Computational Techniques
Module 2: Computation & Error Analysis

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Definition

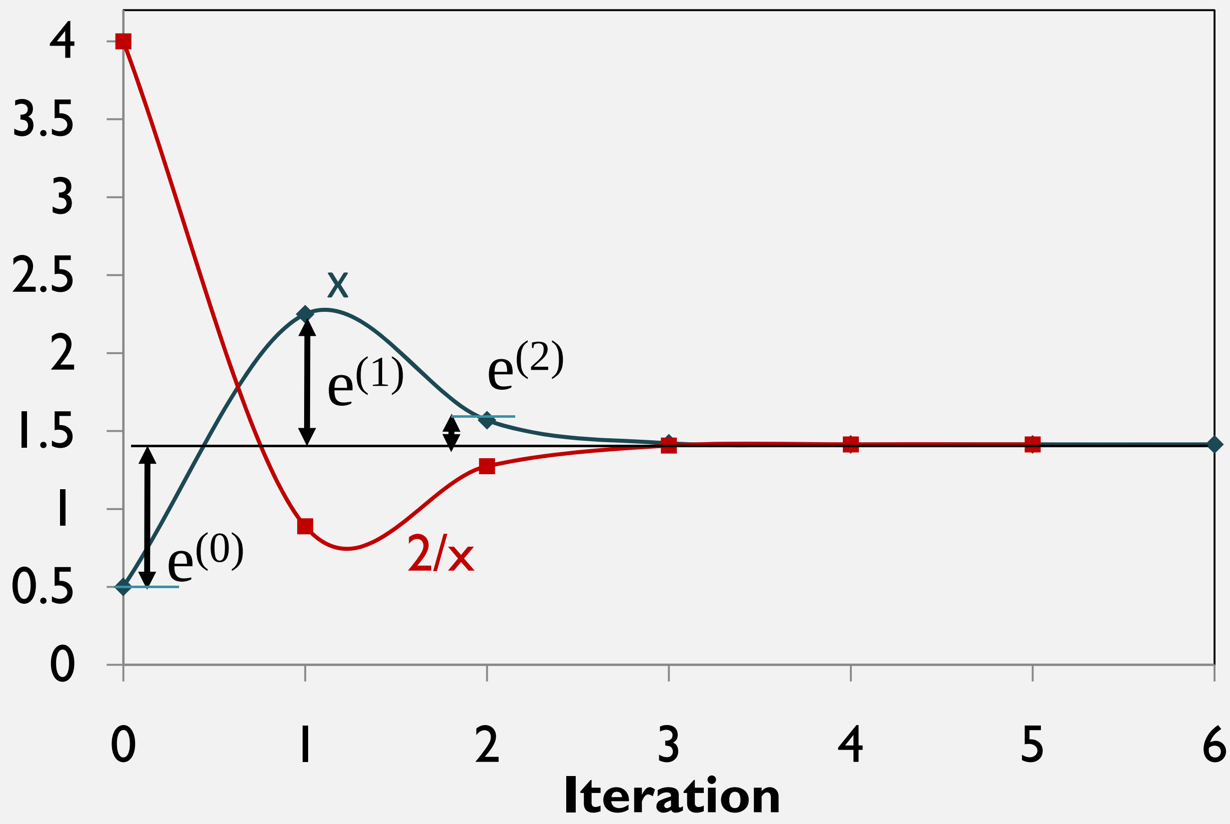
- Numerical methods ➔ Approximate solution
- Error:

  True Solution – Approximate *numerical* solution

- Absolute error: $e = \left| x^* - x^{(a)} \right|$
- Relative error: $\varepsilon = \left| \dfrac{x^* - x^{(a)}}{x^*} \right|$

Square Root Example

$$x^{(i+1)} = \frac{1}{2}\left(x^{(i)} + \frac{2}{x^{(i)}}\right)$$



Square Root Example $x^{(i+1)} = \frac{1}{2}\left(x^{(i)} + \frac{2}{x^{(i)}} \right)$

iteration	x	2/x	Error
0	**0.5**	4	0.914214
1	2.25	0.888889	0.835786
2	1.569444	1.274336	0.155231
3	1.42189	1.406578	0.007677
4	1.414234	1.414193	2.07E-05
5	1.414214	1.414214	1.52E-10
6	**1.414214**		2.22E-16

What if we do not know x^*?

$$e_a^{(i)} = \left| x^{(i)} - x^{(i-1)} \right|$$

$$\varepsilon_a^{(i)} = \left| \frac{x^{(i)} - x^{(i-1)}}{x^{(i-1)}} \right|$$

We use "approximation error" as the difference between the previous and the new values of the variables computed

Causes of Errors

- Precision, Accuracy and Significant Digits
- Truncation Error vs. Round-off Error
  - Taylor's Series Expansion and approximation
  - "Finite-precision"
  - Binary number system
- Example: Infinite series expansion of e^x