Advanced Mathematical techniques in Chemical Engineering

Module XI : Solution of non-homogeneous PDEs by Green's function

Exercises

1. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + x$

Solve the above parabolic non-homogeneous equation subject to the following conditions:

At t=0, u=1; at x=0, u=1 and at x=1, $\frac{\partial u}{\partial x} = 2$

2. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2x^2$

Solve the above parabolic non-homogeneous equation subject to the following conditions:

At t=0, u=1; at x=0, u=1 and at x=1, $\frac{\partial u}{\partial x} + 2u = 2$

3. Consider the PDE $\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) + r^2$

Solve the above parabolic non-homogeneous equation subject to the following conditions:

At t=0, u=1; at r=0, u=finite and at r=1, $\frac{\partial u}{\partial x} + 2u = 2$

4. Consider the PDE $\frac{\partial u}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r} \right) + r^2$

Solve the above parabolic non-homogeneous equation subject to the following conditions:

At t=0, u=1; at r=0, u=finite and at r=1, $\frac{\partial u}{\partial x} = 2$

5. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 3$

Solve the above parabolic non-homogeneous equation subject to the following conditions:

At t=0, u=1; at x=0, u=1 and at x=1, $u = 2$