TUTORIAL - 5 Beams and Grids (Matrix Methods)

$P = 100 + 30$
$= 130\,kN$

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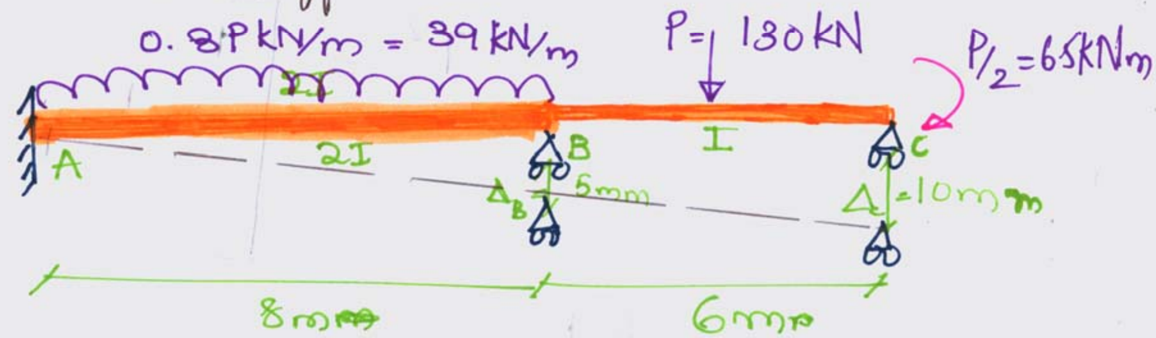
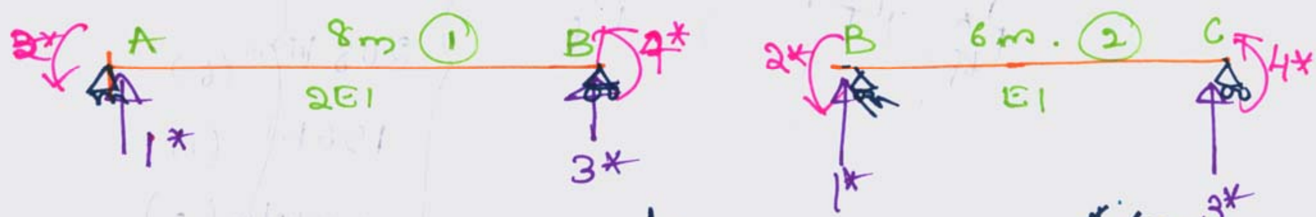
CE09B030

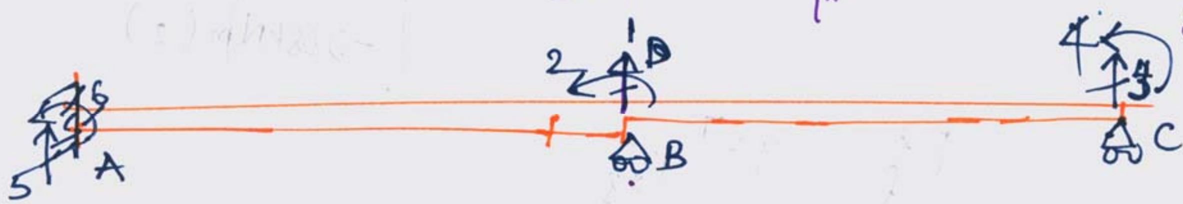
Q.1) (a) Conventional Stiffness Method.

20/20



$EI = 80000\,kNm^2$

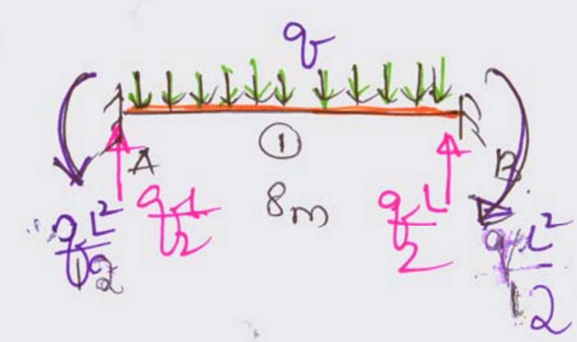




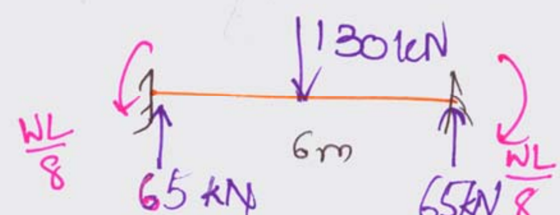
Active Coordinates : 2 and 4.

Element Fixed End forces

$$F_{1f} = \begin{Bmatrix} 156\,kN \\ +208\,kNm \\ 156\,kN \\ -208\,kNm \end{Bmatrix}$$



$$F_{2f} = \begin{Bmatrix} 65\,kN \\ +97.5\,kNm \\ 65\,kN \\ -97.5\,kNm \end{Bmatrix}$$



Element Transformation Matrix $T^i = I$

$D^1_* = T^1 D^1$

$$T^1 = \begin{array}{c} \begin{array}{cccc} 1^* & 2^* & 3^* & 4^* \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} (5) \\ (6) \\ (1) \\ (2) \end{array}$$

$$T^2 = \begin{array}{c} \begin{array}{cccc} 1^* & 2^* & 3^* & 4^* \end{array} \\ \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array}$$

Fixed end forces :

$$F^{1*}_{*f} = T^1 F^1_f \Rightarrow F^1_f = \begin{Bmatrix} 156 \text{ kN} \\ 208 \text{ kNm} \\ 156 \text{ kN} \\ -208 \text{ kNm} \end{Bmatrix} \begin{array}{c} (5) \\ (6) \\ (1) \\ (2) \end{array}$$

$$F^2_f = T^{2T} F^2_{*f}$$

$$F^2_f = \begin{Bmatrix} 65 \text{ kN} \\ 97.5 \text{ kNm} \\ 65 \text{ kN} \\ -97.5 \text{ kNm} \end{Bmatrix} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array}$$

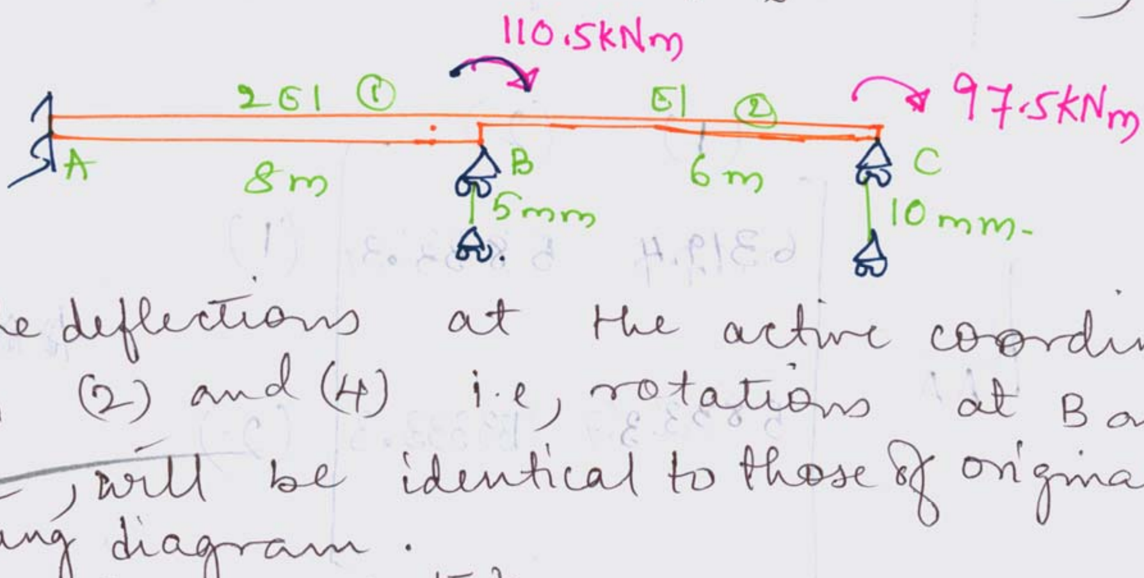
$$F_f = \begin{Bmatrix} 221 \text{ kN} \\ -110.5 \text{ kNm} \\ 65 \text{ kN} \\ -97.5 \text{ kNm} \\ 156 \text{ kN} \\ 208 \text{ kNm} \end{Bmatrix} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{array}$$

Net force Vector (F_{net}):

$$F = \begin{Bmatrix} F_1 \\ 0 \\ F_3 \\ -65\ kNm \\ F_5 \\ F_6 \end{Bmatrix} \qquad F_f = \begin{Bmatrix} 221\ kN \\ -110.5\ kNm \\ 65\ kN \\ -97.5\ kNm \\ 156\ kN \\ 208\ kNm \end{Bmatrix}$$

$$F_{net} = F - F_f = \begin{Bmatrix} F_1 - 221\ kN \\ 110.5\ kNm \\ F_3 - 65\ kN \\ +97.5\ kNm \\ F_5 - 156\ kN \\ F_6 - 208\ kNm \end{Bmatrix}$$

$$F_{net_A} = F_A - F_{fA} = \begin{Bmatrix} 110.5 \\ 97.5 \end{Bmatrix} kNm. \quad \text{Ans.}$$



The deflections at the active coordinates (2) and (4) i.e, rotations at B and C, will be identical to those of original loading diagram.

Element Stiffness Matrices

$$k'_* = \frac{2(EI)}{8} \begin{bmatrix} 12/8^2 & 6/8 & -12/8^2 & 6/8 \\ 6/8 & 4 & -6/8 & 2 \\ -12/8^2 & -6/8 & 12/8^2 & -6/8 \\ 6/8 & 2 & -6/8 & 4 \end{bmatrix}$$

$$k_*^2 = \frac{EI}{6}\begin{bmatrix} 12/6^2 & 6/6 & -12/6^2 & 6/6 \\ 6/6 & 4 & -6/6 & 2 \\ -12/6^2 & -6/6 & 12/6^2 & -6/6 \\ 6/6 & 2 & -6/6 & 4 \end{bmatrix}$$

$EI = 80{,}000\ kNm^2$.

$$k^1 = (T^1)^T k_*^1\, T^1$$

$$k^2 = (T^2)^T k_*^2\, T^2$$

$$k^1 = 2 \times \begin{array}{c} \begin{array}{cccc} (5) & (6) & (1) & (2) \end{array} \\ \begin{bmatrix} 1875 & 7500 & -1875 & 7500 \\ 7500 & 40{,}000 & -7500 & 20000 \\ -1875 & -7500 & 1875 & -7500 \\ 7500 & 20{,}000 & -7500 & 40000 \end{bmatrix} \end{array} \begin{array}{c} (5) \\ (6) \\ (1) \\ (2) \end{array}$$

;

$$k^2 = \begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 4444.44 & 13333.33 & -4444.4 & 13333.3 \\ 13333.33 & 53{,}333.3 & -13333.3 & 26666.7 \\ -4444.44 & -13333.33 & 4444.4 & -13333.3 \\ 13333.33 & 26666.7 & -13333.3 & 53333.3 \end{bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array}$$

$$K_{AA} = \begin{array}{c} \begin{array}{cc} (1) & (2) \end{array} \\ \begin{bmatrix} 8194.3 & -1666.67 \\ -1666.67 & 133333.3 \end{bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \end{array} \quad kNm/rad.$$

$$k_{*}^{2} = \frac{EI}{6} \begin{bmatrix} 12/6^2 & 6/6 & -12/6^2 & 6/6 \\ 6/6 & 4 & -6/6 & 2 \\ -12/6^2 & -6/6 & 12/6^2 & -6/6 \\ 6/6 & 2 & -6/6 & 4 \end{bmatrix}$$

$EI = 80,000 \, kNm^2$.

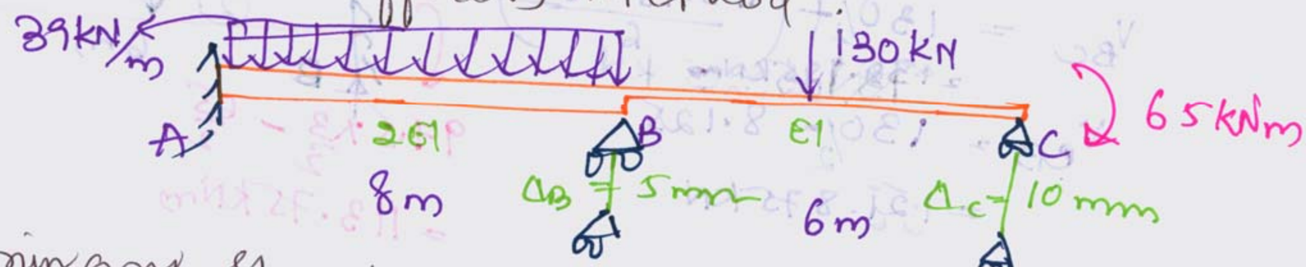
$$k^1 = (T^1)^T k_{*}^{1} T^1$$

$$k^2 = (T^2)^T k_{*}^{2} T^2$$

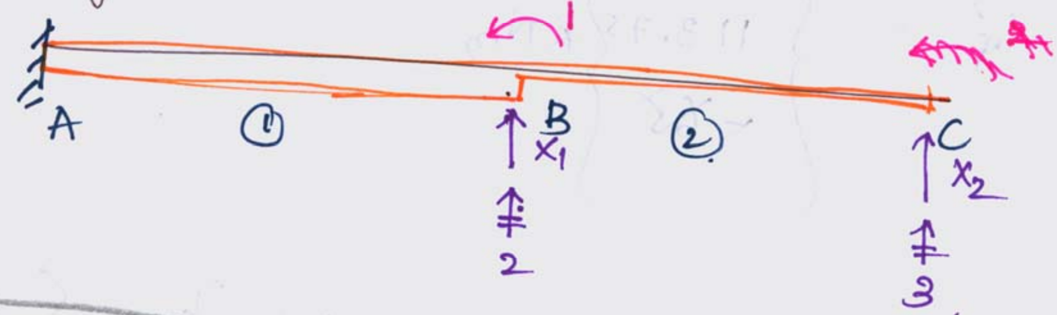
$$k^1 = 2 \times \begin{array}{c} \begin{array}{cccc} (5) & (6) & (1) & (2) \end{array} \\ \begin{bmatrix} 1875 & 7500 & -1875 & 7500 \\ 7500 & 40,000 & -7500 & 20000 \\ -1875 & -7500 & 1875 & -7500 \\ 7500 & 20,000 & -7500 & 40000 \end{bmatrix} \end{array} \begin{array}{c} (5) \\ (6) \\ (1) \\ (2) \end{array} \quad ; \quad k^2 = \begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 4444.44 & 13333.33 & -4444.4 & 13333.3 \\ 13333.33 & 53,333.3 & -13333.3 & 26666.67 \\ -4444.44 & -13333.33 & 4444.4 & -13333.3 \\ 13333.33 & 26666.7 & -13333.3 & 53333.3 \end{bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array}$$

$$K_{AA} = \begin{array}{c} \begin{array}{cc} (1) & (2) \end{array} \\ \begin{bmatrix} 8194.3 & -1666.67 \\ -1666.67 & 133333.3 \end{bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \end{array} \; kNm/rad.$$

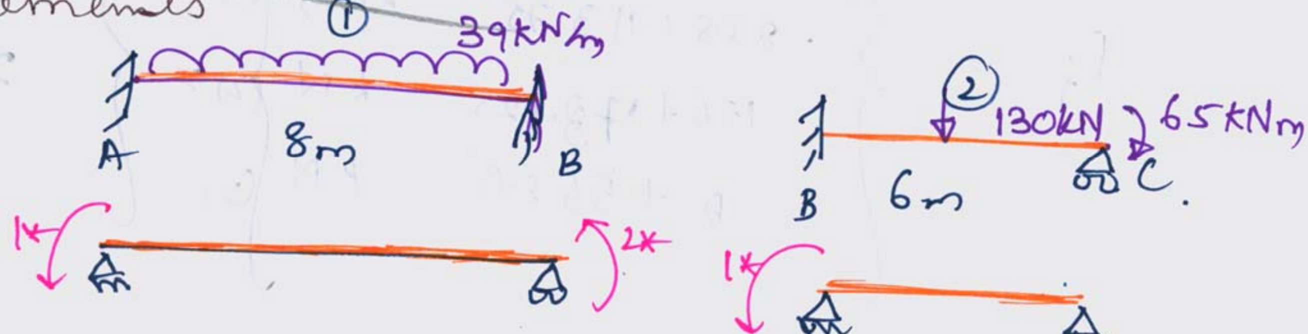
(b) Reduced Stiffness Method



Primary Structure

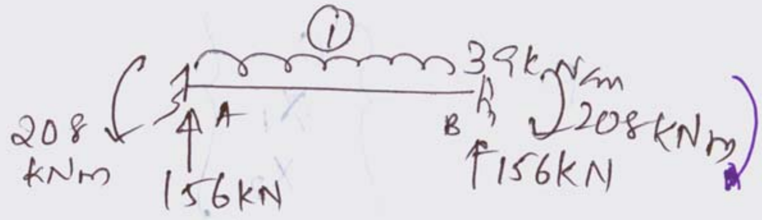


Elements



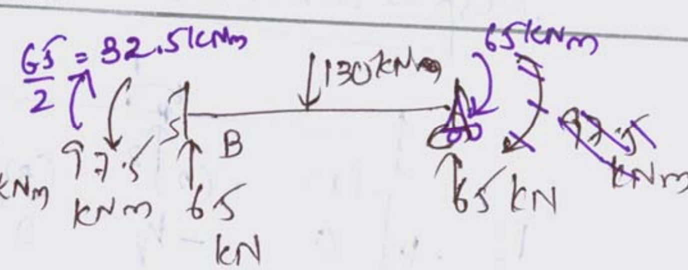
Fixed End forces:

(1)



$$F^1_{*f} = \begin{Bmatrix} +208\ kNm \\ -208\ kNm \end{Bmatrix}$$; Vertical Reactions: $V_{AB} = 156\ kN = V_{BA}$

(2) : $$F^2_{*f} = \begin{Bmatrix} (97.5 - (32.5)) \\ (-97.5 - 65) \end{Bmatrix} kNm = \begin{Bmatrix} 65 \\ -162.5 \end{Bmatrix} kNm$$



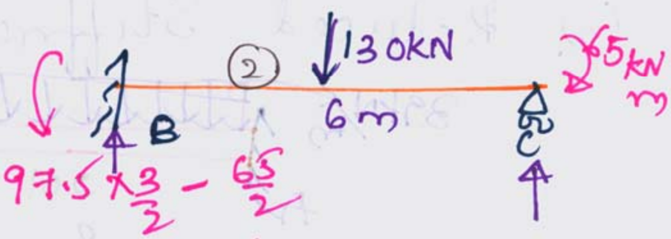
$V_{BC} = V_{CB} = 65\ kN$ $\quad V_{2BC} =$

Fixed End force vector:

$$F^*_f = \begin{Bmatrix} -208 + 65 \\ 156 + 65 \\ 65 \end{Bmatrix} = \begin{Bmatrix} -143.0\ kNm \\ 221\ kN \\ 65\ kN \end{Bmatrix} \begin{matrix} (1) \\ (2) \\ 3 \end{matrix}$$

$$V_{BC} = 130/2 + \left(\frac{113.75-65}{6}\right) = 73.125\ \text{kN}$$

$$V_{CB} = 130/2 - 8.125 = 56.875\ \text{kN}$$



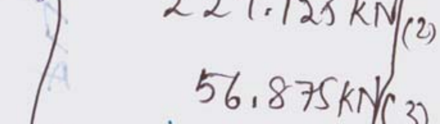
$$97.5\times\frac{3}{2} - \frac{65}{2} = 113.75\ \text{kNm}$$

$$F_f^2 = \begin{Bmatrix} 113.75 \\ \cancel{-65} \end{Bmatrix} \text{kNm}$$

$$F_f = \begin{array}{c} \begin{Bmatrix} -208 + 113.75 & \text{kNm} \\ 156 + 73.125 & \text{kN} \\ 0 + 56.875 & \text{kN} \end{Bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \end{array} = \begin{Bmatrix} -94.25\ \text{kNm} \\ 229.125\ \text{kN} \\ 56.875\ \text{kN} \end{Bmatrix} \begin{array}{c} (1) \\ (2) \\ (3) \end{array}$$

$$F = \begin{Bmatrix} 0 \\ X_1 \\ X_2 \end{Bmatrix}$$

Displacement Transformation Matrices



$$T_D^1 = \begin{array}{c|ccc} & (1) & (2) & (3) \\ \hline 1^* & 0 & -1/8 & 0 \\ 2^* & 1 & -1/8 & 0 \end{array}$$

$$T_D^2 = \begin{bmatrix} 1 & +1/6 & -1/6 \end{bmatrix}$$

Reduced Stiffness Matrixes of Elements

$$\tilde{k}_*^1 = \frac{2EI}{8} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = EI \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}.$$

$$\tilde{k}_*^2 = \frac{\cancel{2}EI}{6} \begin{bmatrix} 3 \end{bmatrix} = EI \begin{bmatrix} 2 \end{bmatrix}$$

$$\tilde{k}_* = EI \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{array}{cc} T_D = & \begin{bmatrix} 0 & -1/8 & 0 \\ 1 & -1/8 & 0 \\ 1 & 1/6 & -1/6 \end{bmatrix} \\ & \begin{array}{ccc} (1) & (2) & (3) \end{array} \end{array}$$

Stiffness Vector at global / structural level.

$$\tilde{K} = T_D^T \, \tilde{k}_* \, T_D$$

$$\begin{array}{cc} & \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ = EI & \left[\begin{array}{c|cc} 240000 & 11666.67 & -26666.67 \\ \hline 11666.67 & 8194.44 & -4444.44 \\ -26666.67 & -4444.44 & 4444.44 \end{array}\right] \end{array}$$

Displacement Vector.

Support Displacements $D_R = \begin{Bmatrix} -0.005 \\ -0.010 \end{Bmatrix} \begin{array}{c} (2) \\ (3) \end{array}$ m.

$$F - F_f = \tilde{k} \, D = \tilde{k} \begin{Bmatrix} D_A \\ D_B \end{Bmatrix}$$

$$D_A = k_{AA}^{-1}\left[(F_A - F_{fA}) - k_{AR}D_R\right]$$

$$= \frac{1}{|k_{AA}|}\left[+94.25 - k_{AR}D_R\right]$$

$$= \frac{(94.25 - 208.333)}{24.0000}$$

$$\boxed{D_1 = \cancel{+0.4754}\ -4.7536\times10^{-4}\ \text{radians}}$$

$$F_R = F_{fR} + k_{RA}D_A + k_{RR}D_R$$

$$= \begin{Bmatrix} 229.125 \\ 56.875 \end{Bmatrix} + \begin{bmatrix} 11666.67 \\ -26666.67 \end{bmatrix}(-4.7536\times10^{-4})$$

$$+ \begin{bmatrix} 8194.44 & -4444.4 \\ -4444.44 & +4444.4 \end{bmatrix}\begin{Bmatrix} -0.005 \\ -0.010 \end{Bmatrix}$$

$$= \begin{Bmatrix} 229.125 \\ 56.875 \end{Bmatrix} + \begin{Bmatrix} -5.546 \\ +12.676 \end{Bmatrix} + \begin{Bmatrix} 3.422 \\ -22.222 \end{Bmatrix}$$

$$F_R = \begin{Bmatrix} 227.1 \\ +\cancel{352.7}\ 47.331 \end{Bmatrix} kN$$

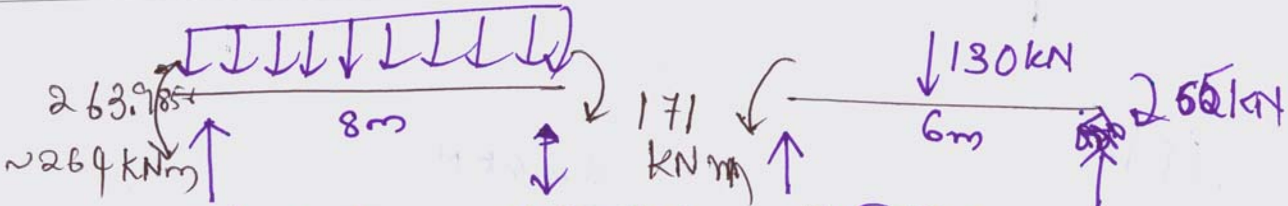
$$F^{i}_{*} = F^{i}_{*f} + \tilde{k}^{i}_{*}\, T^{i}_{D}\, D \qquad D = \begin{Bmatrix} -4.7536\times10^{-4}\ \text{rad} \\ -0.05\ \text{m} \\ -0.010\ \text{m} \end{Bmatrix}$$

$$F^{1}_{*} = \begin{Bmatrix} 208 \\ -208 \end{Bmatrix} + EI \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1/8 & 0 \\ 1 & -1/8 & 0 \end{bmatrix} \begin{Bmatrix} D \end{Bmatrix}$$

$$= \begin{Bmatrix} 263.9856 \\ -171.0288 \end{Bmatrix} \text{kNm}$$

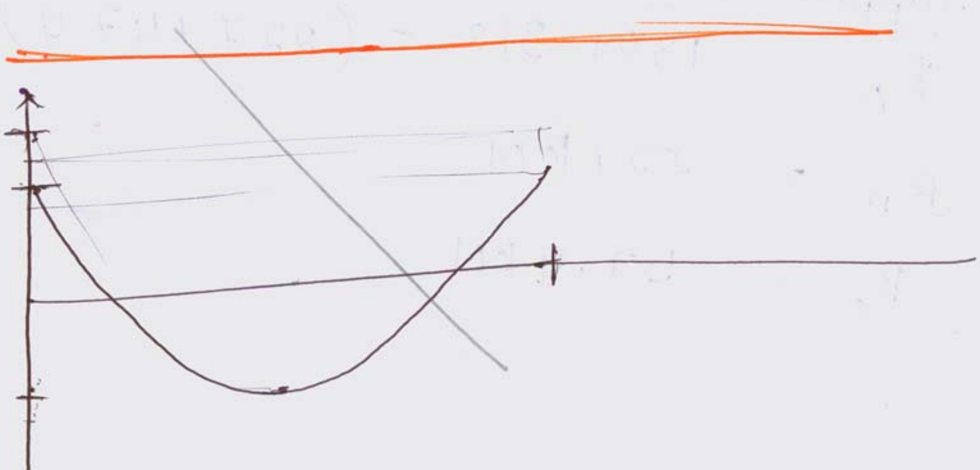
$$F^{2}_{*} = \begin{Bmatrix} 113.75 \end{Bmatrix} + EI(2)\times \begin{bmatrix} 1 & 1/6 & -1/6 \end{bmatrix} \begin{Bmatrix} D \end{Bmatrix}$$

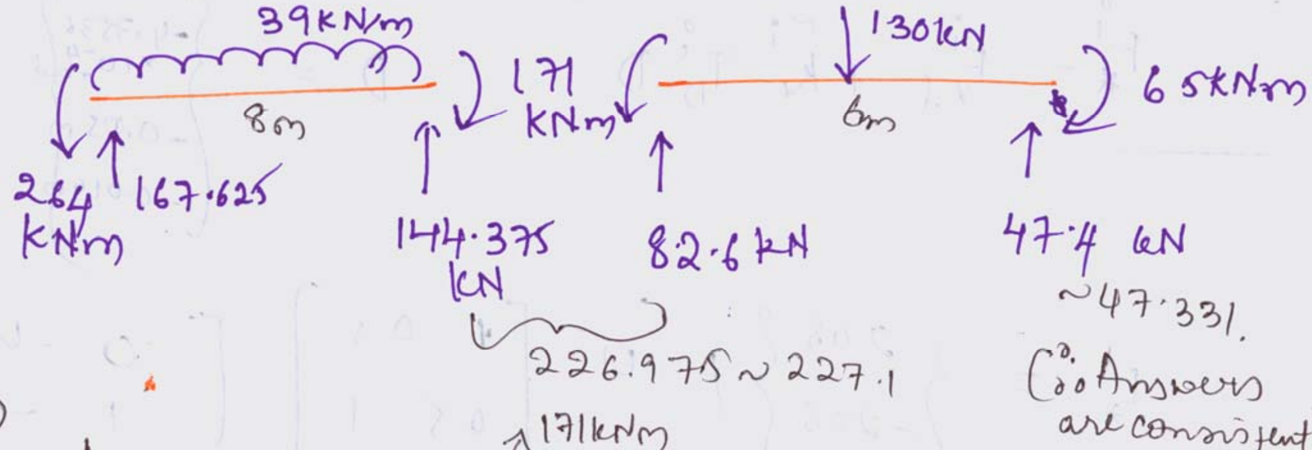
$$= 171.0256\ \text{kNm}$$

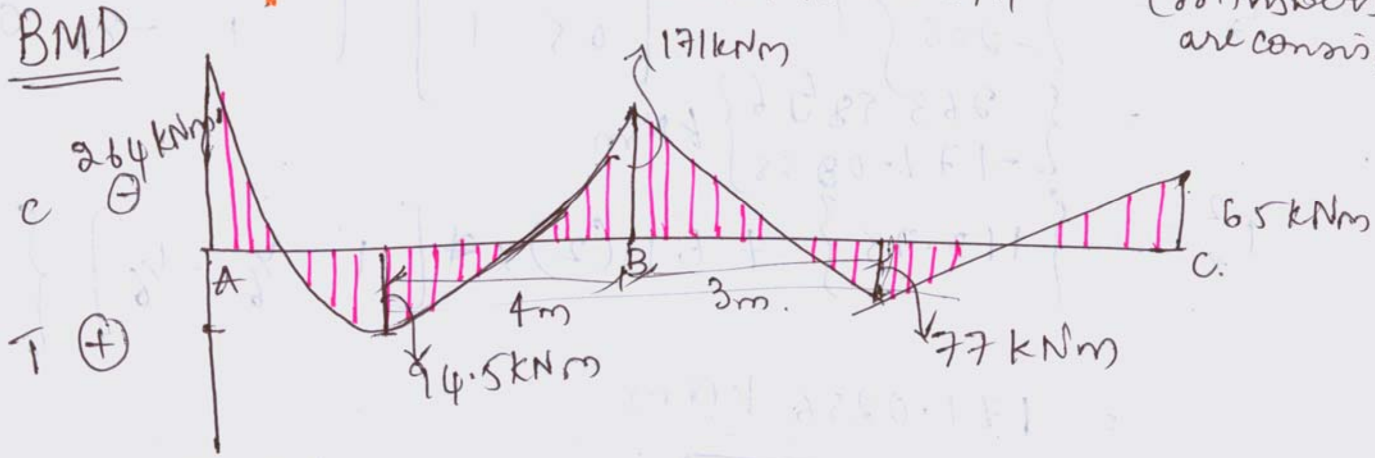


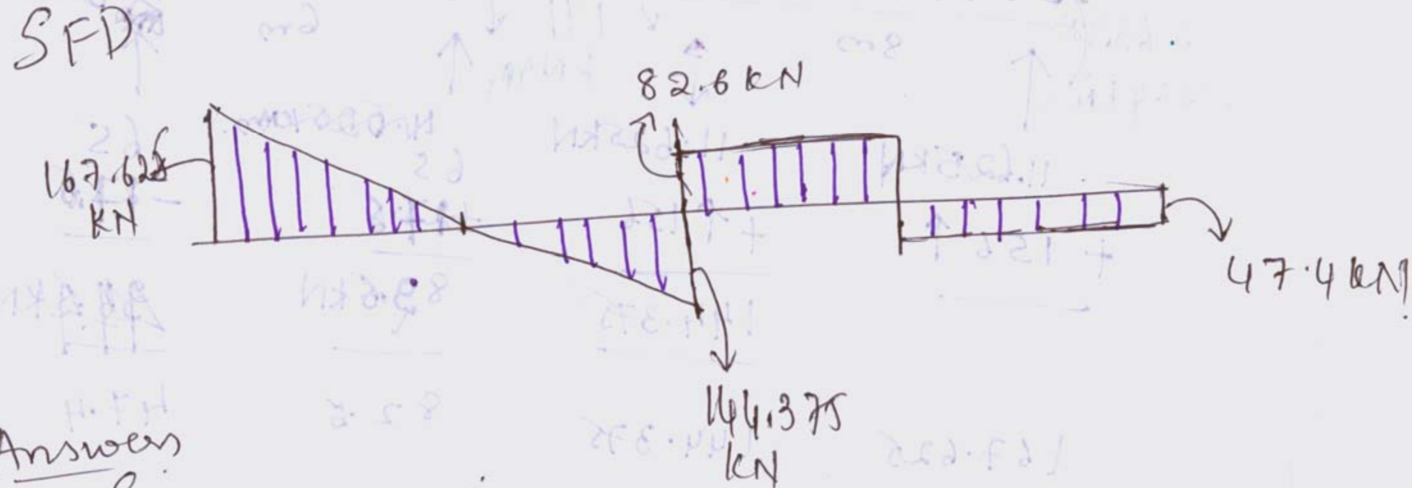
+ 156 ↑	+ ↑156	65	65
		+ 17.6	− 17.6
	144.375	82.6 kN	47.4 kN
167.625	144.375	82.6	47.4

(kN)









Answers

Support reactions

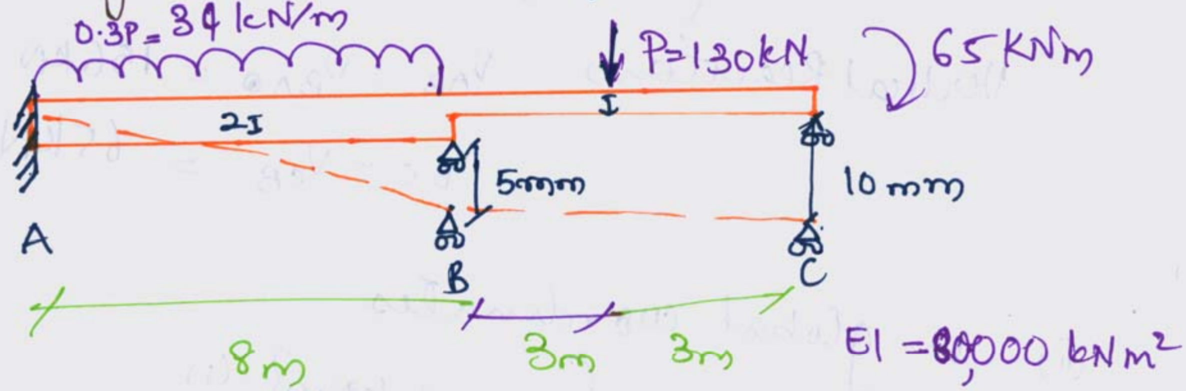
$R_A = 130 + 312 - (227 + 47.4) = 167.6$ kN.

$R_B = 227$ kN

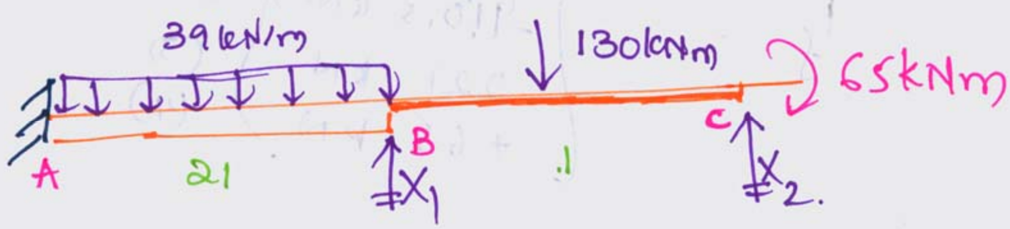
$R_C = 47.4$ kN

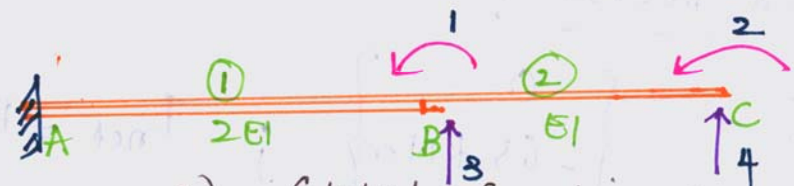
Q.2. Flexibility Method :

Primary Structure of



is :

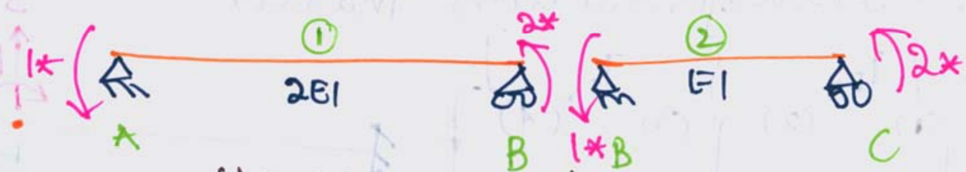




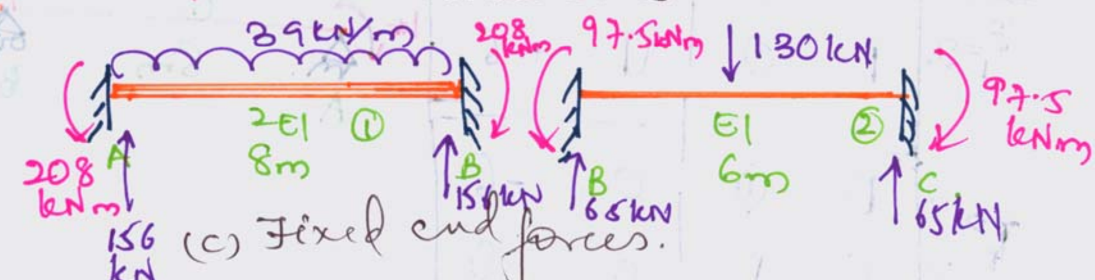
a) Global Coordinates

We donot reduce the degrees of 2nd element from 2 to 1, like, the Reduced Element Method.

1 and 2 : Active, 3 and 4 : redundant.



(b) Local coordinates



(c) Fixed end forces.

Fixed End forces :(i) In local coordinates

$$F_{1*f} = \begin{Bmatrix} 208 \\ -208 \end{Bmatrix} \text{ kNm}, \quad F_{2*f} = \begin{Bmatrix} 97.5 \\ -97.5 \end{Bmatrix} \text{ kNm}$$

Vertical Reactions $V_{AB} = V_{BA} = 156 \text{ kN}$

$V_{BC} = V_{CB} = 65 \text{ kN}$

(ii) In global coordinates

$$F_f = \begin{Bmatrix} -110.5 \text{ kNm} \\ -97.5 \text{ kNm} \\ 221 \text{ kN} \\ +65 \text{ kN} \end{Bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

Net Load Vector :

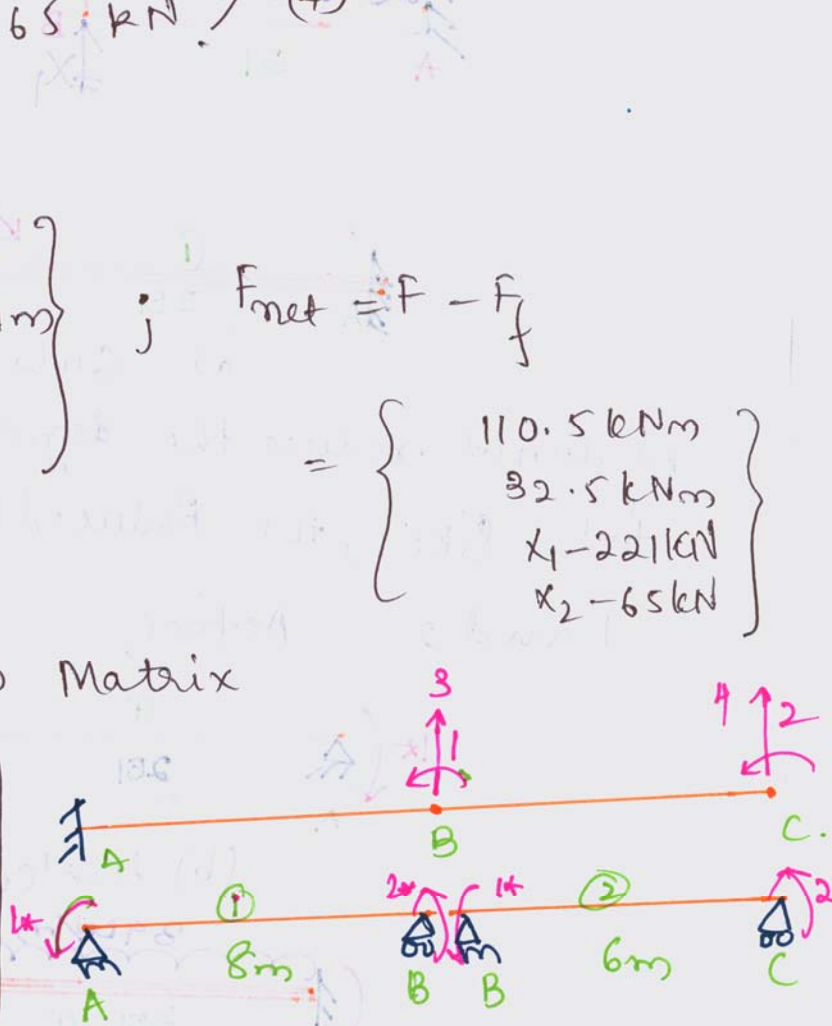
$$F = \begin{Bmatrix} 0 \\ -65 \text{ kNm} \\ X_1 \\ X_2 \end{Bmatrix} ; \quad F_{net} = F - F_f$$

$$= \begin{Bmatrix} 110.5 \text{ kNm} \\ 32.5 \text{ kNm} \\ X_1 - 221 \text{ kN} \\ X_2 - 65 \text{ kN} \end{Bmatrix}$$

Force Transformation Matrix

$$\left[\, T_{FA} \;\middle|\; T_{FX} \,\right]$$

$$T_F = \begin{array}{c} \\ 1* \\ 2* \\ \\ 1* \end{array} \begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \left[\begin{array}{cc|cc} -1 & -1 & -8 & -14 \\ 1 & 1 & 0 & +6 \\ \hline 0 & -1 & 0 & -6 \end{array}\right] \end{array}$$



Unassembled
Flexibility Matrix : (i) Along element level
Local coordinates

$$f_* = \left[\begin{array}{c|c} \frac{8}{6\times 2EI} \times \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} & 0 \\ \hline 0 & \frac{6}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \end{array}\right]$$

$$f_* = \frac{1}{EI}\begin{bmatrix} 4/3 & -2/3 & 0 & 0 \\ -2/3 & 4/3 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Structure flexibility Matrix :

$$f = T_F^T f_* T_F = \frac{1}{EI}\begin{bmatrix} 4 & 4 & 16 & 40 \\ 4 & 10 & 16 & 58 \\ 16 & 16 & 85.33 & 181.33 \\ 40 & 58 & 181.33 & 493.33 \end{bmatrix}$$

Flexibility Relation

$$\left[\begin{array}{c} D_A \\ \hline D_X \end{array}\right] = \left[\begin{array}{c} D_{A,in.} = 0 \\ \hline D_{X,in.} \end{array}\right] + \left[\begin{array}{c|c} f_{AA} & f_{AX} \\ \hline f_{XA} & f_{XX} \end{array}\right]\left[\begin{array}{c} F_A - F_{fA} \\ \hline F_X - F_{fX} \end{array}\right]$$

$$\left[\begin{array}{c} D_A \\ \hline D_X \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \end{array}\right] + \left[\begin{array}{c} f_{AA}(F_A - F_{fA}) + f_{AX}(F_X - F_{fX}) \\ \hline f_{XA}(F_A - F_{fA}) + f_{XX}(F_X - F_{fX}) \end{array}\right]$$

$$D_x = \begin{Bmatrix} -0.005 \\ -0.010 \end{Bmatrix} \begin{matrix} (3) \\ (4) \end{matrix} \; m.$$

$$D_x = f_{xA}(F_A - F_{fA}) + f_{xx}(F_x - F_{fx})$$

$$EI \begin{Bmatrix} -0.005 \\ -0.010 \end{Bmatrix} = \begin{bmatrix} 16 & 16 \\ 40 & 58 \end{bmatrix} \begin{Bmatrix} +110.5 \\ +22.5 \end{Bmatrix} + \begin{bmatrix} 85.33 & 181.33 \\ 181.33 & 493.33 \end{bmatrix} \begin{Bmatrix} x_1 - 221 \\ x_2 - 65 \end{Bmatrix}$$

$$\begin{Bmatrix} -400 \\ -800 \end{Bmatrix} - \begin{Bmatrix} 2688 \\ 6305 \end{Bmatrix} = f_{xx} \begin{Bmatrix} x_1 - 221 \\ x_2 - 65 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 - 221 \\ x_2 - 65 \end{Bmatrix} = f_{xx}^{-1} \begin{Bmatrix} -2688 \\ -7105 \end{Bmatrix}$$

$$\begin{bmatrix} 0.0538 & -0.0197 \\ -0.0197 & 0.0093 \end{bmatrix} \begin{Bmatrix} -2688 \\ -7105 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -4.0914 \\ -12.8981 \end{Bmatrix} + \begin{Bmatrix} 221 \\ 65 \end{Bmatrix}$$

Support Reactions. $\boxed{\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 216.91 \\ 52.10 \end{Bmatrix} \text{ kN}}$

Member forces

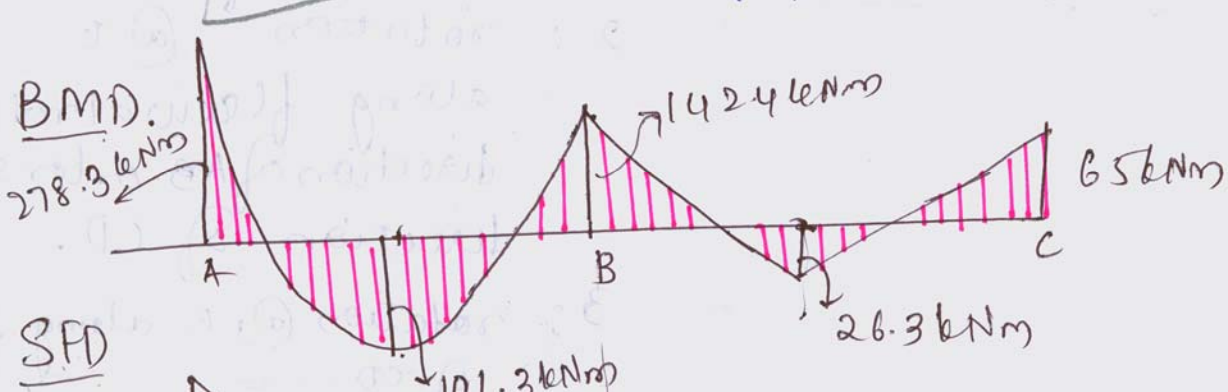
$$F_x = F_{xf} + T_F\, F_{net}$$

$$F_x = \begin{Bmatrix} 208 \\ -208 \\ 97.5 \\ -97.5 \end{Bmatrix} + T_F \begin{Bmatrix} 110.5 \\ 32.5 \\ -4.0914 \\ -12.8981 \end{Bmatrix}$$

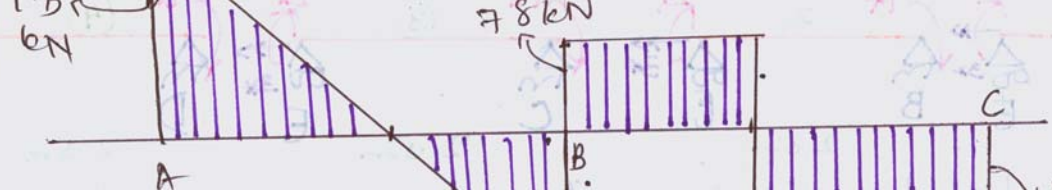
$$= \begin{Bmatrix} 278.3046 \\ -142.3886 \\ 142.3886 \\ -65 \end{Bmatrix} \begin{matrix} 1^* \\ 2^* \\ 1^* \\ 2^* \end{matrix} \begin{matrix} (1) \\ \\ (2) \\ \end{matrix}$$

156	156	65	65
+ 16.99	− 16.99	+ 12.9	− 12.9
173 kN	139 kN	77.9 kN	52.1 kN

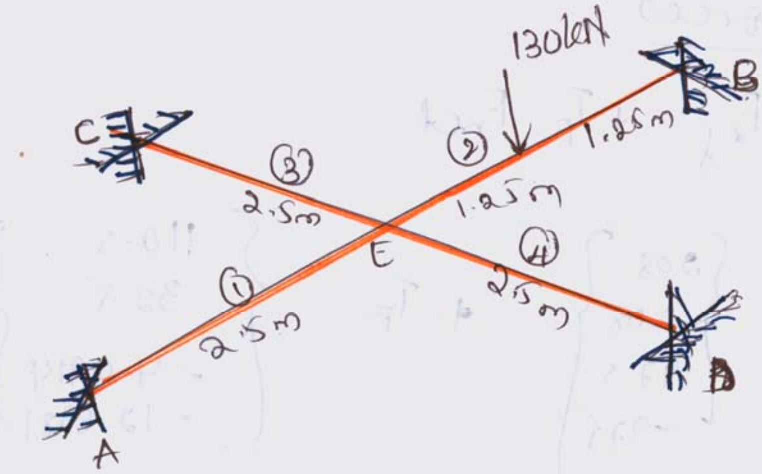
BMD.



SFD



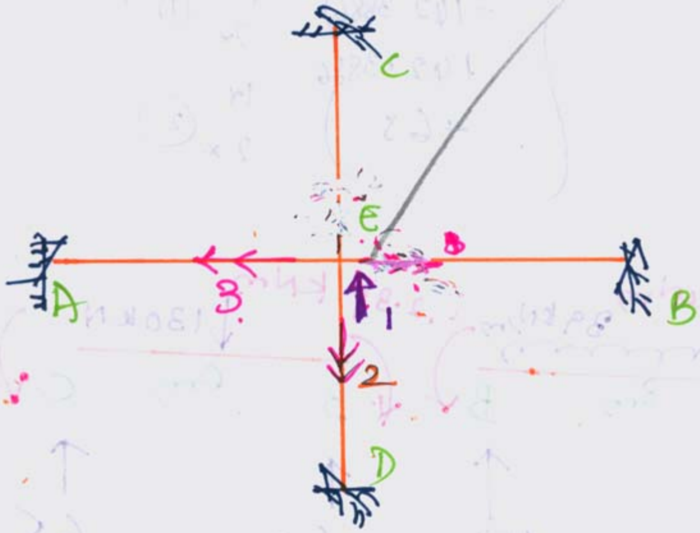
(3.)

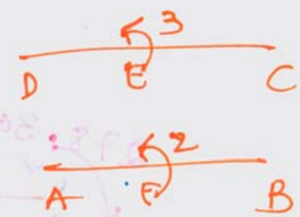


EI = 80000 kNm^2

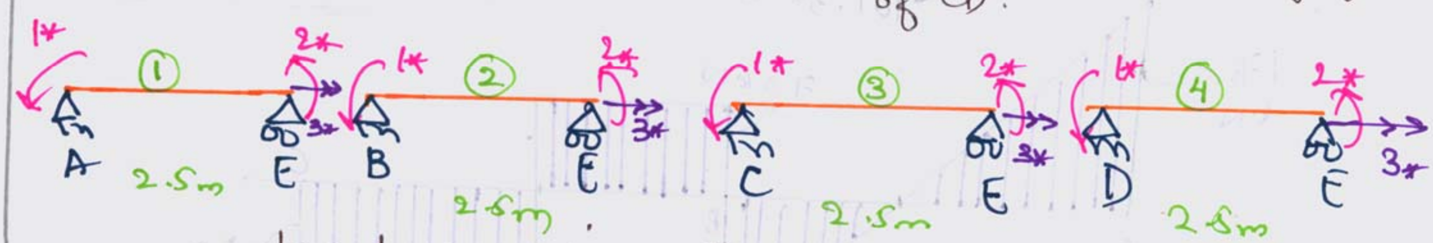
GJ = 0.2EI

L = 5m.





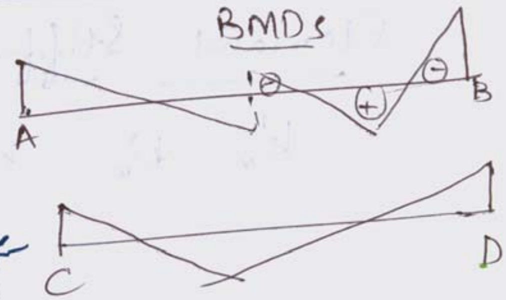
Global Coordinates : 1 : vertical deflection @ E,
2 : rotation @ E along flexural rotation direction of AB or torsional direction of CD.
3 : rotation @ E along flexure of CD.



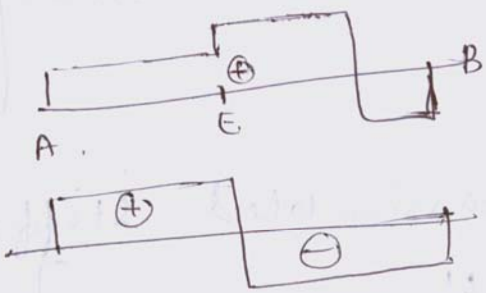
Local Coordinates

a) Predicted

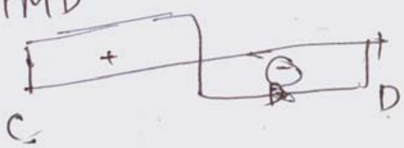
Deflected Shape



SFDs :



; TMD

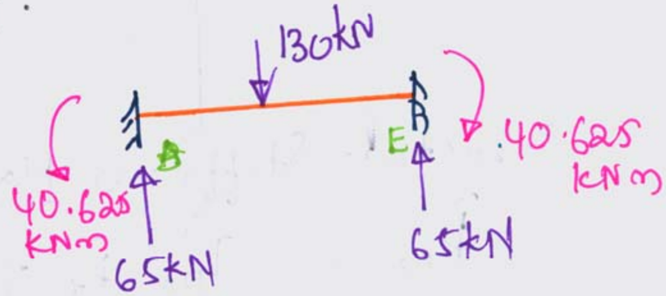


b)

Fixed End forces

Element ② only has F_{*f}.

$$F^2_{*f} = \begin{Bmatrix} 40.625\,kNm \\ -40.625\,kNm \\ 0 \end{Bmatrix}$$



$$F_f = \begin{array}{c} \begin{Bmatrix} 65\,kN \\ +40.625\,kNm \\ 0 \end{Bmatrix} \end{array} \begin{array}{c} (1) \\ (2) \\ (3) \end{array}$$

Displacement Transformation Matrix :

$$T^1_D = \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} & 0 & 0 \\ -0.4 & 1 & 0 \\ -0.4 & 0 & -1 \\ 0 & 0 & 0 \\ -0.4 & 0 & 0 \\ -0.4 & -1 & 1 \\ 0 & 0 & 0 \\ -0.4 & 0 & -1 \\ -0.4 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \left.\begin{array}{l} 1* \\ 2* \\ 3* \end{array}\right\} ① \\ \left.\begin{array}{l} 1* \\ 2* \\ 3* \end{array}\right\} ② \\ \left.\begin{array}{l} 1* \\ 2* \\ 3* \end{array}\right\} ③ \\ 1* \end{array}$$

Element Stiffness Matrix

$$k_*^1 = k_*^2 = k_*^3 = k_*^4 = \frac{EI}{2.5}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

$$= 32{,}000 \begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

$\tilde{k}_*$ = Unassembled Stiffness Matrix.

$$= \begin{bmatrix} k_*^1 & & & \\ & k_*^2 & & \\ & & k_*^3 & \\ & & & k_*^4 \end{bmatrix}$$

Structure Stiffness Matrix

$$\tilde{K}_{A*} = T_{DA}^T \, \tilde{k}_* \, T_{DA}$$

$$= \begin{array}{cc} \begin{bmatrix} 245760 & 0 & 0 \\ 0 & 268800 & 0 \\ 0 & 0 & 268800 \end{bmatrix} & \begin{array}{c} (1) \\ (2) \\ (3) \end{array} \\ \begin{array}{ccc} (1) & \quad (2) & \quad (3) \end{array} & \end{array}$$

Displacement Vector

$$F_A - F_{fA} = \tilde{K}_{AA} D_A \;; \quad F_{net,A} = \begin{Bmatrix} -65\text{ kN} \\ -40.625\text{ kNm} \\ 0 \end{Bmatrix}$$

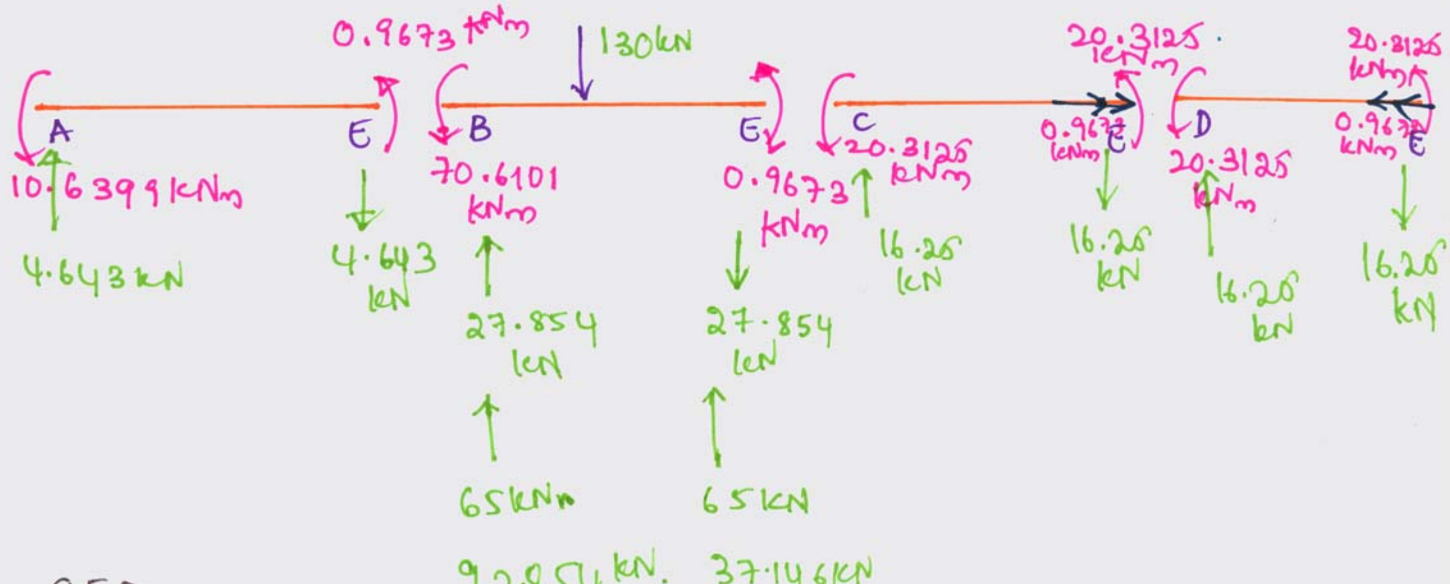
$$D_A = \text{inv}(\tilde{K}_{AA}) \, F_{net,A}$$

$$D_A = 10^{-3}\begin{bmatrix} -0.2645\text{m} \\ -0.1511\text{ rad} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.2645\text{mm} \\ -0.1511\times10^{-3}\text{ rad} \\ 0 \end{bmatrix}$$

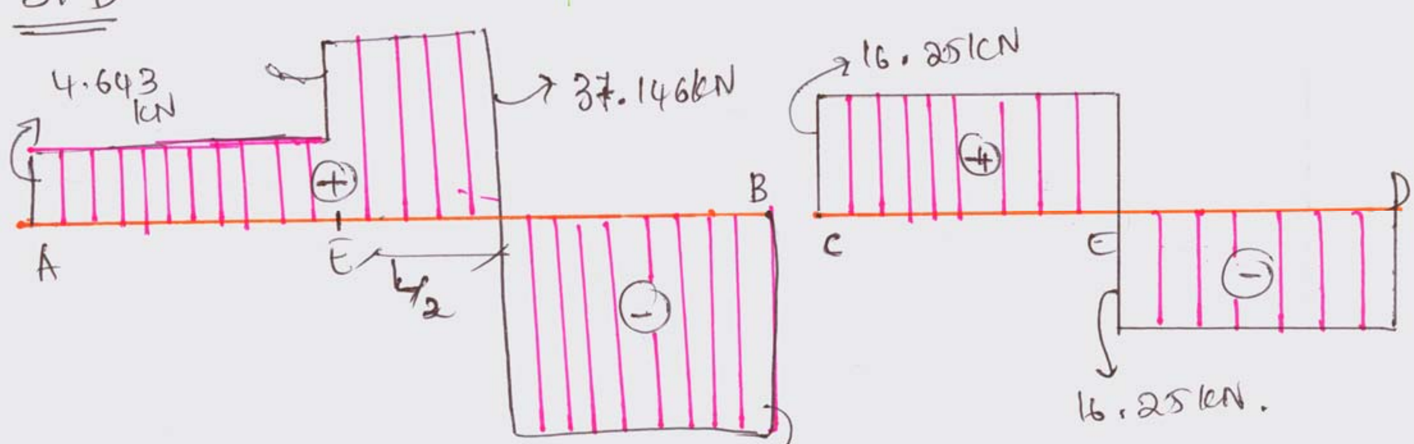
Memberforces

$$f_* = F_{*f} + \tilde{k}_* \, T_{DA} \, D_A$$

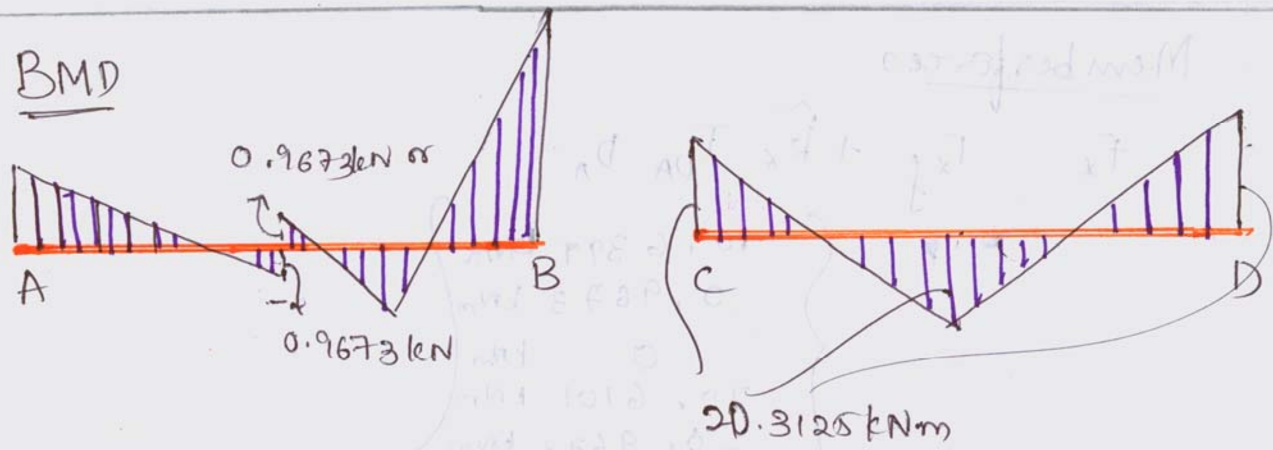
$$= \begin{Bmatrix} 10.6399 \text{ kNm} \\ 0.9673 \text{ kNm} \\ 0 \text{ kNm} \\ 70.6101 \text{ kNm} \\ -0.9673 \text{ kNm} \\ 0 \text{ kNm} \\ 20.3125 \text{ kNm} \\ 20.3125 \text{ kNm} \\ 0.9673 \text{ kNm} \\ 20.3125 \text{ kNm} \\ 20.3125 \text{ kNm} \\ -0.9673 \text{ kNm} \end{Bmatrix}$$



SFD



BMD



TMD (Torsional Moment Diagram)