Clearly state all the assumptions you make

Useful Information

- g = 9.8 m/s^2, density of water = 10^3 kg/m^3, viscosity of water = 10^{-3} Pa s.
- Kinetic energy correction factor α = 2 for laminar flow, α = 1 for turbulent flow in pipes.
- $\frac{d}{dx}\sin^{-1}(x)=\frac{1}{\sqrt{1-x^2}}; \frac{d}{dx}\cos^{-1}(x)=-\frac{1}{\sqrt{1-x^2}}; \frac{d}{dx}\tan^{-1}(x)=\frac{1}{1+x^2}$.
- Loss coefficient K for: entrance = 0.5, elbow (90^0) = 0.9, gate valve (fully open) = 0.3, gate valve (half open) = 5.

1. Consider the gate HA shown in figure 1 which is hinged at H. The gate is 10m wide normal to the plane of the paper. Calculate the force F required at A to hold the gate closed. [6 points]

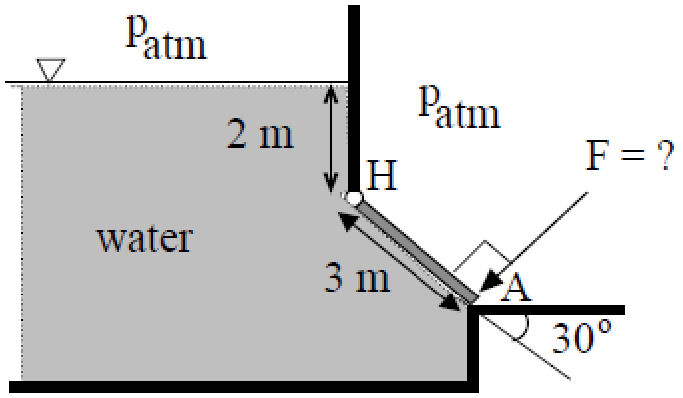


Figure 1: **Problem 1**

2. A pipe of 12 cm diameter containing water flowing at 20 kg/s is capped by an orifice plate (as shown in figure 2) with flange bolts. The water jet exits to the atmosphere, and jet diameter reaches a constant value of 25 mm as shown in the figure. The pressure in the pipe at section 1 is 800 kPa (gage). Assume flow to be uniform both at section 1 and in the jet, and neglect viscous friction at the pipe walls. Calculate the force provided by the flange bolts to hold the orifice plate fixed. [5 points]

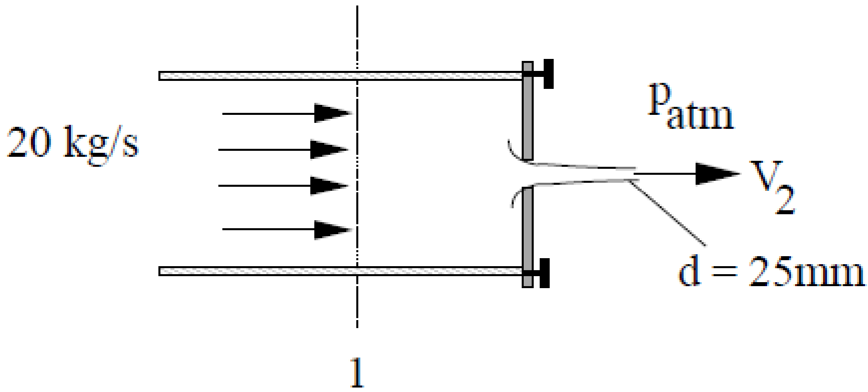


Figure 2: **Problem 2**

3. A turbine (shown in the figure 3) is supplied with $0.6 m^3/s$ of water from a 0.3m diameter pipe; the outlet pipe has 0.4m diameter. Assume flow to be steady, incompressible, and non-viscous. Determine the pressure drop across the turbine if the rate at which work is produced by the turbine is 60kJ/s. [4 points]

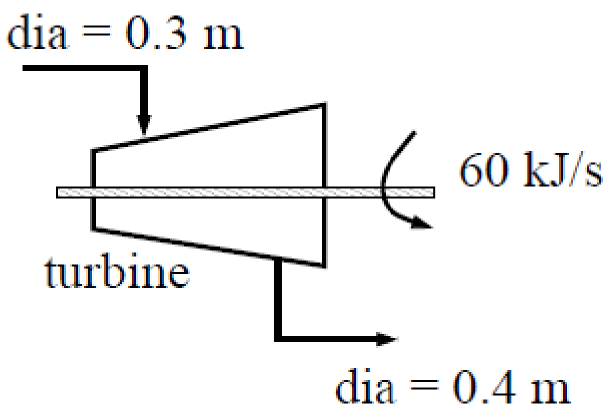


Figure 3: **Problem 3**

4. An infinitely long cylindrical wire of radius R_1 is pulled (with constant velocity *V*) through a circular cylindrical tube of radius R_2 filled with a liquid of viscosity m and density ρ, as shown in figure 4. The wire is placed coaxially in the tube, i.e. their axes of revolution coincide. The cylindrical tube connects two large reservoirs of the liquid maintained at a constant pressure P_0. Determine:

(a) The steady velocity profile in the fully-developed region of the cylindrical tube, neglecting entrance and exit effects. Give brief reasons for why certain terms are neglected in the Navier-Stokes equations (see Navier-Stokes equations in cylindrical coordinates). [4 points]

(b) The force per unit length required to pull the wire through the cylindrical tube. (Neglect contributions to the force from the liquid in the reservoirs.) [3 points]

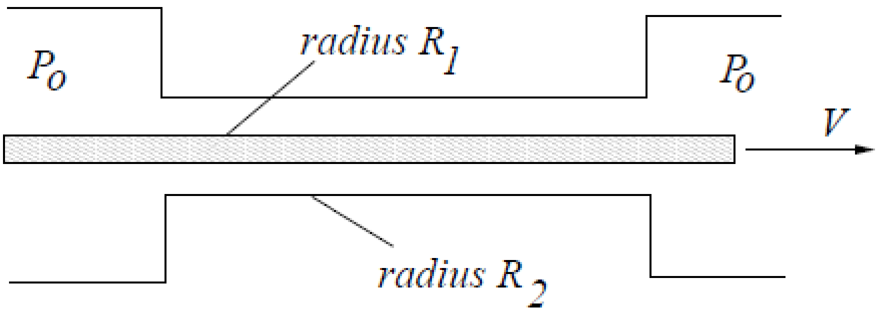


Figure 4: **Problem 4**

5. Consider a water tank which is connected to a pipe system as shown in figure 5. The pipe walls are *smooth*, and the pipe system has a sharp-edged entrance, one 90^0 elbow, and a fully-open gate valve. Water exits from the pipe to atmospheric pressure. The pipe diameter is 5 cm, and the total length of the pipe is 1 m. Calculate the minimum height H of the water in the tank above the pipe system discharge (H shown in the figure), such that the Reynolds number of flow in the pipe is 10^5. [6 points]

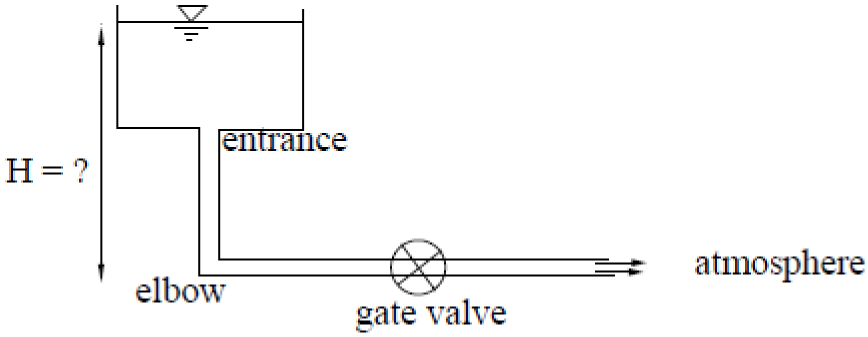


Figure 5: **Problem 5**

6. A swimming pool is approximated by a Rankine half-body shape as shown in figure 6. At point O, which is 0.5 m from the left edge of the swimming pool, there is a line source (into the paper) of water delivering 0.35 m^3/s per meter of depth into the paper. Assuming potential flow, find the coordinates of point B along the axis where the water velocity is 25 cm/s.

[7 points]

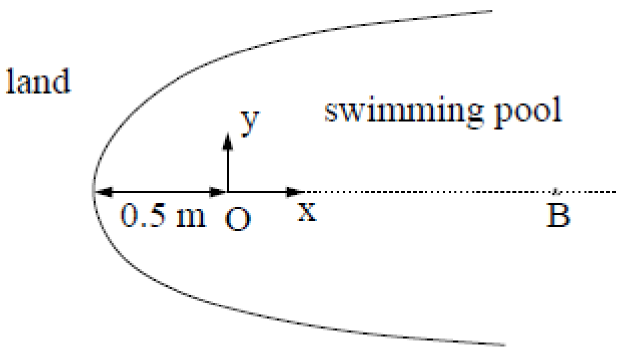


Figure 6: **Problem 6**

7. Two large water tanks are connected with a pipe of diameter D = 5 cm and length 2 m as shown in figure 7. The kinetic energy correction factor for flow in the pipe $\alpha = 1$.

(a) If the frictional loss head (h_l, in meters) for the pipe is given by $5.4\frac{V_{tube}^2}{2g}$ (V_{tube} in m/s, and g in m/s^2), calculate the velocity V_{tube} in the pipe. [2 points]

(b) If a pump is installed in the pipe (with h_l given by the same expression as in part a) to have a velocity V_{tube} = 10 m/s, calculate the rate at which the pump does work on the fluid, if the flow is from A to B. [2 points]

(c) Calculate the rate at which the pump does work on the fluid if the flow is from B to A, for V_{tube} = 10 m/s, with h_l given by the same expression as in part a. [2 points]

(d) What is the inconsistency in using the Bernoulli equation between points 1 and 2 on the free surface of the two tanks? [2 points]

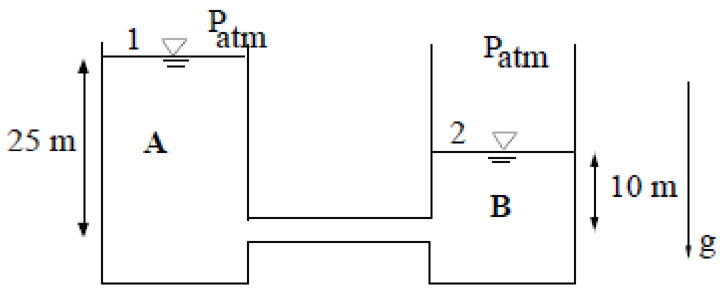


Figure 7: **Problem 7**

8. (a) Derive the equation that governs a streamline using the definition of the streamline. [2 points]

(b) For a 2-D incompressible flow, show that stream function is a constant along a streamline. [3 points]

(c) Show that for a 2-D incompressible flow, the volumetric flow rate (per unit width) between two streamlines is given by the difference in the values of their stream function. [2 points]

9. Consider the axi-symmetric, steady, fully-developed, laminar flow of a liquid film (density ρ and viscosity μ) **outside** a circular tube driven by gravity, as shown in figure 8. The outer radius of the tube is R, and the thickness of the annular liquid film is $(a-1)R$. Use the Navier-Stokes equations in cylindrical coordinates.

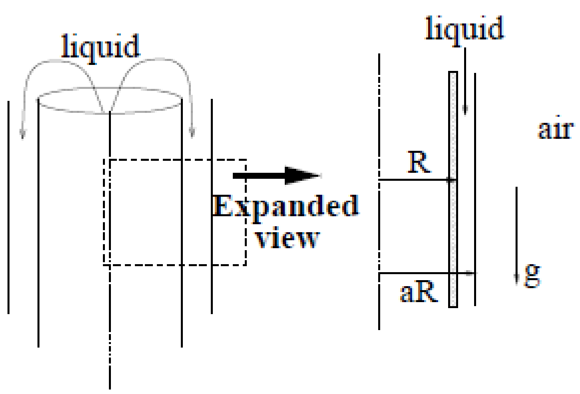


Figure 8: **Problem 9**

(a) Derive the expression for the velocity distribution in the falling liquid film outside the tube. [5 points]

(b) Derive the expression for the shear stress exerted on the outer wall of the tube. [2 points]

10. A water jet of diameter D flows out of a nozzle at a velocity V_0. This jet is used to support a cone-shaped object as shown in figure 9(a). The jet forms a uniform film surrounding the cone. Assume friction-less, steady, incompressible flow. Choose an appropriate control volume (CV) with control surfaces cutting across locations A and B.
(a) Simplify the integral mass balance for the CV.
(b) Simplify the integral momentum balance for the CV.
(c) Apply Bernoulli equation between points A and B.
Using the above three equations, derive an expression for the combined mass (M) of the cone and water in the CV that can be supported by the jet. For V_0 = 10 m/s, H = 1 m, h = 0.8 m, D = 100 mm, $\theta = 30^0$, what is the numerical value of M (in kg)? [8 points]

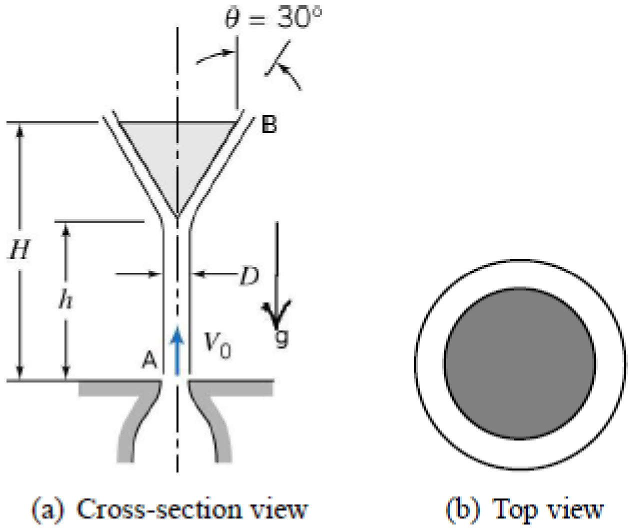


Figure 9: **Problem 10**

Useful Information

• Loss coefficient K for: entrance = 0.78, elbow (90^0) = 0.9, elbow (45^0) = 0.4; gate valve (fully open) = 0.3.

• Vector Identity: $(\mathbf{v}\cdot\nabla)\mathbf{v} = \nabla\left(\frac{1}{2}\mathbf{v}\cdot\mathbf{v}\right) + (\nabla\times\mathbf{v})\times\mathbf{v}$.

11. A large tank containing water (shown in figure 10) has a small smoothly contoured orifice, from which a water jet exits with a velocity of V_{jet}. The height of water in the tank h as shown in the figure is 45.92 m. The diameter of the jet is 100 mm, and this jet moves horizontally to the right, where it is deflected by a cone that moves to the left at V_{cone} = 14 m/s. The thickness t of the liquid film that leaves the cone at the radius R (= 230 mm) is t = 5.434 mm. Neglect effects of gravity for flow around the cone. Determine:
(a) The velocity V_{jet} of the liquid jet that comes out of the orifice, neglecting losses. [4 points]
(b) The velocity of the liquid that leaves the cone at R = 230 mm. [4 points]
(c) The horizontal component of the external force on the cone required to maintain its motion. [6 points]

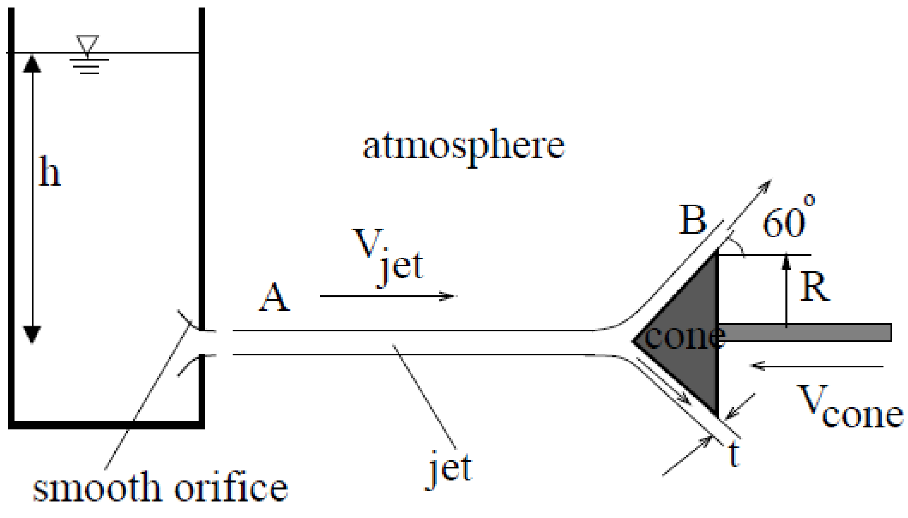


Figure 10: **Problem 11**

12. Water is to be pumped from a reservoir using a pipe system shown in figure 11. The flow rate in the pipe must be 0.038 m3/s and water must leave the exit of the pipe via a nozzle at a velocity V_j = 37 m/s. The dimensions and fittings involved in the pipe system are indicated in the figure. The wall roughness factor e = 0.0015 mm.

Neglect losses at the exit nozzle. Determine the power input (in kW) to the pump required to achieve this flow. [12 points]

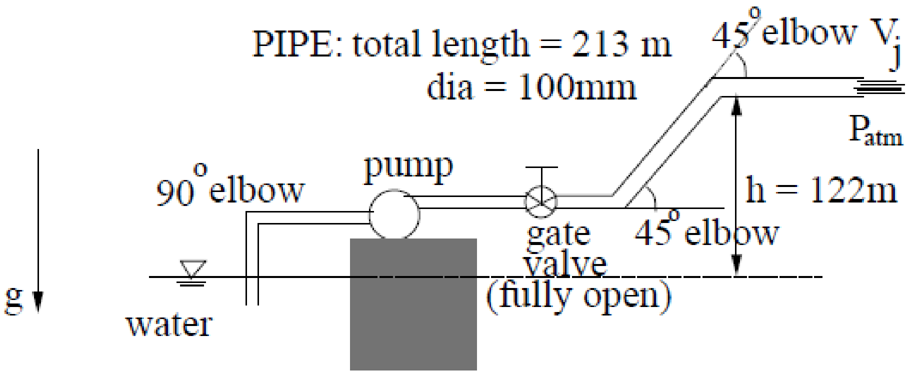


Figure 11: **Problem 12**

13. A belt of width *W* moves at a velocity *V* as shown in figure 12. A very viscous liquid fills the gap *b* between the belt and a stationary plate. Assume steady, fully-developed laminar flow between the belt and the plate, and do not neglect the effect of gravity.

For this system:

(a) State the boundary conditions to solve the problem. [2 points]

(b) Derive the expression for velocity profile in the liquid. [6 points]

(c) Derive the expression for volumetric flow rate of the liquid. [2 points]

(d) If μ = 0.1 Pa-s, ρ = 103 kg/m^3, *b* = 1 cm, θ = 45^0, determine the velocity (in m/s) of the belt above which there is a net volumetric flow rate in the direction of the belt motion. [2 points]

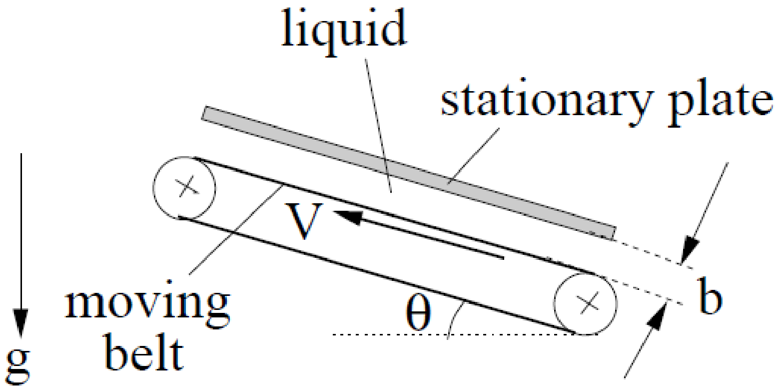


Figure 12: **Problem 13**

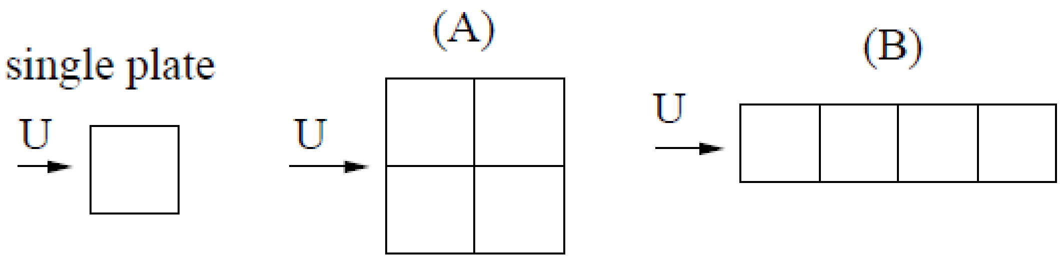
14. (a) Derive the Bernoulli equation for steady incompressible flow from the Euler equation for an inviscid fluid. Between which two points is the derived Bernoulli equation valid?

[7 points]

(b) Prove that for a 2-D incompressible, irrotational flow the streamlines and equipotential lines are always orthogonal. [7 points]

15. (a) Consider the steady laminar boundary layer flow past the top surface of square plate arrangements shown in figure 13. Compared to the friction drag on a single plate (area *A*, drag force *F*1), how much larger is the drag on four plates together as in configurations (A) and (B) shown in figure 5. Will the drag on the two arrangements be the same? Explain.

[7 points]



TOP VIEW
Figure 13: **Problem 15(a)**

(b) A naphthalene ball of diameter 1 cm is suspended in a large room with still pure air at 27^0C and 1 atm. The surface temperature of naphthalene can be assumed to be 27^0C and its vapor pressure at this temperature is 1 mm Hg. Assume naphthalene vapor to be an ideal gas. Determine the steady rate of evaporation of naphthalene (in kg/s). Assume that the radius of the naphthalene ball diameter remains constant in your calculation. Molecular weight of naphthalene is 128 kg/kmol; R = 8314 m^3 Pa/(kmol K); D for naphthalene in air is 5×10^{-6} m^2/s. [7 points]

16. **Integral Momentum Balance:** Two large tanks containing water have smoothly contoured orifices (openings) of equal area (figure 14). A jet of water issues from the orifice of the left tank. Assume that the flow is uniform and **viscous losses are negligible**. The jet impinges on a vertical flat plate covering the opening of the right tank. Choose an appropriate

CV to determine the minimum value of the height h required to keep the plate in place over the orifice of the right tank. [8 points]

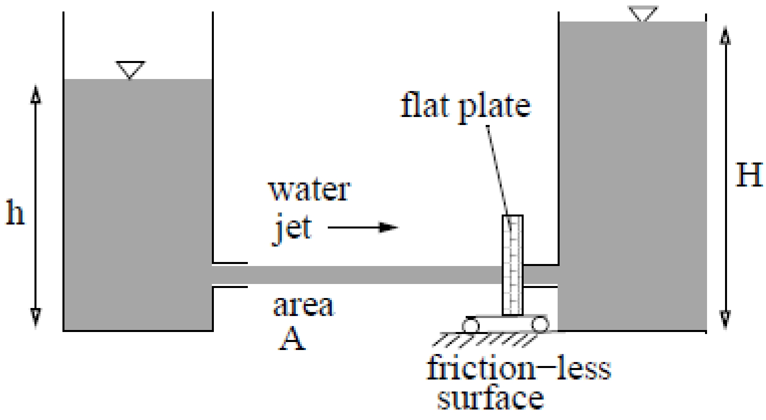


Figure 14: **Problem 16**

17. **Pipe Flows and Losses:** The pipe flow shown in figure 15 is driven by pressurized air (at pressure p_1) in the tank. What gage pressure p_1 is needed to provide water flow rate Q=60m^3/h? Assume the entrance from the tank to the pipe be a sharp entrance, and the two bends to be at an angle of 90^0. The friction factor chart and the minor loss coefficients are provided in a separate data sheet. [10 points]

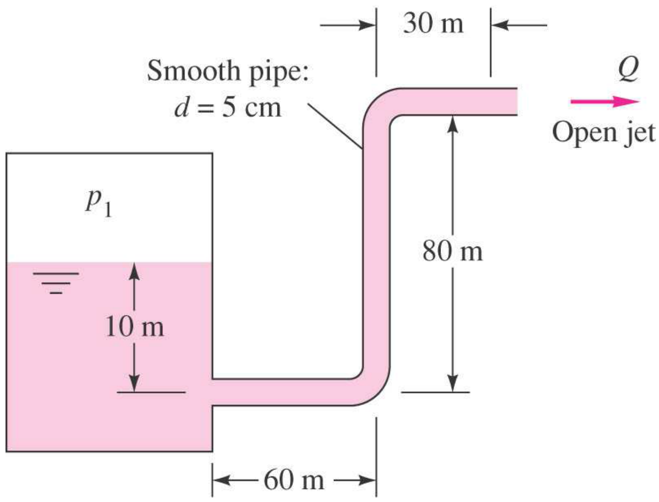


Figure 15: **Problem 17**

18. **Navier-Stokes Equations:** Consider the steady flow of a Newtonian fluid in the annular region between two long cylinders (inner radius κR and outer radius R) as shown in the figure 16. The outer cylinder moves with an angular velocity W, while the inner cylinder is stationary.

(a) Simplify the relevant component of the Navier-Stokes momentum equations in cylindrical coordinates. [2 points]

(b) Solve this differential equation, and obtain the velocity profile in the annular gap after utilizing the boundary conditions. [4 points]

(c) Find the torque exerted by the fluid on the inner cylinder. [4 points]

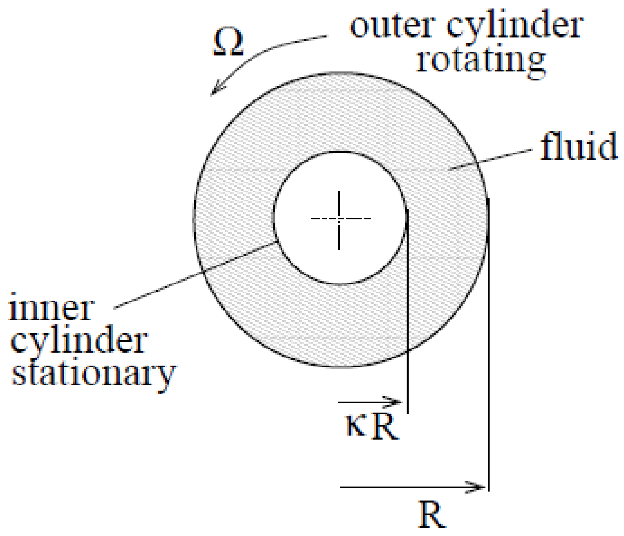


Figure 16: **Problem 18: Top view of the set-up with two concentric cylinders**

19. (a) **Potential Flow:** The velocity potential for a 2-D potential flow is given by $\Phi(r,\theta) = Ar^2 \cos 2\theta$, where A is a constant with appropriate units, and r and θ are the polar coordinate variables. Determine the expressions for the velocity components and stream function for this flow. By considering the streamline with $\psi = 0$, sketch the geometry of the surface. Based on this, qualitatively plot the streamlines of the flow, and comment on what physical situation the potential flow represents. [10 points]

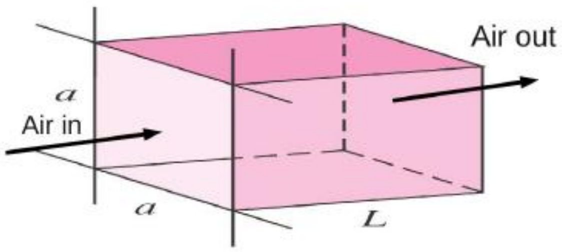


Figure 17: **Problem 19(b): Air flow into a box**

(b) **Boundary Layers:** Consider a box made of four plates with length L = 25 cm and width a = 4 cm as shown in the figure 17. Air flows into this box with a free stream velocity U_0 = 12m/s.

Assuming steady, laminar flat-plate boundary layer flow over the plates, determine the pressure drop required to maintain the flow in the box. Assume that the interaction between the boundary layers at the corners of the box is negligible. The friction coefficient $C_f = 0.664/\mathrm{Re}_x^{1/2}$ for laminar boundary-layer flow over a single flat plate, and Re_x is the Reynolds number based on the distance from the origin of the plate. [8 points]