

Advanced Mathematical techniques in Chemical Engineering

Module VIII : Partial Differential Equations

Exercises

1. State whether following equations are homogeneous or non-homogeneous.

$$(i) u \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$(ii) u \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}$$

$$(iii) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \sin x$$

$$(iv) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \sin y$$

$$(v) \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \exp(-3x) = 0$$

2. State whether following equations are linear or non-linear.

$$(i) u \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$(ii) \left(\frac{\partial u}{\partial t} \right)^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}$$

$$(iii) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \sin x$$

$$(iv) \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u \frac{\partial^2 u}{\partial x \partial y} + 5 \sin y$$

$$(v) \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \exp(-3x)u = 0$$

3. Determine whether the following equations are parabolic, elliptic or hyperbolic

(i) $u \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

(ii) $\left(\frac{\partial u}{\partial t}\right)^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y}$

(iii) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \sin x$

(iv) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + 5 \exp(-3x)u = 0$

(v) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial x^2} + 5 \exp(-3x)u = 0$

4. Give an example of the following boundary conditions

(i) Homogeneous Dirichlet BC

(ii) Non-homogeneous Dirichlet BC

(iii) Homogeneous Neumann BC

(iv) Non-homogeneous Neumann BC

(v) Homogeneous Robin mixed BC

(vi) Non-homogeneous Robin mixed BC

5. Consider the problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$. The initial condition is at $t=0$, $u=u_{01}$; boundary conditions

are at $x=0$, $u=u_{02}$ and at $x=1$, $u=u_{03}$. Breakdown this problem into well-posed sub-problems

using the principle of linear supersposition.