Finite Element Analysis

Assignment 6

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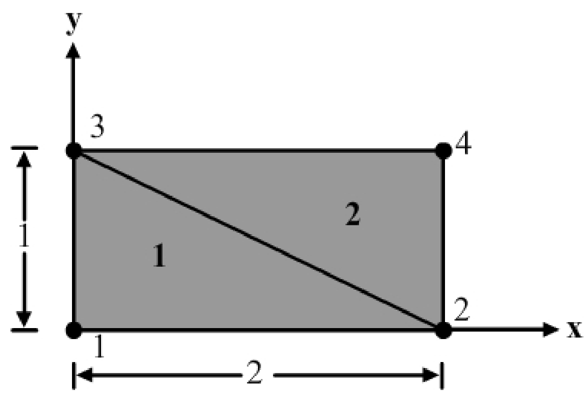
1. Obtain an approximate solution of the following boundary value problem using two linear triangular element as shown in the following figure.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=4 \qquad 0<x<2, \quad 0<y<1$$

with the boundary conditions:

along 1-2: T = 2

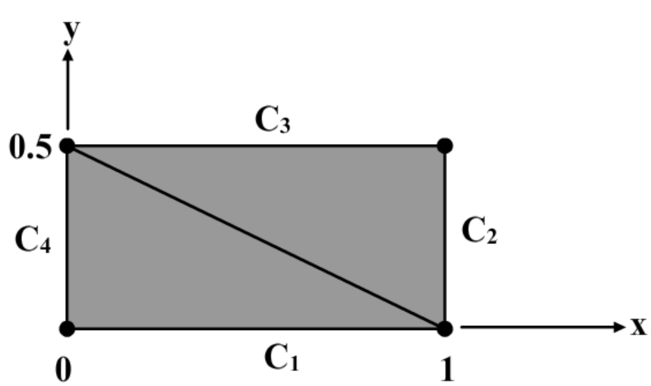
along the other boundaries: $\partial T/\partial n = 2$.



2. Obtain an approximate solution of the following boundary value problem using two linear triangular elements as shown in the following figure.

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+1=0 \qquad 0<x<1, \quad 0<y<0.5$$

with the boundary conditions



$\partial\psi/\partial y = 0$ on C_1

$\psi = 0$ on C_2 and C_3

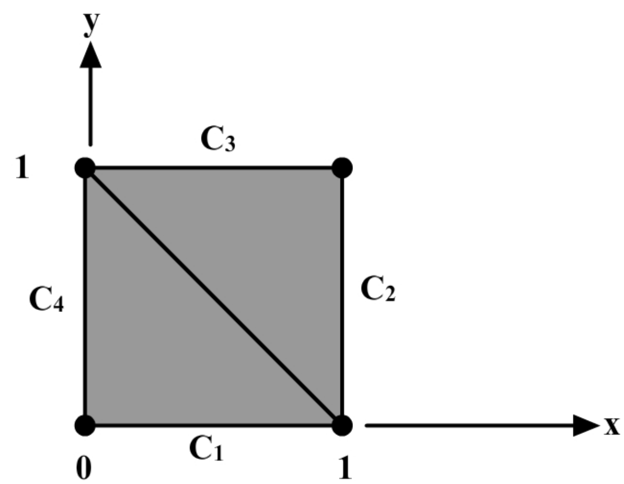
$\partial\psi/\partial x = 0$ on C_4

3. Obtain an approximate solution of the following boundary value problem using two linear triangular elements as shown in the following figure.

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - 2x - 2y + 4 = 0 \qquad 0 < x < 1, \quad 0 < y < 1$$

For non-constant coefficients, use values at element centroids as constant average values for the entire element with boundary conditions

$$\begin{aligned} u &= x^2 \text{ on } C_1 \\ u &= y^2 \text{ on } C_4 \\ \partial u/\partial x &= 2 - 2y - y^3 \text{ on } C_2 \\ \partial u/\partial y &= 2 - 2x - x^3 \text{ on } C_3 \end{aligned}$$



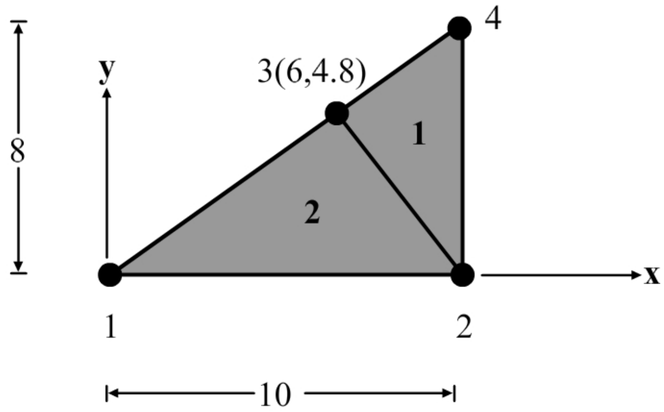
4. Obtain an approximate solution of the following boundary value problem using two linear triangular elements as shown in the following figure.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + 4T + 10 = 0$$

with the boundary conditions:

along 1-2: T = 2

along the other two boundaries: $\partial T/\partial n = 2T$.



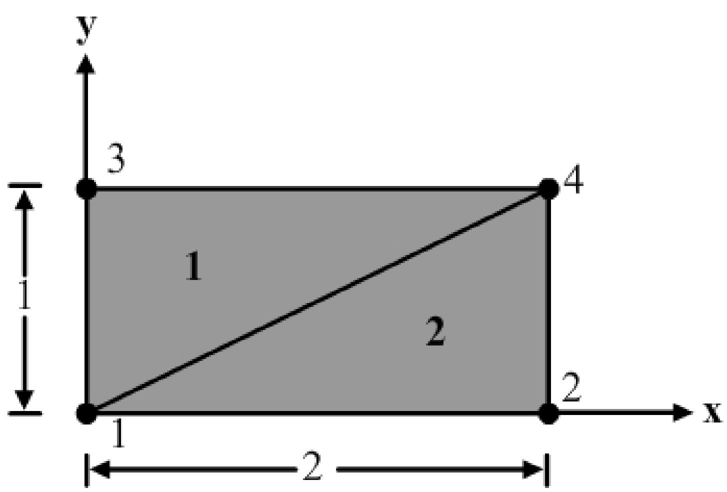
5. Find the lowest eigenvalue λ for the following problem using two linear triangular elements as shown in the following figure.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \lambda T = 0 \qquad 0 < x < 2, \quad 0 < y < 1$$

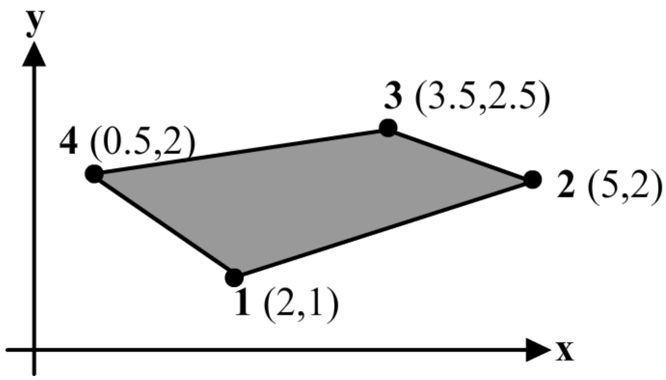
with the boundary conditions:

along 1-2: T = 0

along the other three boundaries: $\partial T / \partial n = 0$.



6. Obtain explicit expressions for isoparametric mapping for the element shown in the following figure. Is the mapping is fine? Compute the derivatives $\partial N_4 / \partial x$, and $\partial N_4 / \partial y$.

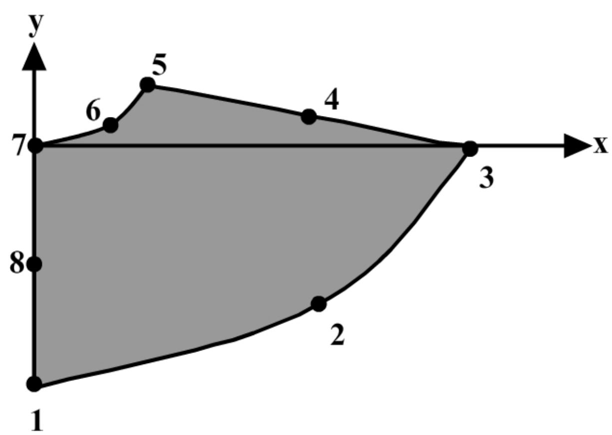


7. For the element shown in the following figure, the solution at the nodes is given as follows:

$$T = \begin{bmatrix} 0 & 10 & 20 & 0 & 0 & 50 & 0 & 0 \end{bmatrix}^T$$

Compute the solution and its x and y derivatives at the point $(1,-1)$. The nodal coordinate vectors are as follows:

$$\boldsymbol{X}_n = \begin{bmatrix} 0.0 & 2.828 & 4.0 & 2.5 & 1.0 & 0.707 & 0.0 & 0.0 \end{bmatrix}^T$$
$$\boldsymbol{Y}_n = \begin{bmatrix} -4.0 & -2.828 & 0.0 & 0.5 & 1.0 & 0.293 & 0.0 & -2.0 \end{bmatrix}^T$$



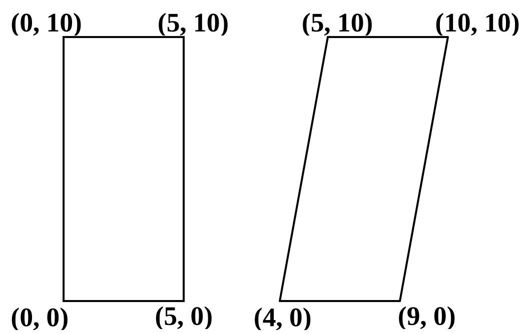
8. Evaluate matrices $\boldsymbol{k}_p = -\iint_A PNN^T \; dA$ and vector $\boldsymbol{r}_\beta = -\int_{S_2} \beta \boldsymbol{N} \; dS$ for the element in Problem 7. Assume $P = 2$ and $\beta = -2$ resulting from a natural boundary condition on side 5−6−7. Use 3×3 integration. Show complete calculations for at least one Gauss point.

9. The state of stress at a point is given as follows:

$$\sigma_{xx} = y^2 + c\left(x^2 - y^2\right)$$
$$\sigma_{yy} = x^2 + c\left(y^2 - x^2\right)$$
$$\sigma_{zz} = \left(x^2 + y^2\right)$$
$$\sigma_{xy} = f(x, y)$$
$$\sigma_{yz} = \sigma_{xz} = 0$$

Determine $f(x,y)$ so that the stress distribution may be in equilibrium in the absence of body forces.

10. Develop a deformation field $u(x,y)$, $v(x,y)$ that describe the deformation of the finite element shown in the following figure. From this determine $\varepsilon_x, \varepsilon_y, \gamma_{xy}$. Interpret your answer.



11. Compute stresses and strains at a point located at (2,2) for the problem shown in the following figure, using only one quadrilateral element. Assume plane strain conditions. E = 20.6842GPa, ν = 0.25, thickness = 0.0254 m.