INDIAN INSTITUTE OF SCIENCE

STOCHASTIC HYDROLOGY

Lecture -29

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Summary of the previous lecture

- Goodness of fit
  - Chi-square test

$$\chi^2_{data} = \sum_{i=1}^{k} \frac{\left(N_i - E_i\right)^2}{E_i} \qquad \chi^2_{data} < \chi^2_{1-\alpha,k-p-1}$$

  - Kolmogorov-Smirnov test

$$\Delta = maximum \left| P\left(x_i\right) - F\left(x_i\right) \right|$$

$\Delta < \Delta_o$

INTENSITY-DURATION-FREQUENCY (IDF) CURVES

IDF Curves

- Hydrologic Designs : First step is the determination of design rainfall
  - e.g., urban drainage system - rainfall intensity used in designs
- Intensity – Duration – Frequency (IDF) relationships (curves) used for the purpose.
- An IDF curve gives the expected rainfall intensity of a given duration of storm having desired frequency of occurrence.
- IDF curve is a graph with duration plotted as abscissa, intensity as ordinate and a series of curves, one for each return period.

IDF Curves

- The intensity of rainfall is the rate of precipitation, i.e., depth of precipitation per unit time
- This can be either instantaneous intensity or average intensity over the duration of rainfall
- The average intensity is commonly used.

$$i = \frac{P}{t}$$

where P is the rainfall depth

t is the duration of rainfall

IDF Curves

- The frequency is expressed in terms of return period (T) which is the average length of time between rainfall events that equal or exceed the given (design) magnitude.
- If local rainfall data is available, IDF curves can be developed using frequency analysis.
- A minimum of 20 years data is desirable.

IDF Curves

Procedure for developing IDF curves:

Step 1: Preparation of annual maximum data series

- From the available rainfall data, rainfall series for different durations (e.g.,1-hour, 2-hour, 6-hour, 12-hour and 24-hour) are developed.
- For each selected duration, the annual maximum rainfall depths are calculated.

IDF Curves

Step 2: Fitting the probability distribution

- A suitable probability distribution is fitted to the each selected duration data series .
- Generally used probability distributions are
  - Gumbel's Extreme Value distribution
  - Gamma distribution (two parameter)
  - Log Pearson Type III distribution
  - Normal distribution
  - Log-normal distribution (two parameter)

IDF Curves

Statistical distributions and their functions:

Distribution	PDF
Gumbel's EV-I	$f(x) = \exp\left\{-(x-\beta)/\alpha - \exp\left[-(x-\beta)/\alpha\right]\right\}/\alpha$ $\quad -\infty \le x \le \infty$
Gamma	$f(x) = \dfrac{\lambda^n x^{\eta-1} e^{-\lambda x}}{\Gamma(\eta)} \quad x, \lambda, \eta > 0$
Log Pearson Type-III	$f(x) = \dfrac{\lambda^\beta (y-\varepsilon)^{\beta-1} e^{-\lambda(y-\varepsilon)}}{x\Gamma(\beta)} \quad \log x \ge \varepsilon$
Normal	$f(x) = \dfrac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right\} -\infty < x < \infty$
Log-Normal	$f(x) = \dfrac{1}{\sqrt{2\pi x\sigma_x}} \mathrm{e}^{-(\ln x-\mu_x)^2/2\sigma_x^2} \quad 0 < x < \infty$

IDF Curves

- Gumbel's Extreme Value Type-I distribution, is most commonly used for IDF relationships.
- The Kolmogorov-Smirnov or the Chi-Square goodness of fit test is used to evaluate the accuracy of the fitting of a distribution.

IDF Curves

Step 3: Determining the rainfall depths

- Using frequency factors, or
- Using the CDF of the distribution (by inverting the CDF).

IDF Curves

Using frequency factors:

- The precipitation depth is calculated for a given return period as

$$x_T = \bar{x} + K_T s$$

where

$\bar{x}$: mean,

s : standard deviation, and

K_T : frequency factor for return period T.

IDF Curves

Using CDF of a distribution:

$$P\left(X \geq x_T\right) = \frac{1}{T}; \qquad 1 - P\left(X < x_T\right) = \frac{1}{T}$$

$$1 - F\left(x_T\right) = \frac{1}{T}; \qquad F\left(x_T\right) = 1 - \frac{1}{T} = \frac{T-1}{T}$$

$$x_T = F^{-1}\left(\frac{T-1}{T}\right)$$

Example – 1

Bangalore rainfall data for 33 years is considered to demonstrate the construction of IDF curves.

Durations of the rainfall : 1-hour, 2-hour, 6-hour, 12-hour and 24-hour.

Return periods : 2-year, 5-year, 10-year, 50-year and 100-year.

Example – 1 (Contd.)

- The rainfall for different durations are obtained from the rain gauge data.
- The rainfall data is converted to intensity by dividing the rainfall with duration (i = P/t).
- The mean and standard deviation of the data for all the selected durations is calculated.
- The frequency factor is calculated for all the selected return periods, based on the selected distribution.
- The design rainfall intensity is obtained by

$$x_T = \bar{x} + K_T s$$

Example – 1 (Contd.)

Max. rainfall (mm) for different durations.

Year	1H	2H	6H	12H	24H
1969	44.5	61.6	104.1	112.3	115.9
1970	37	48.2	62.5	69.6	92.7
1971	41	52.9	81.4	86.9	98.7
1972	30	40	53.9	57.8	65.2
1973	40.5	53.9	55.5	72.4	89.8
1974	52.4	62.4	83.2	93.4	152.5
1975	59.6	94	95.1	95.1	95.3
1976	22.1	42.9	61.6	64.5	71.7
1977	42.2	44.5	47.5	60	61.9
1978	35.5	36.8	52.1	54.2	57.5
1979	59.5	117	132.5	135.6	135.6
1980	48.2	57	82	86.8	89.1
1981	41.7	58.6	64.5	65.1	68.5
1982	37.3	43.8	50.5	76.2	77.2
1983	37	60.4	70.5	72	75.2
1984	60.2	74.1	76.6	121.9	122.4

Year	1H	2H	6H	12H	24H
1986	65.2	73.7	97.9	103.9	104.2
1987	47	55.9	64.8	65.6	67.5
1988	148.8	210.8	377.6	432.8	448.7
1989	41.7	47	51.7	53.7	78.1
1990	40.9	71.9	79.7	81.5	81.6
1991	41.1	49.3	63.6	93.2	147
1992	31.4	56.4	76	81.6	83.1
1993	34.3	36.7	52.8	68.8	70.5
1994	23.2	38.7	41.9	43.4	50.8
1995	44.2	62.2	72	72.2	72.4
1996	57	74.8	85.8	86.5	90.4
1997	50	71.1	145.9	182.3	191.3
1998	72.1	94.6	111.9	120.5	120.5
1999	59.3	62.9	82.3	90.7	90.9
2000	62.3	78.3	84.3	84.3	97.2
2001	46.8	70	95.9	95.9	100.8
2003	53.2	86.5	106.1	106.2	106.8

Example – 1 (Contd.)

The rainfall intensity (mm/hr) for different durations.

Year	1H	2H	6H	12H	24H	Year	1H	2H	6H	12H	24H
1969	44.50	30.80	17.35	9.36	4.83	1986	65.20	36.85	16.32	8.66	4.34
1970	37.00	24.10	10.42	5.80	3.86	1987	47.00	27.95	10.80	5.47	2.81
1971	41.00	26.45	13.57	7.24	4.11	1988	148.80	105.40	62.93	36.07	18.70
1972	30.00	20.00	8.98	4.82	2.72	1989	41.70	23.50	8.62	4.48	3.25
1973	40.50	26.95	9.25	6.03	3.74	1990	40.90	35.95	13.28	6.79	3.40
1974	52.40	31.20	13.87	7.78	6.35	1991	41.10	24.65	10.60	7.77	6.13
1975	59.60	47.00	15.85	7.93	3.97	1992	31.40	28.20	12.67	6.80	3.46
1976	22.10	21.45	10.27	5.38	2.99	1993	34.30	18.35	8.80	5.73	2.94
1977	42.20	22.25	7.92	5.00	2.58	1994	23.20	19.35	6.98	3.62	2.12
1978	35.50	18.40	8.68	4.52	2.40	1995	44.20	31.10	12.00	6.02	3.02
1979	59.50	58.50	22.08	11.30	5.65	1996	57.00	37.40	14.30	7.21	3.77
1980	48.20	28.50	13.67	7.23	3.71	1997	50.00	35.55	24.32	15.19	7.97
1981	41.70	29.30	10.75	5.43	2.85	1998	72.10	47.30	18.65	10.04	5.02
1982	37.30	21.90	8.42	6.35	3.22	1999	59.30	31.45	13.72	7.56	3.79
1983	37.00	30.20	11.75	6.00	3.13	2000	62.30	39.15	14.05	7.03	4.05
1984	60.20	37.05	12.77	10.16	5.10	2001	46.80	35.00	15.98	7.99	4.20
						2003	53.20	43.25	17.68	8.85	4.45

Example – 1 (Contd.)

The mean and standard deviation for the data for different durations is calculated.

Duration	1H	2H	6H	12H	24H
Mean	48.70	33.17	14.46	8.05	4.38
Std. Dev.	21.53	15.9	9.59	5.52	2.86

Example – 1 (Contd.)

K_T values are calculated for different return periods using Gumbel's distribution

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

T (years)	2	5	10	50	100
K_T	-0.164	0.719	1.305	2.592	3.137

Example – 1 (Contd.)

The rainfall intensities are calculated using

$$x_T = \bar{x} + K_T s$$

For example,

For duration of 2 hour, and 10 year return period,

Mean $\bar{x}$ = 33.17 mm/hr,

Standard deviation s = 15.9 mm/hr

Frequency factor K_T = 1.035

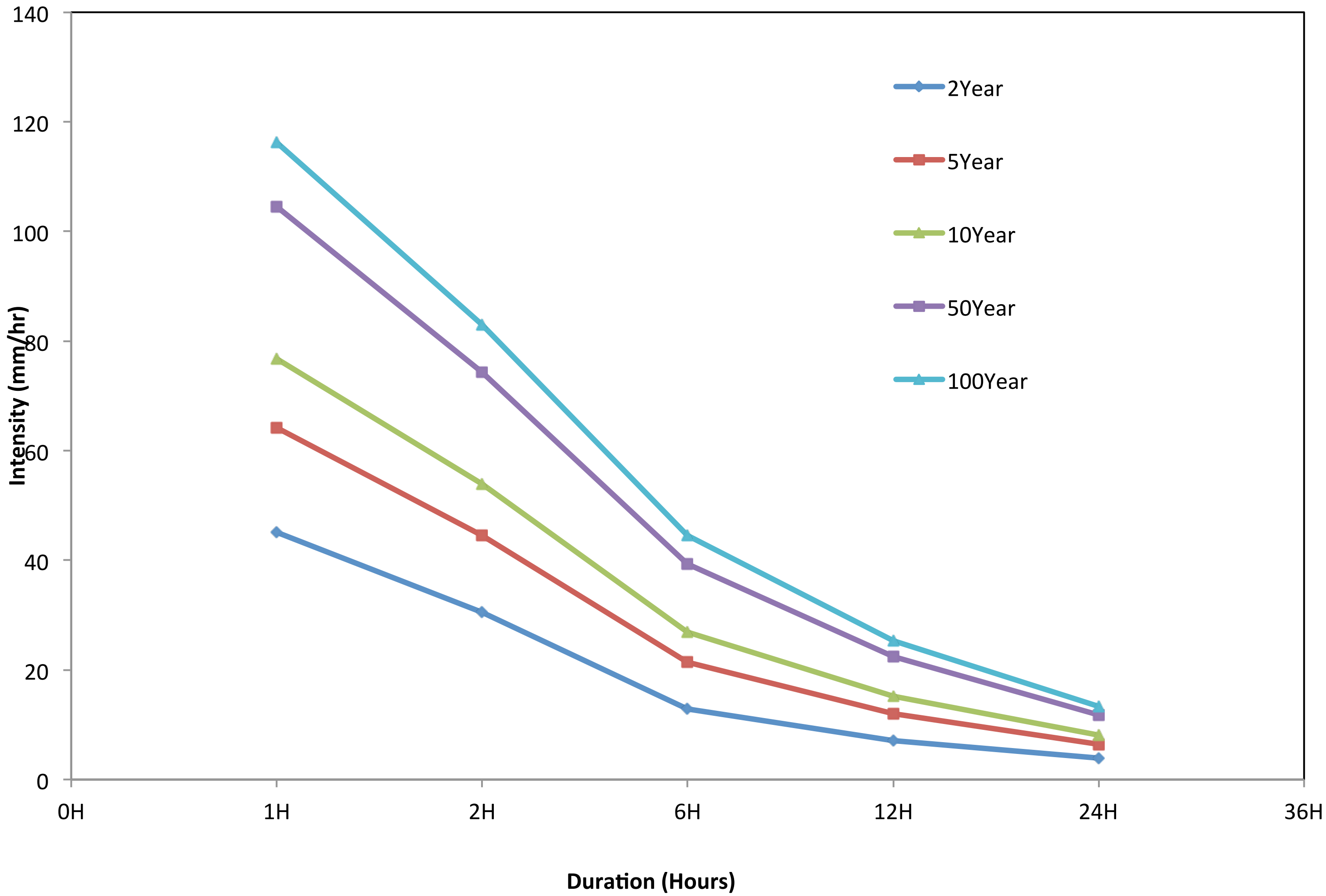
Example – 1 (Contd.)

x_T = 33.17 + 1.035 * 15.9
= 53.9 mm/hr.

The intensities (mm/hr) for other durations are tabulated.

Duration (hours)	Return Period T (Years)				
	2	5	10	50	100
1H	45.17	64.19	76.79	104.51	116.23
2H	30.55	44.60	53.90	74.36	83.02
6H	12.89	21.36	26.97	39.31	44.53
12H	7.14	12.02	15.25	22.36	25.37
24H	3.91	6.44	8.11	11.79	13.35

Example – 1 (Contd.)



IDF Curves

Equations for IDF curves:

- IDF curves can also be expressed as empirical equations

$$i = \frac{c}{t^e + f}$$

$$i = \frac{c\,T^m}{t^e + f}$$

Return Period

where

i is the design rainfall intensity,

t is the duration, and

c, e and f are coefficients varying with location and return period.

Ref: Applied Hydrology by V.T.Chow, D.R.Maidment, L.W.Mays, McGraw-Hill 1998

IDF Curves

IDF Equations for Indian region:

- Rambabu et. al. (1979) developed an equation analyzing rainfall characteristics for 42 stations.

$$i = \frac{KT^a}{\left(t + b\right)^n}$$

where

i is the rainfall intensity in cm/hr,

T is the return period in years,

t is the storm duration in hours, and

K, a, b and n are coefficients varying with location.

Ref: Ram Babu, Tejwani, K. K., Agrawal, M. C. & Bhusan, L. S. (1979) - Rainfall intensityduration-return period equations & nomographs of India, CSWCRTI, ICAR, Dehradun, India

IDF Curves

Coefficients for a few locations are given below

Location	K	a	b	n
Agra	4.911	0.167	0.25	0.629
New Delhi	5.208	0.157	0.5	1.107
Nagpur	11.45	0.156	1.25	1.032
Bhuj	3.823	0.192	0.25	0.990
Gauhati	7.206	0.156	0.75	0.940
Bangalore	6.275	0.126	0.5	1.128
Hyderabad	5.25	0.135	0.5	1.029
Chennai	6.126	0.166	0.5	0.803

Ref: Ram Babu, Tejwani, K. K., Agrawal, M. C. & Bhusan, L. S. (1979) - Rainfall intensityduration-return period equations & nomographs of India, CSWCRTI, ICAR, Dehradun, India

IDF Curves

IDF Equations for Indian region:

- Kothyari and Garde (1992) developed a general relationship for IDF analyzing data from 80 rain gauge stations.

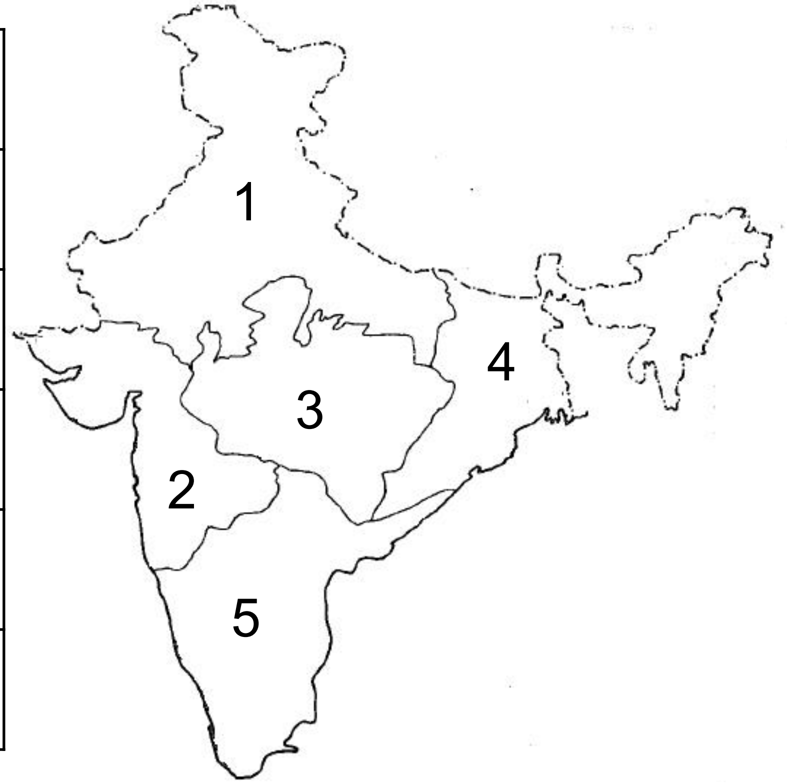
$$i_t^T = C \frac{T^{0.20}}{t^{0.71}} \left(R_{24}^2 \right)^{0.33}$$

i_t^T is the rainfall intensity in mm/hr for T year return period and t hour duration,
C is a constant, and
R_{24}^2 is rainfall for 2-year return period and 24-hour duration in mm.

Ref: Kothyari, U.C., and Garde, R. J. (1992), - Rainfall intensity - duration-frequency formula for India, Journal of Hydraulics Engineering, ASCE, 118(2)

IDF Curves

Zone	Location	C
1	Northern India	8.0
2	Western India	8.3
3	Central India	7.7
4	Eastern India	9.1
5	Southern India	7.1



Ref: Kothyari, U.C., and Garde, R. J. (1992), - Rainfall intensity - duration-frequency formula for India, Journal of Hydraulics Engineering, ASCE, 118(2)

Example – 2

Obtain the design rainfall intensity for 10 year return period with 6 hour duration for Bangalore.

Compare the intensity obtained from the IDF curve derived earlier based on observed data.

Solution:

Rambabu et. al. (1979)

$$i = \frac{KT^a}{\left(t + b\right)^n}$$

Example – 2 (Contd.)

For Bangalore, the constants are as follows

K = 6.275

a = 0.126

b = 0.5

n = 1.128

For T = 10 Year and t = 6 hour,

$$i = \frac{6.275 \times 10^{0.126}}{(6+0.5)^{1.128}} = 1.015 \text{ cm/hr} = 10.15 \text{ mm/hr}$$

Example – 2 (Contd.)

Using Kothyari and Garde (1992) formula,

$$i_t^T = C\frac{T^{0.20}}{t^{0.71}}\left(R_{24}^2\right)^{0.33}$$

C = 7.1 for South India

T = 10 years, t = 6 hr

R_{24}^2 = 93.84 mm

$$i_6^{10} = 7.1\frac{10^{0.20}}{6^{0.71}}\left(93.84\right)^{0.33} = 14.11 \text{ mm/hr}$$

Example – 2 (Contd.)

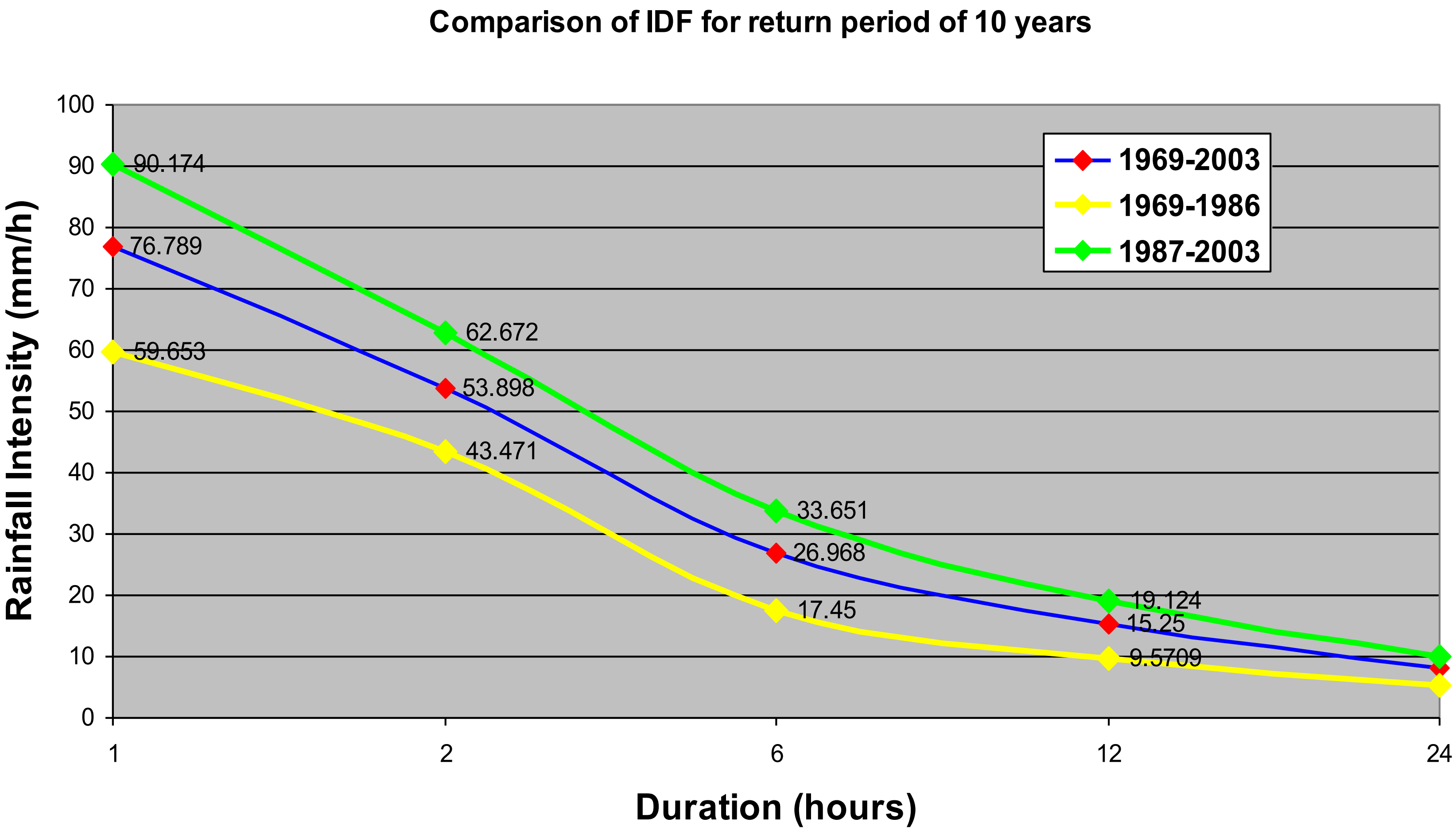
Rainfall intensity:

Rambabu et. al. (1979) :10 mm/hr,

Kothyari and Garde (1992) :14.11 mm/hr

IDF curve from the observed data : 26 mm/hr.

IDF Curves



Comparison of IDF for return period of 10 years
Rainfall Intensity (mm/h)
Duration (hours)
100
90
80
70
60
50
40
30
20
10
0
1
2
6
12
24
1969-2003
1969-1986
1987-2003
90.174
76.789
59.653
62.672
53.898
43.471
33.651
26.968
17.45
19.124
15.25
9.5709

IDF Curves

Design precipitation Hyetographs from IDF relationships:

- Simple way of developing a design hyetograph from an IDF curve.
- Design hyetograph developed by this method specifies the precipitation depth occurring in n successive time intervals of duration Δt over a total duration T_d.
- The design return period is selected and the intensity is read from the IDF curve for each of the durations.

IDF Curves

Alternating block method :

- Simple way of developing a design hyetograph from an IDF curve.
- Design hyetograph developed by this method specifies the precipitation depth occurring in n successive time intervals of duration Δt over a total duration T_d.
- The design return period is selected and the intensity is read from the IDF curve for each of the durations

IDF Curves

- The corresponding precipitation depth found as the product of intensity and duration.
- By taking differences between successive precipitation depth values, the amount of precipitation to be added for each additional unit of time Δt is found.
- The increments are rearranged into a time sequence with maximum intensity occurring at the center of the duration and the remaining blocks arranged in descending order alternatively to the right and left of the central block to form the design hyetograph.

Example – 3

Obtain the design precipitation hyetograph for a 2-hour storm in 10 minute increments in Bangalore with a 10 year return period.

Solution:

The 10 year return period design rainfall intensity for a given duration is calculated using IDF formula by Rambabu et. al. (1979)

$$i = \frac{KT^a}{(t+b)^n}$$

Example – 3 (Contd.)

For Bangalore, the constants are

K = 6.275

a = 0.126

b = 0.5

n = 1.128

For T = 10 Year and duration, t = 10 min = 0.167 hr,

$$i = \frac{6.275 \times 10^{0.126}}{(0.167 + 0.5)^{1.128}} = 13.251$$

Example – 3 (Contd.)

- Similarly the values for other durations at interval of 10 minutes are calculated.
- The precipitation depth is obtained by multiplying the intensity with duration.

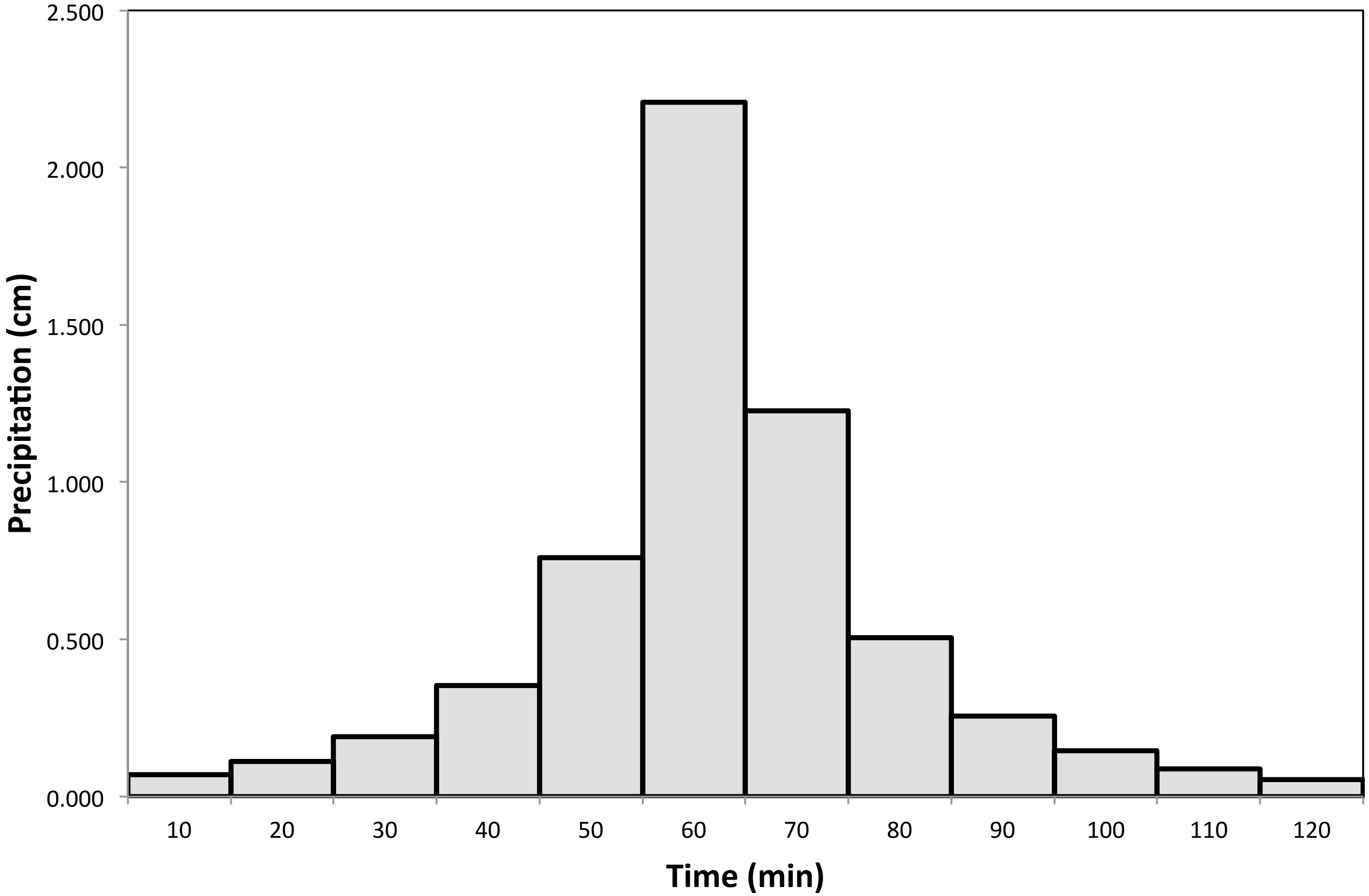
Precipitation = 13.251 * 0.167 = 2.208 cm

- The 10 minute precipitation depth is 2.208 cm compared with 3.434 cm for 20 minute duration, hence the most intense of 20 minutes of the design storm, 2.208 cm will fall in 10 minutes, the remaining 1.226 (= 3.434 – 2.208) cm will fall in the remaining 10 minutes.
- Similarly the other values are calculated and tabulated

Example – 3 (Contd.)

Duration (min)	Intensity (cm/hr)	Cumulative depth (cm)	Incremental depth (cm)	Time (min)	Precipitation (cm)
10	13.251	2.208	2.208	0 - 10	0.069
20	10.302	3.434	1.226	10 - 20	0.112
30	8.387	4.194	0.760	20 - 30	0.191
40	7.049	4.699	0.505	30 - 40	0.353
50	6.063	5.052	0.353	40 - 50	0.760
60	5.309	5.309	0.256	50 - 60	2.208
70	4.714	5.499	0.191	60 - 70	1.226
80	4.233	5.644	0.145	70 - 80	0.505
90	3.838	5.756	0.112	80 - 90	0.256
100	3.506	5.844	0.087	90 - 100	0.145
110	3.225	5.913	0.069	100 - 110	0.087
120	2.984	5.967	0.055	110 - 120	0.055

Example – 3 (Contd.)



ESTIMATED LIMITING VALUES

Estimated Limiting Values

- Estimated Limiting Values (ELV's) are developed as criteria for various types of hydraulic design.

- Commonly employed ELV's for design are
  - Probable Maximum Precipitation (PMP)
  - Probable Maximum Storm (PMS)
  - Probable Maximum Flood (PMF)

Probable Maximum Precipitation (PMP)

- PMP is the estimated limiting value of precipitation.
- PMP is defined as the estimated greatest depth of the precipitation for a given duration that is possible physically and reasonably characteristic over a particular geographic region at a certain time of year.
- PMP cannot be exactly estimated as its probability of occurrence is not known.
- PMP is useful in operational applications (e.g., design of large dams).
- Any allowance should not be made in the estimation of PMP for long term climate change.

Probable Maximum Precipitation (PMP)

- Various methods for determining PMP
- Application of storm models:
- Maximization of actual storms:
- Generalized PMP charts:

Probable Maximum Storm (PMS)

- PMS involves temporal distribution of rainfall.
- Spacial and temporal distribution of rainfall is required to develop hyetograph of a PMS
- PMS values are given as maximum accumulated depths for any specified duration.

For example, given depths for 4h, 8h,…24h represent the total depth for each duration & not the time sequence in which precipitation occurs.

Probable Maximum Flood (PMF)

- PMF is the greatest flood expected considering complete coincidence of all factors that produce the heaviest rainfall and maximum runoff.
- PMF is derived from PMP.
- PMF is used only for selected designs in view of economy

  For example, large spillways whose failure could lead to excessive damage and loss of life.
- A realistic approach is to scale downwards by certain percentage depending on type of structure and the hazard if it fails.

Probable Maximum Flood (PMF)

- PMF is also termed as Standard Project Flood (SPF).
- SPF is estimated from Standard project storm (rainfall-runoff modeling).

ANALYSIS OF UNCERTAINTY

Analysis of uncertainty

- Most uncertainties are not quantifiable.

  For example, the capacity of a culvert with an unobstructed entrance is obtained with small margin of error, but during a flood, its capacity reduces due to the logging of debris near the entrance.
- Three categories of hydrologic uncertainty
  - Natural
  - Inherent
  - uncertainty

Analysis of uncertainty

- Random variability of hydrologic phenomena
  - Model uncertainty: results from approximations made while representing phenomena by equations
  - Parameter uncertainty: results from unknown nature of coefficients in the equations.
- First order analysis of uncertainty is a procedure for quantifying the expected variability of dependent variable calculated as function of one or more independent variables

First order analysis of uncertainty

- Let a variable y is expressed in as a function of x

$$y = f(x)$$

- Two sources of error is possible.
  - The function f may be incorrect or
  - The measurement x may be inaccurate
- It is assumed that there is no bias in the analysis.
- Suppose that f is a correct model, a nominal value of x (denoted as $\overline{x}$) is selected as design input and the corresponding value of y is calculated as

$$\overline{y} = f(\overline{x})$$

First order analysis of uncertainty

- If the true value of x differs from $\overline{x}$, the effect of this discrepancy on y can be estimated by expanding $f(x)$ as a Taylor series around $x = \overline{x}$

$$y = f\left(\overline{x}\right) + \frac{df}{dx}\left(x - \overline{x}\right) + \frac{1}{2!}\frac{d^2 f}{dx^2}\left(x - \overline{x}\right)^2 + ...$$

where the derivatives df/dx, d^2f/dx^2,…are evaluated at $x = \overline{x}$

- First order expression is obtained if second and higher order terms are neglected.

First order analysis of uncertainty

$$y = f\left(\overline{x}\right) + \frac{df}{dx}\left(x - \overline{x}\right)$$

- The first order expression for the error in y is.

$$y = \overline{y} + \frac{df}{dx}\left(x - \overline{x}\right)$$

$$y - \overline{y} = \frac{df}{dx}\left(x - \overline{x}\right)$$

- The variance of this error is

$$s_y^2 = E\left[\left(y - \overline{y}\right)^2\right]$$

First order analysis of uncertainty

$$s_y^2 = E\left[\left(\frac{df}{dx}\left(x - \overline{x}\right)\right)^2\right]$$

$$s_y^2 = \left(\frac{df}{dx}\right)^2 \times E\left[\left(x - \overline{x}\right)^2\right]$$

$$s_y^2 = \left(\frac{df}{dx}\right)^2 s_x^2$$

where $s_x{}^2$ is the variance of x

First order analysis of uncertainty

- The equation gives the variance of dependent variable y as a function of independent variable x, assuming that the functional relationship y=f(x) is correct.
- The value s_y is the standard error of estimate of y.
- If y is dependent on several mutually independent variables $x_1, x_2, x_3 \ldots\ldots\ldots\ldots x_n$,

$$s_y^2 = s_{x_1}^2 \left(\frac{df}{dx_1} \right)^2 + s_{x_2}^2 \left(\frac{df}{dx_2} \right)^2 + \ldots + s_{x_n}^2 \left(\frac{df}{dx_n} \right)^2$$