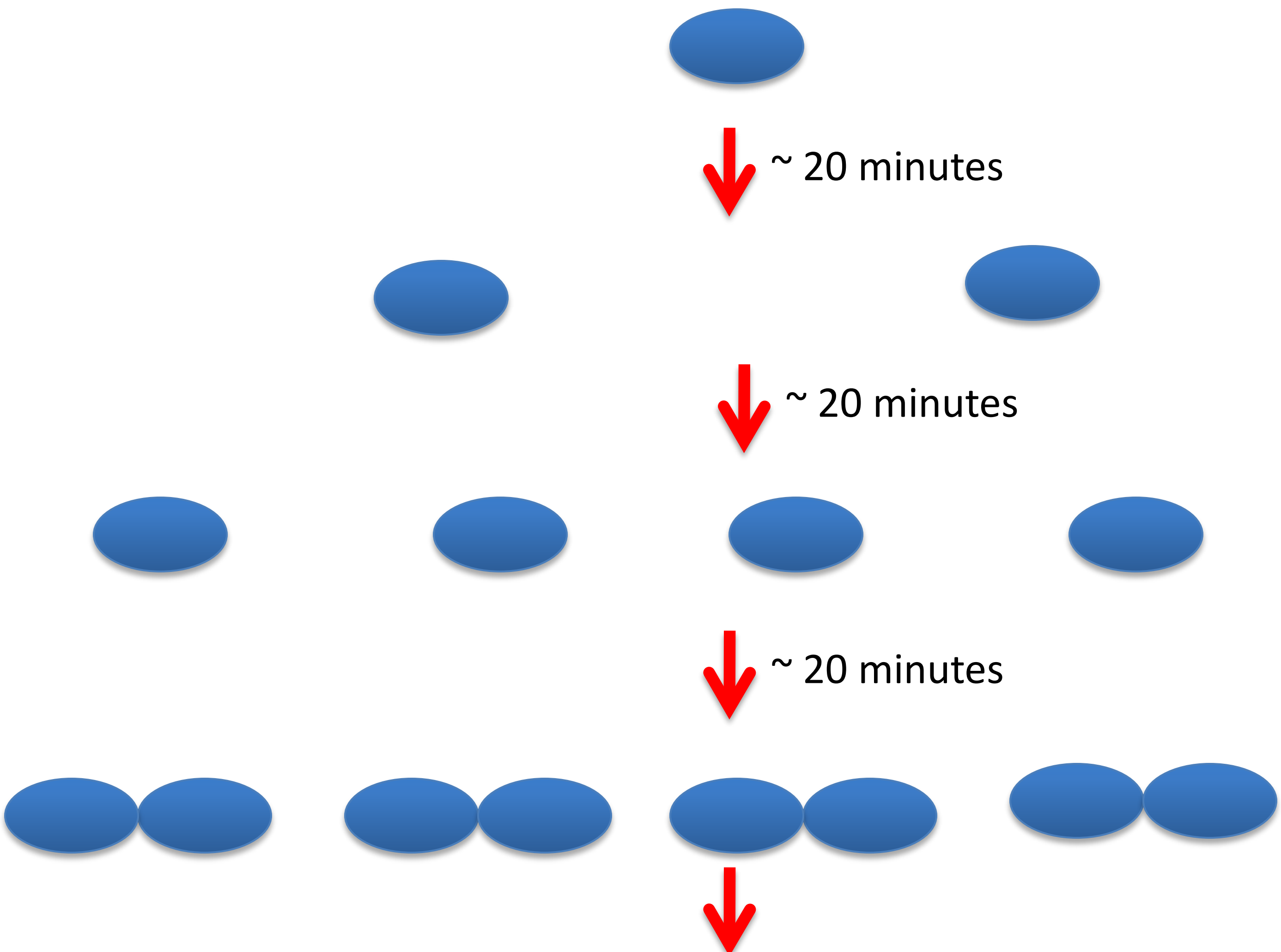
$Y=2^x$



Bacterial growth

Bacterial growth (eg. E. Coli)



$1=2^0$

$2=2^1$

$4=2^2$

$8=2^3$

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Bacterial (eg. E. Coli) growth

Number of bacteria at time t is given by

$$N(t) = 2^{kt}$$

k = rate of cell division; for E-coli, typically $k = \frac{1}{20\min}$

When t=60 minutes, N = 2^3 = 8

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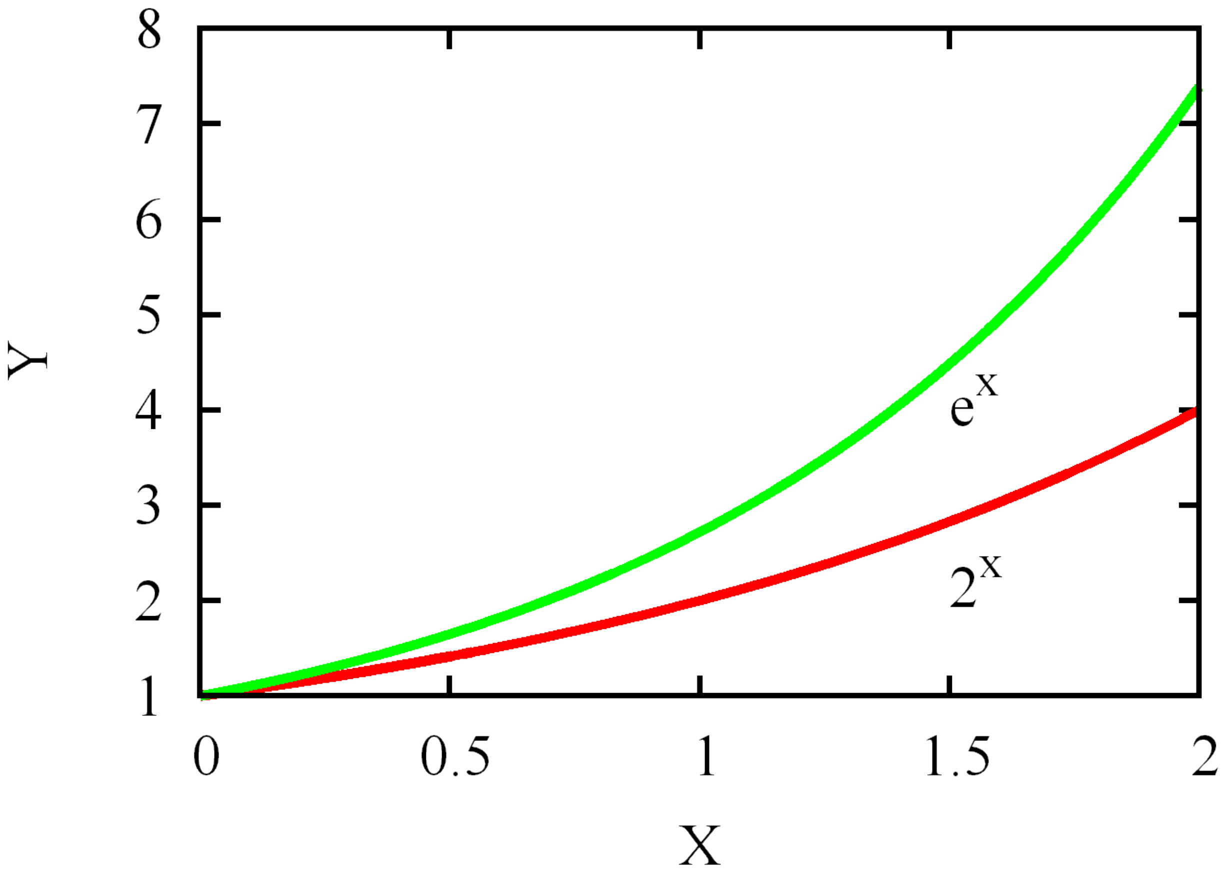
DNA in PCR cycle

Number of DNA molecules in a PCR cycle obey a similar equation

$$N(t) = 2^{kt}$$

Compare: $Y=2^x$ and $Y= e^x$



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Summary

- Periodic events can be represented using Sin(x), Cos(x)
- Functions that we see in biology can be written as combination of simple functions
- Many natural phenomena – Eg. bacterial growth – behave like mathematical functions