Tutorial - 2 Deflection Methods.

19.9.11

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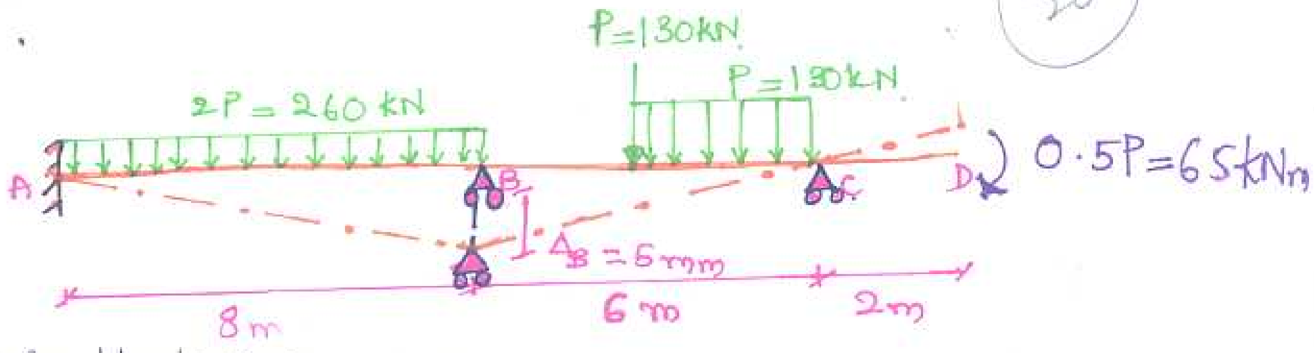
CE09B030

$P = 100 + 30$

$P = 130$

20/20

1.



Single Unknown : θ_B

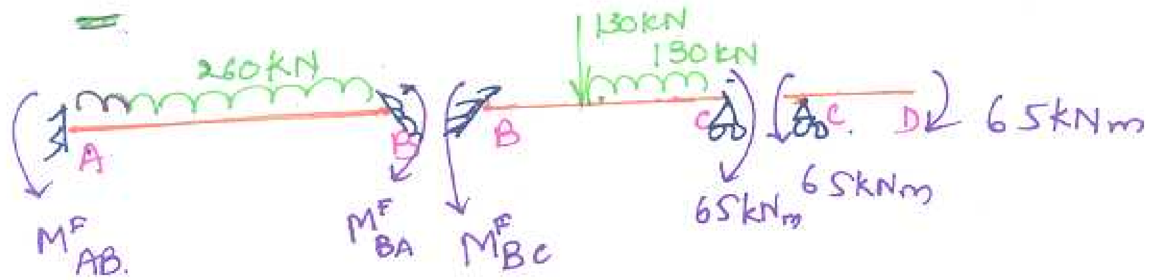
Fixed end moments:

Element AB

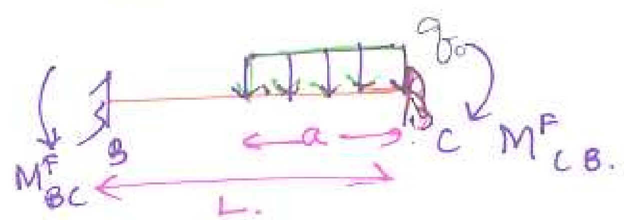
$$M^F_{AB} = -\frac{260 \times 8}{12} = -173.33 \text{ kNm}$$

$$M^F_{BA} = +\frac{260 \times 8}{12} = +173.33 \text{ kNm}$$

$$M^F_{BC} = M^F_{\text{due to conc. load}} + M^F_{\text{due to udl}} + M^F_{\text{due to end moment.}}$$



For M^F_{BC} :



$$M^F_{BC} = -\frac{q_0 a^2}{12L^2}(4L - 3a)$$

$$M^F_{CB} = +\frac{q_0 a^2}{12L^2}\left[2L(3L-4a) + 3a^2\right]$$

∴ for $M^{F_0}_{BC} = M^{F}_{BC} - \frac{1}{2} M^{F}_{CB}$

$M^{F_0}_{BC} = M^{F}_{BC} - \frac{1}{2} M^{F}_{CB}$

$$= -\frac{q_0 a^3}{12L^2}(4L - 3a)$$

$$- \frac{q_0 a^2}{24L^2}\left[2L(3L - 4a) + 3a^2\right]$$

Here $L = 6\,m$, $a = 3\,m$, $q_0 a = 130\,kN$

$M^{F_0}_{BC}$ due to UDL $= -\frac{130 \times 3}{12 \times 6^2}\left[a(4L - 3a) + \frac{1}{2} \times \left\{2L(3L - 4a) + 3a^2\right\}\right]$

$$= \frac{-130 \times 3}{\cancel{12}_{4} \times 6^2} \times \left[3(15) + \frac{1}{2} \times \left\{(12 \times 6) + 27\right\}\right]$$

$$= -\frac{130}{4 \times 36} \times \left(45 + 99/2\right)$$

$$= -130 \times 0.65625$$

$\boxed{M \text{ due to udl} = -85.3125\,kNm}$

∴ $M^{F}_{BC} = \left(-\frac{130 \times 6}{8} \times \frac{3}{2} - 85.3125 + \frac{65}{2}\right) kNm$

$\boxed{M^{F}_{BC} = -199.0625\,kNm}$

Slope Deflection Equations :-

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} M^F_{AB} \\ M^F_{BA} \\ M^{Fo}_{BC} \end{Bmatrix} + \begin{Bmatrix} \frac{2\times2}{L_{AB}} \\ \frac{4\times2}{L_{AB}} \\ \frac{3\times1}{L_{BC}} \end{Bmatrix} EI\theta_B + \begin{Bmatrix} -\frac{6\times2}{L_{AB}^2} \\ -\frac{6\times2}{L_{AB}^2} \\ + \cancel{-}\frac{3\times1}{L_{BC}^2}(-1) \end{Bmatrix} EI\Delta_B$$

$$\left[\because \quad \phi_{AB} = +\frac{\Delta_B}{L_{AB}} \; ; \quad \phi_{BC} = -\frac{\Delta_B}{L_{BC}} \right]$$

$$\therefore \begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -173.33 \\ 173.33 \\ -199.0625 \end{Bmatrix} + \begin{Bmatrix} 1/2 \\ 1 \\ 1/2 \end{Bmatrix} EI\theta_B + \begin{Bmatrix} -3/16 \\ -3/16 \\ 1/12 \end{Bmatrix} \underbrace{EI \times \frac{5}{1000}}_{400}$$

kNm.

Equilibrium :

$$M_B = M_{BA} + M_{BC} = 0$$

$$\Rightarrow EI\theta_B\left(1+\frac{1}{2}\right) - 25.7325 + \left(\frac{1}{12} - \frac{3}{16}\right) \times 400 = 0$$

$$1.5\, EI\theta_B - 25.7325 - 41.67 = 0$$

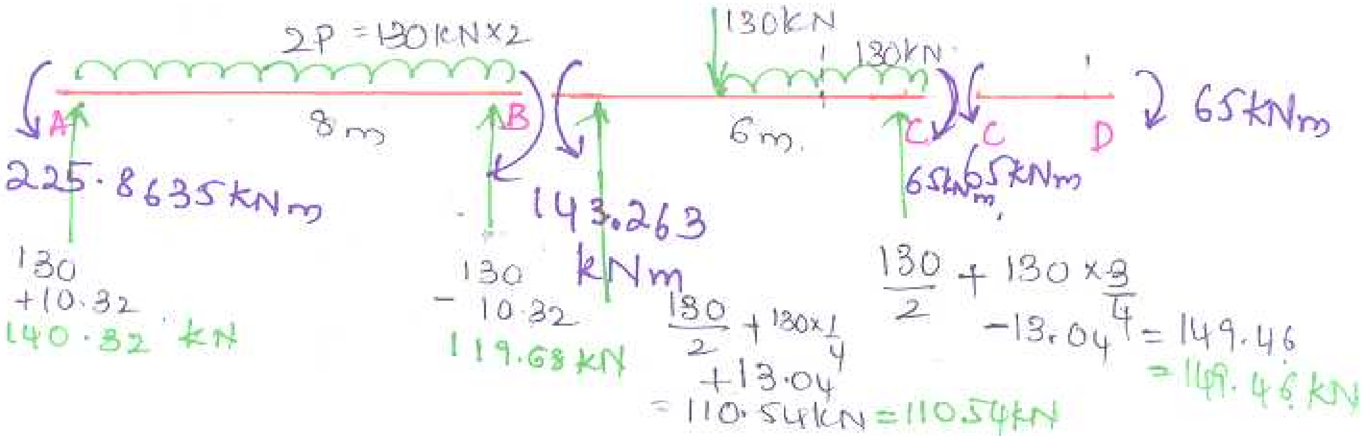
$$EI\theta_B = \frac{67.399}{1.5} = 44.933 \text{ kNm}^2$$

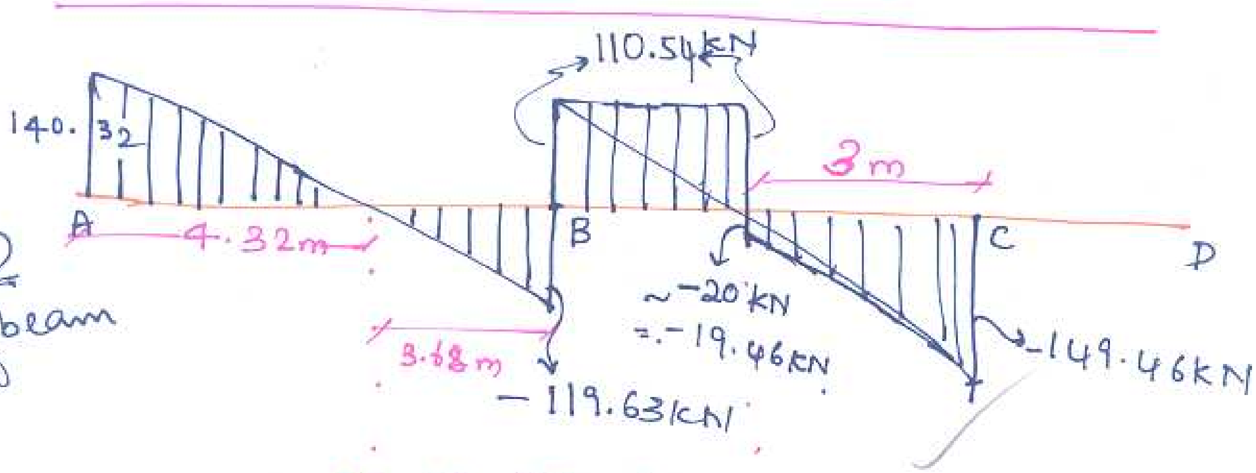
$$\boxed{EI\theta_B = 44.933 \text{ kNm}^2}$$

or $\boxed{\theta_B = 5.617 \times 10^{-4} \text{ rad}}$

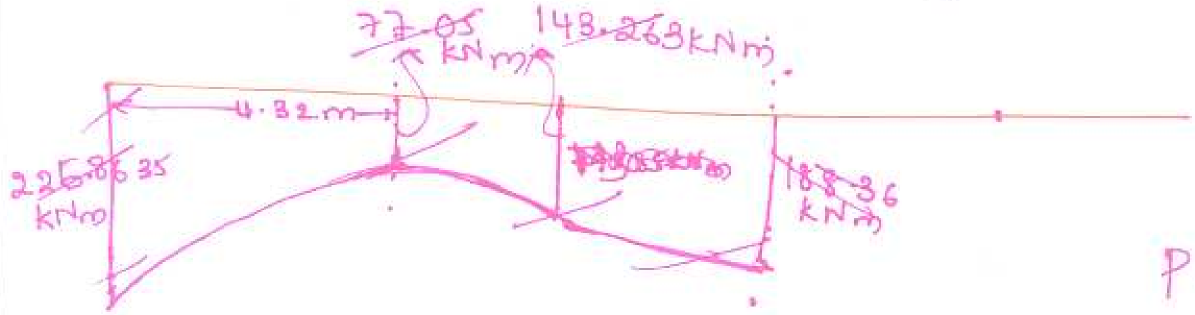
$$\therefore \begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -173.33 \\ 173.33 \\ -199.0625 \end{Bmatrix} + \begin{Bmatrix} 1/2 \\ 1 \\ 1/2 \end{Bmatrix} 44.933 + \begin{Bmatrix} -3/16 \\ -3/16 \\ 1/12 \end{Bmatrix} 400$$

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \end{Bmatrix} = \begin{Bmatrix} -225.8635 \\ 143.263 \\ -143.263 \end{Bmatrix} \text{ kNm}$$

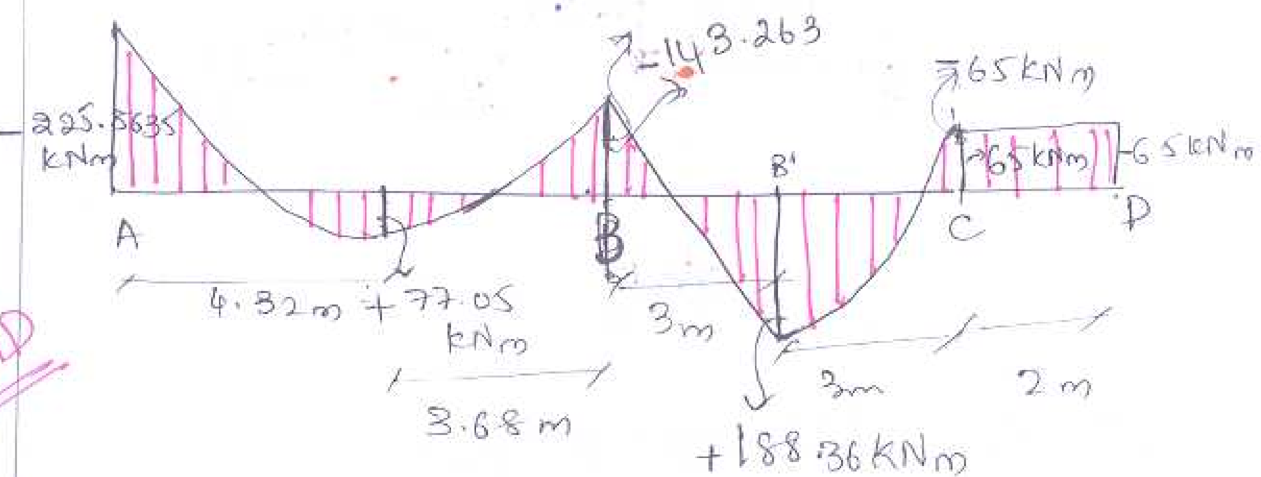




SFD of beam



P.T.O.



BMD

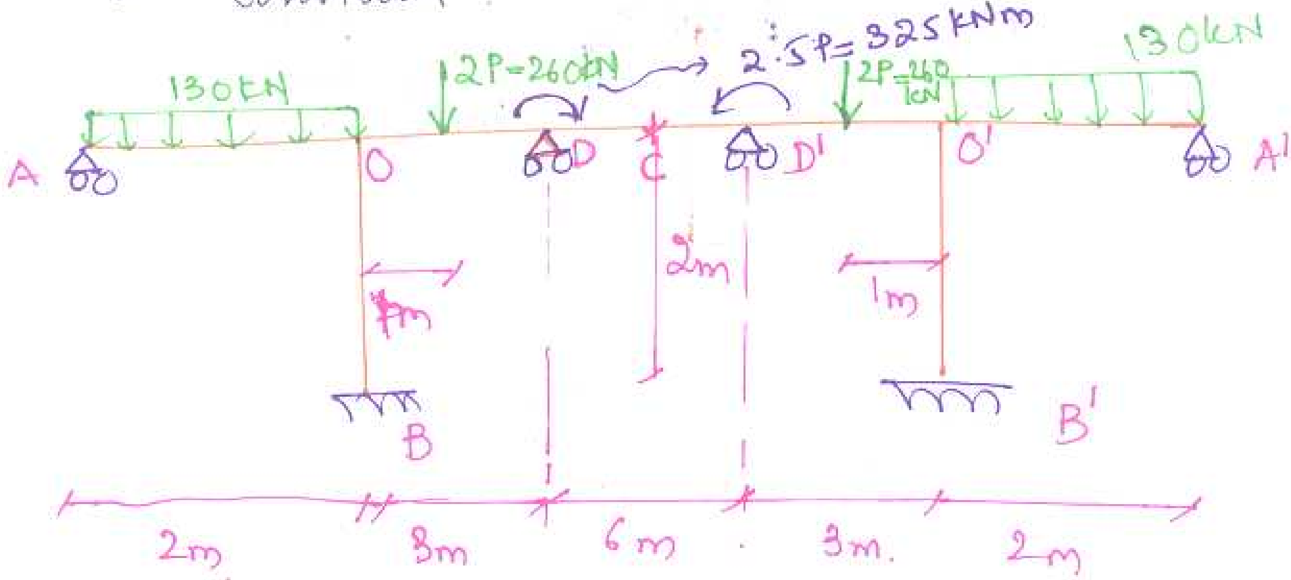
A to B: Quadratic

B to B': Linear

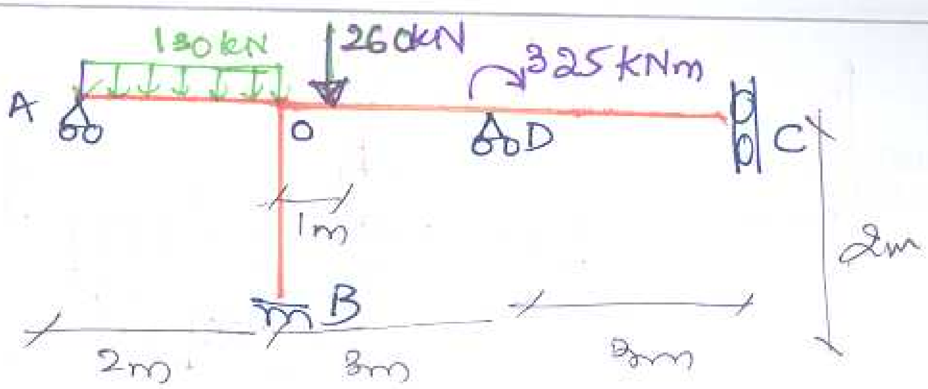
B' to C: Quadratic

C to: Constant

(2)

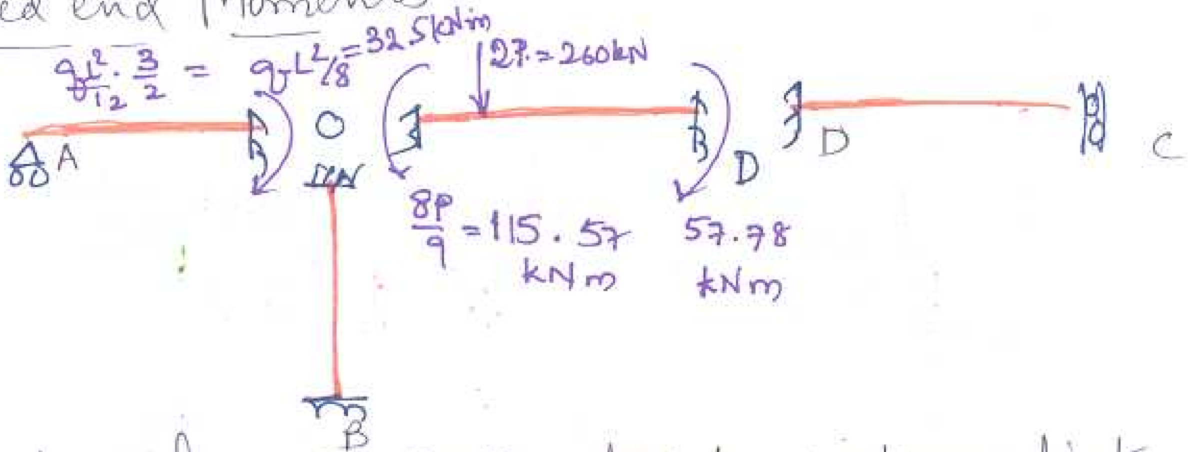


As the structure is symmetric, we can analyse half of it to get all the results. [Note: The loading is also symmetric.]



As there will be a point of contraflexure at C, we take a guided roller at C.

Fixed end Moments:



i.e. Fixed end moments due to intermediate loads only.

Rotational Stiffnesses:

$k_{OB} = \frac{4EI}{2} = 2EI$

$k_{OA} = \frac{3EI}{2} = 1.5EI$

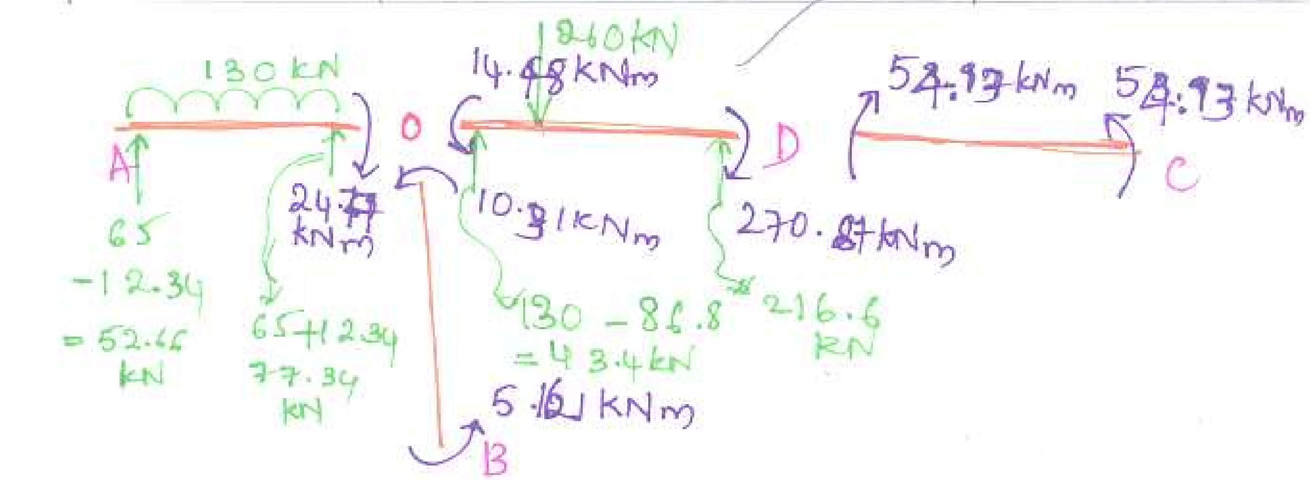
$k_{OD} = \frac{4}{3}EI = 1.33EI$

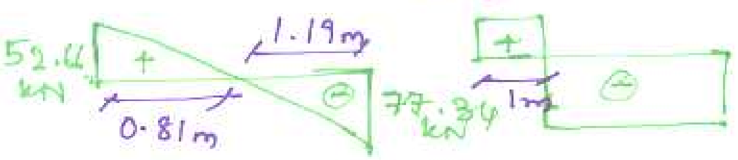
$k_{DO} = \frac{4}{3}EI = 1.33EI$

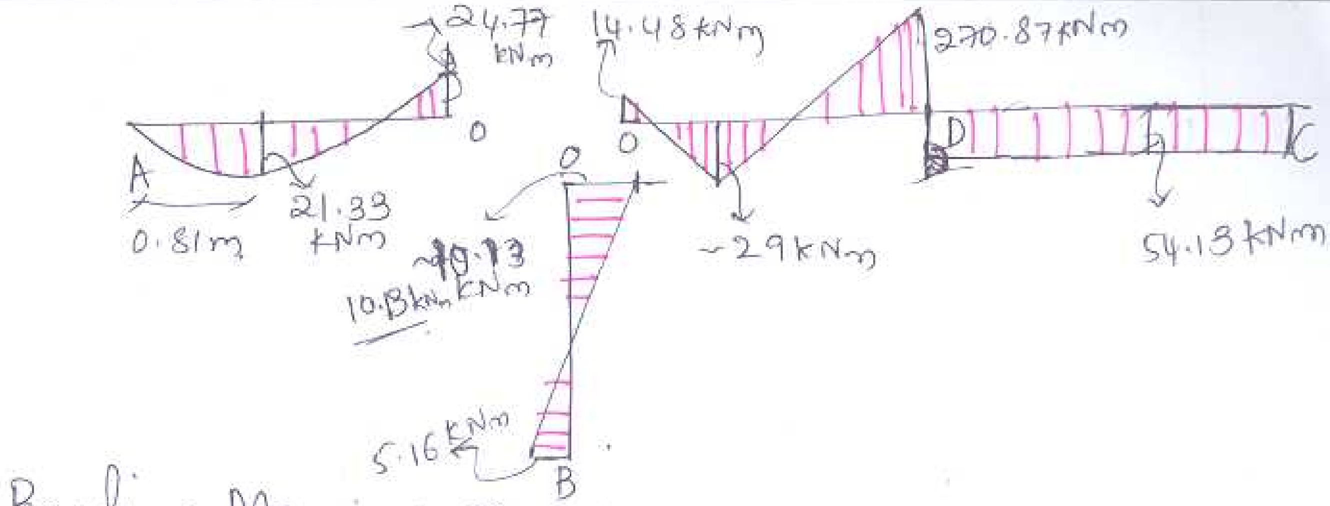
$k_{DC} = \frac{EI}{3} = 0.33EI$

Assuming that there is a rigid beam DA on which the moment is applied.

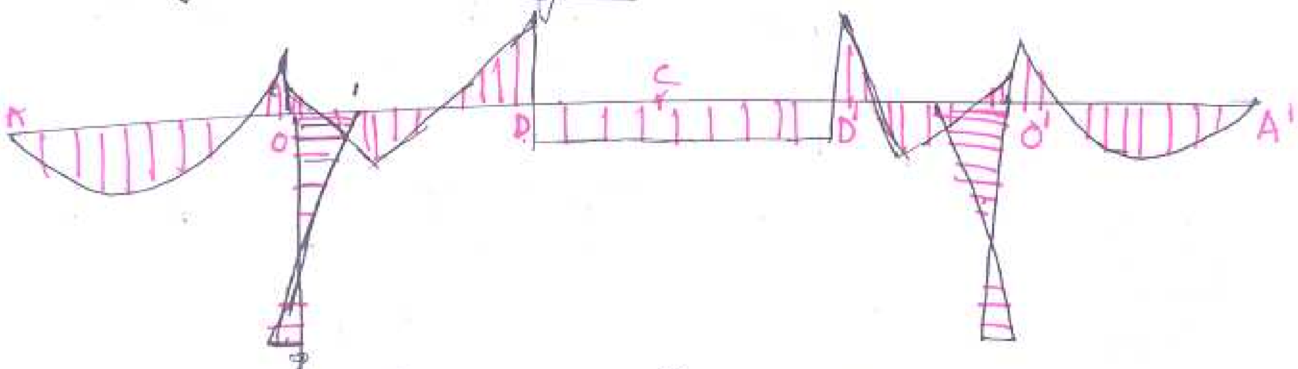
	BO	OB	OA	OD	DO	DL	DC	CD
D.F.		0.4138	0.3103	0.2759	0.8	0	0.2	
CoF.	$\frac{1}{2}$			$\frac{1}{2}$	$\frac{1}{2}$			-1
FEM (kNm)	0	0	32.5	−115.57	57.78	−325	0	0
Bal1 kNm		34.37	25.77	22.92	213.78		53.44	
CoM$_1$	17.18			106.89	11.46			−53.44
Bal2 kNm		−44.23	−33.17	−29.49	−9.17		−2.29	
CoM$_2$	−22.12			−4.58	−14.75			2.29
Bal3 kNm		1.89	1.42	1.26	11.80		2.95	
CoM$_3$	0.95			5.9	0.63			−2.95
Bal4 kNm		−2.44	−1.83	−1.63	−0.50		−0.13	
CoM$_4$	−1.22			−0.25	−0.82			0.13
Bal5	0.051	0.103	0.078	0.069	0.66		0.16	−0.16
kNm Final Moments	−5.21	−10.41	24.69	−14.55	270.21		53.97	−53.97
	−5.16	−10.31	= 24.77	= −14.48	+270.87		54.13	−54.13



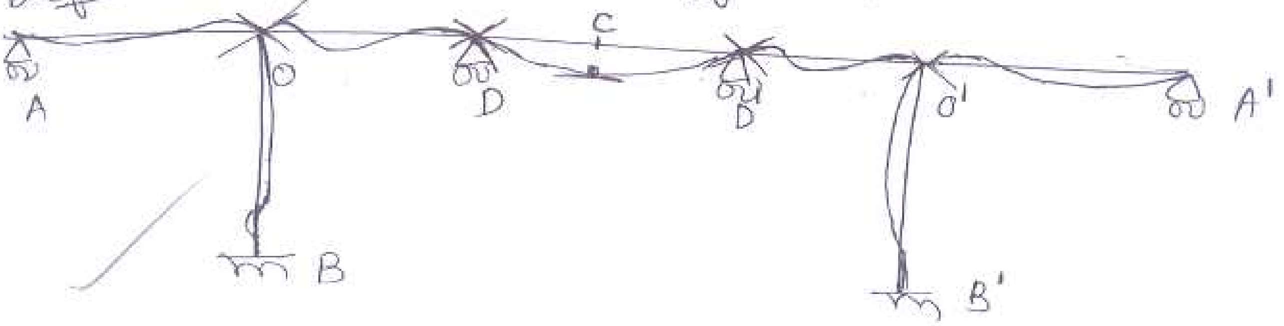




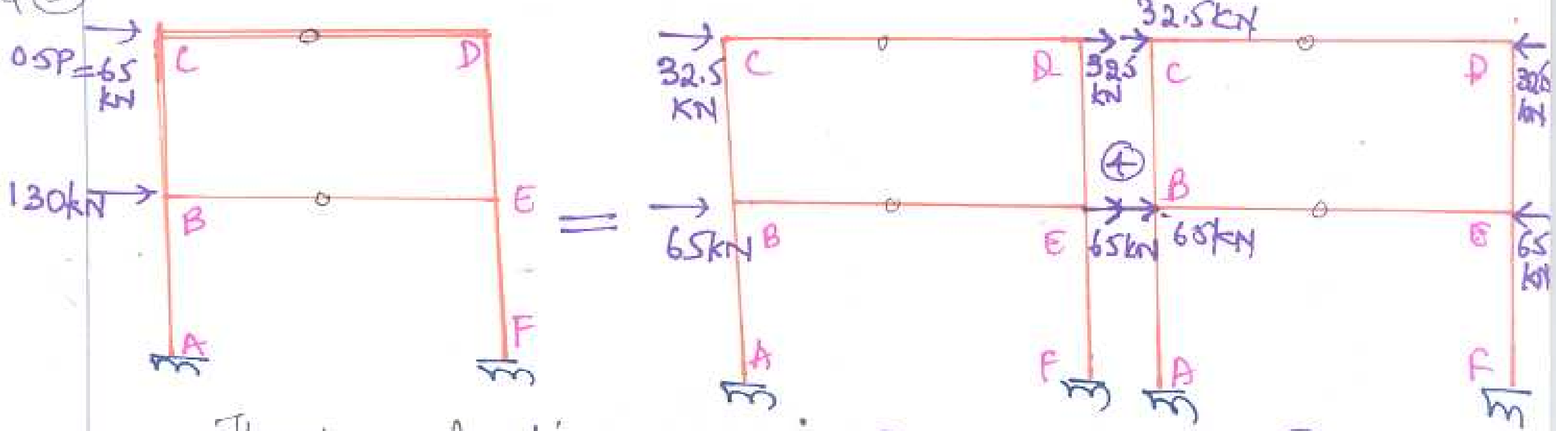
Bending Moment Diagram.



Deflected shape (Probable - Exaggerated)



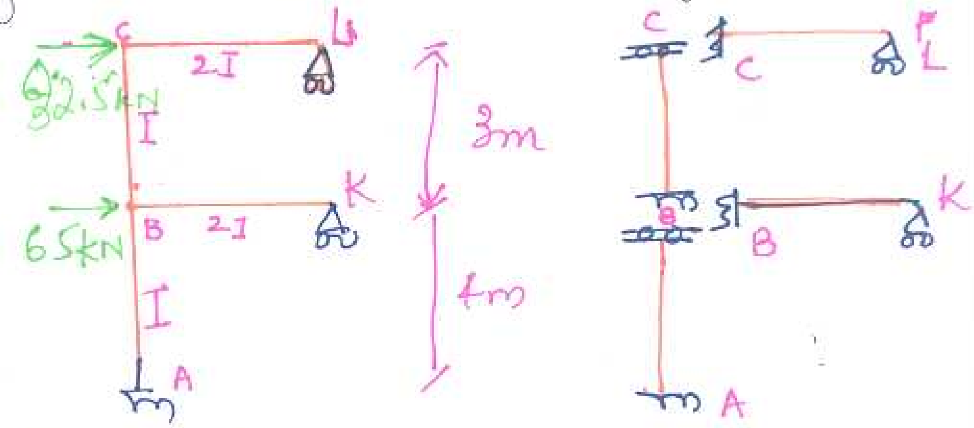
Q(3)



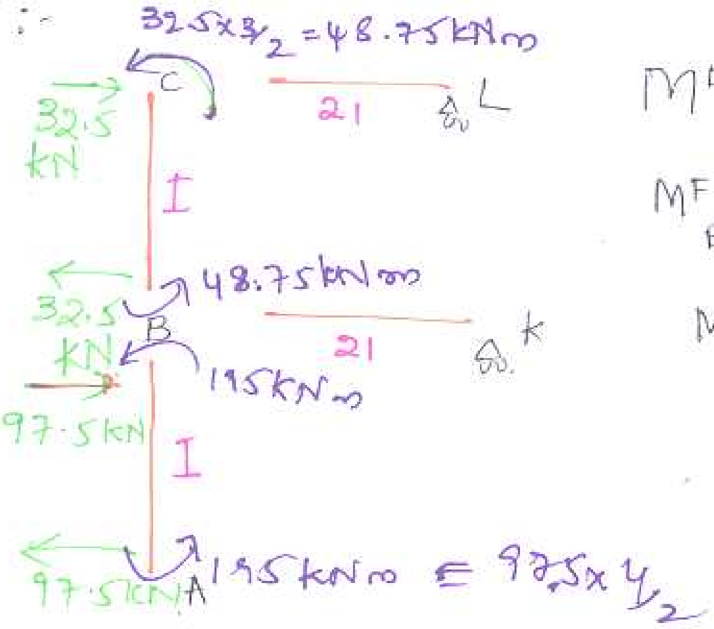
The given loading is a superposition of the two loading diagrams, I and II. ~~the~~ I is anti-symmetric and II is symmetric. Clearly, II doesnot contribute to bending.

So, analyzing (1) is sufficient.

As I is antisymmetric due to the hinges at the centres of the beams.



Fixed End Moments :-



$M^F_{AB} = M^F_{BA} = -195$ kNm

$M^F_{BC} = M^F_{CB} = -48.75$ kNm

$M^F_{CL} = 0$, $M^F_{BK} = 0$

Slope Deflection Equations.

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BK} \\ M_{BC} \\ M_{CB} \\ M_{CL} \end{Bmatrix} = \begin{Bmatrix} -195 \\ -195 \\ 0 \\ -48.75 \\ -48.75 \\ 0 \end{Bmatrix} + EI\theta_B \begin{Bmatrix} -\frac{2}{4} = -\frac{1}{2} \\ 2/4 = \frac{1}{2} \\ \frac{3\times 2}{3} = 2 \\ \frac{1\times 1}{3} = \frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{Bmatrix} + EI\theta_C \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{3} \\ \frac{1}{3} \\ \frac{3\times 2}{3} = 2 \end{Bmatrix}$$

Equilibrium Equations :

$M_B = M_{BA} + M_{BK} + M_{BC} = 0$

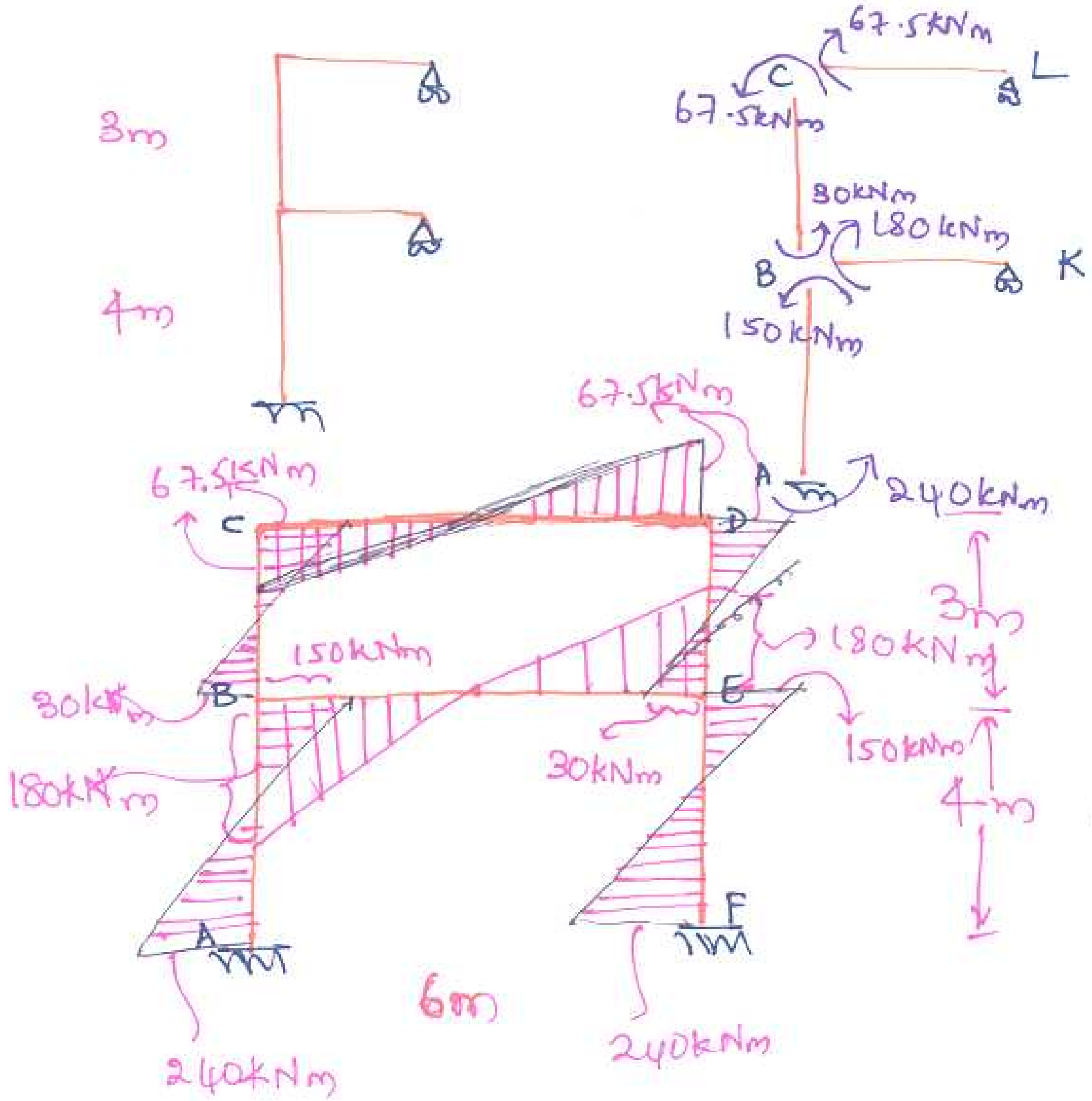
$\Rightarrow -243.75 + \frac{17}{6} EI\theta_B - \frac{1}{3} EI\theta_C = 0$

$M_C = M_{CB} + M_{CL} = 0$

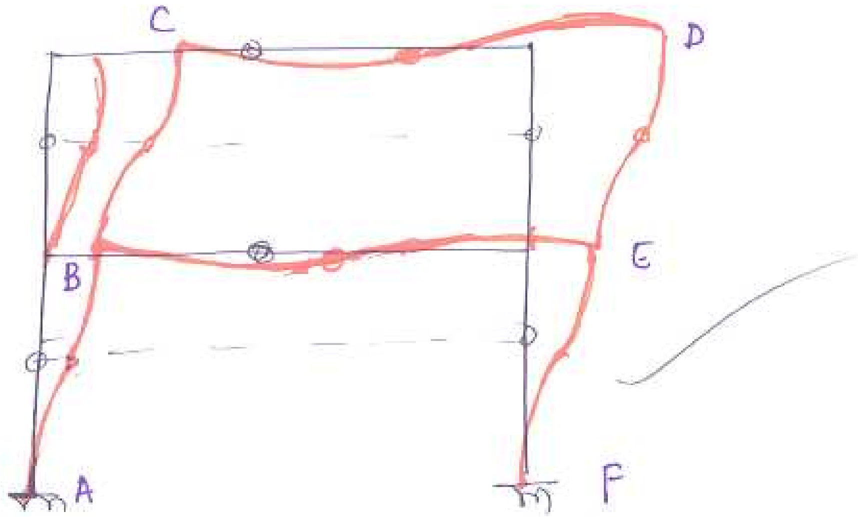
$-48.75 - \frac{1}{3} EI\theta_B + \frac{7}{3} EI\theta_C = 0$

$\Rightarrow$ $\boxed{EI\theta_B = 90\ kNm^2 \ ; \ EI\theta_C = \frac{135}{4} = 33.75\ kNm^2}$

$$\Rightarrow \begin{Bmatrix} M_{AB} \\ M_{BA} \\ M_{BK} \\ M_{BC} \\ M_{CB} \\ M_{CL} \end{Bmatrix} = \begin{Bmatrix} -240 \\ -150 \\ 180 \\ -30 \\ -67.5 \\ 67.5 \end{Bmatrix} \ kNm$$



BMD



Deflection Diagram