INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 16

Course Instructor : Prof. P. P. MUJUMDAR

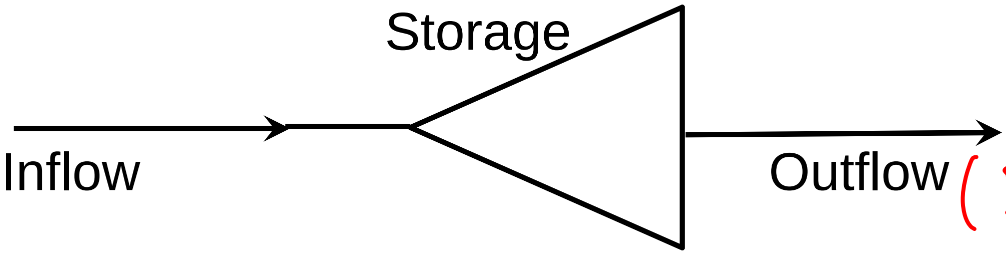
Department of Civil Engg., IISc.

Summary of the previous lecture

- Water allocation problem
- Characteristics of DP
    - A single n-variable problem is divided into n number of single variable problems.
    - Problem is divided into stages, with a policy decision required at each stage; objective function must be separable.
    - Each stage has a number of possible states associated with it.
    - Policy decision transforms the current state into a state associated with the next stage.
    - Recursive relationship identifies the optimal decision at stage n, for state S_n, given the optimal decisions for each state at stage $(n-1)$.
    - Solution moves backward (or forward) stage by stage, till optimal decisions for the last stage are found.
    - Optimal decisions for other stages are traced back from the solutions of those stages.

Dynamic Programming

Reservoir operation problem:

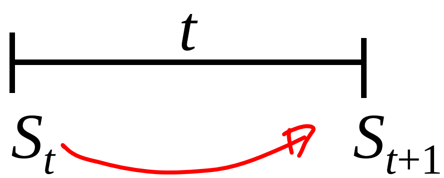


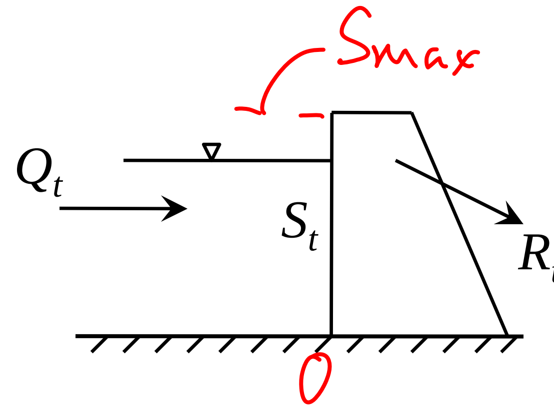
(Release + Spill/overflow)

- Operating policy: Amount of release from the reservoir during a period (e.g., a month, a season etc.), for a given storage level at the beginning of that period.
- Stage: Time period (e.g., month) for which decisions are required.
- State variable: Storage at the beginning of a stage.
- Decision variable: Release from the reservoir during a period.

Dynamic Programming

State transformation:





Storage continuity (Mass balance)

$$S_{t+1} = S_t + Q_t - R_t$$ …... Neglecting losses

where S_t : Storage at the beginning of period t

Q_t : Inflow during period t

R_t : Release during period t

Dynamic Programming

Last time period

Recursive relationship:

	$t = 1$		$t = 2$		$t = 3$		$t = 4$
S_1 (known)	$n = 4$	S_2	$n = 3$	S_3	$n = 2$	S_4	$n = 1$

Diff. discrete values of S_4

Progress of computations

state

$$f_t^n(S_t) = \underset{\substack{0 \le R_t \le (S_t + Q_t) \\ S_t + Q_t - R_t \le S_{\max}}}{\text{Max}} \left[B_t(R_t) + f_{t+1}^{n-1}\big(\underbrace{S_t + Q_t - R_t}_{S_{t+1}}\big) \right]$$

time period

max storage

Total water available in period t

$B_t\ (R_t)$: Benefits associated with release R_t in period t

$0 \le S_t \le S_{max} \qquad \forall \quad t$

S_{max} : Reservoir capacity

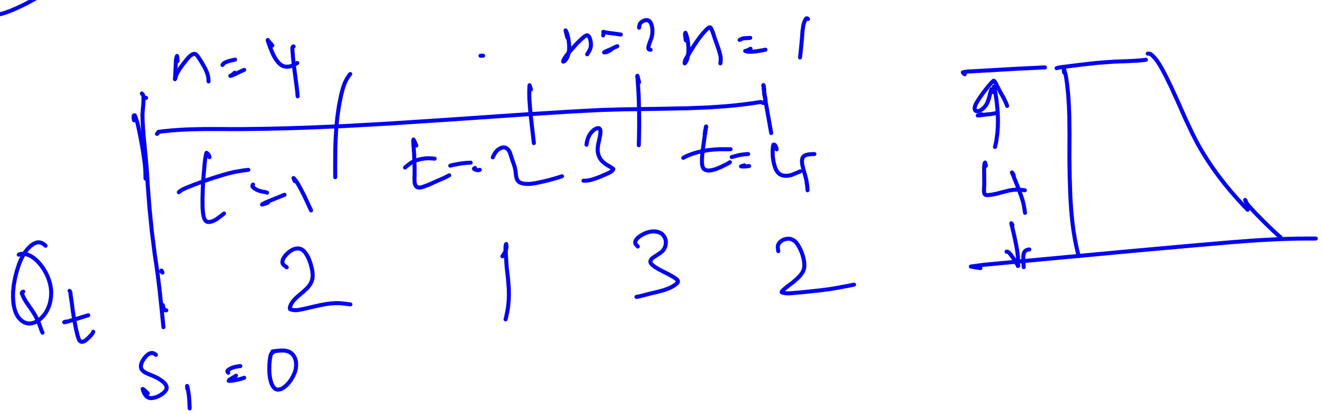
Inflow during time period t

$\{ Q_t \}$ ……… known

Dynamic Programming

Example:

- Inflows during four seasons to a reservoir with storage capacity of 4 units are 2, 1, 3 and 2 units respectively.
- Overflows from the reservoir are also included in the release.
- Reservoir storage at the beginning of the year is 0 units.



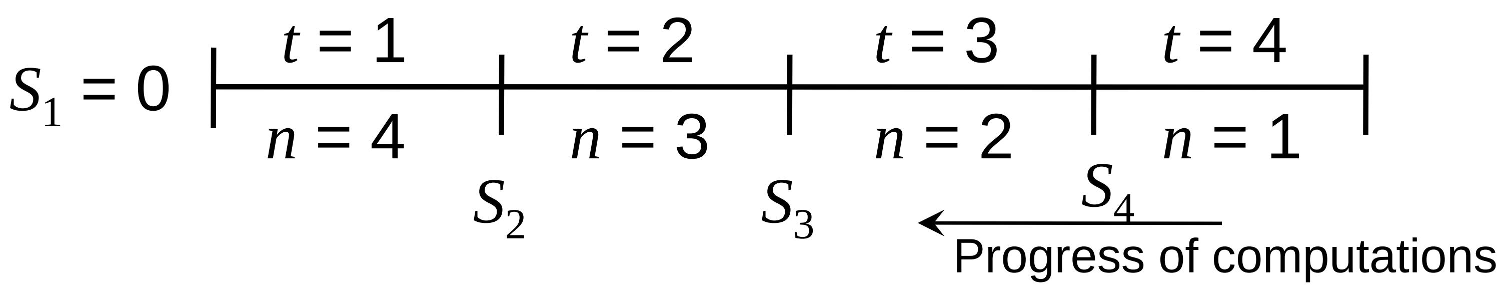
Dynamic Programming

- Release from the reservoir during the season results in the following benefits which are same for all the four seasons.

Release	Benefits
0	-100
1	250
2	320
3	480
4	520
5	520
6	410
7	120

Dynamic Programming

- To obtain the release policy backward recursive equation is used, starting with the last stage.



Stage 1:

$Q_4 = 2$ $\qquad t = 4$ and $n = 1$

$$f_4^1(S_4) = \text{Max}\left[B_4(R_4)\right]$$

$$0 \leq R_4 \leq (S_4 + Q_4)$$

Total water available for release.

$$S_4 + Q_4 - R_4 \leq 4$$

Max. storage.

Dynamic Programming

$Q_4 = 2$

S_4	R_4	$B_4(R_4)$	$f_4^1(S_4) = \text{Max}\left[B_4(R_4)\right]$	R_4^*
0	0	-100	Max: 320	2
	1	250		
	2	320		
1	0	-100	480	3
	1	250		
	2	320		
	3	480		
2	0	-100	520	4
	1	250		
	2	320		
	3	480		
	4	520		

Contd.

Dynamic Programming

$Q_4 = 2$
Contd.

S_4	R_4	$B_4(R_4)$	$f_4^1(S_4) = \text{Max}\left[B_4(R_4)\right]$	R_4^*
3	1	250	520	4,5
	2	320		
	3	480		
	4	520		
	5	520		
4	2	320	520	4,5
	3	480		
	4	520		
	5	520		
	6	410		

Dynamic Programming

Stage 2:

$Q_3 = 3$

$t = 3$ and $n = 2$

Stage

$$f_3^2(S_3) = \text{Max}\left[B_3(R_3) + f_4^1(S_3 + Q_3 - R_3)\right]$$

$$0 \le R_3 \le (S_3 + Q_3)$$

$$S_3 + Q_3 - R_3 \le 4$$

time

Dynamic Programming

$$f_3^2(S_3) = \text{Max}\left[B_3(R_3) + f_4^1(S_3 + Q_3 - R_3)\right]$$

$Q_3 = 3$

$0 \leq R_3 \leq (S_3 + Q_3)$; $S_3 + Q_3 - R_3 \leq 4$

S_3	R_3	$B_3(R_3)$	$S_3 + Q_3 - R_3$	$f_4^1(S_3+Q_3-R_3)$	$B_3(R_3) + f_4^1(S_3+Q_3-R_3)$	$f_3^2(S_3)$	R_3^*
0	0	-100	3	520	420	800	2, 3
	1	250	2	520	770		
	2	320	1	480	800		
	3	480	0	320	800		
1	0	-100	4	520	420	960	3
	1	250	3	520	770		
	2	320	2	520	840		
	3	480	1	480	960		
	4	520	0	320	840		

Contd.