Finite Element Analysis FINAL EXAMINATION (Closed Book)

Answer all questions. Maximum marks: 50

All questions carry equal marks. Time: 3 Hours

Question 1:

Using the Rayleigh-Ritz method solve the following equation in a square region:

$$-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=g_0$$

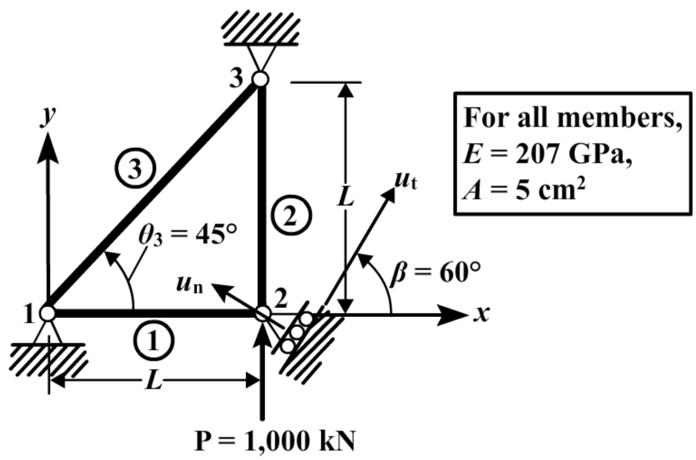
$$T=0 \text{ on sides } x=1 \text{ and } y=1$$

$$\frac{\partial T}{\partial n}=0 \text{ (insulated) on sides } x=0 \text{ and } y=0$$

using a one-parameter approximation of the form $T=c_1\left(1-x^2\right)\left(1-y^2\right)$.

Question 2:

Determine the forces and displacements of points B and C of the structure shown in the figure below.



Question 3:

Derive Hermite interpolation functions for a two node beam element with three primary variables at each node: (w,θ,κ) where $\theta=\frac{dw}{dx}$, and $\kappa=\frac{d^2w}{dx^2}$.

Question 4:

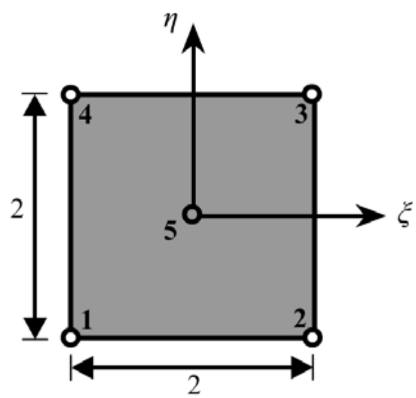
Evaluate the following integrals using the Newton—Cotes and Gauss quadratures when ψ_i are the quadratic interpolation functions.

$$K_{12} = \int_{x_a}^{x_b} (x_0 + x) \frac{d\psi_1}{dx} \frac{d\psi_2}{dx} dx \quad G_{12} = \int_{x_a}^{x_b} (x_0 + x) \psi_1 \psi_2 dx$$

where $\psi_1 = \frac{1}{2}(1-\xi)$, and $\psi_2 = \frac{1}{2}(1+\xi)$. Use the appropriate number of integration points.

Question 5:

Determine the shape functions for the five-node rectangular element shown in the figure below.

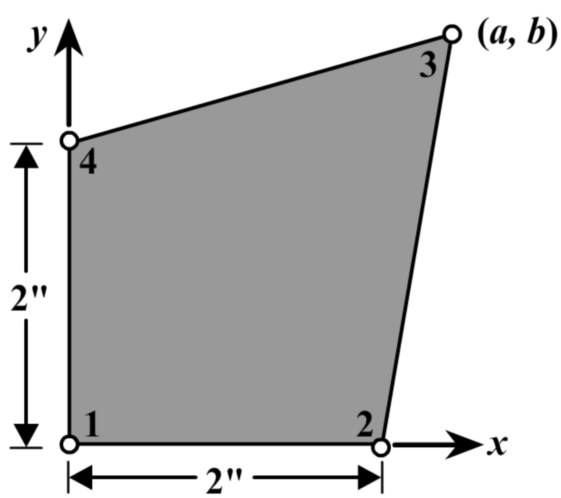


Question 6:

Determine the conditions on the location of node 3 of the quadrilateral element shown in the figure below. Show that the transformation equations are given by

$$x = \frac{1}{4}(1+\xi)\left[2(1-\eta) + a(1+\eta)\right]$$

$$y = \frac{1}{4}(1+\eta)\left[2(1-\xi) + b(1+\xi)\right]$$



$$\int_a^b f(x)\,dx \simeq C_0 h \sum_{i=1}^{n} W_i f(x_i) + C_1 h^{k+1} f^{(k)}(\xi)$$

n	***C*₀**	***W*₁**	***W*₂**	***W*₃**	***W*₄**	***W*₅**	***C*₁**	***k***	**Name**
1	1	1					1/2	1	Rectangle
2	1/2	1	1				–1/12	2	Trapezium
3	1/3	1	4	1			–1/90	4	Simpson
4	3/8	1	3	3	1		–3/80	4	4-point
5	2/45	7	32	12	32	7	–8/945	6	5-point

Gauss Points ($\pm x_i$)	Weights (w_i)
n = 2	
0.57735 02691 89626	1.00000 00000 00000
n = 3	
0.00000 00000 00000	0.88888 88888 88888
0.77459 66692 41483	0.55555 55555 55555
n = 4	
0.33998 10435 84856	0.65214 51548 62546
0.86113 63115 94053	0.34785 48451 37454
n = 5	
0.00000 00000 00000	0.56888 88888 88889
0.53846 93101 05683	0.47862 86704 99366
0.90617 98459 38664	0.23692 68850 56189
n = 6	
0.23861 91860 83197	0.46791 39345 72691
0.66120 93864 66265	0.36076 15730 48139
0.93246 95142 03152	0.17132 44923 79170

n = 7	
0.00000 00000 00000	0.41795 91836 73469
0.40584 51513 77397	0.38183 00505 05119
0.74153 11855 99394	0.27970 53914 89277
0.94910 79123 42759	0.12948 49661 68870
n = 8	
0.18343 46424 95650	0.36268 37833 78362
0.52553 24099 16329	0.31370 66458 77887
0.79666 64774 13627	0.22238 10344 53374
0.96028 98564 97536	0.10122 85362 90376
n = 9	
0.00000 00000 00000	0.33023 93550 01260
0.32425 34234 03809	0.31234 70770 40003
0.61337 14327 00590	0.26061 06964 02935
0.83603 11073 26636	0.18064 81606 94857
0.96816 02395 07626	0.08127 43883 61574
n = 10	
0.14887 43389 81631	0.29552 42247 14753
0.43339 53941 29247	0.26926 67193 09996
0.67940 95682 99024	0.21908 63625 15982
0.86506 33666 88985	0.14945 13491 50581
0.97390 65285 17172	0.06667 13443 08688