Finite Element Analysis

Assignment 2

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1. Find the lowest eigenvalue λ for the following problem using the modified Galerkin method

$$x^2 \frac{d^2u(x)}{dx^2} + 2x\frac{du}{dx} - \lambda u(x) = 0 \qquad 1 < x < 3$$

$$u(1) = 0 \qquad\qquad \frac{du(3)}{dx} = 0$$

(a) Use two linear elements. (b) Use three linear elements.

2. Find the lowest eigenvalue λ for the following problem using the Rayleigh-Ritz method

$$\frac{d^2u(x)}{dx^2} + \lambda u(x) = 0 \qquad 0 < x < 1$$

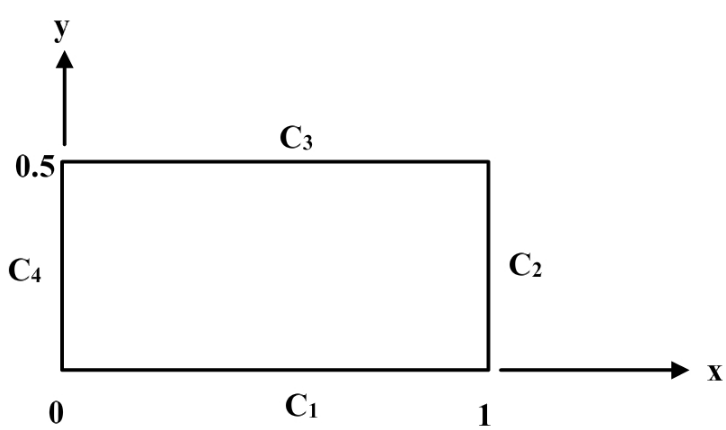
$$u(0) = 0 \qquad 2\frac{du(1)}{dx} - u(1) = 0$$

(a) Use two linear elements. (b) Use three linear elements.

3. Find an approximate solution of the following boundary value problem defined over a rectangular domain

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + 1 = 0 \quad 0 < x < 1, \quad 0 < y < 0.5$$

with the boundary conditions shown in the following figure. Use the modified Galerkin method and try one or two parameter trial solution that is admissible.



$\partial\psi/\partial y = 0$ on C_1

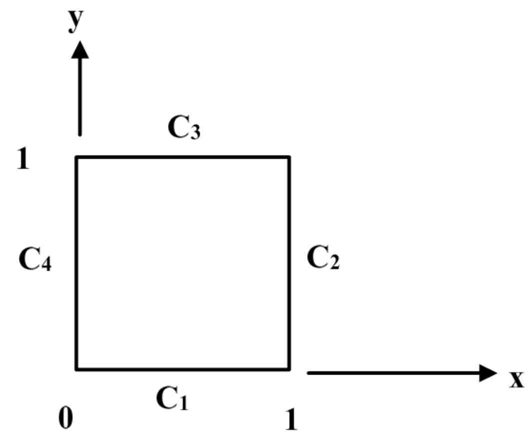
$\psi = 0$ on C_2 and C_3

$\partial\psi/\partial x = 0$ on C_4

4. Find an approximate solution of the following boundary value problem defined over a square domain

$$-\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}-12x-2y+4=0 \quad 0<x<1, \quad 0<y<1$$

Use the Rayleigh-Ritz method and try one or two parameter trial solution that is admissible.



$u = x^2$ on C_1

$u = y^2$ on C_4

$\partial u/\partial x = 2 - 2y - y^3$ on C_2

$\partial u/\partial y = 2 - 2x - x^3$ on C_3

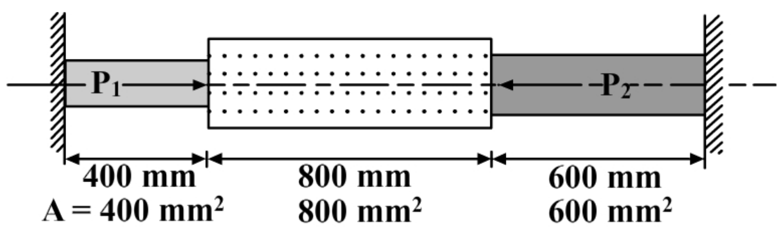
5. Determine the value of λ for which the following system has a non-trivial solution:

$$3x_1 + x_2 - \lambda x_3 = 0; \quad 4x_1 - x_2 - 3x_3 = 0; \quad 2\lambda x_1 + 3x_2 + \lambda x_3 = 0.$$

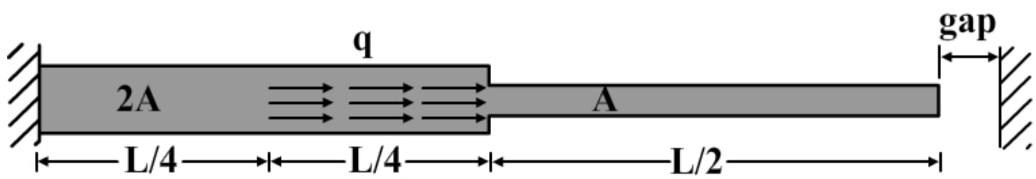
6. Is the following matrix positive definite?

$$K = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \ (k_i > 0, i = 1,2,3).$$

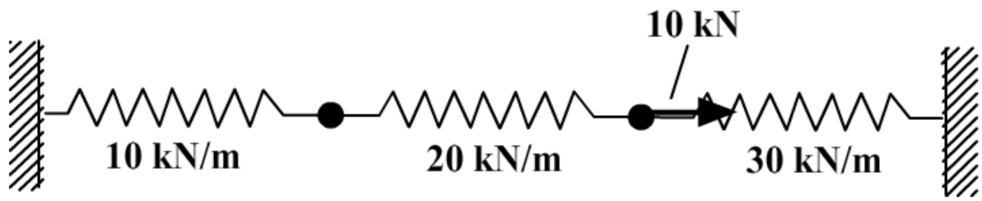
7. Using three elements determine the maximum stress in a bar loaded as shown in the following figure. Assume E = 72 GPa.



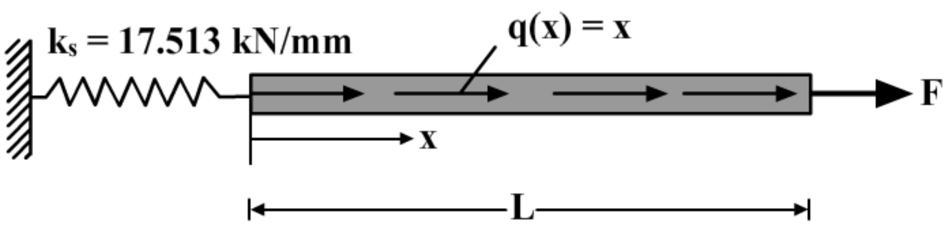
8. An axially loaded bar carries a distributed load over one quarter of its length and has a small gap at one end as shown in the following figure. Divide the bar into appropriate number of elements and compute displacements and stresses in the bar. Note that if the load is large enough to close the gap then the gap can be treated as a known displacement boundary condition. Assume L = 500 mm, A = 25 mm^2, E = 20,000 N/mm^2, q = 400 N/mm, (a) gap = 1 mm, (b) gap = 20 mm.



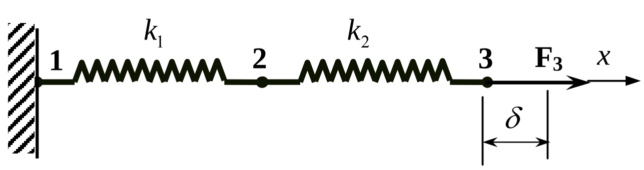
9. Three springs are joined together as shown in the following figure. Find joint displacements and forces in the springs.



10. A uniform bar carries a distributed axial load, and a concentrated load at its tip is supported by a spring at its support, as shown in the following figure. Using three linear elements, obtain stress distribution in the bar. Use E = 689.48 MPa, $A = 6.45\times10^{-4}$ m^2, L = 2.540 m, and F = 44.48 N.

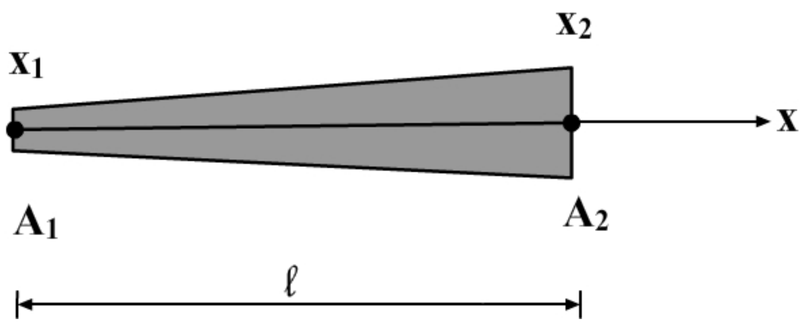


11. For the spring assemblage shown in the following figure, determine the displacements at node 2 and the forces in each spring element. Also determine the force F_3. Given: Node 3 displaces by an amount $\delta = 0.0254$ m in the positive x direction because of the force F_3 and $k_1 = k_2 = 175.126\, kN/mm$.



12. (a) Derive stiffness matrix for an axially loaded bar whose area of cross-section varies linearly as shown in the following figure.

$$\mathrm{A}(x) = -\frac{x - x_2}{\ell}\mathrm{A}_1 + \frac{x - x_1}{\ell}\mathrm{A}_2$$



(b) Using two elements developed in part (a) find maximum stress in a tapered bar subjected to concentrated load at its tip as shown in the following figure. Assume L = 0.508 m, E = 68.947 GPa, F = 4.448 kN, area at tip = 0.00129 m^2 and area at the fixed end = 0.00258 m^2.