BIOMATHEMATICS

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Differentiation and its application in Biology

Two Formulae

$$\frac{d}{dx}\left(x^{n}\right)=nx^{n-1}$$

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{df(x)}{dx}+\frac{dg(x)}{dx}$$

One more formula

Product rule

$$\frac{d}{dx}\left[f(x)\times g(x)\right]=\frac{df(x)}{dx}g(x)+f(x)\frac{dg(x)}{dx}$$

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$$\frac{d}{dx}\left[f(x)\times g(x)\right]=\frac{df(x)}{dx}g(x)+f(x)\frac{dg(x)}{dx}$$

E.g. Let: $f(x)=x$

$$g(x)=x$$

$$\frac{d}{dx}\left[x\times x\right]=\frac{dx}{dx}x+x\frac{dx}{dx}$$

$$=1.x+x.1=2x$$

Derivative of Exponential function

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \ldots$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

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Derivatives of Sin(x), Cos(x)

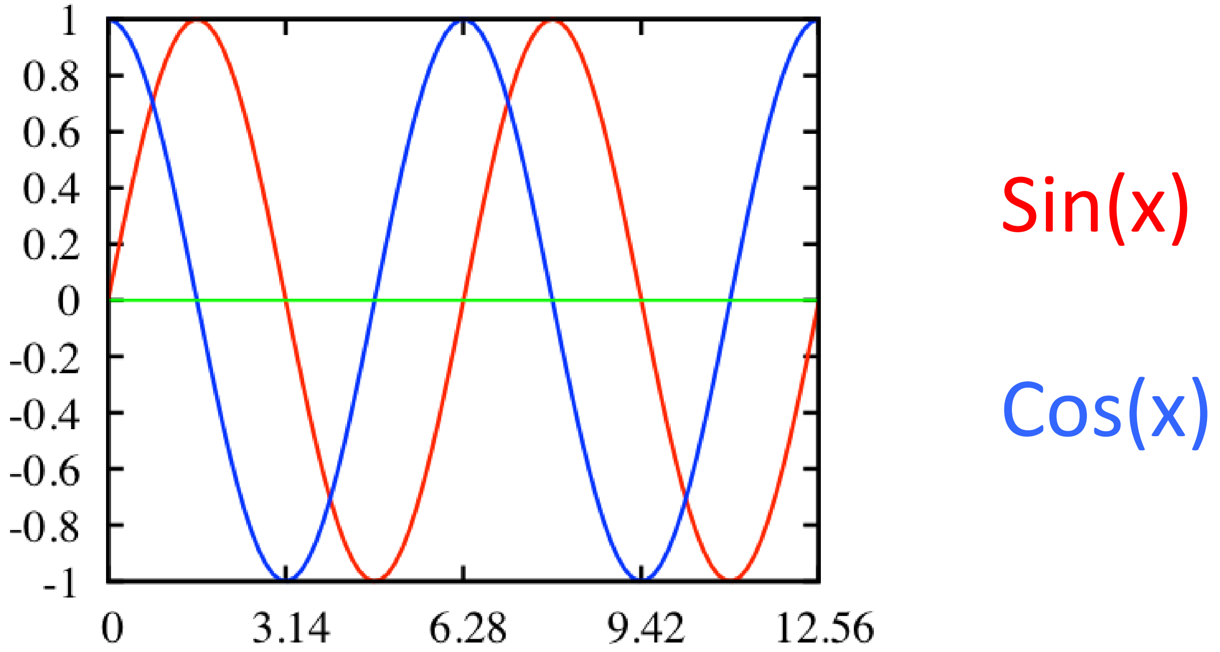
$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} \ldots$$

$$\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$$

$$\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$$

Derivatives of Sin(x), Cos(x)



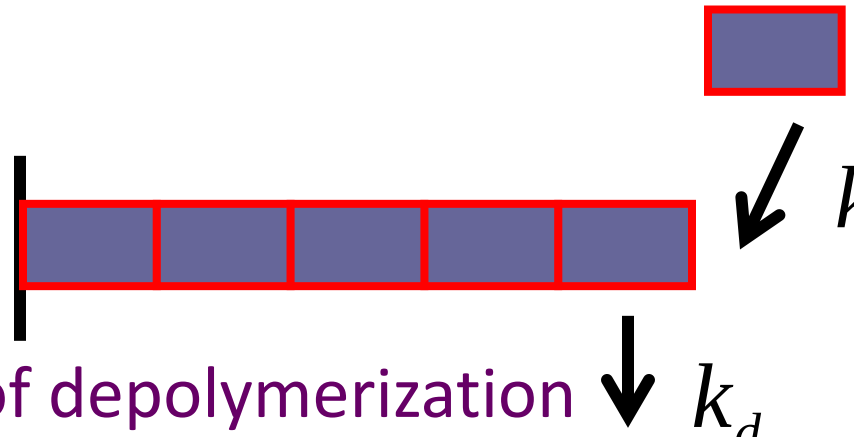
$$\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$$

$$\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$$

Applications in Biology

Example 1: Polymerization of filaments
(Growth of actin filaments)

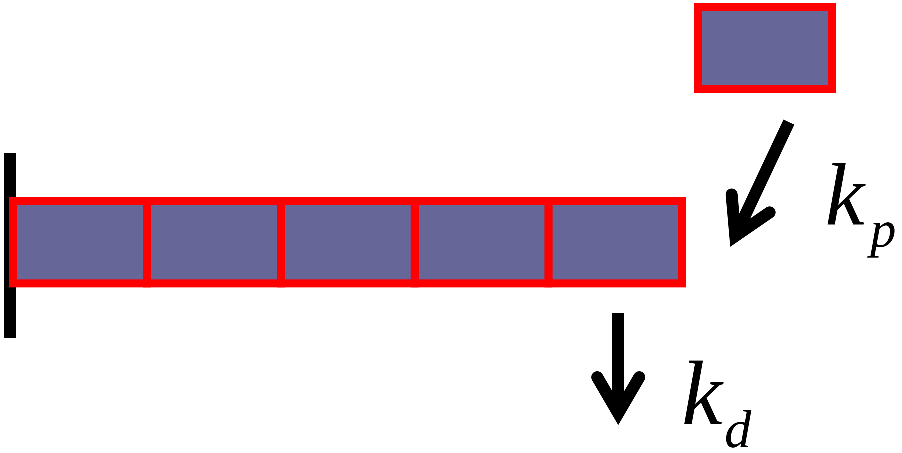
Polymerization of actin



$$k_p = k_0[C]$$

[C] : unpolymerized G-actin monomer concentration

Polymerization of actin

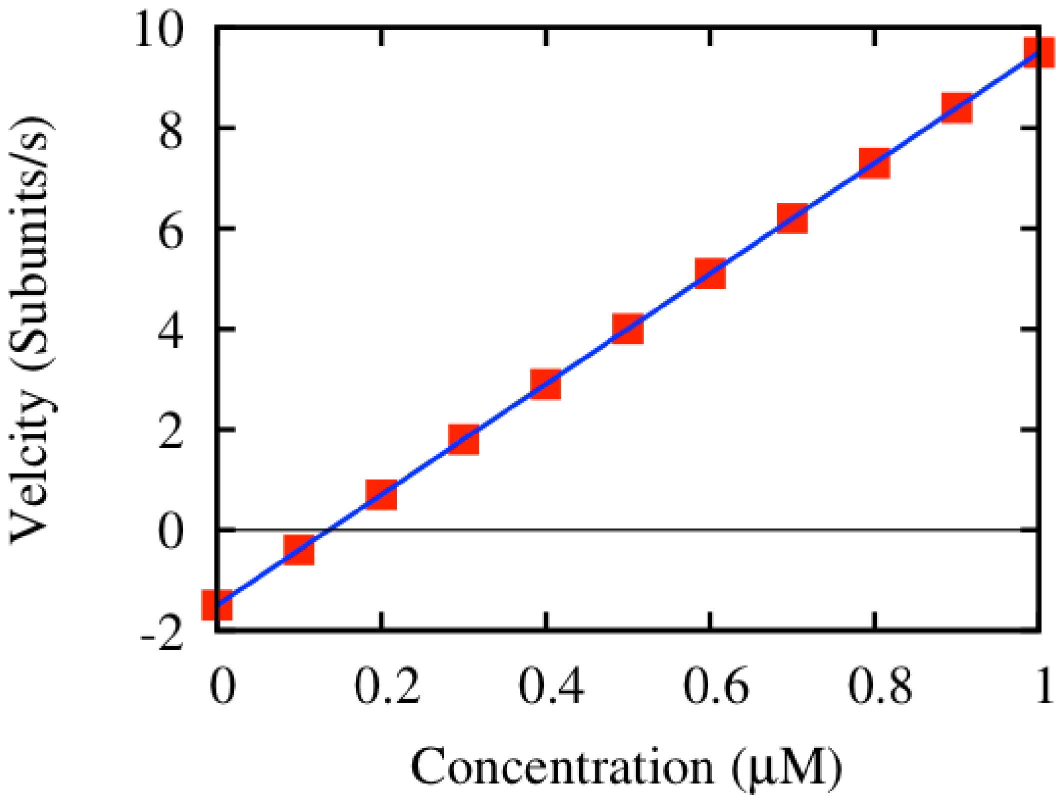


$$k_p = k_0[C]$$

Growth speed $v = k_p - k_d$

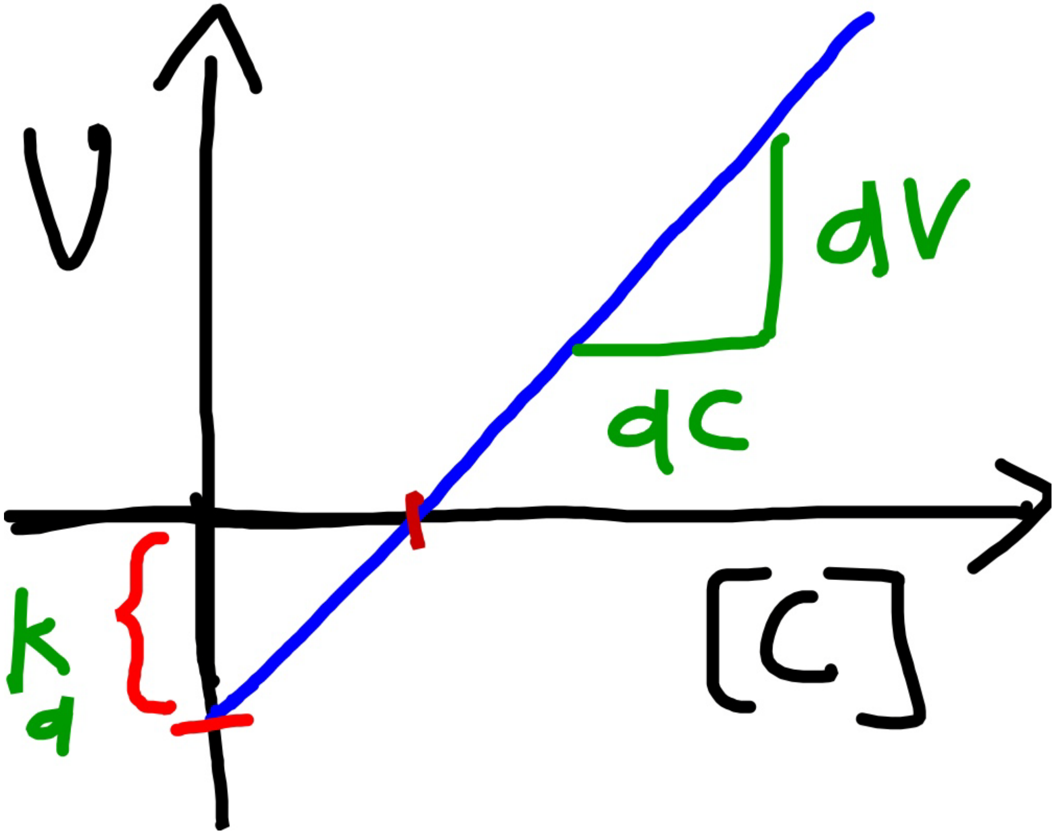
$$v = k_0[C] - k_d$$

Polymerization of actin



$$v = k_0[C] - k_d$$

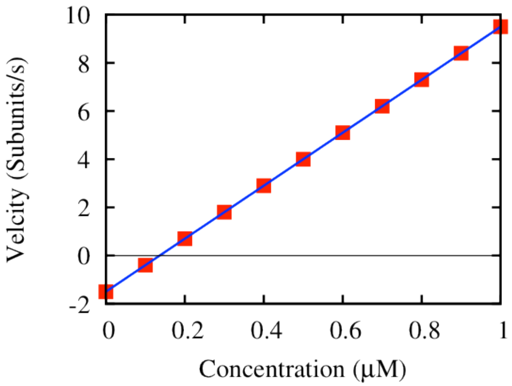
$$v = k_0[C] - k_d$$



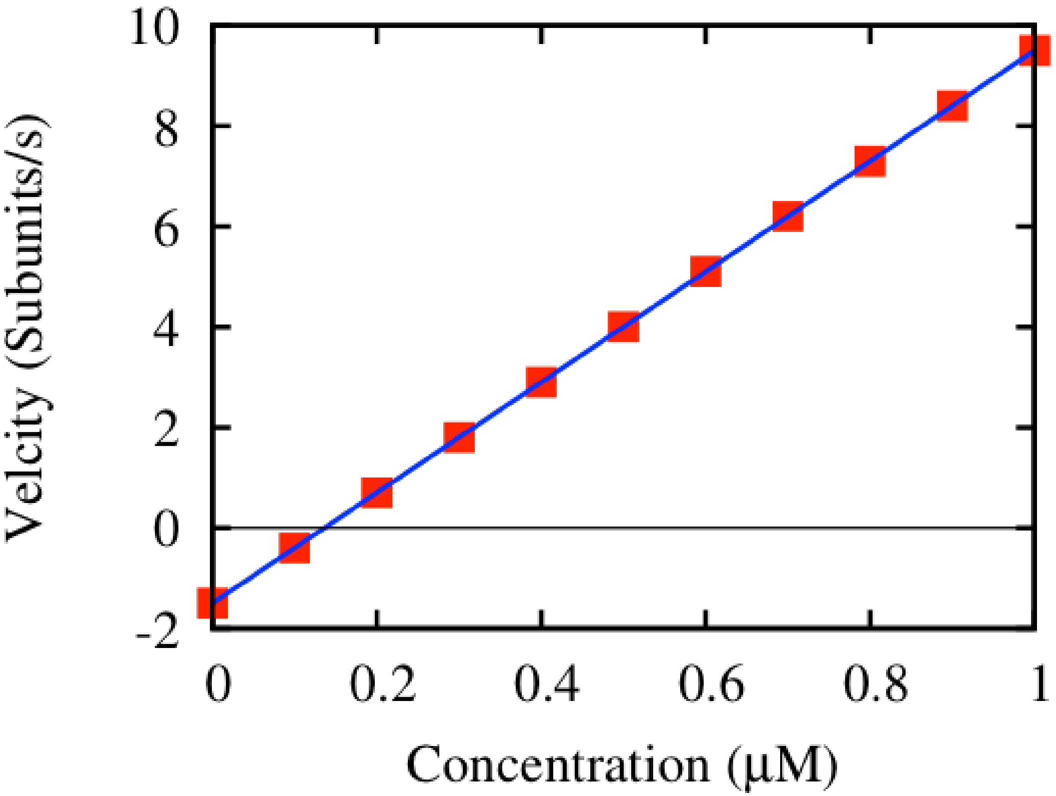
Slope $\frac{dv}{dc} = k_0$

For any polymer

1. Draw Velocity vs concentration data
2. Find the derivative (slope). This gives k_0
3. Find the Y-intercept. This gives k_d



For actin



$$k_0 = 11 \mu M^{-1} s^{-1}$$

$$k_d = 1.5 s^{-1}$$

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Applications in Biology

Example 2: Enthalpy and Entropy of a chemical reaction

$$A \rightleftharpoons B$$

$$K_{eq} = \frac{[B]_{eq}}{[A]_{eq}}$$

Enthalpy, Entropy

$$\Delta G_0 = -RT\ln\left(K_{eq}\right)$$

$$\Delta G_0 = \Delta H_0 - T\Delta S$$

$$\Rightarrow -RT\ln\left(K_{eq}\right) = \Delta H_0 - T\Delta S$$

$$\Rightarrow \ln\left(K_{eq}\right) = -\frac{\Delta H_0}{RT} + \frac{\Delta S}{R}$$

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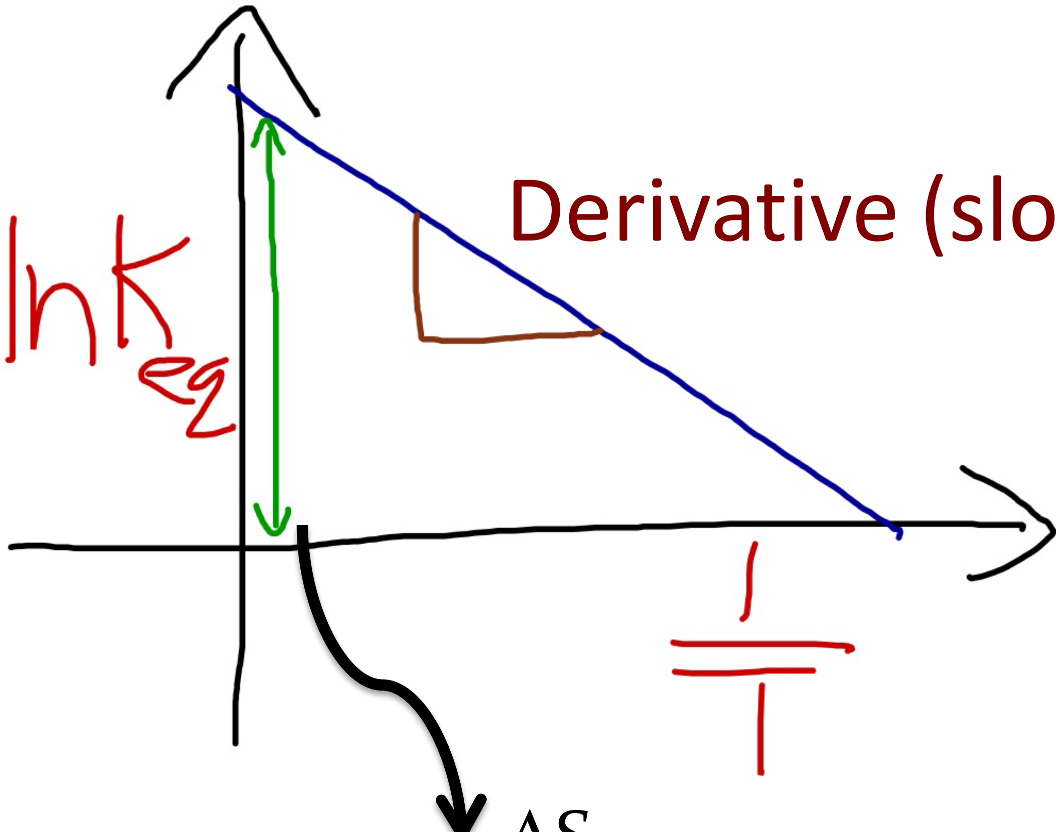

Derivative (slope)= $\dfrac{dy}{dx}$

$$= \frac{d\ln(k_{eq})}{d(1/T)} = \frac{\Delta H_0}{R}$$

Y intercept = $\dfrac{\Delta S}{R}$

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