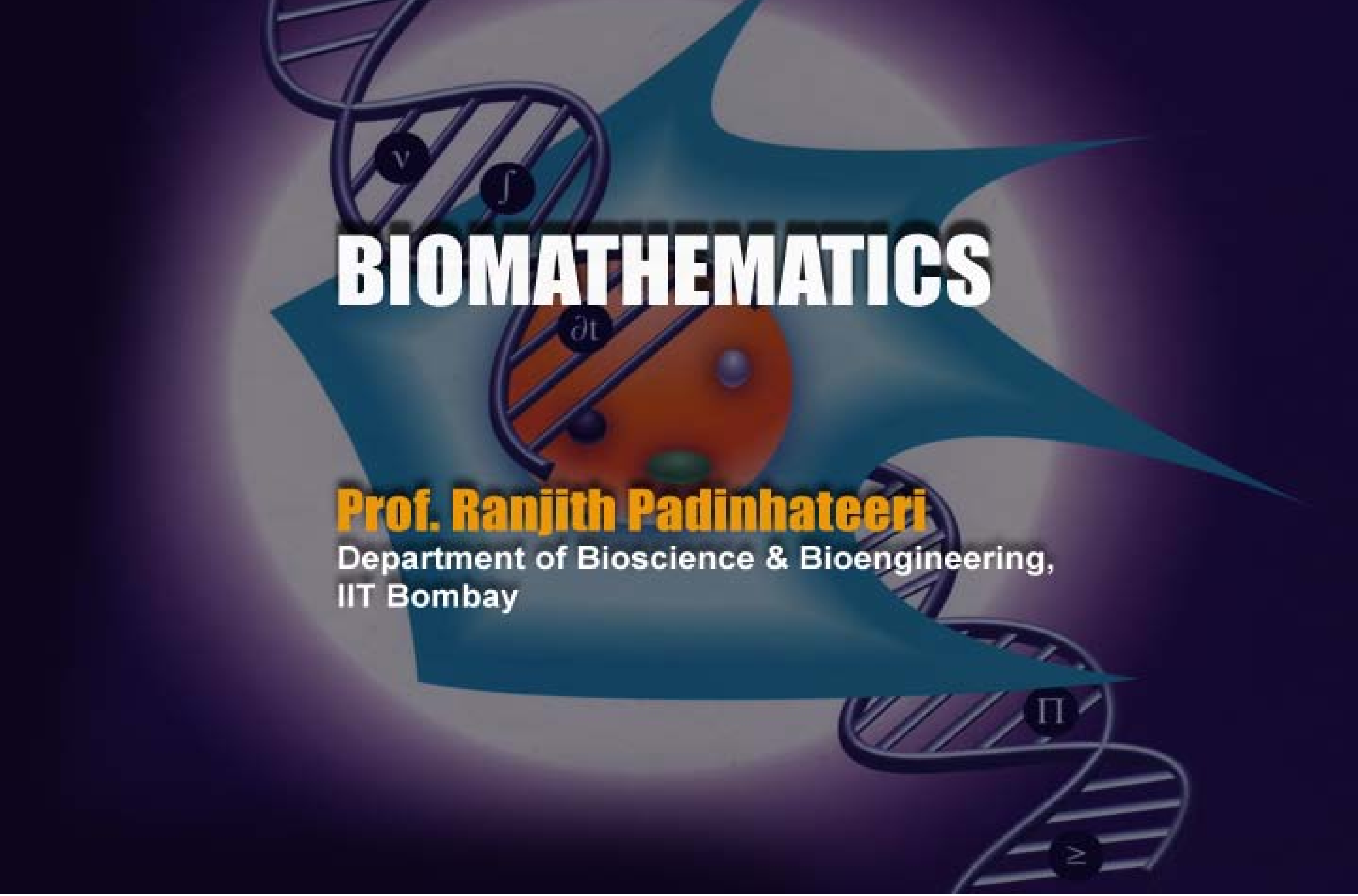
BIOMATHEMATICS
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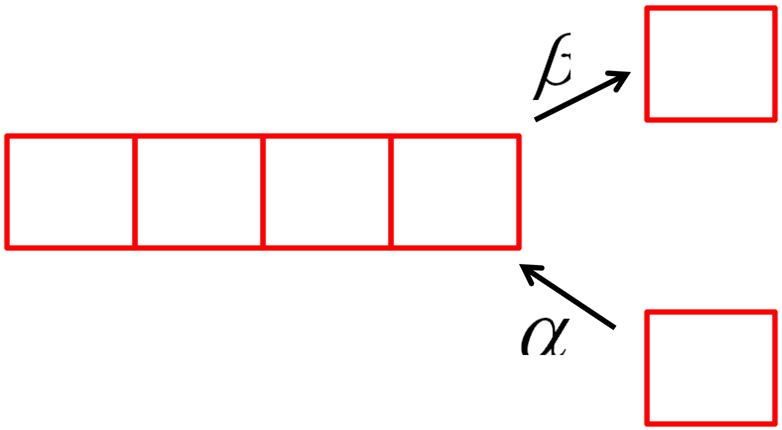
Lecture 30

Master equation: Polymerization dynamics, Molecular motor motion

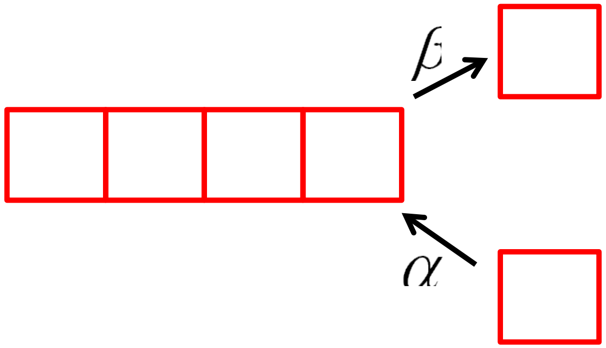
Polymerization and De-polymerization Dynamics



$P(n,t):$ Probability to have n monomers at time t

Polymerization and De-polymerization Dynamics

Master equation,



$$\frac{dP(n,t)}{dt} = \alpha P(n-1,t) + \beta P(n+1,t) - (\alpha+\beta)P(n,t)$$

$$\tilde{P}(k,t) = \sum_{n} P(n,t) e^{-kn}$$

$$-\frac{\partial \tilde{P}(k,t)}{\partial k} = -\frac{\partial}{\partial k}\sum_{n} P(n,t)e^{-kn}$$

$$-\frac{\partial \tilde{P}(k,t)}{\partial k} = \sum_{n} nP(n,t)e^{-kn} = \langle n \rangle$$

$$\frac{d}{dt}\left(-\frac{\partial \tilde{P}}{\partial k}\right)_{k=0} = \frac{d\langle n \rangle}{dt} = v$$

$$\tilde{P}(k,t) = \sum_{n} P(n,t) e^{-kn}$$

$$\frac{d\tilde{P}}{dt} = \alpha \tilde{P} e^{-k} + \beta \tilde{P} e^{k} - (\alpha + \beta)\tilde{P}$$

$$\frac{d\tilde{P}}{dt} = \left[\alpha\left(e^{-k} - 1\right) + \beta\left(e^{k} - 1\right)\right]\tilde{P}$$

$$\tilde{P}(k,t) = Ae^{\left[\alpha\left(e^{-k}-1\right)+\beta\left(e^{-k}-1\right)\right]t}$$

Using normalization condition,

$$\sum_{n} \tilde{P}(n,t) = 1$$

$$\tilde{P}(k=0,t) = 1 \Rightarrow A = 1$$

$$v = \frac{d\langle n \rangle}{dt} = \frac{d}{dt}\left(-\frac{\partial \tilde{P}}{\partial k} \right)_{k=0}$$

$$= \left[-\frac{\partial}{\partial k}\left(\frac{d\tilde{P}}{dt} \right) \right]_{k=0}$$

$$v = \alpha - \beta$$

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