Stochastic Structural Dynamics

Lecture-11

Random vibrations of sdof systems-3

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$\boxed{\text{Recall}}$

$$f(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow x(t) = \int_0^t h(t-\tau) f(\tau) d\tau$$

$$F(\omega) \longrightarrow \boxed{\text{LTI}} \longrightarrow X(\omega) = H(\omega) F(\omega)$$

$$\boxed{\begin{array}{l} f(t) \Leftrightarrow F(\omega) \\ x(t) \Leftrightarrow X(\omega) \\ h(t) \Leftrightarrow H(\omega) \end{array}}$$

$$\delta(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(t)$$

$$\exp(i\omega t) \longrightarrow \boxed{\text{LTI}} \longrightarrow H(\omega) \exp(i\omega t)$$

Input-output relations for linear time invariant systems

Uncertainty propagation in LTI systems

$$f(t) \longrightarrow \boxed{\begin{matrix} h(t) \\ H(\omega) \end{matrix}} \longrightarrow x(t)$$

Samples of $f(t), x(0), \dot{x}(0)$

Mean

Covariance (PSD)

$\vdots$

Higher order moments

I order pdf

II order pdf

$\vdots$

n^{th} order pdf

Samples of $x(t)$

Mean

Covariance (PSD)

$\vdots$

Higher order moments

I order pdf

II order pdf

$\vdots$

n^{th} order pdf

SDOF system under random excitations

$$m\ddot{x} + c\dot{x} + kx = \bar{f}(t)$$

$$x(0) = x_0; \dot{x}(0) = \dot{x}_0$$

Let $\langle \bar{f}(t) \rangle = m_f(t)$.

Introduce $f(t)$ such that

$\bar{f}(t) = m_f(t) + f(t)$ so that $\langle f(t) \rangle = 0$

$\Rightarrow$

$$m\langle \ddot{x} \rangle + c\langle \dot{x} \rangle + k\langle x \rangle = \langle \bar{f}(t) \rangle$$

$$\langle x(0) \rangle = x_0; \langle \dot{x}(0) \rangle = \dot{x}_0$$

$$\left\langle x(t)\right\rangle = \exp(-\eta\omega t)\left[x_0 \cos\omega_d t + \frac{\dot{x}_0 + \eta\omega x_0}{\omega\sqrt{(1-\eta^2)}}\cos\omega_d t \right] + \int_0^t h(t-\tau)\left\langle f(\tau)\right\rangle d\tau$$

$$m_x(t) = \exp(-\eta\omega t)\left[x_0 \cos\omega_d t + \frac{\dot{x}_0 + \eta\omega x_0}{\omega\sqrt{(1-\eta^2)}}\cos\omega_d t \right] + \int_0^t h(t-\tau) m_f(\tau)\, d\tau$$

$$m_f(t) \longrightarrow \boxed{h(t)} \longrightarrow \left\langle x(t)\right\rangle = m_f(t) * h(t)$$

Let $x(t) = \langle x(t) \rangle + y(t)$ with $\langle y(t) \rangle = 0$

$$m\ddot{x} + c\dot{x} + kx = \overline{f}(t)$$

$$x(0) = x_0; \dot{x}(0) = \dot{x}_0$$

$$\Rightarrow$$

$$\underline{m\langle \ddot{x}(t) \rangle} + m\ddot{y} + \underline{c\langle \dot{x}(t) \rangle} + c\dot{y} + \underline{k\langle x(t) \rangle} + ky = \underline{m_f(t)} + f(t)$$

$$\Rightarrow$$

$$m\ddot{y} + c\dot{y} + ky = f(t)$$

Also

$$\langle x(0) \rangle + y(0) = x_0 \Rightarrow y(0) = 0$$

$$\langle \dot{x}(0) \rangle + \dot{y}(0) = \dot{x}_0 \Rightarrow \dot{y}(0) = 0$$

For systems starting from rest, Duhamel's integral provides the complete solution.

$\Rightarrow$

$$y(t) = \int_0^t h(t-\tau) f(\tau)\, d\tau$$

$$\langle y(t) \rangle = \int_0^t h(t-\tau) \langle f(\tau) \rangle\, d\tau = 0$$

$$\langle y(t_1)\, y(t_2) \rangle = \left\langle \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1) f(\tau_1) h(t_2-\tau_2) f(\tau_2)\, d\tau_1 d\tau_2 \right\rangle$$

$$\Rightarrow R_{yy}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1) h(t_2-\tau_2) \langle f(\tau_1) f(\tau_2) \rangle\, d\tau_1 d\tau_2$$

$$R_{yy}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1) h(t_2-\tau_2) R_{ff}(\tau_1,\tau_2)\, d\tau_1 d\tau_2$$

$$R_{yy}\left(t_1,t_2\right)=\int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)R_{ff}\left(\tau_1,\tau_2\right)d\tau_1 d\tau_2$$

Let $t_1 = t_2 = t$

$$R_{yy}\left(t_1,t_2\right)=\sigma_y^2\left(t\right)=\int_0^{t}\int_0^{t} h(t-\tau_1)h(t-\tau_2)R_{ff}\left(\tau_1,\tau_2\right)d\tau_1 d\tau_2$$

$$\left\langle y\left(t_1\right)y\left(t_2\right)y\left(t_3\right)\right\rangle=\left\langle \int_0^{t_1}\int_0^{t_2}\int_0^{t_3} h(t_1-\tau_1)f\left(\tau_1\right)h(t_2-\tau_2)f\left(\tau_2\right)h(t_3-\tau_3)f\left(\tau_3\right)d\tau_1 d\tau_2 d\tau_3\right\rangle$$

$$=\int_0^{t_1}\int_0^{t_2}\int_0^{t_3} h(t_1-\tau_1)h(t_2-\tau_2)h(t_3-\tau_3)\left\langle f\left(\tau_1\right)f\left(\tau_2\right)f\left(\tau_3\right)\right\rangle d\tau_1 d\tau_2 d\tau_3$$

In general for LTI systems, the
knowledge of n^{th} order moment of input is adequate
to determine the n^{th} order moment of the response process.

MOMENT EQUATIONS ARE CLOSED FOR LTI SYSTEMS

Note: this is not true for nonlinear systems

$$M\ddot{x} + c\dot{x} + Kx + \alpha \underline{\underline{x^3}} = f(t)$$

$$M\langle \ddot{x} \rangle + c\langle \dot{x} \rangle + K\underline{\underline{\langle x \rangle}} + \alpha \underline{\underline{\langle x^3 \rangle}} = \langle f(t) \rangle$$

SDOF system under Gaussian white noise excitation

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$x(0) = 0; \dot{x}(0) = 0$$

$$\langle f(t) \rangle = 0; \langle f(t_1) f(t_2) \rangle = I\delta(t_2 - t_1)$$

$$x(t) = \int_0^t h(t-\tau) f(\tau) d\tau$$

$$\Rightarrow$$

$$\langle x(t) \rangle = \int_0^t h(t-\tau) \langle f(\tau) \rangle d\tau = 0$$

$$\left\langle x(t_1)\,x(t_2)\right\rangle = \left\langle \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)\,f(\tau_1)\,h(t_2-\tau_2)\,f(\tau_2)\,d\tau_1 d\tau_2 \right\rangle$$

$$R_{xx}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)\left\langle f(\tau_1)\,f(\tau_2)\right\rangle d\tau_1 d\tau_2$$

$$R_{xx}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)R_{ff}(\tau_1,\tau_2)\,d\tau_1 d\tau_2$$

$$= \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)I\delta(\tau_2-\tau_1)\,d\tau_1 d\tau_2$$

$$= \int_0^{t_2} Ih(t_1-\tau_2)h(t_2-\tau_2)d\tau_2$$

$$R_{xx}\left(t_1,t_2\right)=\int_0^{t_2} Ih(t_1-\tau)h(t_2-\tau)d\tau$$

$$=\int_0^{t_2} I\frac{1}{m\omega_d}\exp\left[-\eta\omega(t_1-\tau)\right]\sin\left[\omega_d(t_1-\tau)\right]\frac{1}{m\omega_d}\exp\left[-\eta\omega(t_2-\tau)\right]\sin\left[\omega_d(t_2-\tau)\right]d\tau$$

$$=\frac{I}{4\eta\omega^3m^2}\exp\left[-\eta\omega(t_2-t_1)\right]\chi(t)$$

$$\chi\left(t\right)=\left[\frac{\exp\left(-2\eta\omega t_1\right)}{1-\eta^2}\left\{\eta^2\cos\omega_d\left(t_1+t_2\right)-\eta\sqrt{1-\eta^2}\sin\omega_d\left(t_1+t_2\right)-\cos\omega_d\left(t_2-t_1\right)\right\}\right]$$

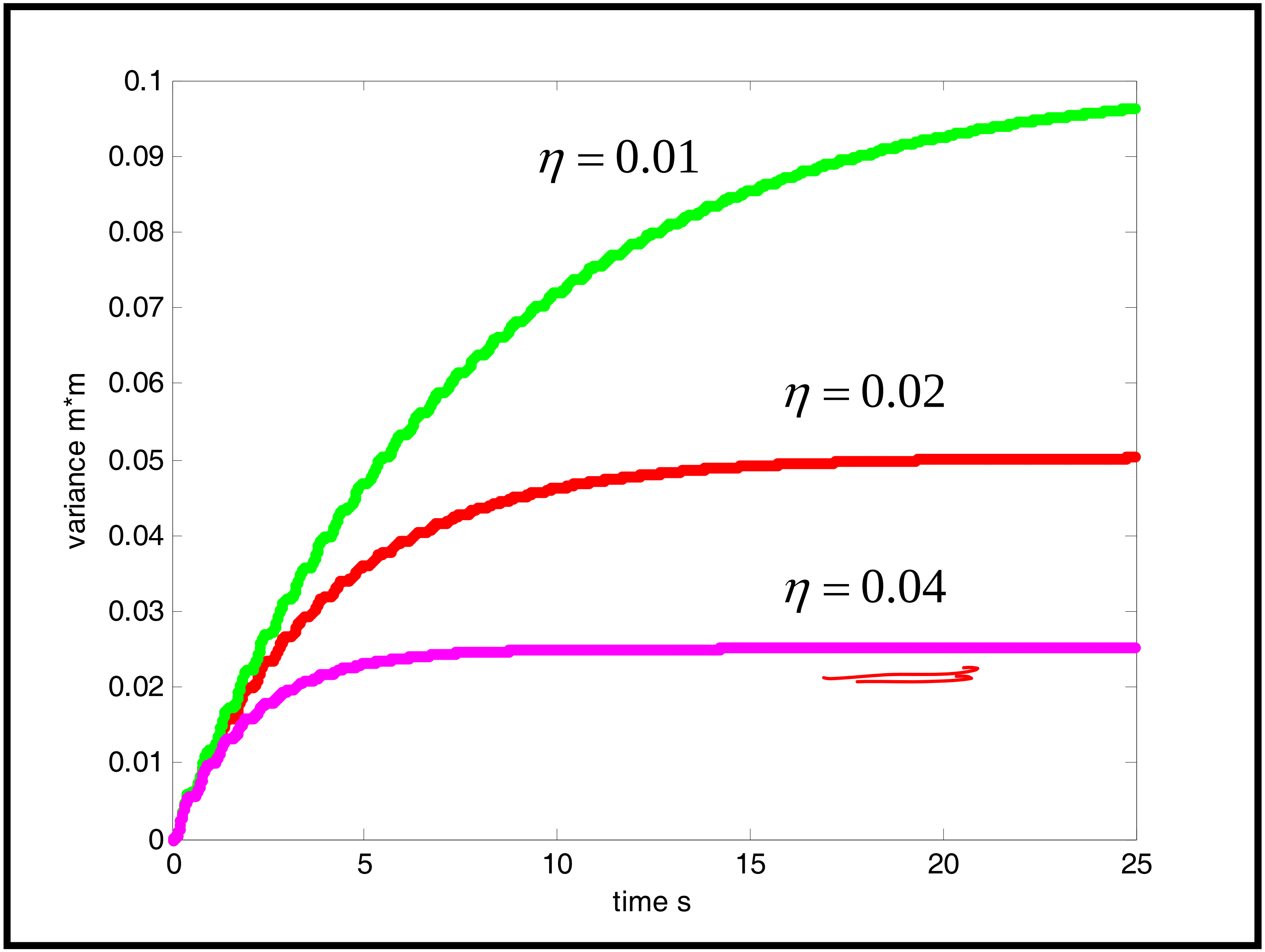
$$+\left[\cos\omega_d\left(t_2-t_1\right)+\frac{\eta}{\sqrt{1-\eta^2}}\sin\omega_d\left|\left(t_2-t_1\right)\right|\right]$$

$$R_{xx}(t,t)=\sigma_x^2(t)=\int_0^t Ih^2(t-\tau)d\tau$$

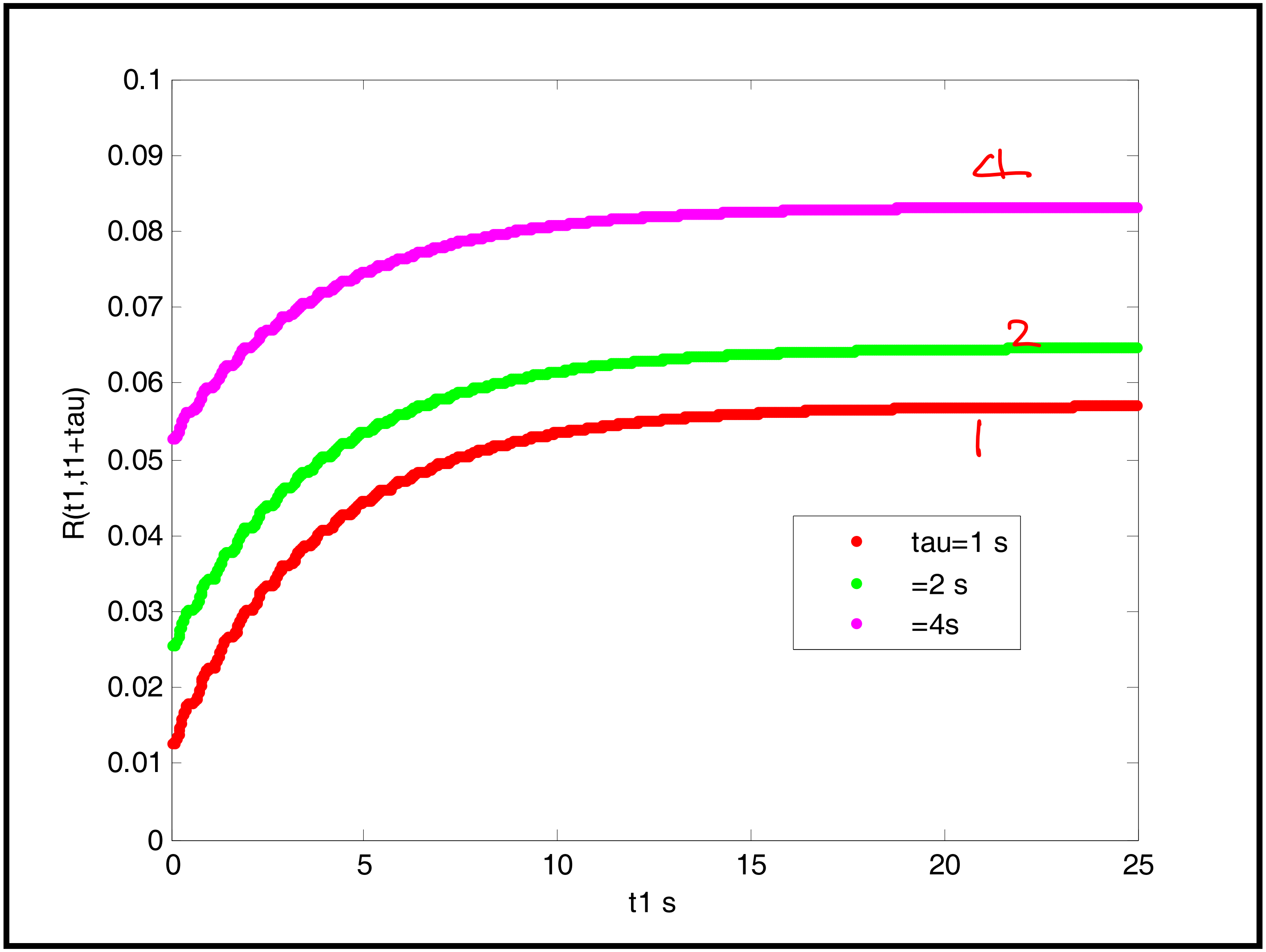
$$=\frac{I}{4\eta\omega^3 m^2}\left[\frac{\exp(-2\eta\omega t)}{1-\eta^2}\left\{\eta^2\cos 2\omega_d t-\eta\sqrt{1-\eta^2}\sin 2\omega_d t-1\right\}+1\right]$$

$$\lim_{\substack{t_1\to\infty\\ t_2\to\infty\\ t_2-t_1=\tau}} R_{xx}(t_1,t_2)\to\frac{I}{4\eta\omega^3 m^2}\exp\left[-\eta\omega|\tau|\right]\left[\cos\omega_d\tau+\frac{\eta}{\sqrt{1-\eta^2}}\sin\omega_d|\tau|\right]$$

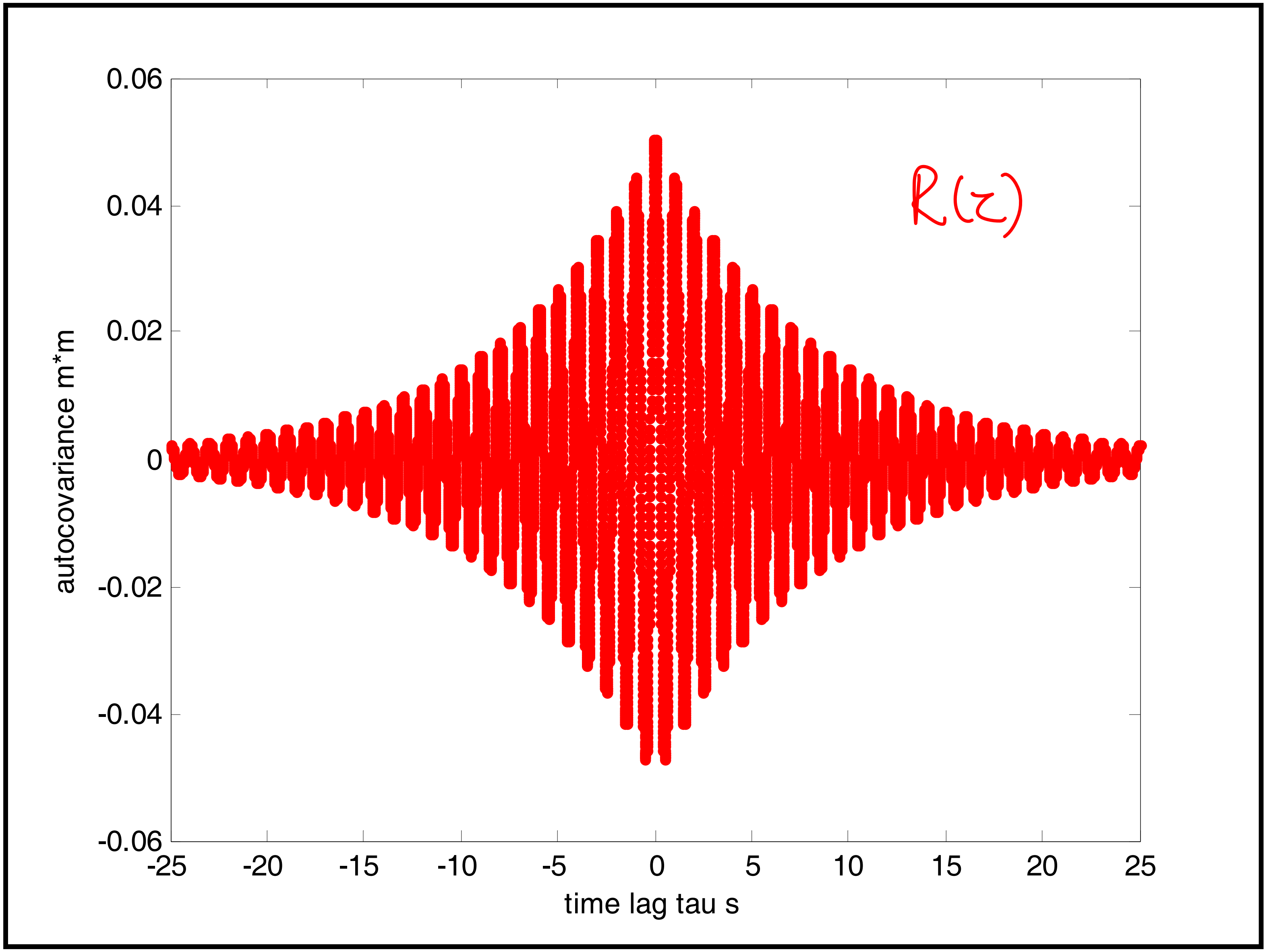
$$\lim_{\substack{t_1\to\infty\\ t_2\to\infty\\ t_2-t_1=0}} R_{xx}(t,t)=\sigma_x^2\to\frac{I}{4\eta\omega^3 m^2}$$



$\eta = 0.01$
$\eta = 0.02$
$\eta = 0.04$
variance m*m
time s
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0
5
10
15
20
25



0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
R(t1,t1+tau)
0
5
10
15
20
25
t1 s
tau=1 s
=2 s
=4s
4
2
1



0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-25
-20
-15
-10
-5
0
5
10
15
20
25
autocovariance m*m
time lag tau s
R(τ)

Remarks

- For small times, the response is non-stationary
  - Covariance is a function of t_1 and t_2
  - Variance is a function of time

For large times, the response becomes stationary

- Covariance is a function of time lags
- Variance becomes time invariant

Note: In the present case, mean=0

We say that the system reaches a stochastic steady state as time becomes large.

If damping=0, the system fails to reach steady state.

Exercise

Discuss the nature of transient and steady state responses of the system governed by

$$m\ddot{x} + c\dot{x} + kx = P\cos\lambda t + f(t)$$

$$x(0) = x_0;\ \dot{x}(0) = \dot{x}_0$$

- P & λ are deterministic
- $f(t)$ is a zero mean Gaussian white noise process

with $\langle f(t_1) f(t_2) \rangle = I\delta(t_2 - t_1)$

Discuss the cases of $c \to 0$ and $\lambda \to \omega = \sqrt{\frac{k}{m}}$

SDOF system under Gaussian modulated white noise excitation

$$m\ddot{x} + c\dot{x} + kx = e(t)\, f(t)$$

$$x(0) = 0; \dot{x}(0) = 0$$

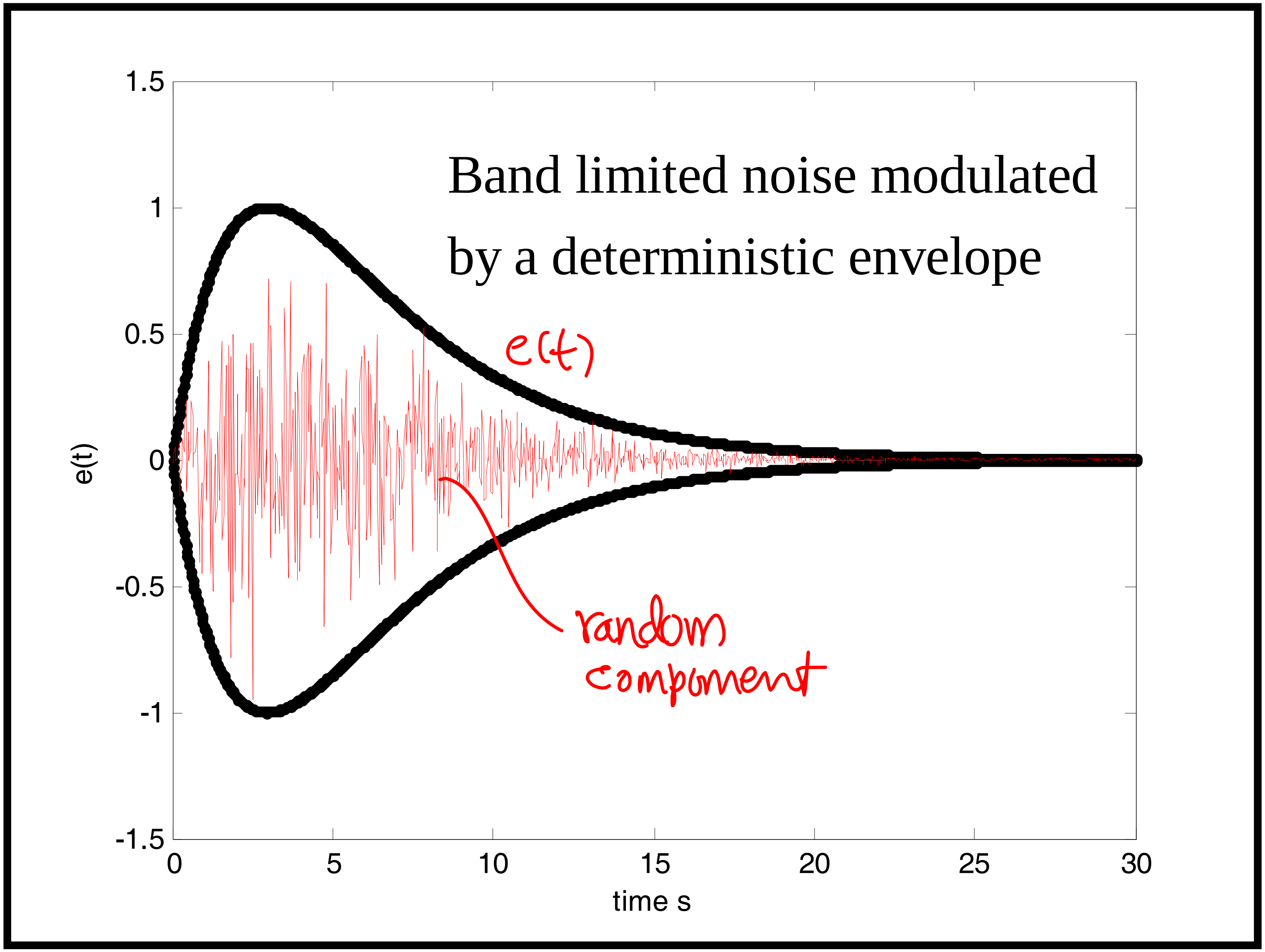
$$\langle f(t)\rangle = 0; \langle f(t_1)\, f(t_2)\rangle = I\delta(t_2 - t_1)$$

$$e(t) = A\left[\exp(-\alpha t) - \exp(-\beta t)\right]$$

$$x(t) = \int_0^t h(t-\tau) e(\tau)\, f(\tau)\, d\tau$$

$$\Rightarrow$$

$$\langle x(t)\rangle = \int_0^t h(t-\tau) e(\tau) \langle f(\tau)\rangle d\tau = 0$$


Band limited noise modulated
by a deterministic envelope
e(t)
random
component
e(t)
time s
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30

$$\left\langle x(t_1)\,x(t_2)\right\rangle = \left\langle \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)\underline{e(\tau_1)}\,\underline{f(\tau_1)}h(t_2-\tau_2)\underline{e(\tau_2)}\,\underline{f(\tau_2)}\,d\tau_1 d\tau_2 \right\rangle$$

$$R_{xx}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)e(\tau_1)e(\tau_2)\underline{\left\langle f(\tau_1)\,f(\tau_2)\right\rangle}\,d\tau_1 d\tau_2$$

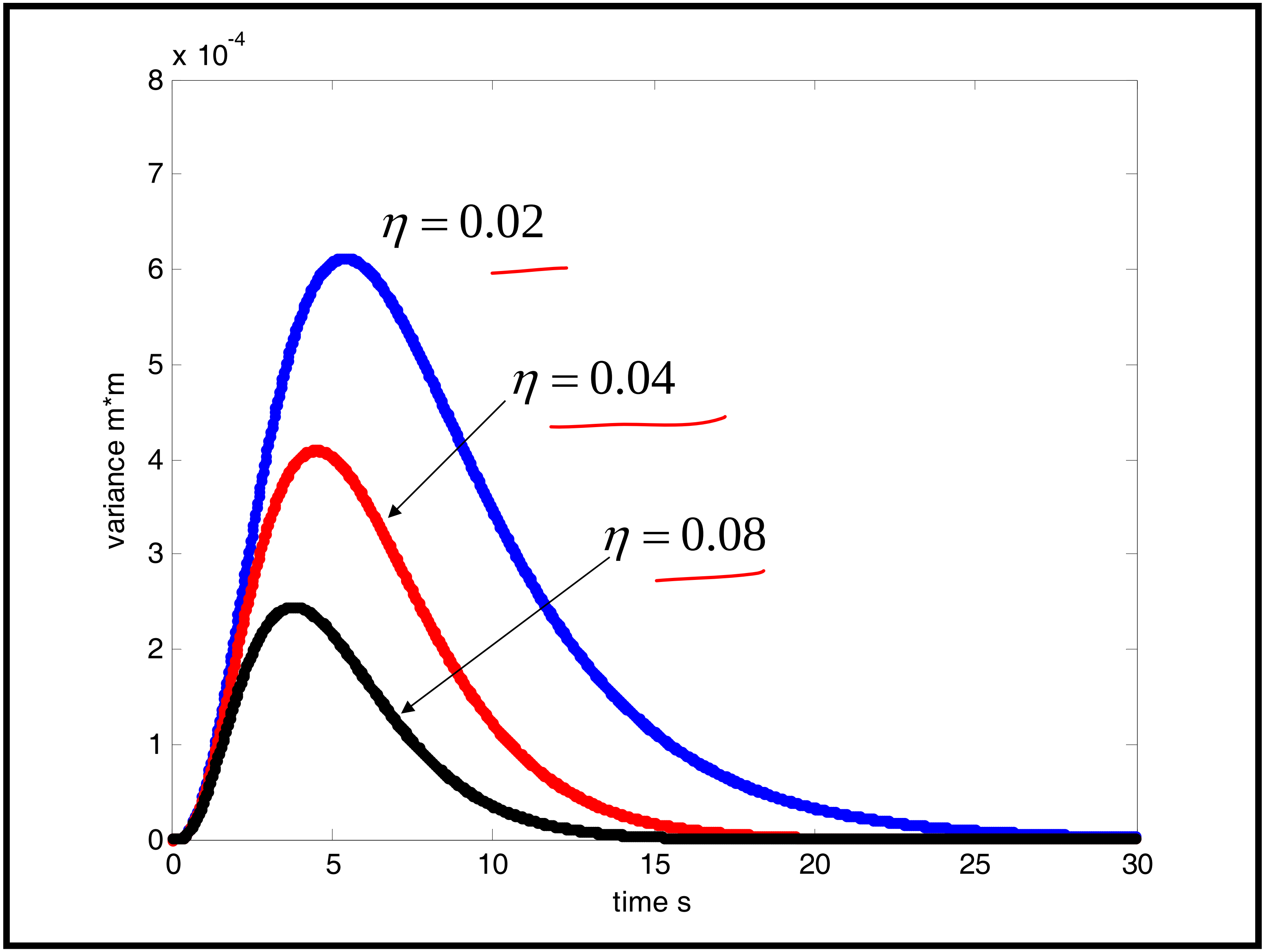
$$R_{xx}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)e(\tau_1)e(\tau_2)\underline{\underline{R_{ff}(\tau_1,\tau_2)}}\,d\tau_1 d\tau_2$$

$$= \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)e(\tau_1)e(\tau_2)\underline{\underline{I\delta(\tau_2-\tau_1)}}\,d\tau_1 d\tau_2$$

$$= \int_0^{t_2} I h(t_1-\tau_2)h(t_2-\tau_2)e^2(\tau_2)\,d\tau_2$$

$$\sigma_x^2(t) = \int_0^t \int_0^t h(t-\tau_1) h(t-\tau_2) e(\tau_1) e(\tau_2) I \delta(\tau_2 - \tau_1) \, d\tau_1 d\tau_2$$

$$= \int_0^t I h^2(t-\tau) e^2(\tau) \, d\tau$$


x 10^-4
8
7
6
5
4
3
2
1
0
variance m*m
0
5
10
15
20
25
30
time s
$\eta = 0.02$
$\eta = 0.04$
$\eta = 0.08$

Input - output relations for LTI systems

driven by random excitations

Frequency domain relations

$$m\ddot{x}+c\dot{x}+kx=f(t);x(0)=0,\dot{x}(0)=0$$

Let $f(t)$ be a stationary random process with zero mean, autocovariance $C_{ff}(\tau)$, and psd $S_{ff}(\omega)$.

In the steady state x(t) becomes a stationary random process.

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By definition

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$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{1}{T} \left\langle \left| X_T(\omega) \right|^2 \right\rangle$$

$x_T(t) = x(t)$

$0 < t < T$

$= 0$ otherwise

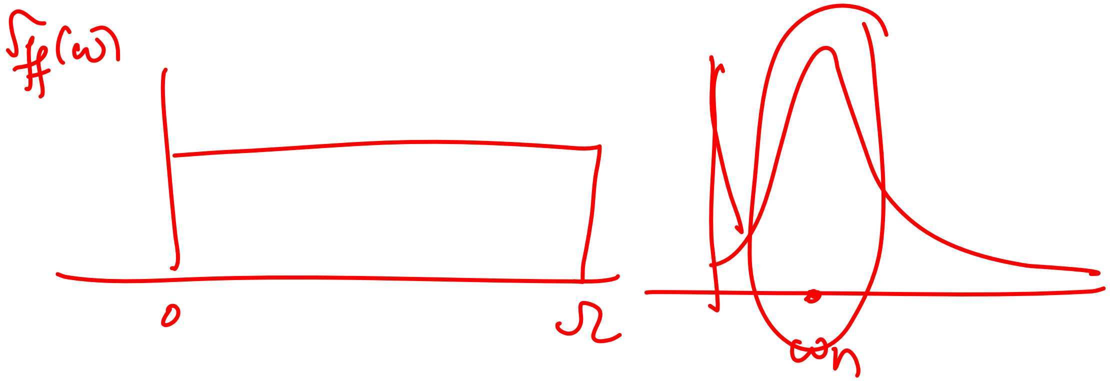
We also have $X_T(\omega) = H(\omega) F_T(\omega)$

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{1}{T} \left\langle H(\omega) F_T(\omega) F_T^*(\omega) H^*(\omega) \right\rangle$$

$\Rightarrow$

$$S_{XX}(\omega) = \left| H(\omega) \right|^2 S_{ff}(\omega)$$

25



$$S_{ff}(\omega) \longrightarrow \boxed{|H(\omega)|^2} \longrightarrow S_{XX}(\omega) = |H(\omega)|^2 S_{ff}(\omega)$$

Linear dynamical system as a filter

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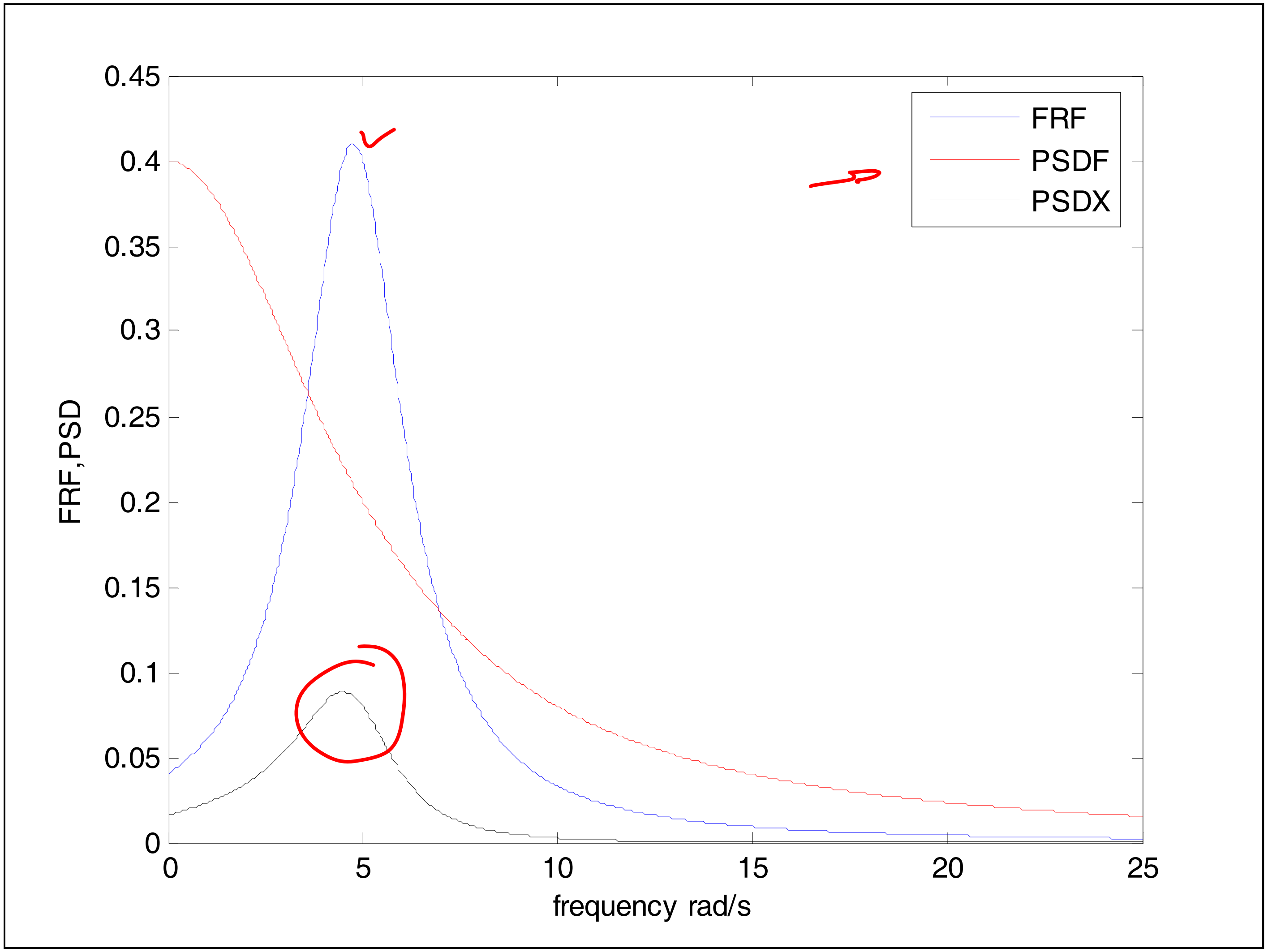
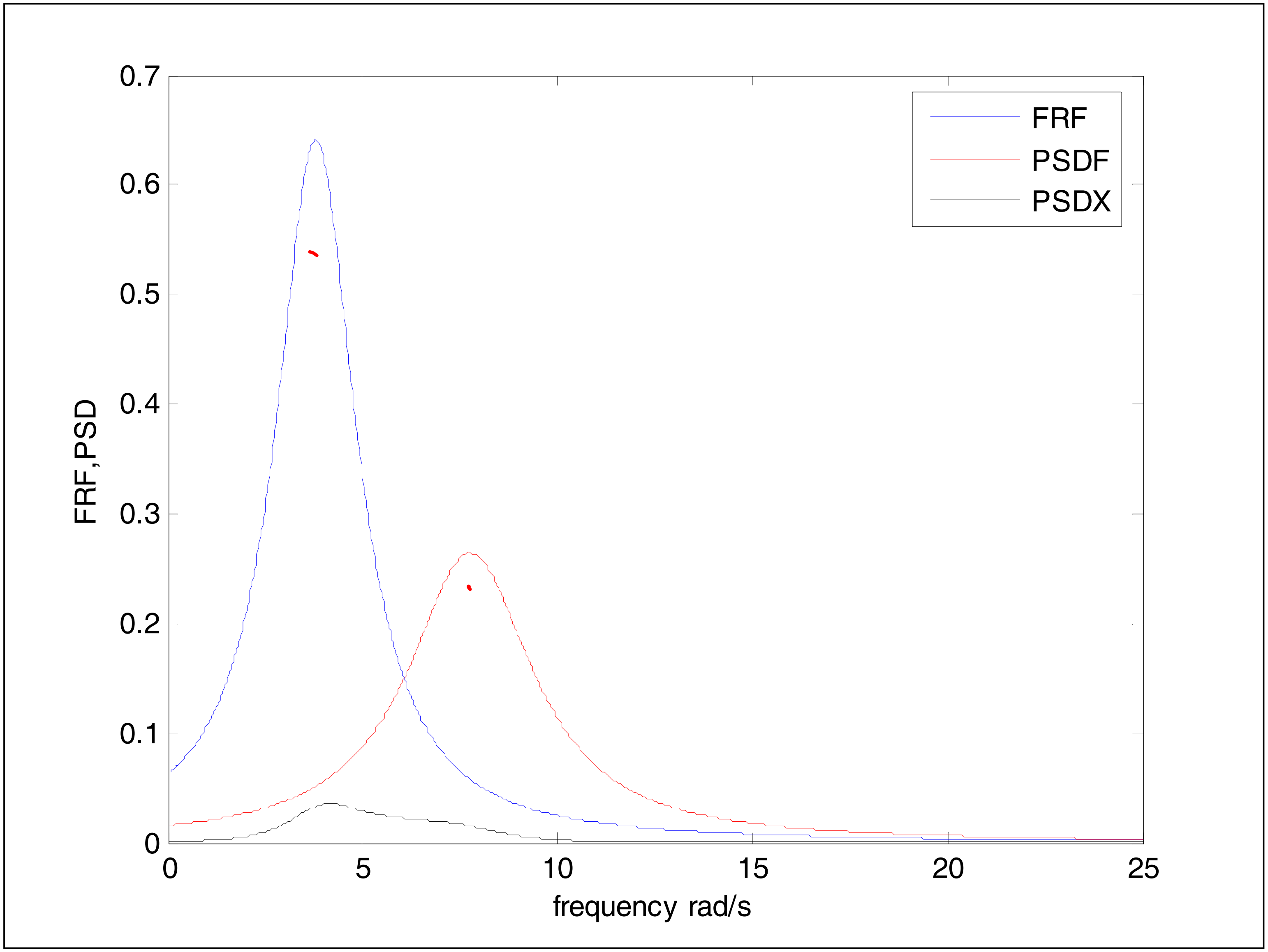
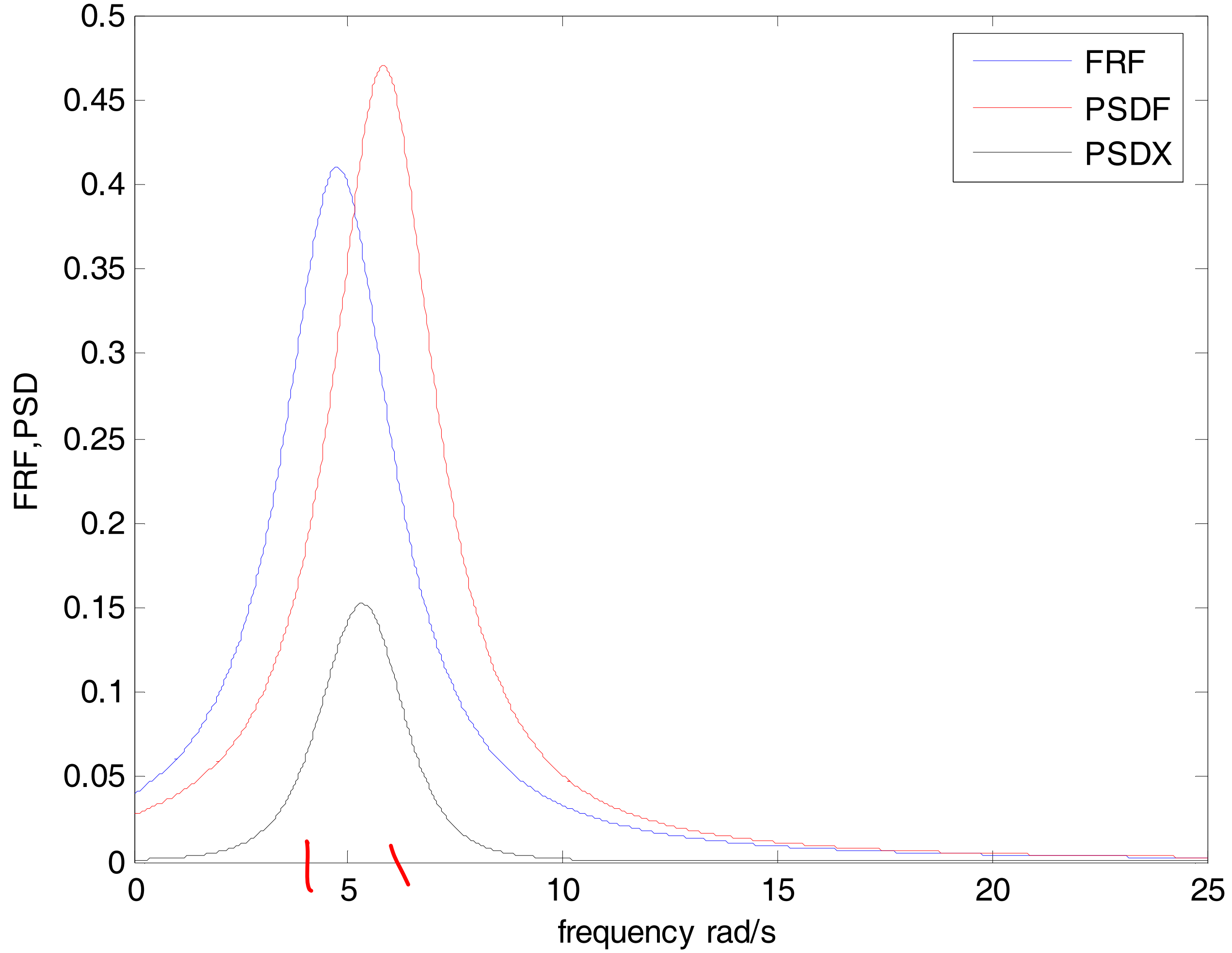

0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
FRF,PSD
0
5
10
15
20
25
frequency rad/s
FRF
PSDF
PSDX



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
FRF,PSD
0
5
10
15
20
25
frequency rad/s
FRF
PSDF
PSDX



FRF
PSDF
PSDX
FRF,PSD
frequency rad/s
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25

Description of Derivative Processes

Recall

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\exp(i\omega\tau)\,d\tau$$

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\exp(-i\omega\tau)\,d\omega$$

$$R_{\dot{x}\dot{x}}(\tau) = -\frac{d^2 R_{xx}(\tau)}{d\tau^2}$$

$$= -\frac{d^2}{d\tau^2}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\exp(-i\omega\tau)\,d\omega\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 S_{XX}(\omega)\exp(-i\omega\tau)\,d\omega$$

$$\Rightarrow$$

$$S_{\dot{x}\dot{x}}(\omega) = \omega^2 S_{XX}(\omega)$$

$$R_{\ddot{x}\ddot{x}}(\tau) = -\frac{d^2 R_{\dot{x}\dot{x}}(\tau)}{d\tau^2}$$

$$= -\frac{d^2}{d\tau^2}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^2 S_{XX}(\omega)\exp(-i\omega\tau)\,d\omega\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^4 S_{XX}(\omega)\exp(-i\omega\tau)\,d\omega$$

$$\Rightarrow$$

$$S_{\ddot{x}\ddot{x}}(\omega) = \omega^2 S_{\dot{x}\dot{x}}(\omega) = \omega^4 S_{xx}(\omega)$$

SDOF system under Gaussian white noise excitation

$$m\ddot{x}+c\dot{x}+kx= f(t)$$

$$x(0)=0;\dot{x}(0)=0$$

$$\langle f(t)\rangle=0;\langle f(t_1)f(t_2)\rangle=I\delta(t_2-t_1)$$

$$S_{XX}(\omega)=|H(\omega)|^2 I$$

$$H(\omega)=\frac{1/m}{(\omega_n^2-\omega^2)+i2\eta\omega\omega_n}$$

Recall

$$R_{xx}(\tau)=\frac{I}{4\eta\omega^3m^2}\exp\left[-\eta\omega|\tau|\right]\left[\cos\omega_d\tau+\frac{\eta}{\sqrt{1-\eta^2}}\sin\omega_d|\tau|\right]$$

$$\sigma_x^2=\frac{I}{4\eta\omega^3m^2}$$

Show that

$$S_{xx}(\omega)\Leftrightarrow R_{xx}(\tau)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{xx}(\omega)\,d\omega=\frac{I}{4\eta\omega^3m^2}$$

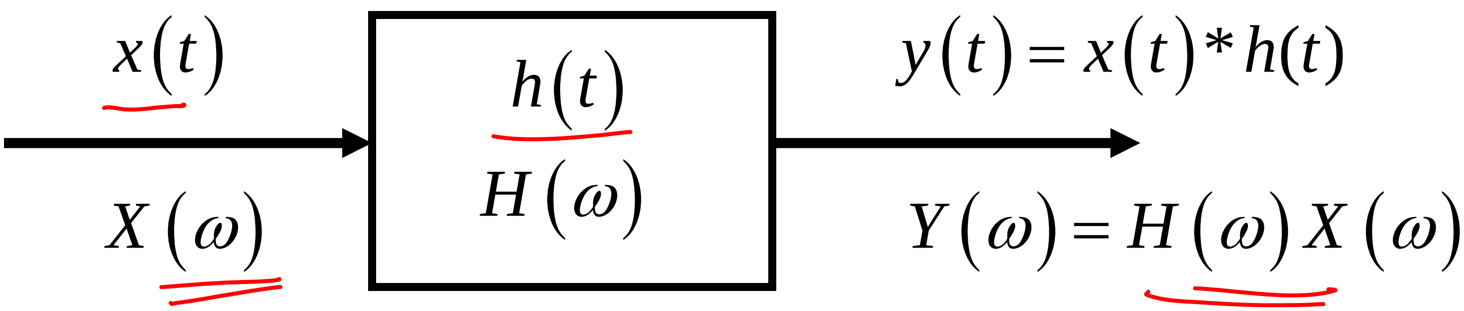
(Hint: Use residue theorem)

(More on this in the later)

Random vibrations

- Study of failure of structures under loads such as those due to earthquakes, wind, road roughness,…
- Major tools for measurement of dynamic characteristics of engineering structures in laboratory and field conditions

Measurement of FRF-s in laboratory



$$\boxed{\begin{aligned} S_{YX}(\omega) &= \lim_{T\to\infty} \frac{1}{T} \left\langle Y_T(\omega) X_T^*(\omega) \right\rangle \\ &= \lim_{T\to\infty} \frac{1}{T} \left\langle H(\omega) X_T(\omega) X_T^*(\omega) \right\rangle \\ &= H(\omega) S_{XX}(\omega) \end{aligned}}$$

$$\boxed{H_1(\omega) = \frac{S_{YX}(\omega)}{S_{XX}(\omega)}}$$

$$
\boxed{\begin{aligned}
S_{YY}(\omega) &= \lim_{T\to\infty}\frac{1}{T}\left\langle Y_T(\omega)Y_T^*(\omega)\right\rangle \\
&= \lim_{T\to\infty}\frac{1}{T}\left\langle H(\omega)X_T(\omega)Y_T^*(\omega)\right\rangle \\
&= H(\omega)S_{XY}(\omega)
\end{aligned}}
$$

$$
\boxed{H_2(\omega) = \frac{S_{YY}(\omega)}{S_{XY}(\omega)}}
$$

$$
\boxed{\begin{aligned}
S_{YY}(\omega) &= \left|H(\omega)\right|^2 S_{XX}(\omega) \\
\left|H_a(\omega)\right|^2 &= \frac{S_{YY}(\omega)}{S_{XX}(\omega)}
\end{aligned}}
$$

Coherence function

$$\boxed{\begin{aligned}\gamma_{XY}^{2}(\omega) &= \frac{\left|S_{XY}(\omega)\right|}{S_{XX}(\omega)S_{YY}(\omega)} \\ &= \frac{S_{XX}(\omega)H(\omega)S_{XX}(\omega)H^{*}(\omega)}{S_{XX}(\omega)\left|H(\omega)\right|^{2}S_{XX}(\omega)} \\ &= 1\end{aligned}}$$

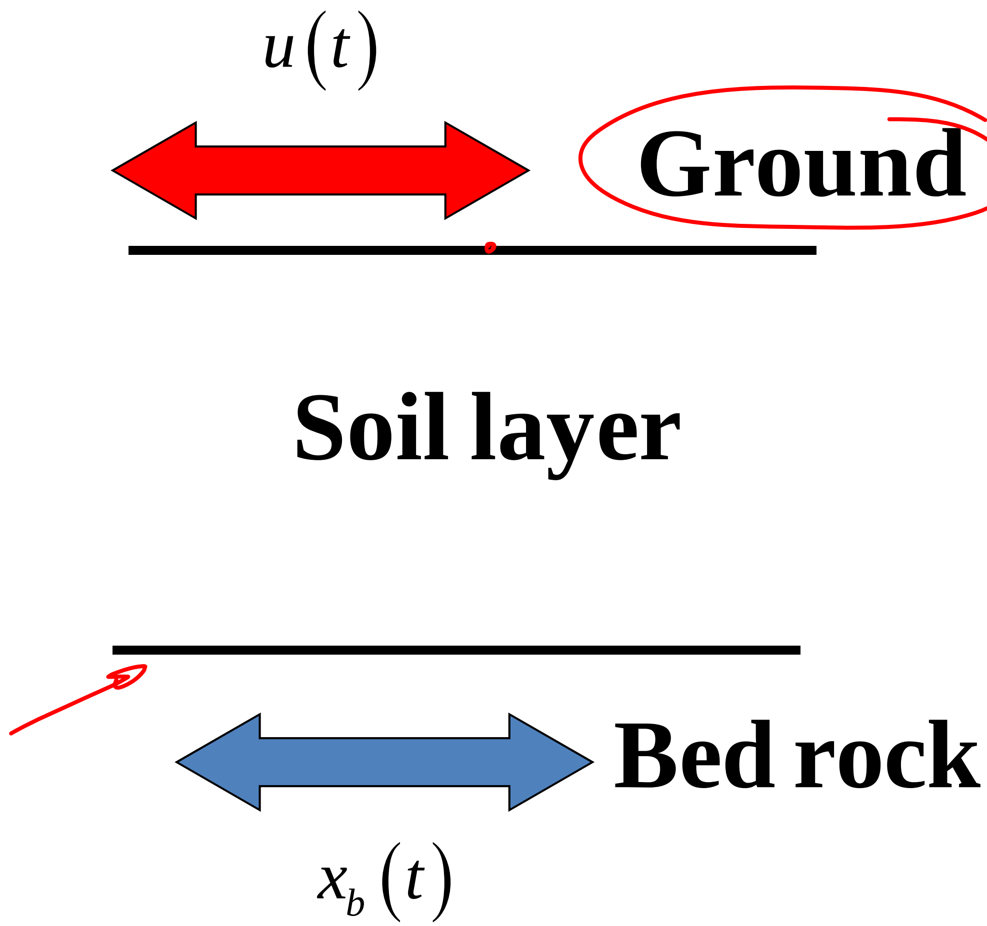
Exercise: Show that

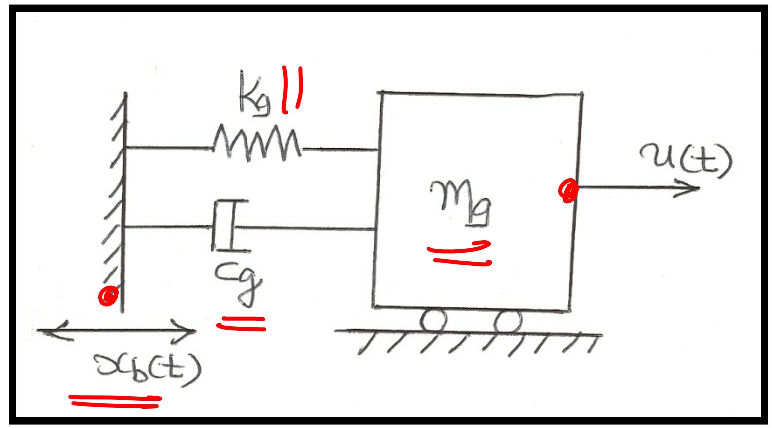
$$\boxed{\gamma_{XY}^{2}(\omega) = \frac{H_{1}(\omega)}{H_{2}(\omega)}}$$

Importance of coherence in FRF measurements

- Measurement of FRF-s is adversely affected by several factors such as
  - Structural nonlinearities
  - Electronic noise
  - Signal processing issues (leakage, time delays,..)
- Coherence serves as a valuable tool in assessing quality of measurements (greater the departure from 1 poorer is the quality of measurements)

Kanai – Tajimi Power spectral density function model for free field earthquake ground acceleration





$$m_g\ddot{u}+c_g\left(\dot{u}-\dot{x}_b\right)+k_g\left(u-x_b\right)=0$$

$$\ddot{u}=-2\eta_g\omega_g\left(\dot{u}-\dot{x}_b\right)-\omega_g^2\left(u-x_b\right)$$

$$\frac{c_g}{m_g}=2\eta_g\omega_g$$

$$\frac{k_g}{m_g}=\omega_g^2$$

Let $v=u-x_b$

$$\Rightarrow \ddot{v}+2\eta_g\omega_g\dot{v}+\omega_g^2 v=-\ddot{x}_b$$

$$\ddot{u}=-2\eta_g\omega_g\dot{v}-\omega_g^2 v$$

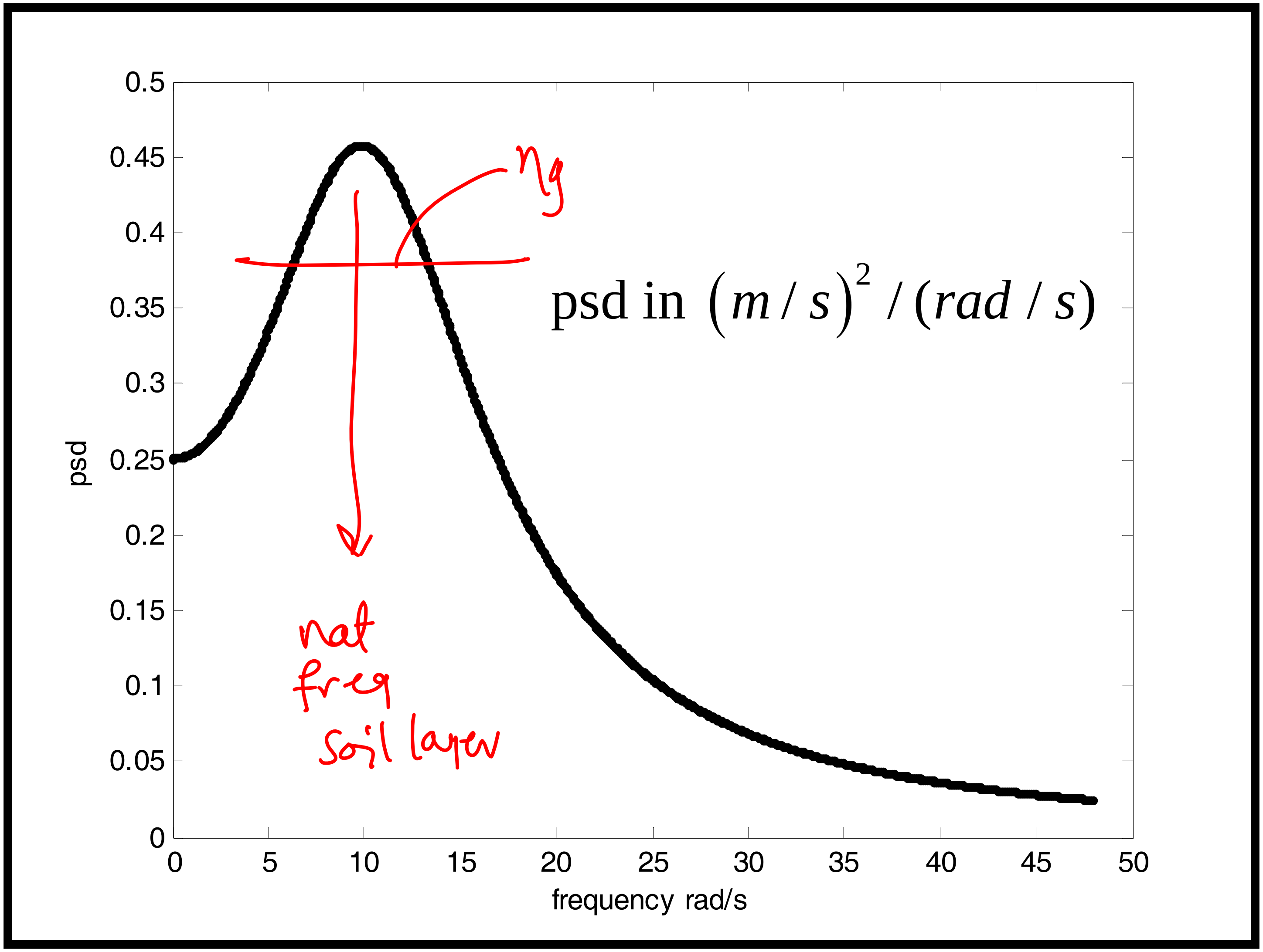
$$\ddot{U}_T\left(\omega\right)=-\left(i2\eta_g\omega_g\omega+\omega_g^2\right)V_T\left(\omega\right)$$

$$=\left(i2\eta_g\omega_g+\omega_g^2\right)\frac{\ddot{X}_{bT}\left(\omega\right)}{\left(\omega_g^2-\omega^2\right)+i\left(2\eta_g\omega_g\omega\right)}$$

$$S\left(\omega\right)=\lim_{T\to\infty}\frac{1}{T}\left\langle\left|\ddot{U}_T\left(\omega\right)\right|^2\right\rangle$$

$$S\left(\omega\right)=I\frac{\left(\omega_g^4+4\eta_g^2\omega_g^2\omega^2\right)}{\left(\omega^2-\omega_g^2\right)^2+4\eta_g^2\omega_g^2\omega^2}$$

Typical plot of a Kanai-Tajimi PSD function for
free field ground acceleration



Remarks

(a) Effective model to capture ground resonance effects

(b) Easy to use in random vibration analysis

(c) Limitations:

- Does not allow for transient nature of earthquake ground accelerations.
- Treats soil as a SDOF system

How to allow for nonstationary nature of ground accelerations?

Strategy: Use a deterministic modulating function.

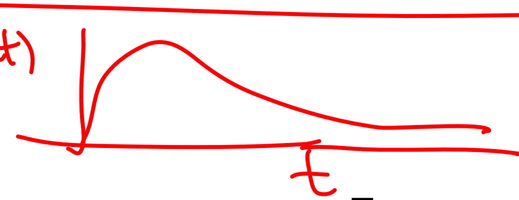
$$\ddot{X}_g(t) = e(t)S(t)$$

$e(t) =$ deterministic envelope function

$S(t)$=zero mean stationary Gaussian random process

Example: $S(t)$ could have the Kanai-Tajimi PSD

$e(t)$



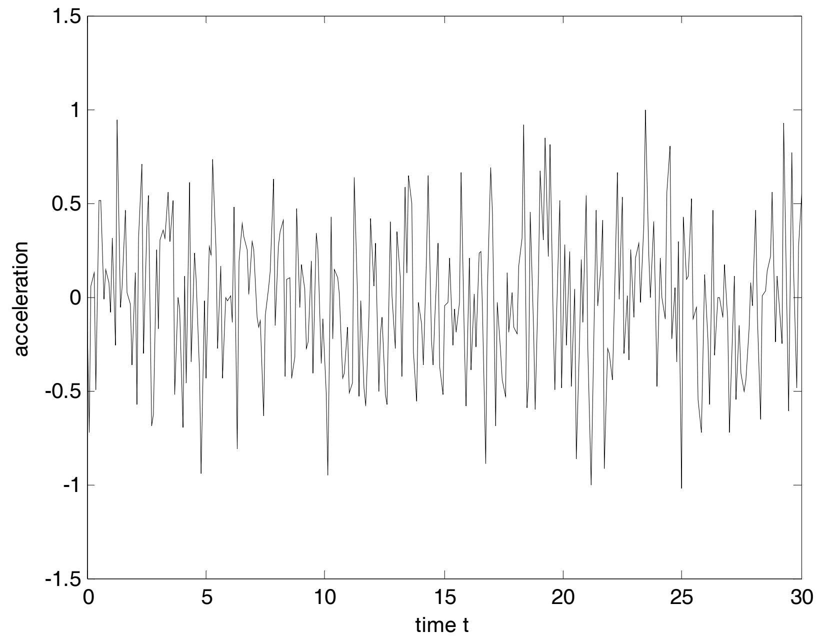
te^{-t}

Examples

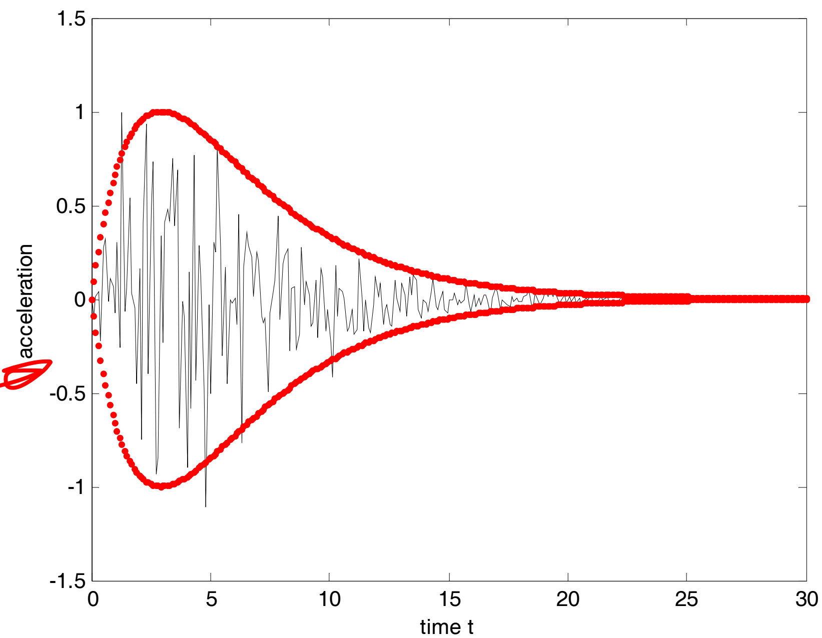
$$e(t) = A_0\left[\exp(-\alpha t) - \exp(-\beta t)\right]; \alpha > \beta > 0$$

$$e(t) = (A_0 + A_1 t)\exp(-\alpha t)$$

S(t)



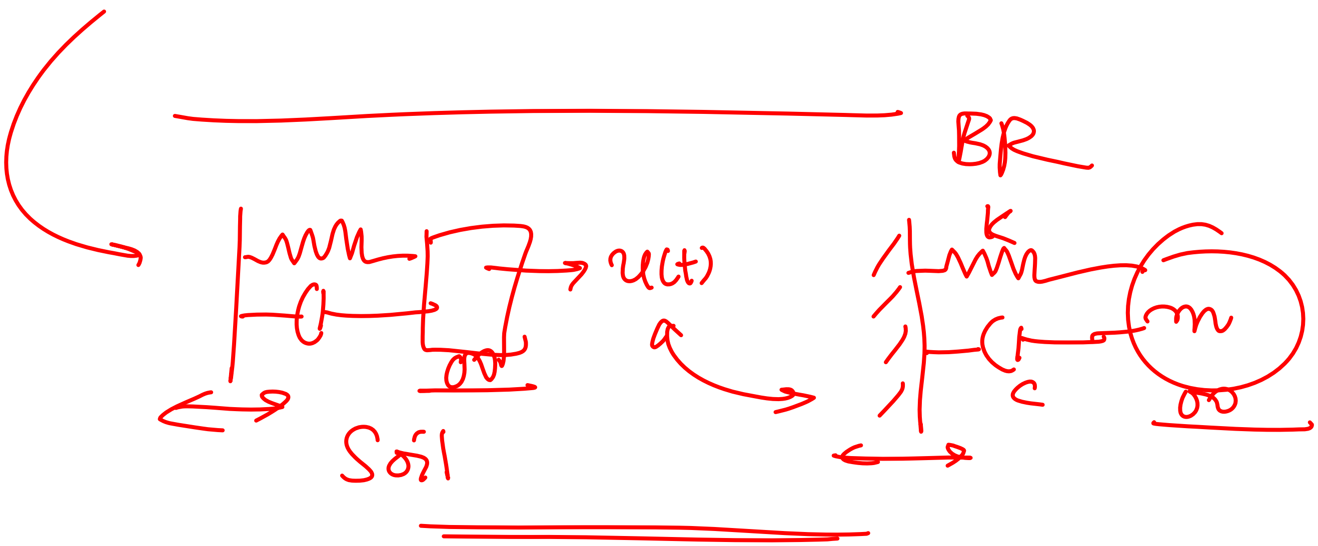
1.5
1
0.5
0
-0.5
-1
-1.5
acceleration
0
5
10
15
20
25
30
time t



1.5
1
0.5
0
-0.5
-1
-1.5
acceleration
0
5
10
15
20
25
30
time t

Structure under earthquake support motions


Soil
Str
Ground


BR
u(t)
k
m
c
Soil

Response of a sdof system to KT earthquake excitaiton

$$m\ddot{x}+c\left(\dot{x}-\dot{u}\right)+k\left(x-u\right)=0$$
$$m_g\ddot{u}+c_g\left(\dot{u}-\dot{x}_b\right)+k\left(u-x_b\right)=0$$

$$S_{XX}\left(\omega\right)=I\left|H_{soil}\left(\omega\right)\right|^2\left|H_{structure}\left(\omega\right)\right|^2$$