Advanced Mathematical techniques in Chemical Engineering

Module XV : Solution of PDEs by Fourier transformation

Exercises

1. Find the complete solution in (x,t) of the transient heat conduction problem, defined as,

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

in a semi-infinite domain of x. The initial condition is u=0 at t=0. The system is supplied with a constant heat flux at x=0, i.e., at this boundary, $\frac{\partial u}{\partial x}=u_0$ and at x= ∞, u=0. It can also be safely assumed that $\frac{\partial u}{\partial x}=0$ at x= ∞.

2. Solve the following equation using Fourier transform

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$. At t=0, u=sinx and u=0= $\frac{\partial u}{\partial x}$ at $x=\pm\infty$

3. Solve the following equation using Fourier transform

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$. At t=0, u=0 and at x=0, u=2; at $x=\infty$, $\frac{\partial u}{\partial x}=0$

4. Solve the following equation using Fourier transform

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$. At t=0, u=0 and at x=0, $\frac{\partial u}{\partial x}$ =2; at $x=\infty$, $\frac{\partial u}{\partial x}=0$

5. Solve the following equation using Fourier transform

$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$. At t=0, u=1 and at x=0, u=2; at $x=\infty$, $\frac{\partial u}{\partial x}=0$