Finite Element Analysis

Assignment 5

1. Evaluate following integrals using Gaussian Quadrature. Compare results by using 2,3 and 4 points.

$$\text{(a) } I=\int_{-1}^{1} e^{x^2}\ln(2-x)\,dx \qquad \text{(b) } I=\int_{-1}^{1} x\sin x\,dx$$

$$\text{(c) } I=\int_{0.2}^{0.8} e^{-2x}\tan x\,dx \qquad \text{(d) } I=\int_{0}^{1}\left(x^7+2x^2-1\right)dx$$

$$\text{(e) } I=\int_{1}^{3}\frac{dx}{\left(x^4+1\right)^{1/2}} \qquad \text{(f) } I=\int_{-2}^{2}\frac{dx}{1+x^2}$$

$$\text{(g) } I=\int_{0}^{1} x\exp\left(-3x^2\right)dx$$

2. Evaluate the following integrals by using 2,3 and 4 Gauss-Laguerre points.

$$(a)\int_{0}^{\infty} e^{-x}\cos x\,dx \quad (b)\int_{0}^{\infty}\frac{e^{-x}}{x+4}dx$$

3. Evaluate the following integrals using appropriate Gaussian quadrature.

$$(a)\int_{-1}^{1}\int_{-1}^{1} x\sin\left(x+y^2\right)dx\,dy \qquad (b)\int_{-1}^{1}\int_{-1}^{1}\left(t^3+s^2\right)ds\,dt$$

$$(c)\int_{1}^{2}\int_{3}^{4} f(x,y)\,dx\,dy \qquad (d)\int_{0}^{1}\int_{0}^{1}\exp\left(-x^2\right)y^2\,dx\,dy$$

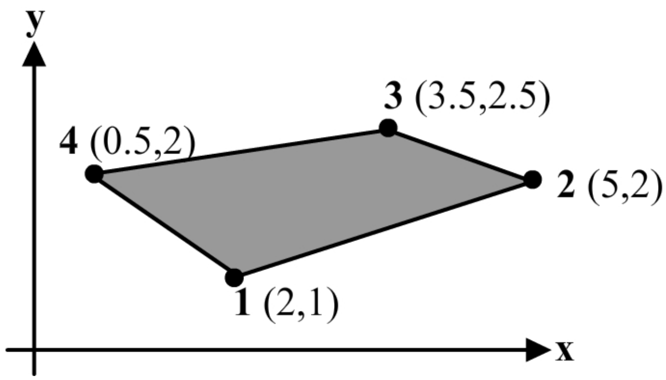
where $f(x,y)=(1)\ xy,(2)\ x^2y,(3)\ x^3y,(4)\ x^4y,$

$$(e)\int_{1}^{2}\int_{0}^{3} xy^3\,dx\,dy \qquad (f)\int_{-2}^{0}\int_{0}^{1} e^x\sin y\,dx\,dy$$

4. Evaluate the following integrals over a right triangle using 3 point integration

$$(a)\int_{0}^{1}\int_{0}^{1-t} t\sin\left(s+t^2\right)ds\,dt \quad (b)\int_{0}^{1}\int_{0}^{1-t}\left(t^3+s^2\right)ds\,dt$$

5. Obtain explicit expressions for isoparametric mapping for the element shown in the following figure. Is the mapping is fine? Compute the derivatives $\partial N_4/\partial x$, and $\partial N_4/\partial y$.

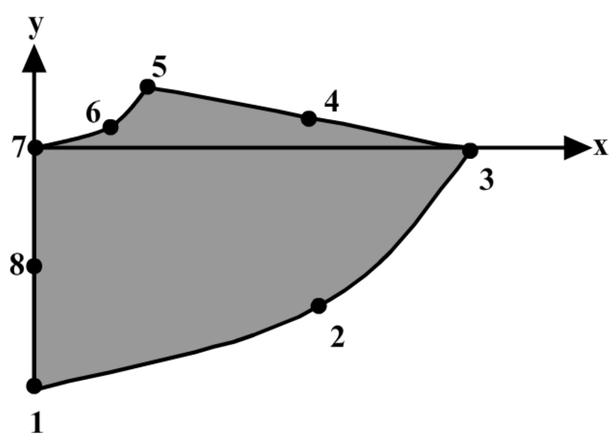


6. For the element shown in the following figure, the solution at the nodes is given as follows:

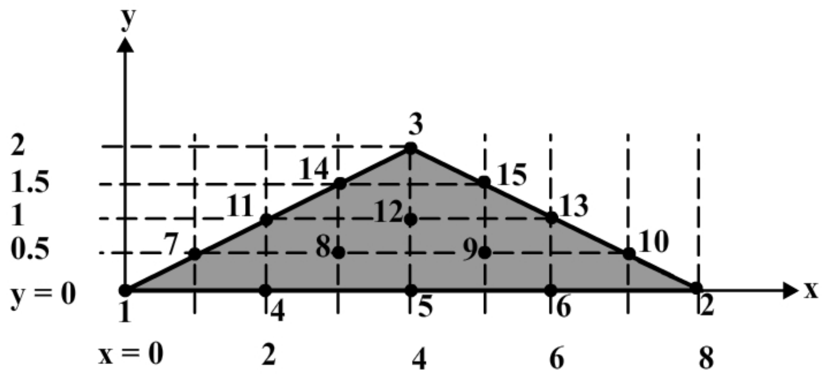
$$T = \begin{bmatrix} 0 & 10 & 20 & 0 & 0 & 50 & 0 & 0 \end{bmatrix}^T$$

Compute the solution and its x and y derivatives at the point $(1,-1)$. The nodal coordinate vectors are as follows:

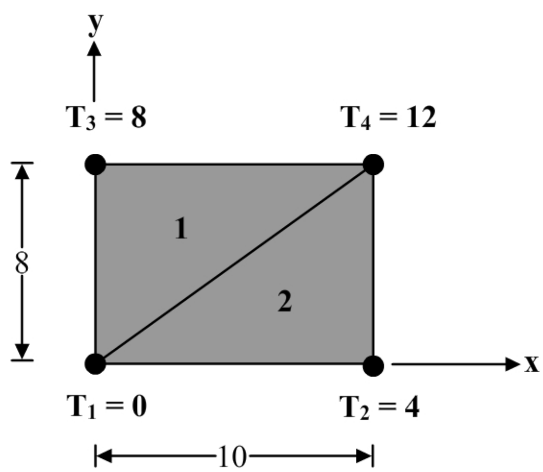
$$\boldsymbol{X}_n = \begin{bmatrix} 0.0 & 2.828 & 4.0 & 2.5 & 1.0 & 0.707 & 0.0 & 0.0 \end{bmatrix}^T$$
$$\boldsymbol{Y}_n = \begin{bmatrix} -4.0 & -2.828 & 0.0 & 0.5 & 1.0 & 0.293 & 0.0 & -2.0 \end{bmatrix}^T$$



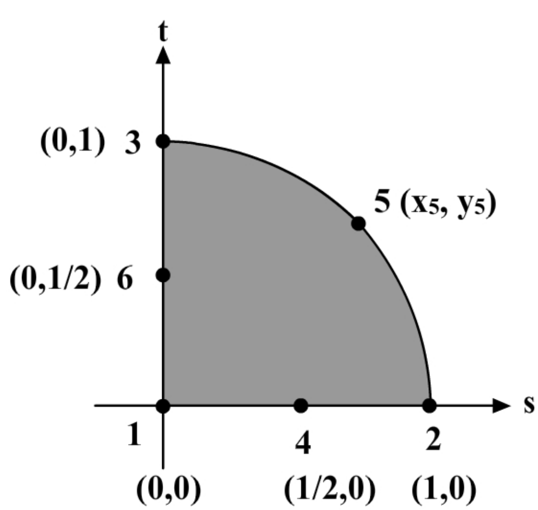
7. Develop shape functions N_1, N_4, N_{12} and N_{13} for the triangle element shown in the following figure. Compute the derivative $\partial N_{12}/\partial y$ using the isoparametric mapping and the chain rule. Express N_{12} explicitly in terms of x, y and verify its derivative with respect to y by direct differentiation.



8. A boundary value problem is solved using a mesh of triangular elements as shown in the following figure. The nodal values of the solution are indicated in the figure. Determine the complete solution $T(x, y)$ and its x and y derivatives.



9. A triangular element, with two sides straight and third a ¼ circle, is shown in the following figure. Show that the isoparametric mapping for the element is valid only if node 5 is placed such that $x_5 > 1/4$ and $y_5 > 1/4$.



Gauss Points ($\pm x_i$)	Weights (w_i)
n = 2	
0.57735 02691 89626	1.00000 00000 00000
n = 3	
0.00000 00000 00000	0.88888 88888 88888
0.77459 66692 41483	0.55555 55555 55555
n = 4	
0.33998 10435 84856	0.65214 51548 62546
0.86113 63115 94053	0.34785 48451 37454
n = 5	
0.00000 00000 00000	0.56888 88888 88889
0.53846 93101 05683	0.47862 86704 99366
0.90617 98459 38664	0.23692 68850 56189
n = 6	
0.23861 91860 83197	0.46791 39345 72691
0.66120 93864 66265	0.36076 15730 48139
0.93246 95142 03152	0.17132 44923 79170
n = 7	
0.00000 00000 00000	0.41795 91836 73469
0.40584 51513 77397	0.38183 00505 05119
0.74153 11855 99394	0.27970 53914 89277
0.94910 79123 42759	0.12948 49661 68870
n = 8	
0.18343 46424 95650	0.36268 37833 78362
0.52553 24099 16329	0.31370 66458 77887
0.79666 64774 13627	0.22238 10344 53374
0.96028 98564 97536	0.10122 85362 90376

n = 9	
0.00000 00000 00000	0.33023 93550 01260
0.32425 34234 03809	0.31234 70770 40003
0.61337 14327 00590	0.26061 06964 02935
0.83603 11073 26636	0.18064 81606 94857
0.96816 02395 07626	0.08127 43883 61574
n = 10	
0.14887 43389 81631	0.29552 42247 14753
0.43339 53941 29247	0.26926 67193 09996
0.67940 95682 99024	0.21908 63625 15982
0.86506 33666 88985	0.14945 13491 50581
0.97390 65285 17172	0.06667 13443 08688

Number of points	**Degree of accuracy**	**Integration Points**	**Weights**	
		s	t	
1	1	1/3	1/3	1/2
3	2	1/6	1/6	1/6
		2/3	1/6	1/6
		1/6	2/3	1/6
4	3	1/3	1/3	-9/32
		1/5	1/5	25/96
		3/5	1/5	25/96
		1/5	3/5	25/96

Gauss-Leguerre Integration Points & Weights

n	x_i	w_i
1	1.0	1.0
2	0.58578644 3.41421356	0.85355339 0.14644661
3	0.41577456 2.29428036	0.71109301 0.27851773

	6.28994508	0.01038926
4	0.32254769 1.74576110 4.53662030 9.39507091	0.60315410 0.35741869 0.03888791 0.00053929