Advanced Mathematical techniques in Chemical Engineering

Module IX : Special ODEs and Adjoint operators

Exercises

1. Prove that Zeroth order Bessel functions are orthogonal functions.

2. Prove that two dimensional Laplacian operator $L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ with the boundary operator Bu=0 (where, u is a dummy dependent variable) is a self adjoint operator.

3. Prove that two dimensional Laplacian operator $L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ with the boundary conditions at x=0, $\frac{\partial u}{\partial x} = u_{01}$; at x=1, $u=u_{02}$; at y=0, $u=u_{03}$; at y=1, $u=u_{04}$ is a self adjoint operator, where, u is a dummy dependent variable.

4. Prove that two dimensional Laplacian operator $L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}$ with the boundary conditions at x=0, $\frac{\partial u}{\partial x} = u_{01}$ and at x=1, $\frac{\partial u}{\partial x} + \beta u = u_{02}$ at y=0, $u=u_{03}$; at y=1, $u=u_{04}$ is a self adjoint operator, where, u is a dummy dependent variable.

5. Prove that two dimensional operator $L = \frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}$ with the boundary conditions at x=0, $u=u_{01}$ and at x=1, $\frac{\partial u}{\partial x} + \beta u = u_{02}$; at y=0, $u=u_{03}$; at y=1, $u=u_{04}$ is not a self adjoint operator, where, u is a dummy dependent variable.