Assignment – Module 1

1. Check whether the function

$$f(x) = \frac{3(1 - x^2)}{4} \qquad\qquad -1 < x < 1$$

$$= 0 \qquad\qquad\qquad \text{elsewhere}$$

is a probability density function (pdf). If the function is a pdf, obtain $P[0.25 \le X \le 0.5]$

2. The pdf of a random variable X is given as

$$f(x) = ce^{-2x} \qquad\qquad x \ge 0$$

    i.   Obtain the value of 'c',

    ii.   Obtain the CDF

    iii.   Obtain the value of x if $P[X \le x] = 0.5$

3. The joint pdf of two random variables X and Y is given as

$$f(x,y) = c(2x+y) \qquad\qquad 0 < x < 1, \, 0 < y < 1$$

$$= 0 \qquad\qquad\qquad \text{elsewhere}$$

Obtain the constant 'c' and obtain $P[X > 0.5, Y > 0.5]$

4. Obtain the marginal density function of X and the marginal density function of Y for the joint pdf in problem no.3.

Also obtain $P[X > 0.5]*P[Y > 0.5]$. Is this probability the same as $P[X > 0.5, Y > 0.5]$? Explain the reason.

5. The joint pdf of two random variables X and Y is given as

$$f(x,y) = cxy \qquad\qquad 0 \le x \le 1, \, 1 \le y \le 2$$

$$= 0 \qquad\qquad\qquad \text{elsewhere}$$

    i.   Obtain the value of 'c',

ii.   Obtain P[0.5 < X < 0.75, 0.75 < Y < 1.25]

iii.   Obtain P[X+Y < 2]

6. Check whether the random variables X and Y are dependent for the joint pdf in problem no.5.

7. The joint pdf of two random variables X and Y is given as

$$f(x, y) = \frac{6}{5}(x^2 + y) \qquad 0 \le x \le 1, 0 \le y \le 1$$

$$= 0 \qquad \text{elsewhere}$$

   i.   Obtain the conditional density function of X given Y,

   ii.   Obtain the conditional density function of Y given X

   iii. Obtain P[0.5 < X < 0.75 | Y = 0.25]

8. Obtain the pdf of Y, related to random variable X as $Y = X^2$, the pdf of X is

$$f(x) = 3e^{-3x} \qquad x \ge 0$$

9. Consider the joint pdf of X and Y

$$f(x, y) = \frac{2x + y}{4} \qquad 0 \le x \le 1, 0 \le y \le 2$$

$$= 0 \qquad \text{elsewhere}$$

   If $U = X + Y$ and $V = Y$, obtain the joint pdf of U and V

10. Consider the joint pdf of X and Y

$$f(x, y) = xy \qquad 0 \le x \le 1, 0 \le y \le 2$$

$$= 0 \qquad \text{elsewhere}$$

   i.   Obtain E(X), E(Y), E(X^2), E(Y^2)

   ii.   Check if E(X+Y) = E(X) + E(Y) and E(XY) = E(X)E(Y).

11. Obtain the sample estimates of mean, standard deviation, coefficient of variation, coefficient of skewness and kurtosis for the following observed data of daily maximum rainfall (mm) for 30 years

Year	1	2	3	4	5	6	7	8	9	10
Daily rainfall (mm)	116	93	99	65	90	153	95	72	62	58

Year	11	12	13	14	15	16	17	18	19	20
Daily rainfall (mm)	136	89	69	77	75	122	104	68	449	78

Year	21	22	23	24	25	26	27	28	29	30
Daily rainfall (mm)	82	147	83	71	51	72	90	191	121	91