1. Which term(s) in the Navier-Stokes equation can be neglected for 'creeping flow' in macroscale?

$$\rho \frac{D\vec{w}}{Dt} = \rho \left(\frac{\partial}{\partial t} + \vec{w} \cdot div \right) \vec{w} = -\ grad\ p + \mu \nabla^2 \vec{w} + \vec{k}$$

Is it advisable to drop the term(s) for microscale flow as well?

For creeping flow, the convective terms (second term of the LHS of the equation) may be safely dropped from the equation, owing to the very small velocities encountered in creeping flow.

The situation is different in the case of microscale flow, e.g.; flow in a microchannel. For flow in small systems, the effect of body forces are quite small compared to the surface forces and hence the last term on the right hand side of the equation can be neglected. Additionally, the non dimensional form of the equation for flow in a microchannel is

$$\frac{1}{Re^2} \left[v^* \frac{\partial\ v^*}{\partial y^*} + w^* \frac{\partial\ v^*}{\partial z^*} \right] = -\frac{\partial\ p}{\partial y^*} +$$
$$+ \frac{1}{Re^2} \left[\frac{\partial}{\partial y^*} \left(\eta^* \frac{\partial\ v^*}{\partial y^*} \right) + \frac{\partial}{\partial z^*} \eta^* \frac{\partial\ v^*}{\partial z^*} \right]$$

One can see that owing to the small values of Reynolds number, the convective terms may not be neglected for microscale flows.

2. Starting with Einstein-Smoluchovski equation, $x^2 = 2\ D\ t$, explain the enhanced selectivity and high yield of chemical reactions in microreactors.

Small channels allow short transport lengths for heat and mass transfer. This results in high transfer rates, as described for diffusive mass transfer with the mean transport length from the Einstein-Smoluchovski equation $x^2 = 2\ D\ t$. Microstructured devices with typical length scales from 100 µm to 1 mm provide fluid structures with length scales of approx. 1 µm. These small fluid structures lead to mixing times shorter than 100 µs in gases and approx. 1 ms in liquids. This is the main reason for the enhanced selectivity and high yield of chemical reactions in microreactors.

3. Cite the main reasons for faster transport processes in microscale devices.

The main reasons can be listed as

- Smaller length scales, leading to small values of transfer lengths for heat and mass transfer

- In general, the shorter the length, the shorter the characteristic time for transport processes will be, and the higher the transformation frequencies.
- System sizes are of the order of the boundary layers, thereby causing appreciable gradients
- Additional effects may start to dominate the transport processes
- The high surface-to-volume ratio is also responsible for the fast heat transfer in microchannels.
- Additionally, a high surface-to-volume ratio is beneficial for surface reactions such as heterogeneous catalysis, emulsification or transport-limited processes

4. (a) It is proposed to conduct three reactions separately in a microchannel. The kinetics of reaction 1 is relatively slow, whereas the second reaction suffers from a low mass transfer coefficient. The third reaction is exothermic in nature. Explain for which reaction(s), a microchannel reactor would be beneficial.

The slow kinetics will not yield any different results in a microreactor. Whereas, the mass transfer coefficient can be enhanced as well as heat transfer will be more efficient in microscale.

(b) Starting with the definition of Bond number, explain how the relative importance of surface forces can be evaluated for a water droplet of diminishing size.

$$Bo = \sqrt{g\,\Delta\rho R^2 / \sigma_{lv}}$$

Bond number represents the ratio of body and surface forces. Here g is O (10), $\Delta\rho$ = O (1000), σ = O (0.1). Therefore, g $\Delta\rho$/ σ = O (10^5). Thus as the value of R becomes less than 10^{-4} m, the value of Bo becomes too small signifying importance of surface forces.

5. Flow of a gas in two microchannels of equal cross section, temperature and inlet pressure is being studied. Microchannel 1 has a length of 2 cm, while that of channel 2 is 6 cm. In which case, compressibility of the gas needs to be taken into account and why?

To estimate the influence of compressibility, the speed of sound $c = \sqrt{(\kappa RT)}$ for a perfect gas is compared with the mean flow velocity of a system. This dimensionless number is also known as the Mach number.

$$Ma = \frac{w}{c} = \frac{w}{\sqrt{\kappa RT}}$$

In a channel with constant cross section $A_1 = A_2$, the continuity equation can be written as

$$Ma_2 = Ma_1 \frac{p_1}{p_2} \sqrt{\frac{T_2}{T_1}}.$$

For small Ma number at the channel inlet, the Ma number along the channel increases for high pressure losses $(p_1 \gg p_2)$. A temperature increase has less influence due to the square root. For $Ma_2 > 0.3$, the flow can be reconsidered as compressible. Thus for flow in the longer channel (6 cm in this case) with the possibility of having a larger pressure drop, the flow may turn out to be compressible towards the end of the channel.

6. For two gases, namely hydrogen and methane, which one will start showing presence of a slip velocity first under identical operating conditions?

The mean molecular diameter of hydrogen is smaller than that of methane. Thus the mean free path of hydrogen will be larger than that of methane.

$$\lambda = \frac{kT}{\sqrt{2}\, p\sigma}$$

The parameter that denotes the presence of slip is Knudsen number. (Knudsen number = mean free path/characteristic length and higher the value of Kn (>0.1)), higher will be the slip. Therefore for hydrogen the value of Kn will be more and therefore more chances of the presence of slip velocity.

7. In the proposed NASA Space Flight experiment, flow visualizations of liquid flow in a microchannel will be made. As part of a preliminary experiment on earth, the pressure drop of water flowing with a velocity of 1 cm/s ($\rho = 10^3$ kg/m^3, $\mu = 1$ cP) through a rectangular microchannel of dimension (50 μm x 50 μm) and length 2cm is measured. The scientist would like to project his results for the proposed experiments in microgravity (space) situation. Can you help the scientist?

Body forces are not important for flow in microchannels and the effect of absence of gravity will not play any significant role in the results of proposed experiments in microgravity. The calculation can be made of the ratio of the body and surface force to demonstrate the same.

8. The same experiment is performed on earth's surface with twice the velocity (keeping other parameters unchanged). What would be the change in pressure drop in this case, compared to the first?

The pressure drop in microchannel flow depends strongly on the flow velocity.

$$\Delta p = \sum_i \left(\lambda_i \frac{l_i}{d_{h,i}} + \zeta_i \right) \cdot \frac{\rho}{2} w_{ref,i}^2$$

Where $w_{ref,i}$ is the reference velocity and must be determined for each channel element i. This shows the strong dependence of pressure drop on velocity. Thus with increase in velocity the pressure drop would be four times than before.

9. Starting with the Augmented Young-Laplace equation, prove that for an evaporating, curved thin film the flow towards the interline (from the capillary meniscus region) is sustained by capillary pressure gradient and disjoining pressure.

At equilibrium situation, where no evaporation or condensation is taking place, the Augmented Young-Laplace equation can be written as

$$P_l - P_V = -\sigma K - \Pi$$

Where, K is the curvature of the system, Π is the disjoining pressure, equal to B/δ^4, where δ represents the film thickness, σ the surface tension and B, a modified Hamaker constant (B < 0 for wetting systems). The Y-L equation can be written for two points, one in the capillary region and the other in the transition region and the pressure difference between the capillary region and the transition region can be written as –

$$P_{l,CAP} - P_{l,TR} = \sigma \left(K - K_\infty \right) - \frac{B}{\delta^4}$$

The curvature at the thicker portion of the meniscus (K_∞) is nearly constant and is smaller than the curvature at the transition region. Thus considering the fact that B < 0 for wetting systems, it is clear that the flow towards the interline (from the capillary meniscus region) is sustained by capillary pressure gradient and disjoining pressure.

10. For heat transfer in a straight channel with laminar flow, the characteristic time, t_h is defined as

$$t_h = \frac{\rho c_p d_h^2}{3.65 \pi \lambda}$$

Utilizing this relation and associated concepts answer and/or comment on the following questions/statements, citing clear reasons in each case.

(i)   The equation suggests that there is a clear advantage of using microchannels in heat transfer. Is it true always?

(ii)   Does this equation in any way help in choosing a coolant liquid?

(iii)   Based on this equation and related concepts can you justify using microchannels for any specific type of mass transfer operations as well?

This is true but since the area covered by the microchannels is rather small, the total amountof heat transfer may be quite small even though the heat flux can be large.

A liquid with high latent heat, e.g., water, and high heat capacity will be a good choice. However, a high density liquid is generally not chosen because of the problems associated with the flow of a high density liquid in microchannels.

Microchannels are ideal for reactions that are highly exothermic since the characteristics time for a properly designed microchannel is quite small and hence heat dissipation would not be a problem.

11. Using the square-cube law, demonstrate <u>quantitatively</u> how the surface forces become important compared to gravitational forces in a micro-system.

In general, properties (p) that are a function of the area of interaction (A) decrease more slowly than properties that depend on the volume (V), as expressed by the *"square-cube" law.* $\dfrac{p_1(A)}{p_2(V)} \propto \dfrac{L^2}{L^3} \propto \dfrac{1}{L}$, L is the characteristic dimension of the microdevice. This has interesting application in microdevices. The characteristic dimension of a microdevice is of the order of 10^{-6} - 10^{-5} m, and hence the effect of surface force dominates over the gravity force, that depend on the volume.

12. The relevant relations for electrowetting are given below:

$$\Delta P = \sigma_{lv}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \sigma_{lv}\, K \qquad \cos\theta_Y = \frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}} \qquad \sigma_{sl}^{eff}(U) = \sigma_{sl} - \frac{\varepsilon_0 \varepsilon_d}{2d} U^2$$

Using the relations, wherever necessary, answer the following questions:

i)   Why is EWOD the more preferred method over EW?

ii)   Calculate the value of the applied voltage that would cause transition from partial to complete wetting for a droplet having the following properties: $\theta_Y = 110^o$, d = 5 μm,

$\varepsilon_d = 10$, and $\sigma_{lv} = 0.07$ N/m, $\varepsilon_o = 8.85 \times 10^{-12}$ F/m. Do you anticipate any problem with this calculation?

iii)   The claim for a specific surface coating is that it can make a surface close to hydrophobic. It was used on a specific substrate, completely covering it. However, due to certain problems in the coating and subsequent curing process, the coverage was microscopically uneven. The substrate was used continuously in a reactor and the process slowly made the surface more even. It was observed during the recalibration process, that the contact angle has reduced considerably compared to its initial value. Give possible reasons.

Even though the voltage required is much more to cause a change in contact angle in EWOD, it is still the preferred method as in EW electrolysis takes place whenever the voltage is above a few millivolts. On the other hand, due to the presence of the dielectric layer between the electrode and electrolyte large voltages (few hundred volts) can be used leading to substantial change in the contact angle.

The electrowetting equation is $\cos\theta = \cos\theta_Y + (\varepsilon_o\,\varepsilon_d)/(2\,d\,\sigma_{lv})\,U^2$

Putting the values of the constants given in the problem in the equation above, we can get for complete wetting ($\theta = 0$), the potential required is U = 103V.

It is not possible to increase the voltage to the extent so that a system can shift from non-wetting to complete wetting due to contact angle saturation phenomena where the contact angle can not be reduced by application of potential difference beyond a certain point.

This may be due to the slow shift of the surface characteristics from the Cassie state to the Wenzel state with the gradual erosion of the imperfections which act as pillars as in a Cassie surface. Thus the behavior of the liquid over an uneven surface will slowly turn towards Wenzel state with enhanced wetting.

13. Comment on the following statements/questions with precise, short reasons –

(i) In two capillary tubings of identical material but with different radii, a well wetting liquid moves from the wider to the more narrow side.

(ii) A lumped capacitance model is more practical during the cooling of a microdroplet (dimensions of one tenth of a mm and below) compared to that of a large drop (of sizes roughly equal to centimeters).

(iii) Identify the process described in the adjacent figure and comment whether it involves a top down or a bottom up approach?
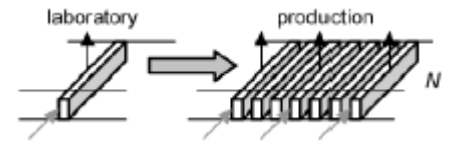
(iv) Since surface tension forces play a crucial role in the operation of micro heat pipes (MHP), it will be advisable to use high surface tension liquids in MHPs.
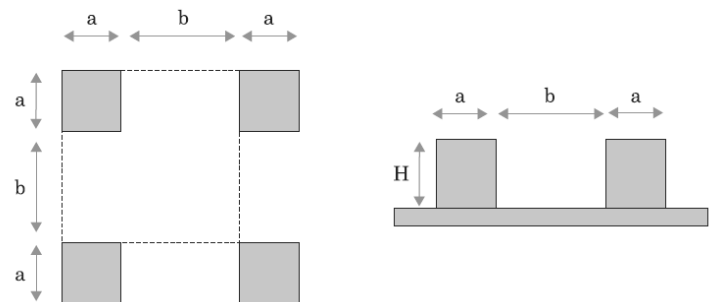
This is due to the higher curvature of the liquid meniscus in the narrow tube and consequent lower pressure in the liquid as predicted by Young-Laplace equation.

The possibility of Biot number ($=hL/k_s$) to be small such that lumped capacitance model is applicable will b more for a small sized droplet.

Numbering up process. Bottom up approach.

High surface tension liquids are less wetting in nature and hence should not be used in MHPs.

14. Transition from a Cassie droplet to a Wenzel droplet can be obtained by electrowetting actuation. It has been shown that a droplet sitting only on the top of the pillars in a Cassie regime, sinks down as soon as electrowetting actuation is turned on. Take the case of a lattice of square pillars along a square grid. Suppose the surface is constituted of square pillars of size a×a, height H and spacing b as shown in the Figures. Consider only one motif, as shown by the dashed line in the first figure.

Starting with the basic equations (e.g., definition of 'r', 'f', relevant equations for the different droplet states etc.), find the relation between the angle (i.e., the Young contact angle θ_i) and the geometric parameters, i.e., 'a', 'H' and 'B' for the shift from Cassie to Wenzel regime.

Use the relation to predict the effect of the following in terms of the behavior of the droplet:

i)   Reducing the height of the pillars

ii) When the pillars are thin

iii)   When the pillars are very close to one another

By considering one "motif" in Fig., the roughness r is determined by

Where,
$$r = \frac{S_{\text{total}}}{A = \left(\frac{a}{a+b}\right)^2} = \frac{(a+b)^2 + 4aH}{(a+b)^2} = 1 + \frac{4A}{(a/H)}$$

The Cassie coefficient f is given by

$$f = \frac{S_{\text{top}}}{S_{\text{horizontal}}} = \frac{a^2}{(a+b)^2} = A$$

Substituion yields leads to the Wenzel and Cassie formulas for square pillars in a square "motif"

$$\cos\theta_W = \left[1 + \frac{4A}{(a/H)}\right]\cos\theta$$

$$\cos\theta_C = -1 + A(1 + \cos\theta)$$

Using the above equations, the switch from Cassie to Wenzel state ($\theta_W = \theta_C$) can be obtained, after simplification as

$$\cos\theta_i = \frac{-1}{\left[1 + \frac{4aH}{b^2 + 2ab}\right]}$$

Cases

i)   Reducing the height of the pillars (H close to 0) would lead to r $\longrightarrow$ 1 and cosθ $\longrightarrow$ -1

Thus roughness induced by pillars favors the Wenzel state

ii)   When the pillars are very thin

A tends to 0 and this A tends to zero as well. Thus Wenzel state predominates

iii)   When the pillars are very close to each other – b tends to zero. Thus A and f tend to 1, which leads to (from Cassie law) $\theta = \theta_c$. Thus Cassie state is preferred.

15. The Bernoulli equation or mechanical energy equation can be written as

$$\frac{w_2^2}{2} + \frac{p_2}{\rho} + g\, y_2 = \frac{w_1^2}{2} + \frac{p_1}{\rho} + g\, y_1 + w_{t12} - \varphi_{12}$$

The last term on the right hand side denotes viscous dissipation. State with clear reasons whether this term be negligible for a microchannel flow.

The viscous dissipation term refers to the generation of heat due to friction, generally for high speed flows, whereas the flow inside a microchannels are rather slow flows. However, the viscous dissipation is a function of the square of the velocity gradient and since the velocity gradient can be quite large (even though the velocities are small), the term cannot be neglected in microchannel flows.

16. The linear relationship between the cause and effect type of laws (e.g. Newton's law) shows departure as the system size becomes progressively smaller. Name a few situations where the relationship is not valid and cite the changes that this would give rise to. Do you think that Soret and Dufour effects would be more pronounced for processes at microscale? Explain with proper reasons.

For high force gradients, the linear relations must be enlarged by terms of higher order – ncluding relaxation time of fast processes. Important for high energy density processes, such as laser processing or fast chemical reactions (explosions). Surface effects have to be considered and can be dominant opposed to volume effects such as gravity or momentum. Not only the main transport processes (mass, momentum, and energy) increase due to higher gradients, less predominant processes and coupled processes may gain importance. Transport coefficients may change as well.

Since the coefficients involved in Soret and Dufour effects are rather small, these effects are likely to be of less importance in macroscale systems. However, due to the large gradients generally encountered in microscale processes, these effects will start to become more pronounced.

17. The thermal energy equation can be expressed as

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{1}{Pr} \frac{\partial}{\partial y^*} \left[\lambda^* \frac{\partial T^*}{\partial y^*} \right] + \\ + \frac{1}{PrRe^2} \frac{\partial}{\partial z^*} \left[\lambda^* \frac{\partial T^*}{\partial z^*} \right] + \frac{Ec}{Re^2} \Phi^*$$

Where $Ec = w_r^2 / c_p \Delta T$. Explain the implications of applying this equation for a microchannel.

Explain how different flow regimes can be encountered for flows with constant Reynolds numbers but different Dean Numbers [$= Re\ (D/R_c)^{1/2}$]

The presence of Reynold's number in the denominator of the terms on the right hand side ensures that none of these terms can be neglected for flow in microscale systems' for which the values of the Reynold's number is of the order of 1. The last term in the energy equation refers to the dissipation term. It is appreciable only for high speed flows in macroscale systems. However, in microscale systems even though the flow velocity is small, the gradients are quite large and hence the dissipation term cannot be neglected.

With increase in Dean number for flow in a curved system, the stabilizing effects of viscous forces diminished leading to the creation of low and high pressure zones as well as secondary flows in and around a bend in the flow path. The effect becomes more pronounced at higher values of Dean number and Reynold's number. This can also lead to better mixing between different layers of liquids moving side by side along a microchannel.

18. Mention the names of polymeric materials that are used for molding and casting of microfluidic device.

Molding – PDMS; Casting – PMMA.

19. Define Knudsen number and explain its significance.

Knudsen number is the ratio of mean free path of the molecules to the characteristic dimension of the channel. When Knudsen number is small (< 0.1), the continuum solution for bulk flow can be extrapolated to the wall surface. In that case, the average velocity and the temperature of the gas on the wall is continuous and same as the velocity and the temperature of the wall.

20. If the tissue response leads to inflammation, will you consider the implant biocompatible?

If the inflammation, arising from a tissue response persists for a long time, and destroys local cells – the material will not be considered biocompatible. Otherwise, if the tissue response attracts giant cells for encapsulation that does not affect the normal functioning of the implant, the material will be considered biocompatible.

21. What is the importance of capillary force and Van der Waals force in fabrication of microfluidic devices?

Capillary force pulls the polymeric liquid through the network of capillaries that are formed in the space between the mold with relief features and the surface. After solidification of the polymer, the mold is removed to complete the patterning. Van der Waals force becomes important for thin polymer films of thickness < 100 nm. Because of this attractive force, the film breaks into islands, causing dewetting of substrate in places.

22. Define capillary number and discuss its significance.

Capillary number ($\mu u/\sigma$) represents the ratio of viscous force to interfacial force, and decides the importance of these forces in bubble/droplet formation and movement. Here μ is the viscosity of the liquid or the continuous phase, u is the velocity of the same phase, and σ is the interfacial tension.

23. Define the two regimes in droplet generation in T-joint.

The two regimes in droplet generation are shear regime and squeezing regime. When the capillary number is higher than a critical value, the droplet formation is determined by the balance between the shear-force of the carrier flow and the interfacial tension at the injection port. This is referred as shear regime. On the other hand, when the capillary number is smaller than the critical value, the dispersed phase completely fills the channel. Squeezing that results from the continuous injection of carrier fluid leads to the neck formation and droplet release. The latter regime is referred as squeezing regime.

24. As the feature size is reduced, which regime of droplet generation in a T-joint dominates?

As the feature size is reduced, squeezing regime of droplet generation in a T-joint dominates.

25. What type of electric field is required for Dielectrophoresis?

Non-uniform electric field is required here. Since the net charge on the particle is zero, the force on the particle arises from ∇E, which has to be non-zero.

26. How the electric field for Dielectrophoresis created?

Non-uniform electric field is created by the set of a planar electrode and a point electrode.

27. Is the physical dry etching isotropic? State the reason.

Physical dry etching is anisotropic. The reflected ions make an angle to the substrate, leading to the anisotropy.

28. Is the chemical dry etching isotropic? State the reason.

Chemical dry etching is isotropic. The diffusion of a chemical species occurs at same rate in all directions.

29. Is the wet etching of silicon isotropic? State the reason.

Wet etching of silicon is anisotropic. The crystalline structure of silicon has highest density of atoms in [1 1 1] plane, along which the etching rate is slowest.

30. Is the wet etching of glass isotropic? State the reason.

Wet etching of glass is isotropic. Glass is an amorphous material, and does not have any particular plane with higher atomic density.

31. Is Taylor dispersion beneficial to the chromatographic separation in microfluidic channel?

Taylor dispersion leads to the broadening of solute pulse at the outlet. This may mask other peaks.

32. Will the Taylor dispersion of a slug at the outlet of the channel increase due to the increase in length of the channel?

Increase in length implies longer residence time for the slug. Accordingly, the dispersion increases.

33. Will the Taylor dispersion increase due to increase in diameter of the channel?

Increase in diameter leads to increase in dispersion, provided the velocity is maintained same as it was for smaller diameter. $E_z \propto D^2$.

34. Will the Taylor dispersion increase due to increase in flow rate through the channel?

Increase in flow rate slides one layer more against the other, and increases the dispersion. $E_z \propto v^2$.

35. For computation of temperature profile in multichannel stack, what is the importance of single temperature field?

In a simpler model, thermal equilibrium between the fluid and the solid phases are assumed, so that a single temperature field can be used. In a more rigorous model, the temperatures of solid phase and the fluid are considered different at a point in the reactor, with heat exchange taking place between the fluid and the solid phases.

36. Describe a method by which a Kovar tube can be bonded to a silicon wafer.

Kovar tube can be fitted to the silicon wafer by placing glass beads around the junction and local heating so that the glass melts and flows around the joint. The sealing becomes complete upon solidification of glass.

37. What is the use of vacuum in spin-coating process?

Vacuum holds the wafer on the chuck.

38. What is the difference between anodic bonding and fusion bonding of two wafers?

Anodic bonding employs high voltage across the interface of the silicon and glass wafers in addition to the application of moderately elevated temperature. High voltage induces interpenetration of ions at the interface. Fusion bonding employs only the temperature at a highly elevated level.

39. What is the difference between the Guoy Chapman layer and the Stern layer?

In an electric double layer, the space immediately adjacent to the solid surface, where counter-ions are held firmly on the surface, and considered immobile is referred as Stern layer. The space, little away from the surface, where the counter-ions are still in higher concentration than co-ions, and the counter-ions are mobile due to diffused nature of holding by the surface is referred as Guoy Chapman layer.

40. What is the utility of microinjection molding in making plastic microfluidic devices?

Microinjection molding is used for serial production. This is ideally suited for industrial manufacturing in large scale.

41. What is the difference between mixing quality and mixing effectiveness?

Mixing quality refers to the deviation or variance of the local concentration from the average or fully mixed concentration, summed over the entire cross-section. Mixing quality signifies the extent of mixing in 0 to 1 scale defining the non-mixed state to fully mixed state in the co-flowing streams. Mixing effectiveness refers to the revenue, earned in mixing from a given effort of pressure loss. A passive mixer will have higher mixing effectiveness, if the mixer achieves higher extent of mixing at lower energy consumption or lower pressure loss.

42. A circular conduit of diameter D and length L is used to transport two reagents one immediately after the other using fully developed laminar flow with average velocity of V. Assuming the two reagents are miscible with diffusion coefficient of D, obtain the fraction of length of the conduit that will contain the mixed reagents at the end of the tube. How this fraction is affected if a) the length is doubled, b) the diameter is doubled, c) the average velocity is doubled, and d) the diffusivity is doubled?

$\frac{\partial C}{\partial t} = E_z \frac{\partial^2 C}{\partial z^2}$; Here C is the concentration, averaged over the entire cross-section. The solution of the above equation is $\frac{C-C_0}{C_\infty-C_0} = \text{erf}\left\{\frac{z}{\sqrt{4E_z t}}\right\}$

The concentration change is considered significant when $C=C_\infty$, or the left hand side of the above equation attains a value of 1.0.

For $x \geq 1$, $\text{erf}(x) \approx 1.0$

This implies $\frac{z}{\sqrt{4E_z t}} = 1$, where $E_z = \frac{r^2 v^2}{48D}$; That is $\frac{z}{L} = 2r\sqrt{\frac{v}{48LD}}$, which represents the fraction of length of the conduit that will contain the mixed reagents at the end of the tube.

If the length is doubled, $\frac{z}{L}$ is multiplied by a factor of $\frac{1}{\sqrt{2}}$. If the diameter is doubled and the velocity is retained same, the $\frac{z}{L}$ is also doubled. If average velocity is doubled, $\frac{z}{L}$ is multiplied by $\sqrt{2}$.

43. Draw the electric potential $\phi(z)$ and the concentration of ions and counter-ions in the Debye layer. Under what assumption the Poisson-Boltzmann equation of the following form is valid?

$$\partial_z^2 \phi(z) = \frac{\phi(z)}{\lambda_D^2}$$

What is the name of this assumption?

The electric potential ϕ and concentration of co-ions C_+ and counter-ions C_- are plotted as follows

Debye-Hü z approximation is required. This approxima z s valid when $Ze\xi \ll k_B T$ such that $sinh\left\{\frac{Ze}{k_B T}\phi(z)\right\}$ can be approximated as $\frac{Ze}{k_B T}\phi(z)$. Here Ze is the charge on the ion, k_B is Boltzmann constant, T is the temperature in absolute scale, and ξ refers to the ξ-potential.

44. Write down the first-order surface-slip condition and draw the velocity profile for flow between two infinite parallel plates with top wall moving at constant velocity U, the lower surface stationary, and separation between two plates as h. In the drawing, compare the velocity profile with the same for no-slip boundary condition.

First order surface slip condition is given by

$$u = U_w + \frac{2-\sigma_v}{\sigma_v}\lambda\frac{\partial u}{\partial y}$$
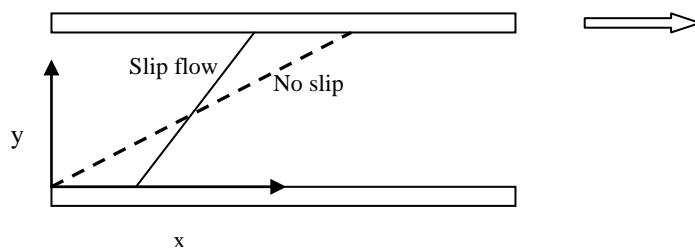
45. A polymeric formulation when deposited at the center of the disk, spinning at 1000 rpm provided an equilibrium thickness of 20 nm. What thickness would you expect if the disk had rotated at 10,000 rpm.

The equilibrium thickness of the coating $\propto \dfrac{1}{\omega^{\frac{2}{3}}}$

The new thickness $= 20(1000/10000)^{2/3}$ nm $= 4.3$ nm.

46. Consider a particle with radius 96 nm and zeta potential = -5 mV in aqueous solution with 10^{-3} molar KCl solution at room temperature. Assuming Debye-Huckel approximation to be valid, calculate the Debye layer thickness. How does the thickness compare with the radius of the particle? What would be the velocity of the particle under an electric field of 100 Volt/cm?

$$\partial_z^2 \phi(z) = 2 \frac{(z_e)^2 C_0}{\varepsilon k_B T} \phi(z)$$

$$\lambda_D = \sqrt{\frac{\varepsilon k_B T}{2(Ze)^2 C_0}} .$$

Here, $\varepsilon = 80 \times 8.85E\text{-}12$ coulomb^2/(N-m^2)

   $k_B = 1.38E\text{-}23$ J/K

   $e = 1.6E\text{-}19$ coulomb

   T=298 K

   Z=1

   $C_0 = 10^{-3}$ moles/litre = 1 mole/m^3

   $\xi = $-0.005 volt

   $\mu = 10^{-3}$ Pa-s for water

   E=ΔV/L=100volt/cm

   This implies $\lambda_D \approx 9.6\ nm \ll 100\ nm$, that is the particle diameter can be considered large compared to the Debye thickness. Therefore

$$u_{ep} = \frac{\varepsilon \xi E}{\mu} = 35.40\ \mu m/s$$

47. Do you expect broadening of analyte slug when the motive force is electro-osmotic (with thin EDL approximation)? If you expect the broadening, what would be the mechanism and how is it different from Taylor dispersion?

Broadening is expected due to diffusion from the slab. No Taylor dispersion is possible.

48. A horizontal capillary of internal diameter d is to be filled by a liquid of viscosity µ, and density ρ solely by capillary force. Do you expect the velocity of the liquid front in the tube to increase or decrease with time / progression? Write down the momentum conservation equation in terms of a balance between the surface tension force and wall viscous force. Assume the surface tension and contact angle as σ and θ respectively. Obtain the distance travelled by the liquid meniscus, and the fluid velocity (averaged over the cross-section) – both as function of time, and σ, θ, ρ, µ, d.

Velocity of the liquid front will decrease with time. The volumetric flow rate

$\frac{dV}{dt} = \frac{\pi r^4 \Delta P}{8\mu z}$, where $\Delta P = \frac{2\sigma Cos\theta}{r}$, and $\frac{dv}{dt} = \pi r^2 \frac{dz}{dt}$

This implies that $z \propto \sqrt{t}$, and velocity decreases with time.

49. Discuss the significance of Weber number.

Weber number represents the ratio of inertial stress to interfacial stress

$$Weber\ Number = \frac{\rho u^2 L}{\sigma}$$

A large Weber number indicates elongated jet that breaks up into droplets far away from the nozzle. A small Weber number indicates that the interfacial tension dominates. Instead of pronounced jet, the dripping of droplets near the nozzle will occur in the second case.

50. Discuss the significance of Bond number.

$Bond\ Number = \frac{\rho L^2 g}{\sigma}$ compares the gravitational force on the fluid with the interfacial force. A large Bond number implies that the gravity dominates over interfacial forces. This implies a flat puddle of liquid. On the other hand, a small Bond number implies that the interfacial force dominates over gravity. This results in a spherical shape of the fluid element.

51. Draw a rough sketch of the velocity profile for electro osmotic flow through a tube against a back pressure.

Velocity profile arising from the electro osmotic flow is flat in most of the cross-section, except near the wall. The back pressure creates a parabolic dent to the velocity profile.

Back Pressure