Problem Sheet 02

Problem 1: Round-off Errors

The roots of the equation $ax^2 + bx + c = 0$ can be computed in the following two ways:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ or } x_1, x_2 = \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}.$$

Use five-digit arithmetic with chopping (i.e., the sixth digit is discarded) to compute the roots of the equation $x^2 - 5000.002x + 10 = 0$ from the two formulae given above.

Problem 2: Taylor's Series for Finite and Infinite Series

Consider the following functions. It is known that $f(1) = 0$ for both these cases. Use zero to fourth order Taylor's series expansion to obtain $f(3)$. The two functions are:

$$f(x) = 18x^4 + 25x^3 - 6x^2 + 7x - 44 \text{ and } f(x) = \ln(x).$$

Comment on the relative error of the two cases, as more terms are included from the Taylor's series expansion.

Problem 3: Taylor's Series and Effect of Step Size

- Using the Taylor's series expansion:

  $$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \cdots + \frac{h^n}{2!} f^{n}(a) + \cdots$$

  Obtain the infinite series expansion of e^x and $\cos(x)$. [Assume $f(0)$ is known and $h = x$]

- For each of these functions, calculate the function values at $x = \pi/4$, each time increasing the number of terms used by 1 until the relative error reduces to 10^{-5} (i.e., report the relative error when 1, 2, 3, … terms are included from the infinite series). Compute the true relative error at each stage. Observe that the truncation error reduces as the number of terms increase. How many terms are required to reduce the relative error to 10^{-5}?

- Alternatively, we can truncate the infinite series at the first order term $hf'(x)$. Compute the value of $\cos(\pi/4)$ using this first-order approximation, using $h = \pi/4, \pi/8, \pi/16, \ldots, \pi/64$. Plot error vs. step size (h) on a log-log plot and comment about its slope.