BIOMATHEMATICS

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Differentiation and its applications

Applications in Biology

Example 2: Enthalpy and Entropy of a chemical reaction

$$A \rightleftharpoons B$$

$$K_{eq} = \frac{[B]_{eq}}{[A]_{eq}}$$

Enthalpy, Entropy

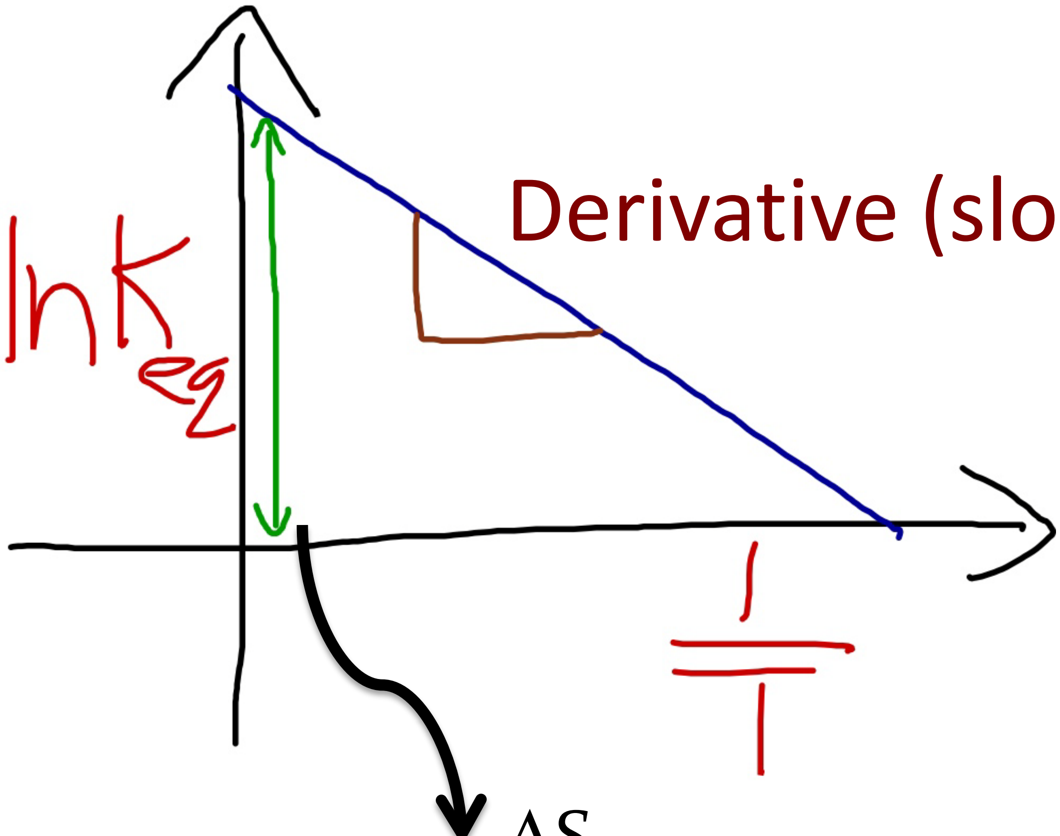
$$\Delta G_0 = -RT\ln\left(K_{eq}\right)$$

$$\Delta G_0 = \Delta H_0 - T\Delta S$$

$$\Rightarrow -RT\ln\left(K_{eq}\right) = \Delta H_0 - T\Delta S$$

$$\Rightarrow \ln\left(K_{eq}\right) = -\frac{\Delta H_0}{RT} + \frac{\Delta S}{R}$$

$$\ln\left(K_{eq}\right) = -\frac{\Delta H_0}{RT} + \frac{\Delta S}{R}$$



Derivative (slope)= $\dfrac{dy}{dx}$

$$= \frac{d\ln(k_{eq})}{d(1/T)} = \frac{-\Delta H_0}{R}$$

Y intercept = $\dfrac{\Delta S}{R}$

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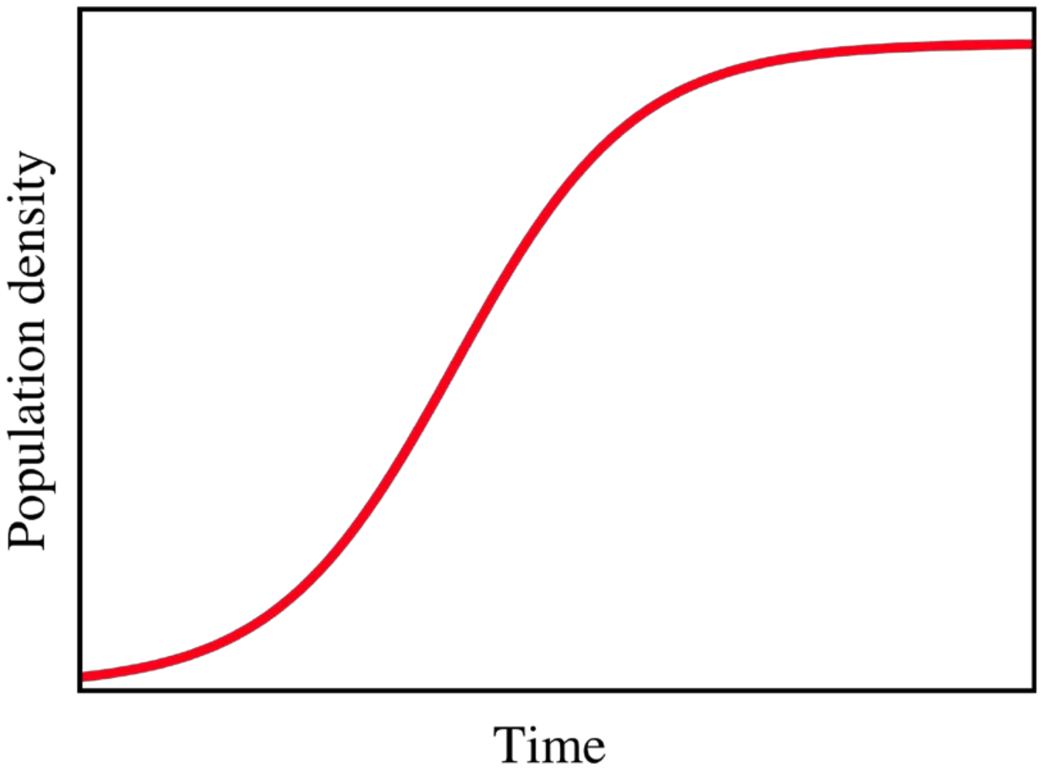
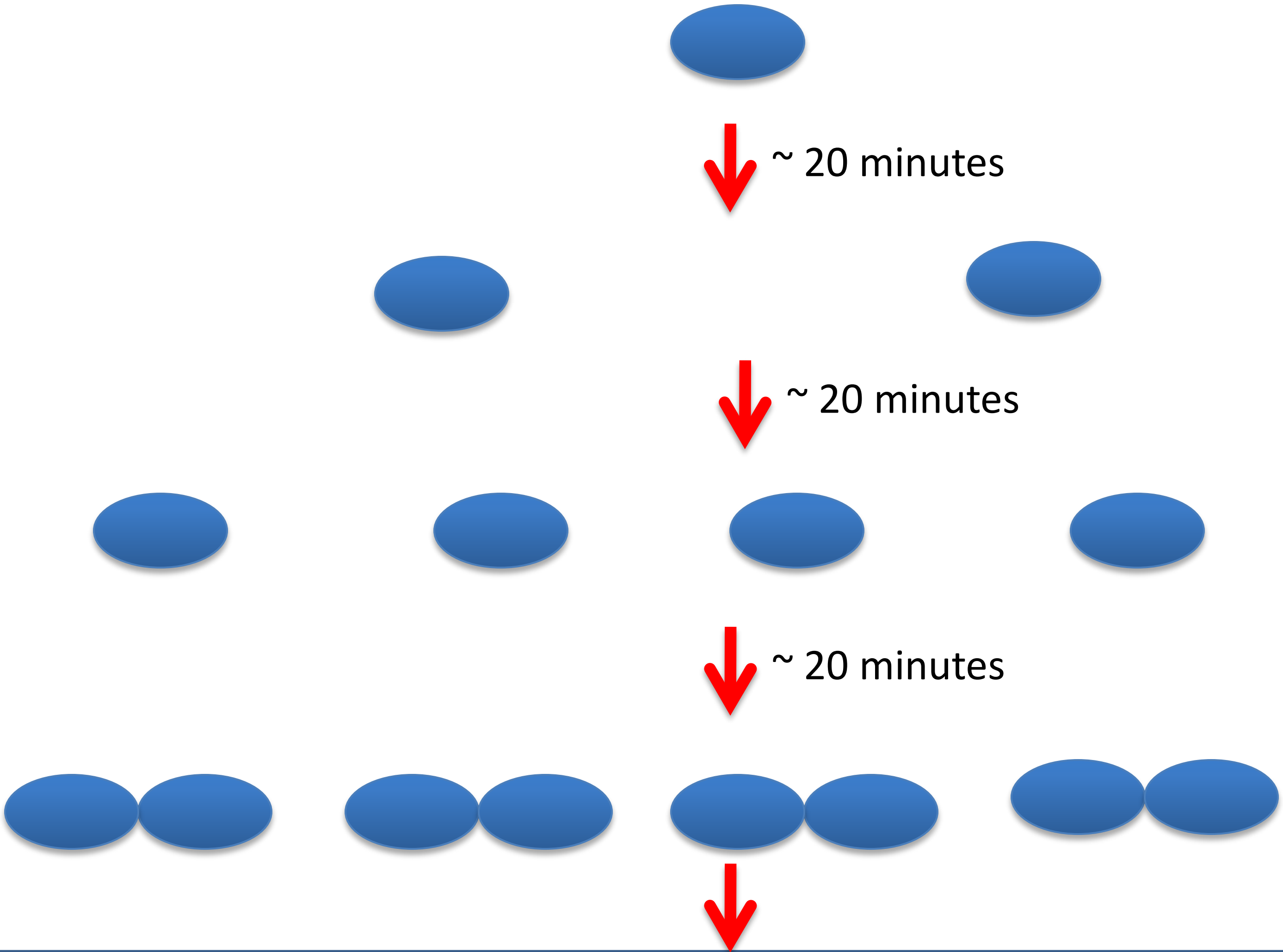
Growth of fish population



At what population density, we will get maximum yield ?

When the derivative (slope) is maximum, growth is fastest

Bacterial growth (eg. E. Coli)



$1=2^0$

$2=2^1$

$4=2^2$

$8=2^3$

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Bacterial (eg. E. Coli) growth

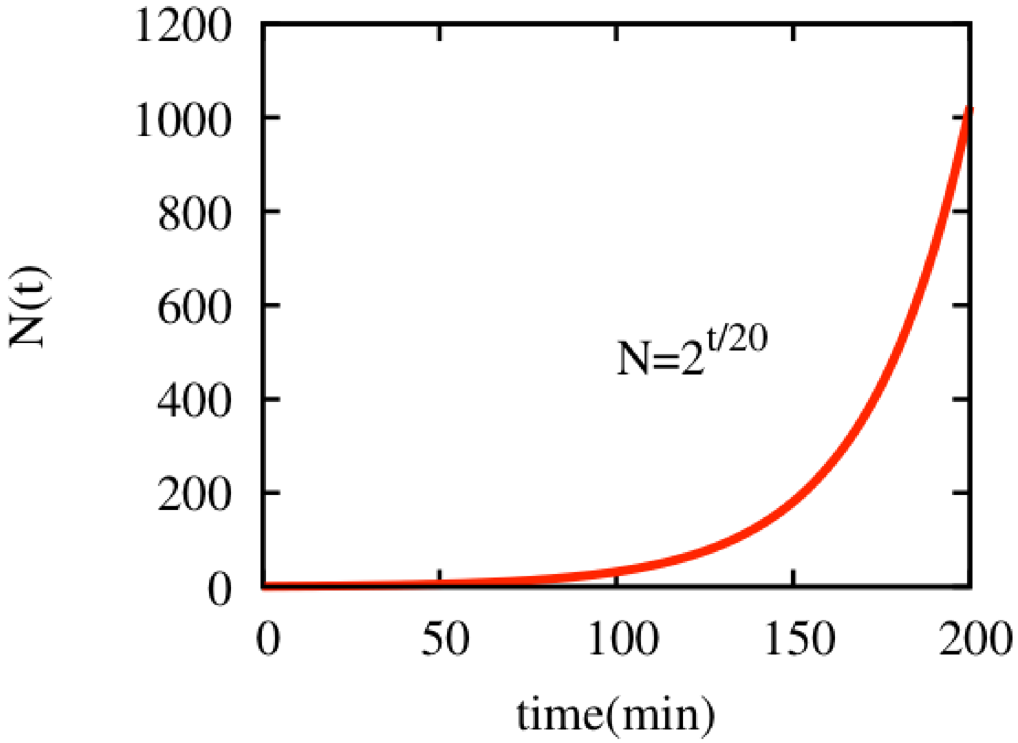
Number of bacteria at time t is given by

$$N(t) = 2^{kt}$$

k = rate of cell division; for E-coli, typically $k = \frac{1}{20\min}$

When t=60 minutes, N = 2^3 = 8

Bacterial (eg. E. Coli) growth



$$k = \frac{1}{20\,\text{min}}$$

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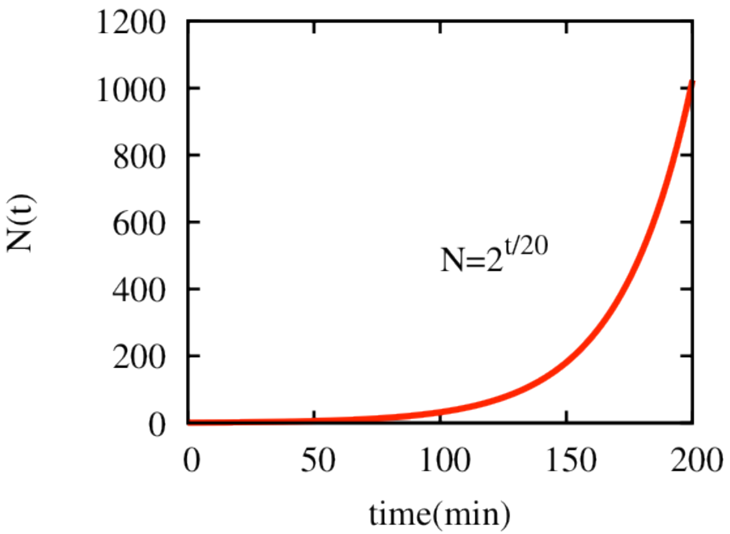
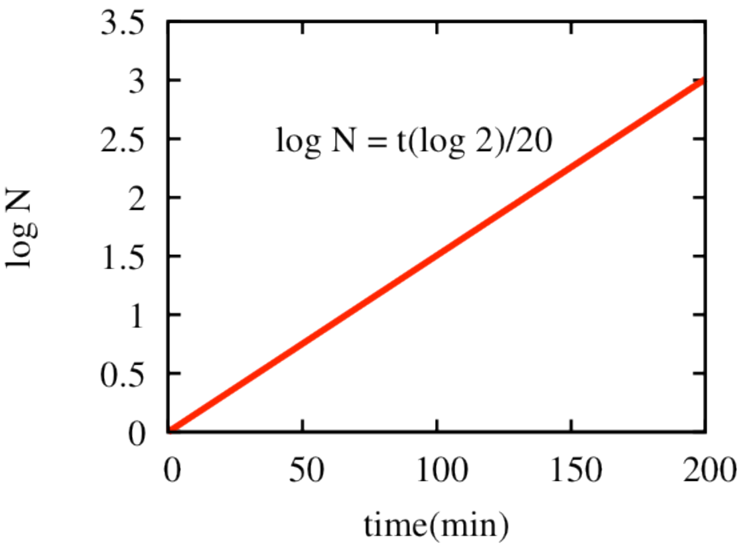
Bacterial growth : Log phase





$$k = \frac{1}{20\,\mathrm{min}}$$

Slope=(log2)/20

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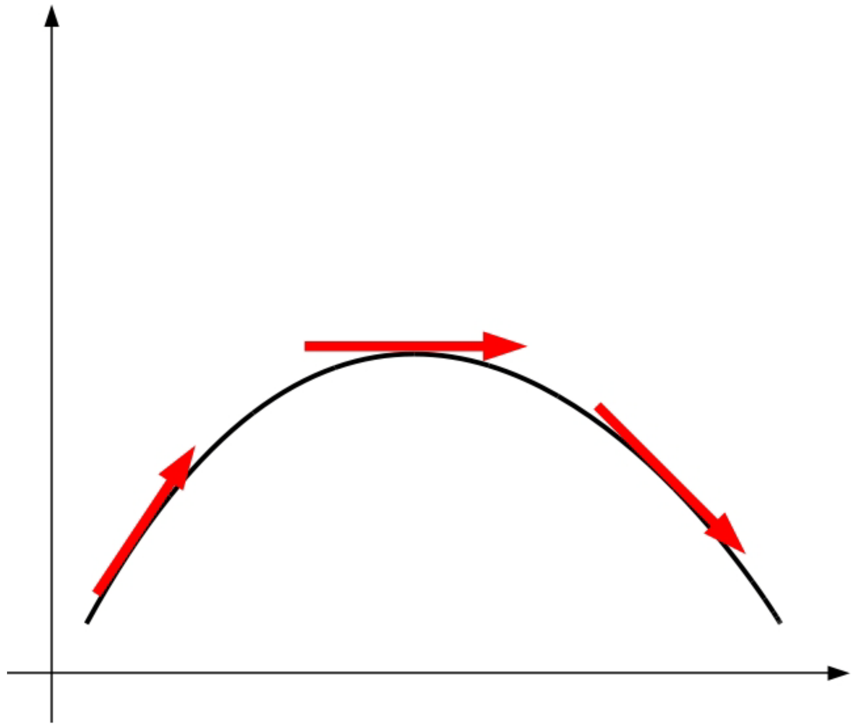
Curvature: Change in slope



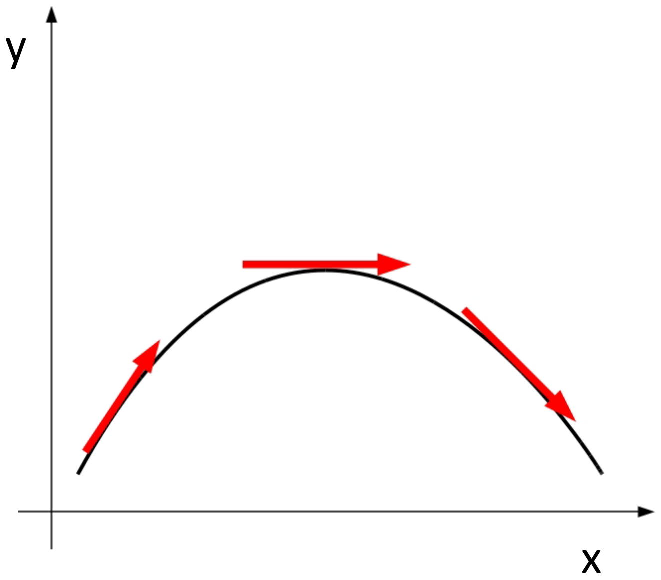
Positive slope

Zero slope

Negative slope

Slope decreases as we go along x (convex)

Curvature, C(x)



$$m(x) = \frac{dy}{dx}$$

$$C(x) = \frac{d}{dx}(m(x)) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

C(x) is negative

Slope decreases as we go along x

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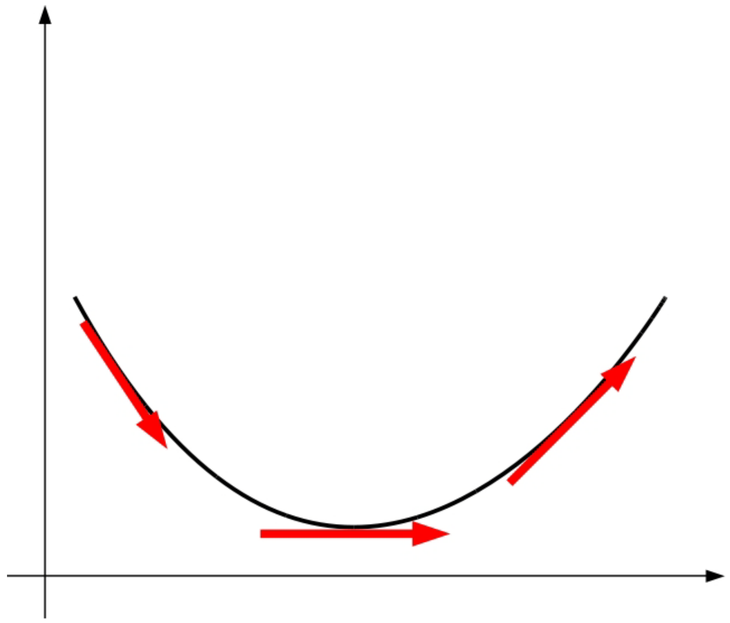
Curvature: Change in slope



Negative slope

Zero slope

Positive slope

C(x) is positive

Slope increases as we go along x (concave)