

## Advanced Mathematical Techniques in Chemical Engineering

### Module I : Introduction of vector space

#### Exercises

1. There are two functions  $f(x,y)=2xy-3y^2+2y+4$  and  $g(x,y)=3x^2-5xy+2y^2-2$  in the range of  $0 < x, y < 1$ . Check whether  $f$  and  $g$  are orthogonal to each other .\

2. There are two functions  $f(x,y,z)=2xyz-3y+2z$  and  $g(x,y,z)=3xyz$  in the range of  $0 < x, y, z < 1$ . Check whether  $f$  and  $g$  are orthogonal to each other.

3. For a continuous function,  $f(x,y)=ax-y$  where,  $0 \leq x, y \leq 1$ , evaluate the value of  $a$  such that  $f$  is orthonormal?

4. Consider two continuous functions  $f=(x+y)$  and  $g=xy$  such that  $0 \leq x \leq 1$  and  $1 \leq y \leq 2$ . If the following equation is satisfied,

$$d^2(f, g) + 2\langle f, g \rangle + 3\|f\|^2 - k\|g\|^2 = 0$$

Find the value of  $k$  ?

5. Consider the vectors  $X=[1 \ 2 \ 2]^T$  and  $Y=[-1 \ b \ 3]^T$ . What is the value of  $b$  such that  $X$  and  $Y$  are orthogonal.

6. Consider two vectors,  $A=(1 \ 2 \ 3)^T$  and  $B=(1 \ 0 \ -1)^T$

Find (i) metric between  $A$  and  $B$ ; (ii) Norm of  $A$  and norm of  $B$ ; (iii) Inner product of  $A$  and  $B$  .